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An Analysis of Problem-Posing Tasks in Chinese and US Elementary Mathematics Textbooks

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Abstract This paper reports on 2 studies that examine how mathematical problem posing is integrated in Chinese and US elementary mathematics textbooks. Study 1 involved a historical analysis of the problem-posing (PP) tasks in 3 editions of the most widely used elementary mathematics textbook series published by People's Education Press in China over 3 decades. Study 2 compared the PP tasks in Chinese and US elementary mathematics textbooks. This allows for the examination of PP tasks from an international comparative perspective, which provides one point of view about the kinds of learning opportunities that are available to students in China and the USA. We found evidence that the inclusion of PP tasks in the Chinese textbook series reflected, to some degree, changes in the curricular frameworks in China. However, the distribution of PP tasks across grade levels and content areas, as well as the variety of types of PP tasks included, suggest a need for greater intentionality in the design and placement of PP tasks in both the Chinese and US textbook series. Findings from the 2 studies reported in this paper not only contribute to our understanding about the inclusion of PP tasks in curriculum both historically and internationally, but also suggest a great need to systematically integrate PP activities into curriculum and instruction. The fact that both Chinese and US curriculum standards have heavily emphasized PP in school mathematics, despite there being only a small proportion of PP activities in both Chinese and US elementary mathematics curricula, suggests the existence of challenges that are delaying the implementation of reform ideas such as problem posing in school mathematics.

Keywords Mathematics curriculum · Mathematics education reform · Problem posing · Textbook analysis · China · United States

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In recent years, interest in incorporating problem posing in school mathematics instruction has grown steadily among mathematics education researchers and practitioners (e.g. Cai, Hwang, Jiang, & Silber, 2015; Singer, Ellerton, & Cai, 2013, 2015). Although, historically, problem solving has been more central than problem posing in school mathematics and mathematics education research, over the past several decades, curriculum reforms in many countries around the world have begun to raise the profile of problem posing at different educational levels (e.g. Brown & Walter, 1993; Chinese Ministry of Education, 1986, 1992, 2000a, b, 2001a, 2011; Hashimoto, 1987; Kilpatrick, 1987; National Council of Teachers of Mathematics [NCTM], 2000; Silver, 1994). In part, this has been reflective of a growing recognition that problem-posing activities can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interest and curiosity (Cai et al., 2015; NCTM, 1991). In addition, because problem posing and problem solving are often interwoven activities and success in one has been shown to be associated with success in the other (Cai & Hwang, 2002; Silver & Cai, 1996), it is natural to consider how problem posing can be integrated into school mathematics curriculum and instruction.

For problem posing to play a more central role in mathematics classrooms, teachers must have access to resources for problem-posing activities. In particular, mathematics curriculum materials should feature a good representation of problem-posing activities (Cai, Jiang, Hwang, Nie, & Hu, 2016). It is important to have problem-posing activities in the curriculum materials that teachers regularly use, as curriculum can be a powerful agent for instructional change (Ball & Cohen, 1996; Cai & Howson, 2013). Thus, the significance of including productive and robust problem-posing activities in curriculum materials should not be overlooked. Yet, there is at present, a lack of research that focuses on problem posing in the mathematics textbooks that students and teachers actually use, as opposed to the curriculum standards on which those textbooks are based (Cai et al., 2016). How has the inclusion of problem posing in curriculum standards played out in real textbooks? Given the variety of ways to engage students in one form or another of problem posing, how exactly do textbooks include problem posing? What kinds of choices have textbook writers and curriculum developers made in creating existing materials? This paper reports the results obtained from two studies to answer these research questions.

Study 1 involved a historical analysis of the problem-posing tasks in three editions of the most widely used elementary textbook series in China over three decades. It allows for the examination of the impact of curriculum standards on the inclusion of problem-posing tasks in elementary textbooks in China. China has engaged in curriculum reform over the past three decades, and problem posing has been explicitly included in the reform documents that have guided the reforms and related curriculum development. Curriculum standards do not only determine what students are taught but, with respect to the design of textbooks, also convey the ideas underlying the educational reforms. Thus, it seemed fruitful to conduct a historical analysis of the Chinese textbooks developed over three decades. Study 2 compared the problem-posing tasks in Chinese and US elementary textbooks. This allows for the examination of problem-posing tasks from an international comparative perspective, which provides one point of view about the kinds of learning opportunities that are available to students in China and the USA.

Background

Mathematical Problem Posing and Student Learning

A primary goal of research in mathematics education, including problem posing, is to improve student learning. Researchers have noted the potential for problem posing to benefit student learning, both in mathematics (Cai et al., 2013; English, 1998; Lavy & Shriki, 2010; Silver, 1994; Toluk-Uçar, 2009) and in other areas such as reading (Rosenshine, Meister & Chapman, 1996). Problem-posing activities are often cognitively demanding tasks (Cai & Hwang, 2002) that require students to stretch their thinking beyond problem-solving procedures to improve their understanding by reflecting on the deeper structure and goal of the task. As tasks with different cognitive demands are likely to induce different kinds of learning (Doyle, 1983), the high cognitive demand of problem-posing activities can provide intellectual contexts for students' rich mathematical development.

In particular, because problem posing involves the generation of new problems and questions aimed at exploring a given situation as well as the reformulation of a problem during the process of solving it (Silver, 1994), encouraging students to generate problems is likely to foster both student understanding of problem situations and the development of more advanced problem-solving strategies. Indeed, using eight open-ended problem-solving tasks, Silver and Cai (1996) found a high correlation between students' mathematical problem-solving performance and their problem-posing performance. More successful problem solvers were those who generated more and more complex problems. Similarly, Cai and Hwang (2002) found links between students' strategy use in problem solving and the types of problems students posed. Clearly, the relationships between problem posing and problem solving provide a rationale for recommendations to incorporate problem posing into school mathematics at different educational levels (Chinese Ministry of Education, 1986, 2001a, 2003, 2011; NCTM, 2000). An analysis of the problem-posing tasks in textbooks would provide one point of view about the kinds of learning opportunities students are provided in China and the USA.

Problem Posing, Mathematics Curricula, and Curriculum Reform

Given the potential positive impact of including problem-posing activities in the mathematics classroom, it is useful to consider how curriculum might support such activities. Curriculum has historically been seen as a powerful agent for instructional change in the face of changing societal demands on the education system (Cai & Howson, 2013; Howson, Keitel, & Kilpatrick, 1981). For example, a number of countries including China and the USA have been undertaking similar mathematics education reforms. The overarching goals of the reforms have been to improve students' learning of mathematics and to nurture students' innovation and creativity (Chinese Ministry of Education, 2001b, 2011; NCTM, 2000). In the USA, NCTM (1991) suggested that "students should be given opportunities to formulate problems from given situations and create new problems by modifying the conditions of a given problem" (p. 95). Ten years later, NCTM has called for students to "formulate interesting problems based on a wide variety of situations, both within and outside of mathematics" (NCTM, 2000, p. 258).

In China, students' thinking and reasoning has also been emphasized in mathematics education reform. One of the six objectives of the new curriculum reforms is for students to be actively involved in inquiry-based activities in order to develop their abilities to collect and process information, to attain new knowledge, to analyze and solve problems, and to communicate and cooperate (Chinese Ministry of Education, 2001b, 2011). At the 9-year compulsory education stage, students learn how to pose problems from mathematical perspectives, how to understand problems, and how to apply their knowledge and skills to solve problems so as to increase their awareness of mathematical applications (Chinese Ministry of Education, 2001a, 2011). The high school mathematics curriculum is intended to enhance students' abilities to pose, analyze, and solve problems from mathematical perspectives, to express and communicate mathematically, and to attain mathematical knowledge independently. An additional goal is for students to change their learning styles from passive to active through being engaged in problem posing and problem solving (Chinese Ministry of Education, 2003).

Yet if, as these curriculum reform documents advocate, problem-posing activities are to become a more central part of mathematics classrooms, there must be resources ready for problem-posing activities (Cai & Nie, 2007). Particularly, in countries like China in which teachers carefully study the textbooks to guide and improve their teaching, the inclusion of problem-posing resources in those textbooks should be particularly powerful influences on classroom practice. While the current mathematics textbooks have been designed to implement reform curriculum standards, it is not so clear where and how they include problem posing (e.g. related to what mathematical content), nor do we know the extent to which these textbooks embody the stances of the reform standards toward problem posing (Cai et al., 2016). In this paper, we refer to problem-posing tasks as those requiring students to generate new problems based either on a given situation or on a mathematical expression or diagram. In contrast to problem-solving tasks, in a problem-posing task, students are not necessarily asked to solve a given problem.

In addition to examining the percentage of tasks in textbooks that are problem-posing tasks, it is useful to consider how those problem-posing tasks that do appear are presented in various types of mathematical content. For example, problem-solving research has examined how students use different types of representations in their solutions (e.g. Cai, 2000; Cai & Hwang, 2002). It seems a natural extension to ask similar questions in the realm of problem posing. What kinds of representations, including visual and symbolic, are used in problem-posing tasks? How are these representations used?

Similarly, the inclusion of sample problems in problem-posing tasks may provide a window into the intent of textbook designers. In earlier versions of Chinese mathematics curricula, problem posing was not included as a topic in its own right. Rather, problem posing was treated as an intermediate step in problem solving (Cai et al., 2016). Newer, reform-oriented revisions of the curricula have included problem posing as a learning goal. To that end, mathematics textbook designers might have incorporated materials that can guide students through the process of posing problems. One way to do this is to include sample problems in problem-posing tasks for students to emulate.

Thus, the degree to which problem-posing tasks in mathematics textbooks include sample problems can be an indicator of how intentional textbook designers were in building problem posing into the curriculum.

There are several types of problem-posing tasks that have been identified in research on problem posing. Based on works by Stoyanova (1998), Silver (1994), and Christou, Mousoulides, Pittalis, Pitta-Pantazi, and Sriraman, (2005) describe five such types defined by the nature of the problem students are asked to pose: a problem in general (free situations), a problem with a given answer, a problem that contains certain information, questions for a problem situation, and a problem that fits a given calculation. These are the five main categories of mathematical problem-posing tasks used in the two studies reported here. In addition, different problem-posing tasks may present given information to students in several ways, including the use of visual and symbolic modes of representation, which may or may not be influenced by and consonant with other design and pedagogical choices for a given textbook. Different types of tasks thus reflect different qualities and priorities in problem-posing task design, such as the degree to which the task is constrained for students (e.g. Stoyanova, 1998) or the role the task may play in relationship with problem solving (e.g. Silver, 1994). Therefore, the manner in which different types of problem-posing tasks are incorporated into textbooks can provide further information about the degree to which these materials systematically integrate problem posing into the curriculum and to which they aim to develop particular aspects of problem posing for students.

As yet, there has not been a substantial body of research examining whether and how curricular materials incorporate problem posing (Cai et al., 2015, 2016). The studies contained in this paper intend to address this gap. Specifically, we address the following research question: How are different problem-posing tasks included in Chinese and US mathematics textbooks? the two studies shall provide researchers and curriculum developers with information about the kinds of problem-posing tasks in existing curricula as well as insights about how to incorporate problem posing into school mathematics.

Methods

Selection of Textbook Series

In study 1, three series of elementary mathematics textbooks published by People's Education Press (PEP) were analyzed (PEP, 1990s, 2000s, 2010s). PEP was the only textbook publisher in China from 1949 to 1988 (Zhang, Sun, & Powell, 2015) and is now the most influential textbook publisher in China. The textbooks published by PEP are the most widely used textbooks in China (over 65 % of the market).

In study 2, we compared the latest series of elementary mathematics textbooks published by PEP (2010s) and two series used in the USA. The two US series selected were: *Everyday Mathematics,* developed by the University of Chicago School Mathematics Project (UCSMP, 2012a, b), and *Investigations in Number, Data, and Space* (hereafter shortened as *Investigations*), published by TERC, Cambridge, MA (TERC, 2008a, b, c, d, e, f). In all cases, the textbooks represent the most widely adopted elementary mathematics curriculum materials in their respective countries. The two US series were considered reformed textbooks (Senk & Thompson, 2003). The latest edition of the PEP textbook series is also considered to be a reformed textbook series in China. Through this comparison, we are able to understand how the reform ideas of problem posing are reflected in the mathematics textbooks in China and the USA.

Task Analysis

We first checked every problem in the five textbook series to identify those that were problem-posing tasks. A problem-posing task is an activity which requires students to generate new problems based either on a given situation or on a given mathematical expression or diagram. In contrast to problem-solving tasks, in a problem-posing task, students are not necessarily asked to solve a given problem. According to the nature of the problem students are asked to pose (Christou et al., 2005), we categorized the problem-posing tasks: a problem in general (free situations), a problem with a given answer, a problem that contains certain information, questions for a problem situation, and a problem that fits a given calculation. All of the identified problem-posing tasks in the five textbook series were included in the analysis. We then analyzed each problem-posing task in terms of its (a) grade level, (b) content area, (c) use of various representations for the given information and whether there were sample questions that students could imitate, and (d) types of problem-posing tasks.

With respect to the types of problem-posing tasks, we classified each problem-posing task according to what it required students to do, relative to the information provided in the task, consistent with Christou et al. (2005). These types were specified based on a holistic analysis of the requirements in a problem-posing task. Special attention was paid to whether a problem poser needed to provide information as givens and whether there was a sample question that a problem poser could emulate to reproduce similar ones. Four types of problem-posing tasks were identified. We describe these types below, roughly ordered from the problem-posing task types that are the most mathematically constrained to those that are least mathematically constrained.

1. *Posing a problem that matches the given arithmetic operation(s)*. Students are asked to make up a story or a word problem that can be solved with a given arithmetic operation. Tasks of this type provide the student with an explicit arithmetic operation, and the student is expected to provide a context and pose a problem that matches the operation. For example: “Make up a word problem orally for 14 + 8 = ?”
2. *Posing variations of a question with similar mathematical relationship or structure*. Given a sample problem or problem situation (it is not necessary for the sample to include a question), students are asked to pose a similar problem with given information and question. The student can change the context, the specific numbers, or even which quantity is the unknown quantity, but the fundamental mathematical relationship or structure must mirror the sample. For example: “The distance between two cities, A and B, is 2590 km. A plane is flying from A to B at a speed of 650 km/h. Another plane is flying from B to A at a speed of 645 km/h. If they set out at the same time from their respective airports, after how many hours will they meet up? First solve it, then change it to a problem with meeting time as one of the givens and the distance of the two cities as the unknown” (PEP, 1990, p. 62).

3. *Posing additional questions based on the given information and a sample question*. Students are asked to pose additional problems after solving a given problem with sample question(s). The additional problems are expected to involve the given information, but are not required to mirror a particular mathematical relationship. Although students may choose to provide additional information, they may not change the given information. For example: "On weekends, a father and his son went climbing. The distance from the ground to the top of the mountain is 7.2 km. It took them 3 hours to climb up and 2 hours to walk down. What are the speeds going up and going down? Can you pose additional mathematical questions?" (PEP, 2000, p. 20).
4. *Posing questions based on given information*. Students are provided with a problem context and information, but no sample problem. They are expected to generate questions based on the given information. For example: "The average floor areas per person in three cities are shown in the following table, what problems can you pose? Can you solve them?" (PEP, 2010, p. 100).

Cities	A	B	C
Floor area (m^2)	14.6	16.7	17.6

The coding of the types of problem-posing tasks is the most challenging aspect of this analysis. Thus, we first established inter-rater reliability for the coding of the types of problem-posing tasks. A total of 24 problem-posing tasks from the Chinese textbook series (PEP, 1990) were randomly selected and coded by two coders whose native tongue is Chinese; they reached 96 % in agreement with respect to the types of problem-posing tasks. We then checked inter-rater agreement on how to count the number of problems on each page; what can be counted as problem-posing tasks; and what we are going to look at with respect to the content areas, problem-posing types, and whether they are presented with sample questions and/or pictures, figures, and tables. The two raters separately coded all the problems and problem-posing tasks for the 2010s Chinese textbook series. For 92 % of the total 1391 pages, the two raters counted the same numbers of problems. For the other 8 %, the differences in counted problems were only one or two problems apart. For the problem-posing tasks, the two coders reached the following levels of agreement in each of the categories: (a) content area (89 %), (b) types of problem-posing tasks (89 %), and (c) use of various representations for the given information (e.g. with/without graphs, figures, tables, etc.; 98 %) and whether there were sample questions that students could imitate (99 %). The discrepancies were resolved through discussion.

Our coding of the US textbooks followed the same procedure as that used for coding the Chinese textbooks, without encountering any challenges. To ensure coding reliability, 26 problem-posing tasks from the US textbooks were randomly selected and coded by the two coders. The two coders reached the following levels of agreement in each of the categories: (a) content area (89 %), (b) types of problem-posing tasks (77 %), and (c) use of various representations for the given information (e.g. with/without graphs, figures, tables, etc.; 88 %) and whether there were sample questions that students could imitate (81 %). Again, discrepancies were resolved through discussion.

Results

Study 1: Historical Analysis of Three Editions of Chinese Elementary Textbook Series

Number of problem-posing tasks at different grade levels. The total number of problems (n) and the percentage of those that were classified as problem-posing tasks in the Chinese textbook series published over three decades are shown in Table 1. Overall, the percentages of problem-posing tasks were quite small across the three textbook series. However, there were some differences across the series. The percentages of problem-posing tasks in the two latest textbook series were greater than that in the 1990s series, although the differences between the latest two series did not reach a significant level. Similarly, the percentages of problem-posing tasks across all six grade levels, except for grade 3, in the latest two series were greater than those in the 1990s series. For grade 3, the percentages of problem-posing tasks in the 1990s series were slightly but not significantly higher than that in the 2000s series, with both significantly lower or marginally lower than that in the 2010s series (1990s vs. 2010s: $z = 2.18$, $p < .05$; 2000s vs. 2010s: $z = 1.90$, $p = .06$). The percentages of problem-posing tasks were very different across different grade levels. No grade had the largest percentage of problem-posing tasks across the three series, and indeed the percentages rose and fell from grade to grade within each of the three series. We also compared the percentages of problem-posing tasks at each grade level between the 1990s and the 2000s series and between the 2000s and the 2010s series. For the 1990s to 2000s series, the percentages of problem-posing tasks were significantly higher for grades 2–6. For the 2000s to 2010s series, the differences in percentages of problem-posing tasks were significant only for grade 4, for which the percentage was lower.

Number of problem-posing tasks in different content areas. We classified the problem-posing tasks in the three textbook series by the content area in which they were situated: number and operations, algebra, geometry, measurement, and data

Table 1 Total number of problems and percentages of problem-posing (PP) tasks in the three editions of textbook series published by People's Education Press over three decades

Grade	1990s		2000s		2010s		1990s vs. 2000s	2000s vs. 2010s
	n	% PP	*n*	% PP	*n*	% PP		
1	851	2.47	527	3.61	669	3.74	−1.22	−.12
2	1278	4.23	565	6.73	711	5.06	−2.27*	1.26
3	1298	3.54	589	3.40	694	5.62	.16	−1.90
4	1344	.37	621	4.83	699	1.57	−6.95***	3.40***
5	1299	.38	659	2.12	821	2.80	−3.71***	−.83
6	1110	.63	627	1.75	745	2.01	−2.22*	−.35
Total	7180	1.92	3588	3.68	4339	3.43	−5.50***	.59

The numbers in the last two columns are the z scores

*$p < .05$; **$p < .01$; ***$p < .001$

analysis and probability, following the content areas used by NCTM (2000; Table 2). We also compared the percentages of problem-posing tasks in each content area between the 1990s and 2000s series and between the 2000s and 2010s series. Across the three textbook series, the majority of the problem-posing tasks were related to number and operations. The percentage of problem-posing tasks in number and operations in the 1990s series was significantly decreased in the 2000s and 2010s textbook series (1990s vs. 2010s: $z = 4.75$, $p < 0.001$). However, the difference in the percentages of problem-posing tasks in number and operations in the latest two textbook series was not significant. In addition, across the three textbook series, very few problem-posing tasks were related to algebra, geometry, or measurement, with the percentages for these all less than 5 %, meaning the chi-square test could not be conducted to compare across the five content areas. The differences in the percentages of problem-posing tasks between the 1990s and 2000s textbook series in algebra and geometry were significant at the 0.05 level.

The uneven distribution of problem-posing tasks across content areas is mirrored in the way tasks are distributed across grade levels. The distribution of tasks across grades shows a great deal of variability in every series. As shown in Fig. 1, although the percentages of problem-posing tasks in the curricula are generally small, they rise and fall markedly from grade to grade. There does not appear to be any trend toward increasing or decreasing problem posing as students progress through the elementary grades, nor do the textbooks maintain a regular level of problem posing from grade to grade.

For the latest two textbook series, the second highest percentage of problem-posing tasks was related to data analysis and probability. The percentages of problem-posing tasks in data analysis and probability in the 2000s and 2010s textbook series were significantly greater than that in the 1990s series. However, the difference in the percentages of problem-posing tasks in data analysis and probability in the latest two textbook series was not significant.

Types of problem-posing tasks. The problem-posing tasks in the three textbook series were classified into the aforementioned four types. The percentages of problem-posing tasks of each type are shown in Table 3. The data shows large discrepancies among the

Table 2 Percentage distribution of problem-posing tasks in different content areas in the three editions of mathematics textbook series

Content area	1990s ($n = 138$)	2000s ($n = 132$)	2010s ($n = 149$)	1990s vs. 2000s	2000s vs. 2010s
Numbers and operations	97.10	73.48	78.52	5.52***	−.99
Algebra	2.90	0	.67	1.97*	−.94
Geometry	0	3.79	4.03	−2.31*	−.10
Measurement	0	.76	2.01	−1.02	−.89
Data analysis and probability	0	21.97	14.77	−5.83***	1.56

The numbers in the last two columns are the z scores

*$p < .05$; ***$p < .001$

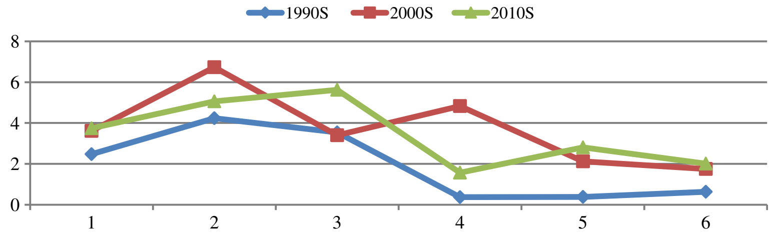

Fig. 1 Percentages of problem-posing tasks in each grade by publication years

three textbook series (χ^2 = 233.21, df = 6, p < .001). Recall that the types of problem-posing tasks were roughly ordered from the most constrained to the least constrained. The percentages in Table 3 suggest that the 1990s textbook series had larger percentages of problem-posing tasks that were at the two ends (most and least constrained), whereas the 2000s and 2010s textbook series had larger percentages of problem-posing tasks that were in the intermediate constrained level.

For the 1990s textbook series, the problem-posing tasks were more evenly distributed among three of the four types of problem-posing tasks, whereas for the 2000s and 2010s textbook series, the majority of the problem-posing tasks required students to pose additional questions for given information after presenting students with sample questions. The percentages of the first two types of problem-posing tasks were drastically decreased to less than 5 % from the 1990s to the 2000s series. Meanwhile, the percentages of the third type of problem-posing task were significantly increased over the three decades, and the percentages of the fourth type of problem-posing task were significantly decreased.

Use of representations and inclusion of sample questions in problem-posing tasks. Table 4 shows the degree to which the three textbook series included sample questions in problem-posing tasks and to which degree they presented information in these tasks using pictures, figures, or tables. Significant differences existed among the three textbook series in both aspects (χ^2 = 350.70, df = 6, p < .001). Specifically, the two latest series (2000s = 66 %, 2010s = 80 %) had significantly higher percentages of problem-posing tasks with sample questions than the 1990s series (3 %).

Table 3 Percentages of types of problem-posing tasks in the three mathematics textbook series

Types of problem-posing tasks	1990s (n = 138)	2000s (n = 132)	2010s (n = 149)	1990s vs. 2000s	2000s vs. 2010s
1. Posing a problem that matches the given/specific kinds of arithmetic operation(s)	26.81	3.79	.67	5.22***	1.80
2. Posing variations on a question with the same mathematical relationship or structure	27.54	0	2.01	6.50***	−1.64
3. Posing additional questions based on the given information and a sample question	2.17	65.91	79.19	−11.11***	−2.50**
4. Posing questions based on given information	43.48	30.30	18.12	2.24*	2.39*

The numbers in the last two columns are the z scores

*p < .05; **p < .01; ***p < .001

Table 4 Percentages of problem-posing tasks with/without sample questions and with/without information presented in pictures, figures, or tables (PFT)

	1990s ($n = 138$)	2000s ($n = 132$)	2010s ($n = 149$)	1990s vs. 2000s	2000s vs. 2010s
With sample questions					
With PFT	0	33.33	72.48	−7.41***	−6.57***
Without PFT	2.17	32.58	7.38	−6.64***	5.35***
Without sample questions					
With PFT	.73	12.88	16.78	−4.00***	−.92
Without PFT	97.10	21.21	3.36	12.72***	4.64***

The numbers in the last two columns are the z scores

*$p < .05$; **$p < .01$. ***$p < .001$

The difference between the two latest series regarding inclusion of sample questions was also significant ($z = 2.64$, $p < .01$). Of the problem-posing tasks included in the 1990s textbook series, only one was presented with information in pictures, figures, or tables. This was drastically changed in the 2000s series (46 %) and kept increasing in the 2010s series (89 %). The difference between the two latest series regarding presentation in pictures, figures, or tables was also significant ($z = 7.79$, $p < .001$).

Study 2: A Comparison of 2010s Chinese Textbook Series and two US Reform Series

Number of problem-posing tasks at different grade levels. The Chinese textbook series and the US *Everyday Mathematics* series were written for children in grades 1 to 6. However, the *Everyday Mathematics* textbooks for children at grades 1 and 2 are combined. The US *Investigations* series was written for children in grades 1 to 5. For each textbook series, the total number of tasks (n) and the percentage of those that were classified as problem-posing tasks are shown in Table 5. Overall, the percentages of problem-posing tasks were quite small for all three textbook series. However, there were some differences across the series. The percentage of problem-posing tasks in the Chinese textbook series was significantly higher than those in the two US textbook series (PEP vs. *Investigations*: $z = 5.52$, $p < .001$; PEP vs. *Everyday Mathematics*: $z = 9.68$, $p < .001$). For the two US textbook series, there was a higher percentage of problem-posing tasks in the *Investigations* textbooks than in the *Everyday Mathematics* textbook series ($z = 2.25$, $p < .05$).

The percentages of problem-posing tasks were also very different across different grade levels. No grade had the largest percentage of problem-posing tasks across the three series, and indeed, the percentage rose and fell from grade to grade within most of the series (although the grade-to-grade fluctuations within *Everyday Mathematics* were comparatively small). For the two US textbook series, we compared the percentage of problem-posing tasks at each grade level. There were no significant differences except at grade 5 ($z = 2.69$, $p < .01$).

Table 5 Total number of problems and percentages of problem-posing (PP) tasks in the Chinese and US mathematics textbooks series from grades 1 to 6

Grade	Chinese		US			
	PEP		Investigations		*Everyday*	
	n	% PP	*n*	% PP	*n*	% PP
1	669	3.74	490	0	–[b]	–
2	711	5.06	741	1.62	1651	1.03
3	694	5.62	832	0.72	1322	1.06
4	699	1.57	760	1.97	1565	1.28
5	821	2.80	726	2.62	1896	1.16
6	745	2.01	–[a]	–	1673	.42
Total	4339	3.43	3549	1.47	8107	.99

[a] *Investigations* does not have grade 6 textbooks

[b] For *Everyday Mathematics* grades 1 and 2, we combined the data because there is only one combined Student Reference Book for the two grades

Number of problem-posing tasks in different content areas. The percentage distribution of problemposing tasks (Table 6) in the five content areas was significantly different across the one Chinese and two US textbook series ($\chi^2 = 19.19$, $df = 8$, $p < .05$). However, no significant difference was found between the two US series. Across the three textbook series, the majority of the PP tasks were related to number and operations. The percentages of number and operations problem-posing tasks in the two US textbook series were higher or marginally higher than those in the Chinese textbook (PEP vs. *Investigations*: $z = -1.90$, $p = .06$; PEP vs. *Everyday Mathematics*: $z = -2.44$, $p < .05$). However, the difference in the percentages of PP tasks in number and operations in the two sets of US textbooks was not significant.

For the Chinese textbook series, the second highest percentage of problem-posing tasks was related to data analysis and probability. The percentages of data analysis problem-posing tasks in the Chinese textbook series were significantly or marginally significantly higher than those in the two US textbook series (PEP vs. *Investigations*:

Table 6 Percentage distribution of problem-posing tasks in different content areas in the Chinese and US mathematics textbook series

Content area	Chinese	US	
	PEP ($n = 149$)	Investigations ($n = 52$)	*Everyday* ($n = 80$)
Numbers and operations	78.52	90.38	91.25
Algebra	.67	5.77	1.25
Geometry	4.03	0	1.25
Measurement	2.01	0	0
Data analysis and probability	14.77	3.85	6.25

$z = 2.09$, $p < .05$; PEP vs. *Everyday Mathematics*: $z = 1.91$, $p = .06$). The difference in the percentages of problem-posing tasks in data analysis and probability in the two US textbook series was not significant. For all three textbook series, very few problem-posing tasks were related to algebra, geometry, or measurement, with the percentages all less than 6 %.

Types of problem-posing tasks. As in study 1, the problem-posing tasks in the three textbook series were classified into four types. The percentages of the problem-posing tasks of each type are shown in Table 7. Data in Table 7 shows large discrepancies between the Chinese and US textbook series ($\chi^2 = 245.21$, $df = 6$, $p < .001$) and between the two US textbook series. Recall that the types of problem-posing tasks were roughly ordered from the most to the least constrained. The percentages in Table 7 suggest that the Chinese textbooks had larger percentages of problem-posing tasks that were comparatively less constrained, whereas the US textbooks had larger percentages of tasks that were comparatively more constrained.

For the Chinese textbook series, the majority of the problem-posing tasks required students to pose additional questions for given information after presenting students with sample questions. The percentage of this type of task in the Chinese textbook series was significantly higher than those in the two US textbook series (PEP vs. *Investigations*: $z = 9.76$, $p < .001$; PEP vs. *Everyday Mathematics*: $z = 10.73$, $p < .001$), but the percentages of problem-posing tasks of this type were not significantly different between the two US textbook series. In contrast, for the two US textbook series, the majority of problem-posing tasks required students to pose problems that matched the given arithmetic operations, and the percentage of problem-posing tasks of this type in *Investigations* was significantly higher than that in *Everyday Mathematics* ($z = 2.06$, $p < .05$). The percentages in the two US textbook series were significantly higher than those in the Chinese textbook series (PEP vs. *Investigations*: $z = -12.50$, $p < .001$; PEP vs. *Everyday Mathematics*: $z = -11.43$, $p < .001$). For the Chinese textbook series, the second most common type of problem-posing task was posing questions based on given information. The percentage of such tasks in PEP was significantly higher than those in the two US textbook series (PEP vs. *Investigations*:

Table 7 Percentages of types of PP tasks in the Chinese and US mathematics textbooks

Types of problem-posing tasks	Chinese	US	
	PEP ($n = 149$)	Investigations ($n = 52$)	*Everyday* ($n = 80$)
1. Posing a problem that matches the given/specific kinds of arithmetic operation(s)	.67	84.62	68.75
2. Posing variations on a question with the same mathematical relationship or structure	2.01	13.46	23.75
3. Posing additional questions based on the given information and a sample question	79.19	1.92	5.00
4. Posing questions based on given information	18.12	0	2.50

$z = 3.30$, $p < .001$; PEP vs. *Everyday Mathematics*: $z = 3.39$, $p < .001$). In the two US mathematics textbooks, there are very few such type of problem-posing tasks.

For the two US mathematics textbook series, the second most common problem-posing task was posing variations on a question with the same mathematical relationship or structure. The percentages in the two US mathematics textbooks were significantly higher than that in the Chinese textbook series (PEP vs. *Investigations*: $z = -3.27$, $p < .01$; PEP vs. *Everyday Mathematics*: $z = -5.32$, $p < .001$). However, the percentages of such type of problem-posing tasks in the two US mathematics textbooks were not significantly different.

Presentation of problem-posing tasks and inclusion of sample questions. Table 8 shows the degree to which the three textbook series included sample questions in problem-posing tasks and to which they presented information in these tasks using pictures, figures, or tables. There were no significant differences between the two US textbook series. Specifically, the Chinese textbook series (PEP 80 %) had higher percentages of problem-posing tasks with sample questions than the US textbook series (*Investigations*: 9.62 %, *Everyday Mathematics* 27.50 %; PEP vs. *Investigations*: $z = 8.97$, $p < .001$; PEP vs. *Everyday Mathematics*: $z = 7.77$, $p < .001$). The differences between the two US (*Investigations* 10 %, *Everyday Mathematics* 28 %) and Chinese textbook (79.86 %) series regarding inclusion of sample questions were also significant (*PEP* vs. *Investigations*: $z = 8.97$, $p < .001$; *PEP* vs. *Everyday Mathematics*: $z = 7.77$, $p < .001$). The difference between the two US textbook series regarding inclusion of sample questions was significant ($z = -2.49$, $p < .05$).

Of the problem-posing tasks included in the US textbooks, less than one fourth were presented with information in pictures, figures, or tables (*Investigations* 7.70 %, *Everyday Mathematics* 23.75 %), which was lower than in the Chinese textbook series (PEP 46.21 %; PEP vs. *Investigations*: $z = 10.87$, $p < .001$; PEP vs. *Everyday Mathematics*: $z = 10.00$, $p < .001$). The two US textbook series were significantly different in their percentages of problem-posing tasks that included information presented in pictures, figures, and tables ($z = .02$, $p < .05$).

Table 8 Percentages of problem-posing tasks with/without sample questions and with/without information presented in pictures, figures, or tables (PFT)

	Chinese	US	
	PEP ($n = 149$)	Investigations ($n = 52$)	*Everyday* ($n = 80$)
With sample questions			
With PFT	72.48	3.85	17.50
Without PFT	7.38	5.77	10.00
Without sample questions			
With PFT	16.78	3.85	6.25
Without PFT	3.36	86.54	66.25

Discussion

Analysis of the intended curriculum has been a line of scholarly inquiry for nearly a century (e.g. Davis, 1962). Studies of the intended curriculum provided new insights into the content and design of mathematics textbooks and afforded new understanding about relationships between written curricular materials and students' opportunities to learn (Cai & Cirillo, 2014; Charalambous, Delaney, Hsu, & Mesa, 2010; Lloyd, Cai, & Tarr, 2016). Curriculum reform has often been viewed as a powerful tool for educational improvement because changes in curriculum have the potential to change classroom practice and student learning (Cai & Howson, 2013). Reform-guided mathematics curricula in China and the USA have put great emphasis on problem posing because of its potential to develop students' creative thinking and innovative abilities in the new century. Consequently, Chinese and US textbook developers have made some effort to integrate problem-posing tasks into curriculum materials. Although our data show that the Chinese and US textbooks do contain problem-posing tasks, the percentage of such tasks in each of the textbook series we examined is still quite low.

The comparatively small representation of problem-posing tasks among a large sea of problem-solving tasks in both Chinese and US series can be interpreted in at least two ways. The first is, to some degree, the relative emphases and placement of problem posing in the reform curriculum guidelines. Problem solving is always one of the objectives of school mathematics; however, problem posing was only first explicitly included in the curriculum standards of the past two decades. Given the strong focus on the role of problem solving in reform mathematics curricula in both China and the USA, it may be the case that problem posing was overshadowed. The second way to interpret these results is based on the longer time it may take to specifically integrate reform ideas such as problem-posing tasks into curriculum and the classroom. The encouraging news is that the historical analysis of the three decades of the three textbook series in China did show increased emphasis on problem posing in textbooks.

It is clear that the distributions of problem-posing tasks across different content areas and different grade levels in the five textbook series are extremely uneven. More specifically, across the five textbook series in the two nations, the problem-posing tasks are heavily concentrated in the number and operations area. While number and operations strand has traditionally been a primary focus of elementary mathematics, the degree of concentration of problem-posing tasks in number and operations exceeds what would be expected based on the content of the textbooks in this study. In particular, the dearth of problem-posing tasks related to algebra, geometry, and measurement is out of proportion to the coverage of these topics in the textbooks. This is somewhat puzzling, given the degree to which geometry, in particular, is amenable to conjecturing and forming hypotheses (Yerushalmy, Chazan, & Gordon, 1990). Again, the good news is that in the 2000s and 2010s Chinese textbooks series we examined, a more substantial percentage of problem-posing tasks are related to data analysis and probability than to algebra, geometry, and measurement. In recent years more emphasis has been put on early algebraization (Cai & Knuth, 2011); however, this trend was not observed in the Chinese and US textbooks with respect to the problem-posing tasks.

The general lack of consistency in the inclusion of problem-posing tasks, both across content areas and across grades, suggests a need for greater intentionality in the planning and design of how problem posing should be embedded in mathematics textbooks in both China and the USA. Although there have clearly been some intentional efforts to incorporate problem posing in these textbook series, the inconsistency of implementation across the content areas and grade levels may suggest a need for more effort to make problem posing a classroom routine.

The distributions of problem-posing tasks into the four types that we identified are also uneven in both the US and Chinese textbooks, but with different patterns. For the Chinese textbooks, in the 1990s, problem-posing tasks were much more evenly distributed than in the 2000s and 2010s series; however, in the latest two series, the majority of the problem-posing tasks are tasks in which the student is given some information and a sample question and is then asked to pose additional questions based on the given information. Although a sample question is provided in these tasks, the student is not necessarily expected to mirror the mathematical structure of the given problem. From the 1990s to the 2010s series, there is a decrease in the proportions of problem-posing tasks in which the student is expected to pose additional questions based on given information but without a sample question. These tasks give the student a great deal of latitude in choosing the mathematical structure of their problem, although the context is fixed. In contrast, slightly more than half of the problem-posing tasks in the 1990s series have much stronger constraints, requiring students to pose problems with solutions that match the given arithmetic operations or to pose problems with the same mathematical relationship or structure. In these problems, the student may choose a context relatively freely, but the mathematical structure of the problem is already fixed. It is not immediately clear why there should be a difference in the level of task constraints between problem-posing tasks in the three textbook series in China. One potential explanation might lie in differences in how teachers and textbook designers view the use of problem-posing tasks for mathematics teaching, such as teaching a new concept versus practicing a newly learned approach. However, this would need to be further investigated with respect to how these problem-posing tasks are actually used in mathematics classrooms in China.

In conclusion, we have observed more similarities than differences between Chinese and US textbooks. The main difference is related to the types of problem-posing tasks included in Chinese and US textbook series. The vast majority of the problem-posing tasks in the Chinese textbook series are those posing additional questions based on the given information and sample questions, while the vast majority of the problem-posing tasks in the US textbook series are those posing a problem that matches the given/specific kinds of arithmetic operation(s). This difference may reflect the general trends of the Chinese and US textbook series: Chinese textbook series include worked out examples, but the two US textbook series do not.

Another noticeable difference between the problem-posing tasks of the Chinese and US textbook series is related to their presentations and sample questions. In this study, we examined the use of pictures, figures, and tables to represent information in the problem-posing tasks from the five textbook series. The data show a clear pattern that, from the 1990s to the 2010s Chinese series, the proportions of problem-posing tasks that include pictures, figures, and tables increased from 0.73 % to 46.2 %, and finally, to 89.2 %. With respect to the inclusion of sample questions, the problem-posing tasks in the latest two Chinese series are again more likely than those in the 1990s series to exhibit this feature. However, in the US textbook series, only a small proportion of the problem-posing tasks involve pictures, figures, and tables. Similarly, only a small proportion of the problem-posing tasks in the US textbook series involve sample questions. Problem-solving research has shown that Chinese textbooks generally have much fewer problems that include information represented in pictures, figures, and tables compared with US textbooks (Zhu, 2003), and that Chinese students are more likely to solve mathematical problems using symbolic rather than visual representations compared to US students (Cai, 1995, 2000). The disjunction between these results and the findings from problem-solving research may be related to the prevalence of tasks in the latest two series that asked students to pose problems that are less mathematically constrained, and thus perhaps may afford greater latitude to employ diverse representations.

As we noted earlier, sample questions may be included in problem-posing tasks as a way to guide students as they learn how to pose their own problems. The Chinese reform curriculum guidelines have made problem posing a learning goal in its own right (Chinese Ministry of Education, 2000a, b, 2011). Thus, it makes sense that textbook designers would intentionally include examples for students to study and emulate as students learn how to formulate their own problems. This might be one explanation for why a large proportion of the problem-posing tasks involve sample questions.

Comparative and historical analyses have often been used in the scholarly inquiry of intended mathematics curriculum (Baker et al., 2010). This study used both historical and comparative analyses to understand the kinds of problem-posing tasks included in elementary textbooks in both China and the USA. The findings of this study show the feasibility and value of comparative and historical analyses to understand the potential learning opportunities Chinese and US mathematics textbook series contain with respect to problem posing. As we discussed before, problem posing has been elevated as an important component of mathematics learning in reform mathematics curriculum documents in both China and the USA. However, our examination of the mathematics textbook series from these two countries shows that there is still a very small proportion of problem-posing tasks built into the materials that students use every day. If curriculum is a major agent of change for the teaching and learning of mathematics, there simply may not be enough problem posing tasks in current curriculum materials to realize the goals stated in the reform documents in both Chinese and US classrooms. More specifically, although curriculum designers have clearly made some efforts to include problem posing in mathematics textbooks, these efforts have resulted in uneven inclusion, both with respect to content area and to grade level. The results of this study suggest that in order to better support teachers as they attempt to fulfill reform recommendations to engage their students in problem-posing activities and to develop their students' mathematical dispositions around problem posing, curriculum developers will need to carefully examine the quantity and types of problem-posing tasks that are included at every grade level. In particular, the dearth of problem-posing tasks related to geometry and measurement is somewhat perplexing and requires attention.

In summary, findings from the two studies reported in this paper not only contribute to our understanding about the inclusion of PP tasks in curriculum both historically and internationally, but also suggest a great need to systematically integrate PP activities into curriculum and instruction. The fact that both Chinese and US curriculum standards have heavily emphasized PP in school mathematics, despite there being only a small proportion of PP activities in both Chinese and US elementary mathematics curriculum, suggests the existence of challenges that are delaying the implementation of reform ideas such as problem posing in school mathematics.

The two studies reported in this paper have also established a framework for and feasibility of analyzing problem-posing tasks in mathematics curriculum. Thus, a natural extension of these two studies is to analyze problem-posing tasks in middle and high school mathematics curriculum in both China and the US. In addition, it should be noted that curriculum operates on several levels. These two studies have focused on the intended curriculum as embodied in mathematics textbooks. Thus far, there have been no studies that have reported on the actual use of problem-posing tasks from this mathematics textbook series in real classrooms. Looking forward, in addition to extending the studies to analyze problem posing in middle and high school mathematics curricula, there are at least three other directions in which the studies can be extended. The first would be to investigate students' thinking involved in engaging in mathematical problem posing in the various problem-posing situations. Different types of problem-posing tasks were identified in Chinese and US textbooks, but we do not know how students approach these different types of situations and pose problems. Second, future studies must also attend to the implemented curriculum, i.e., how problem posing actually happens in classrooms using regular curriculum materials. Finally, studies are needed to investigate the impact of engaging students in problem posing tasks on their learning of mathematics.

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