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The Transition from School to University in Mathematics: Which Influence Do School-Related Variables Have?

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Abstract Particularly in mathematics, the transition from school to university often appears to be a substantial hurdle in the individual learning biography. Differences between the characters of school mathematics and scientific university mathematics as well as different demands related to the learning cultures in both institutions are discussed as possible reasons for this phenomenon. If these assumptions hold, the transition from school to university could not be considered as a continuous mathematical learning path because it would require a realignment of students' learning strategies. In particular, students could no longer rely on the effective use of school-related individual resources like knowledge, interest, or self-concept. Accordingly, students would face strong challenges in mathematical learning processes at the beginning of their mathematics study at university. In this contribution, we examine these assumptions by investigating the role of individual mathematical learning prerequisites of 182 first-semester university students majoring in mathematics. In line with the assumptions, our results indicate only a marginal influence of school-related mathematical resources on the study success of the first semester. In contrast, specific precursory knowledge related to scientific mathematics and students' abilities to develop adequate learning strategies turn out as main factors for a successful transition phase. Implications for the educational practice will be discussed.

Keywords Individual learning processes · Role of learning prerequisites · Study success · Transition school – university

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Introduction

The transition from school to university is experienced as an exciting period by many first-year students. Leaving school and entering university can be considered as the beginning of a new stage of life. For students, this period is often accompanied by increasing personal responsibility and for students enrolled in university programs with courses in higher mathematics in particular, this transition between two educational institutions turns out to be a major challenge. Dieter (2012), for example, reports a dropout rate of up to 40 % among first-year students at German universities. In general, high dropout rates can be considered as a serious problem because it causes individual psychological strain for the involved students on the one hand and economical costs for society on the other hand (Rasmussen & Ellis, 2013). Accordingly, possible reasons for dropout have been discussed and examined for decades (e.g. Bressoud, Mesa, & Rasmussen, 2015; Rasmussen & Ellis, 2013; Tinto, 1975). While for example, sociological reasons comprise problems to finance the study or aspects of family planning in the study phase, educational reasons are concerned with the quality of teaching (cf. Gueudet, 2008), different teaching approaches (cf. Bressoud et al., 2015), and the fit between students' individual learning approaches and academic learning opportunities or academic demands (cf. Eley & Meyer, 2004). In this article, we mainly concentrate on educational reasons for study termination and on students' individual characteristics. This restricted focus on individual student characteristics has pragmatic reasons and stems from the conditions of the empirical study presented below. However, both, students as learners as well as universities as educational institutions, are responsible for the organization of effective academic learning environments. According to person-environment fit theories, an important factor for effective learning is the high degree of matching between an individual learner and the environmental characteristics. Therefore, the responsibility to prevent high dropout rates does not rest with the students (or schools) alone.

Following a cognitive-constructivist perspective on learning, learners must possess adequate learning prerequisites when challenged by new learning opportunities. For studying mathematics at university, students, for example, need appropriate prior mathematics knowledge in order to benefit from academic learning environments. Many empirical studies in the field of higher education examine such variables for individual learning prerequisites but often remain on a domain-independent level and ignore domain-specific features (e.g. Valle et al., 2003). Only a few studies specifically focus on mathematical learning processes at university and connect individual factors of study success with features of the subject academic mathematics (e.g. Hailikari, Nevgi, & Komulainen, 2008). Such a connection is necessary for a deeper interpretation of the identified factors and, consequently, for the subsequent development of ideas to optimize learning processes at university. Especially in the case of mathematics, the specific character of mathematics as a school subject and mathematics as an academic discipline must be taken into account when investigating the transition problems of students. Based on these assumptions, we conducted a study with 182 first semester students in mathematics to analyze (a) which mathematics-related cognitive and affective learning prerequisites students bring from school to university, and (b) how school-related mathematical learning prerequisites influence the individual study success in the first semester. Against the background of the differences between school mathematics and mathematics as an academic discipline, the question of which school-related learning prerequisites matter for study success in the first semester is addressed in particular.

Theoretical Background

Before addressing previous research concerning mathematics learning at university, we shortly introduce the person-environment fit theories as a general framework for our research. Especially the first year at university seems to play an important role for study success in mathematics. It is plausible that the high dropout rates in this phase (cf. Dieter, 2012) are caused by the specific challenges of the transition process from school to university. Although students leave school with a degree, they show serious problems in academic learning processes at the beginning of university study. For the subject mathematics, students are faced with new demands of the academic learning environment which often turn out to be a hurdle in individual learning processes (Clark & Lovric, 2009). A similar type of problem is well known in the field of work and organizational psychology. Here, person-environment fit theories provide frameworks to describe effective and ineffective arrangements in organizations (with respect to job performance, job satisfaction, stress, e.g. Kristof-Brown, Zimmerman, & Johnson, 2005). In these theories, the degree to which individual and environmental characteristics match is considered as a main factor of influence.

Person-environment fit theories have also been used in educational research projects concerning the transition from school to work (Swanson & Fouad, 1999) and from school to university (across domains: Nagy, 2006). The assumption is that in order to generate successful learning processes, a learner's individual characteristics (e.g. prior knowledge, interest, attitudes) and characteristics of the learning environment (e.g. demands, learning opportunities) have to fit together (Lubinski & Benbow, 2000; Swanson & Fouad, 1999). The corresponding models include cognitive variables—e.g. students' knowledge vs. cognitive demands—as well as affective variables—e.g. learners' wishes or expectations vs. the real situation. Following this assumption, unsuccessful learning processes indicate an inappropriate fit between variables of the involved learner and the learning environment. Accordingly, it is necessary to analyze and to describe the current mathematical learning environment at university and the corresponding individual learners' characteristics.

Learning Environment of Studying Mathematics at University

Based on a review of research literature, we postulate two fundamental differences between learning mathematics at school and learning mathematics at university: (1) the character of mathematics that is taught, and (2) the demands of learning opportunities.

School Mathematics vs. Academic Mathematics at University. In many countries, mathematics at school is of a different character as mathematics at university (e.g. Germany: Rach & Heinze, 2011; South Africa: Engelbrecht, 2010; UK: Hoyles, Newman, & Noss, 2001; Hong Kong: Luk, 2005). Although this general observation seems to be true for many countries, the specific differences of mathematics at school and at university might vary between countries due to traditions concerning curricula or learning goals. The subsequent presentation refers to the situation in Germany which was analyzed in empirical studies (Vollstedt, Heinze, Gojdka, & Rach, 2014; Witzke, 2015) and which is relevant for the empirical study we present below. Nevertheless, several of the described features might be true for other countries as well. Mathematics as a school subject, as it appears in the current curricula, is predominantly legitimated from a utilitarian perspective: mathematics concepts and procedures are useful tools for modeling, describing, and explaining real life situations and for solving associated everyday problems (e.g. Hoyles et al., 2001; Witzke, 2015). Accordingly, technical aspects (e.g. manipulating algebraic expressions) as well as modeling and problem-solving activities in real contexts seem to be central. In contrast, propaedeutic aspects toward a scientific perspective (e.g. formal definitions and proofs) are underrepresented and only sporadic (and are mostly implemented in the geometry curriculum, Witzke, 2015). This emphasis of a utilitarian perspective corresponds with the contribution of the school subject mathematics to the goal of general education. Contrary to that, mathematics at university level is predominantly legitimated from a scientific perspective. Mathematics as a scientific discipline (here also denoted as academic mathematics) is characterized by formally defined abstract concepts, logical deduction of mathematical theorems, and formal deductive proofs (Engelbrecht, 2010).

This difference between mathematics at school and at university has consequences, for example, for the introduction of new concepts. At school, teachers lay emphasis on a concept development which pick up and further develop experiences and knowledge students already have. In general, this means that the meaning of a concept is rooted in students' experiences from real life so that a concept image is induced in students' minds (Tall & Vinner, 1981). Depending on the specific school context and related educational standards, a formalized description of the concept (a formal definition) is less important. In contrast, at university, concepts are formally often defined in a de-ontologized sense by their characteristic properties (i.e. the concept definition plays the essential role, Engelbrecht, 2010). In particular, concept images in the sense of specific mental representations are rarely induced by university teachers or academic textbooks (Vollstedt et al., 2014) so that students must apply specific elaboration strategies for an individual construction of the meaning of formally defined concepts. A second consequence of the different characters of school and university mathematics is the different role of proofs. From a utilitarian perspective, it is sufficient that mathematical concepts and rules are reliable (Witzke, 2015). This means, for example, that a person who wants to apply a mathematical rule in a specific context must be sure that this rule provides a result which can be considered as acceptably accurate for this specific context. So, for many mathematical statements in school mathematics, in principle empirical evidence is sufficient to take validity for granted—that may be one reason why there are only rare opportunities to prove a statement in German school textbooks (Vollstedt et al., 2014). In contrast, at university, the scientific mathematical community has developed strict standards whose kind of argument is accepted as evidence for mathematics as a scientific discipline. Moreover, the functions of proofs in mathematics research comprise much more than just providing evidence for the validity of a statement (e.g. Hanna, 2000).

These two examples, definition of concepts and proof of mathematical statements, indicate two differences between mathematics at school and at university. Tall (1991) described the corresponding changes as follows: "The move from elementary to AMT [advanced mathematical thinking] involves a significant transition: that from describing to defining, from convincing to proving in a logical manner based on definitions" (Tall, 1991, p. 20). The specific character of academic mathematics is unfamiliar to most of the students when entering university so that they are faced with great challenges. Several research projects on the so-called advanced mathematical thinking (Tall, 1991) explored specific hurdles and problems. For example, Moore (1994) gave an overview about reasons of students' problems to formally prove mathematical statements. Following these results and the fact that the change of the subject mathematics during the transition from school to university is abrupt, it can be questioned that learner characteristics specifically related to school mathematics play a significant role for a successful transition. In particular, we ask to what extent students' knowledge and skills, interest, and self-concept concerning school mathematics are effective for mastering the unfamiliar mathematical challenges in a first semester mathematics course.

Learning Opportunities at School and at University. A second apparent difference between school and university is the type of learning opportunities and the demands for self-regulated learning. At university, mathematical content is often presented in lectures (Bergsten, 2007). In self-study phases students have to solve mathematical tasks that are also discussed in tutorials (Rach & Heinze, 2011). However, not only the formal organization of learning opportunities (in lectures, tutorials, and self-study phases), but also the teaching process itself changes: at school, mostly educated teachers give lessons, at university, often researchers are responsible for the teaching process (Thomas, Klymchuk, Hong, Kerr, McHardy, Murphy, Spencer, Watson et al., 2010). It is assumed that university researchers know only little about students' prior knowledge and expectations at the beginning of study (Clark & Lovric, 2009). Moreover, mathematical tasks at university are not divided into (easily manageable or well-structured) subtasks as at school (Gueudet, 2008).

Because of this more content- than student-oriented presentation of mathematics (Bergsten, 2007), students need self-regulated learning strategies to make the content accessible. Accordingly, teaching processes at university are criticized as less well didactically prepared as at school (cf. Clark & Lovric, 2009; Gueudet, 2008; Weber, 2004). At school, the structure of a mathematical content is generally organized in a student-oriented manner, following learning trajectories based on mental models and relating mathematical concepts to everyday problems. In contrast, at university, the definition-theorem-proof structure is often used to present mathematical ideas to students. Because of this product orientation, the genesis of mathematical knowledge remains implicit (Engelbrecht, 2010; Gueudet, 2008). Accordingly, students have to use in-depth strategies, e.g. elaboration strategies, to make the content accessible and to connect the new content to their existing knowledge structure.

The described differences in the learning environments at university and at high school cause specific challenges for students' learning activities. In particular, successful learning strategies, students used at school, might fail at university and need to be adapted. Against this background, it is unclear which impact individual school-related cognitive and affective learning prerequisites will have on students' learning in their first semester at university.

Impact of Learning Prerequisites and Learning Strategies for Successful Learning Processes: Findings from Educational Research

In many studies on learning processes at school and at university, affective and cognitive variables are considered as important factors for further learning. This influence of learning prerequisites on the acquisition of mathematical knowledge may be direct or indirect (i.e. mediated by learning strategies, cf. Trigwell, Ashwin, & Millan, 2013). In the following paragraphs, we present a short overview of five selected individual variables (interest, self-concept, specific prior knowledge, prior achievement, and learning strategies). There is empirical evidence that these variables are relevant for successful learning processes so that they were included in our study presented below.

Individual (*subject-specific*) *interest* can be defined as "a relatively enduring predisposition to attend to certain objects and activities and is associated with positive affect, persistence, and learning" (Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005, p. 399). Findings from school-based surveys indicate that interest in mathematics decreases during lower secondary level (Frenzel, Goetz, Pekrun, & Watt, 2010). Concerning the influence of interest in mathematics on mathematics achievement, many studies report only moderate correlations (e.g. Malmivuori, 2006). Longitudinal studies addressing the upper secondary level (e.g. Köller, Baumert, & Schnabel, 2001) report small direct effects of mathematical interest on mathematics achievement from grade 10 to grade 12 and indirect effects on course selection in upper secondary level. We assume a similar (mediated) relationship between interest and achievement for learning processes in mathematics at university. Learning processes should be more successful with a high degree of interest because the concentration on the learning content does not use extra capacities to maintain learning motivation.

Academic self-concept (or domain-specific self-concept of ability) can be defined as the mental model of one person about their knowledge (Trautwein, Lüdtke, Köller, & Baumert, 2006). The development of self-concept mainly originates from experience of competence. As empirical studies show, the correlation between mathematical self-concept and cognitive learning progression at school are substantial (Chen, Yeh, Hwang, & Lin, 2013; Köller, Trautwein, Lüdtke, & Baumert, 2006; Seaton, Parker, Marsh, Craven, & Yeung, 2014). However, the empirical results of Hailikari et al. (2008) in a mathematics course with 139 students do not support the influence of academic self-concept on study success. Thus, the role of mathematical self-concept for learning progression is empirically not as clear as it is on the theoretical level. Furthermore, mathematics self-concept decreases during school (Hannula, Maijala, & Pehkonen, 2004; Köller et al., 2006) which is also expected during the transition from school to university in mathematics. On the one hand, this can be explained with the big-fish-little-pond effect because students compare their performance with a different social group (e.g. Marsh, Trautwein, Lüdtke, Baumert, & Köller, 2007). On the other hand, the high demands at university give students the feeling of a lower performance at university in comparison to their performance at school.

Content knowledge in the sense of prior knowledge is presumed to be an important factor for successful learning processes in mathematics. The cumulative character of mathematical knowledge is often held accountable for this result, which is backed by striking empirical evidence (Hailikari et al., 2008; Halverscheid & Pustelnik, 2013; Köller et al., 2006). Following cognitive-constructivist learning theories, new knowledge is integrated into existing knowledge which extends and deepens student's individual knowledge. Accordingly, a higher level of prior knowledge provides a better opportunity for this integration. However, a necessary condition is that the prior knowledge fits to the new content. Several researchers conclude, mainly from theoretical analyses, that many students enter university with insufficient learning prerequisites in mathematics (Selden, 2005).

Another important predictor of learning progression which is related to prior knowledge but less subject-specific is the *prior academic achievement* or *previous study success*. Empirical studies at the beginning of university indicate that the school leaving grade (examination) predicts study success at university (e.g. Hailikari et al., 2008; Richardson, Abraham, & Bond, 2012). In the study of Schiefele, Streblow, and Brinkmann (2007), the prior academic achievement is found to predict the time frame for students' dropout: students, who already left university after the first or second semester, reported a worse prior achievement than students who dropped their study after their third semester or later.

As already mentioned in the "Introduction", learning prerequisites may influence the learning progression directly as well as indirectly, mediated by the use of learning strategies. *Learning strategies* can be described as "behaviors and thoughts that a learner engages in during learning and that are intended to influence the learner's encoding process" (Weinstein & Mayer, 1986, p. 315). The use of learning strategies is assumed to be a substantial mediator between learners' prerequisites and the learning achievement (Fenollar, Román, & Cuestas, 2007). A special learning strategy is the use of self-explanations which was found successful in many studies (e.g. Chi, de Leeuw, Chiu, & Lavancher, 1994). By using self-explanations, learners probably elaborate the learning content in a deeper way. However, there are also studies which question a general positive effect of learning strategies (e.g. the use of a deep approach for undergraduate students: Trigwell et al., 2013).

Research Questions

There are many empirical studies that confirm the influence of various affective and cognitive variables as well as the use of specific learning strategies when learning mathematics at school. However, mathematics taught at school differs substantially from mathematics taught at university. Hence, it is not clear, whether these results also hold for the transition phase school-university. There are some empirical studies addressing learning processes at university and focusing on learning prerequisites and the use of learning strategies (e.g. Valle et al., 2003). However, these studies are often cross-sectional studies including different subjects or study programs. Therefore, these studies lack the subject-specific analysis of the challenges accompanied with the school-university transition, which might reduce the explanatory power of the variables that predict study success. In particular, it is unclear to what extent and how school-related learner characteristics (like overall school achievement, achievement in school mathematics, self-concept and interest related to school mathematics) contribute to a successful transition school-university in mathematics. Accordingly, our first research question is as follows:

1. Which individual learning prerequisites related to school mathematics influence the individual study success in the first semester of a mathematics university program?

To examine research question 1, we examine the role of several individual cognitive and affective variables which are specific for school mathematics (i.e. these variables are related to the school content as it is described in the curriculum and realized in mathematics lessons).

Depending on the mathematics teacher and on the internal school program it might happen that students in some schools are offered additional learning opportunities with a stronger emphasis on a scientific view of mathematics (e.g. toward formal proofs, formal definitions, exploration of mathematical structures or relations instead of finding and applying rules). Hence, a certain group of students acquires specific mathematical abilities already in school which can serve as a preparation for the scientific mathematics as it is taught at university. Though these mathematical abilities are acquired in school, we want to distinguish these in the following from learning prerequisites related to school mathematics because they are in a certain sense related to scientific mathematics and are not prescribed by the German upper secondary curriculum. Here, the question arises whether there is a connection between cognitive and affective learning prerequisites related to scientific mathematics and cognitive and affective learning prerequisites related to school mathematics:

2. How strong are the connections between individual learning prerequisites related to school mathematics and corresponding learning prerequisites related to scientific mathematics?

In particular, we are interested in the variables knowledge, self-concept, and interest.

Finally, we want to compare the influence of both kinds of individual learning prerequisites on study success. Hence, the third research question is as follows:

3. How strong are the effects of individual learning prerequisites related to school mathematics on study success in comparison to that of individual learning prerequisites related to scientific mathematics?

Based on the analysis presented in the previous sections, we hypothesize that the influence of individual learning prerequisites related to school mathematics is weak. As a consequence, students' self-concept and interest related to scientific mathematics is significantly lower than their self-concept and interest related to school mathematics. Moreover, we assume that prior knowledge and learning strategies related to scientific mathematics, which to a certain extent might already be acquired during school, have a much stronger influence on students' study success in scientific mathematics.

Methods

Sample and Design

The sample consisted of 182 first-semester students (95 female, 87 male) of a university in a medium city in Germany. All students started to study mathematics as a major. The sample covered all students of the first-semester course "Analysis 1".

There were three data collection time points in the first semester either during lectures or tutorials. At the beginning of semester (t1), data for cognitive and affective learning prerequisites students' acquired in school (overall school achievement, achievement in school mathematics, interest and self-concept related to school mathematics) was collected. Moreover, we administered a test on prior knowledge which is specifically related to scientific mathematics. In the middle of the semester (t2), students reported their interest and self-concept related to scientific mathematics and their individual use of self-explanation strategies. At the end of the semester (t3), data on students' study success was collected. Overall, a moderate dropout rate during the first semester was observed (see Table 1).

Instruments

For measuring students' interest and self-concept related to school or scientific mathematics, we used validated instruments that were adapted for this study (interest in mathematics from Pekrun, Goetz, Titz, & Perry, 2002; mathematics self-concept from Köller et al., 2006). We used the same questionnaires at t1 to collect data related to school mathematics and at t2 to collect data related to scientific mathematics. For both scales, interest and self-concept, the students were asked to assess statements on a four-point Likert scale (see Table 1).

To measure prior knowledge related to scientific mathematics, we used a validated test instrument addressing a specific precursory knowledge for the first semester Analysis 1 course (Wagner, 2011). The test consists of 11 multiple choice and open-ended items—item formats which are familiar to high school students in Germany. It covers, for example, formal presentation of mathematics, the use of properties of mathematical concepts, deep conceptual knowledge, formal proofs etc. as it is described for the advanced mathematical thinking (e.g. Tall, 1991). Though the test items address

Table 1 Overview of the instruments

Variable	Sample item	Data collection time point
Prior knowledge in scientific mathematics (11 items)	Let f be $f: \mathbb{R} \to \mathbb{R}, f(x) = \lvert x \rvert$. Give a mathematical proof that f is not differentiable at zero.	t1 ($N = 182$)
Interest in mathematics (6 items, Likert scale: 0 = disagree, 3 = agree)	(1) Mathematics for me is an important subject.	t1 ($N = 182$)
	(2) I am interested in mathematics.	t2 ($N = 141$)
Mathematics self-concept (4 items, Likert scale: 0 = disagree, 3 = agree)	(1) As far as mathematics is concerned, I am pretty fit.	t1 ($N = 181$)
	(2) I am good in mathematics.	t2 ($N = 139$)

mathematical concepts which also occur in German school curricula, the perspective in most of the items is unfamiliar to school students. In particular, tasks of this type are hardly implemented in the regular mathematics classroom. The test was checked by an upper secondary mathematics teacher who confirmed that only 3 of the 11 items might be considered as standard in the regular mathematics classroom, whereas the other items are advanced for the students. A sample item is presented in Table 1, another sample item can be found in the electronic supplemental materials (ESMs) or in Rach & Heinze, 2011. Each item was scored dichotomously, i.e. one point for a solution which was accepted as correct and zero points for other solutions or missing.

As a second variable for a learning prerequisite related to scientific mathematics, we collected data for the applied learning strategies at t2 in the middle of the first semester. We focused on *students' individual use of self-explanations* and standardized the context to students' work on the weekly exercises. Students were asked to report which of the following three learning types fits best their own behavior in these self-study phases: *reproducing type* (I study the exercises intensively and try to solve them. I try to comprehend the solutions of other students. I rarely give self-explanations.), *self-explanation type* (I study the exercises intensively and try to solve them. I try to comprehend the solutions of other students. I explain the solution to myself and/or to other students even if I rarely find solutions myself.) or *self-solver type* (Often, I can solve the exercises or I find ideas for solutions. Then I explain the solution to myself and/or other students.). The *reproducing* and the *self-explanation type* only differ in the level of using self-explanations while working on mathematical tasks: the reproducing type uses this elaboration strategy only rarely, whereas the self-explanation type uses it more often. The *self-solver type* was used to identify the high-achieving students who start their study with good learning prerequisites (see Rach & Heinze, 2011). Students of the self-solver type also use self-explanations in their self-study phases but find solutions on their own.

Finally, the *study success* in the first semester as dependent variable was defined by the success in the essential first semester course "Analysis 1". In this course, the newly enrolled students are faced with mathematics as a scientific discipline (i.e. with formal definitions of abstract concepts, mathematical theorems, and deductive proofs) for the first time. Accordingly, this course is very important for the whole course of studies (see Weber, 2008). To be successful in this course, students had to solve weekly mathematical tasks and two attempts to pass a written examination were allowed at the end of the first semester. We categorized the students of the study into three groups: *successful students*, who passed the examination in the first or second attempt, *failing students*, who participated in the examinations but were not successful, and *dropout students*, who ended this course during the semester and did not participate in one of the examinations.

Results

Descriptive Data Concerning Learning Prerequisites

Table 2 provides descriptive data for the instruments. The reliabilities range from acceptable to good. The test on prior knowledge in scientific mathematics shows the weakest reliability (α = 0.62) which can be considered as acceptable since the test measures a broad construct.

Table 2 Descriptive data for the different instruments

Variable	Data collection time point	Mean (SD)	Cronbach's α
Prior knowledge in scientific mathematics (11 items)	t1 ($N = 182$)	4.80 (2.28)	0.62
Interest in mathematics (6 items) (Likert scale: 0 = disagree, 3 = agree)	t1 ($N = 182$)	2.14 (0.44)	0.76
	t2 ($N = 141$)	1.96 (0.50)	0.81
Mathematics self-concept (4 items) (Likert scale: 0 = disagree, 3 = agree)	t1 ($N = 181$)	1.85 (0.46)	0.78
	t2 ($N = 139$)	1.57 (0.55)	0.83

Since the test on prior knowledge in scientific mathematics comprised tasks which are unfamiliar to the students, we got a broad range of answers to the open-ended items. In the ESM, we insert a typical student's answer to the sample item asking for a proof that $f: \mathbb{R} \to \mathbb{R}, f(x) = |x|$ is not differentiable at zero. The student provided the graph of the absolute value function and pointed to the kink—an argument which is often accepted in school but which was not accepted in our test to distinguish between school mathematics and scientific mathematics. There were also several students providing the argument that the left and right derivative are different at $x = 0$ which was accepted as correct.

Data for the overall school achievement was collected by asking the students for their school leaving examination marks (in a sense the grade point average in Germany). The theoretical range of this scale reaches from 1.0 (very good) to 4.0 (sufficient) and for our sample a mean of $M = 2.33$ ($SD = 0.66$) was identified. For their achievement in school mathematics, students were asked for their final mark in mathematics as an indicator. The theoretical range of this scale varies from 0 (very poor) to 15 points (very good). For our sample, a mean of $M = 11.53$ ($SD = 2.50$) was identified.

Influence of School-Related Learning Prerequisites on Study Success

In order to answer the first research question, the influence of school-related learning prerequisites on the study success at the end of the first semester was examined. We conducted a logistic regression analysis with the dichotomous criterion *study success* (passing the examination vs. failing/dropout) as the dependent variable (see model 1 in Table 3). Only the overall school achievement ($\mathrm{Exp}(B) = 0.35$) and students' achievement in school mathematics ($\mathrm{Exp}(B) = 1.40$) turned out to be significant predictors of the study success. The affective school-related variables, i.e. interest and self-concept related to school mathematics, did not have an additional impact on study success. Only 29 % of the variance (Nagelkerke R^2) in the study success at the end of the first semester could be explained by the overall school achievement and students' achievement in mathematics.

Connections Between School-Related and University-Related Learning Prerequisites

For the second research question, we have analyzed the correlations between learning prerequisites related to school mathematics and the corresponding learning prerequisites related to scientific mathematics (see Table 4).

Table 3 Learning prerequisites for study success: mean values (standard deviations) and results of the logistic regression analyses

	Successful ($n = 47$)	Not successful ($n = 135$)	Model 1	Model 2	Model 3
Overall school achievement	1.90 (0.50)	2.48 (0.64)	Wald(1) = 6.90, $p < 0.01$, Exp(B) = 0.35	Wald(1) = 4.80, $p < 0.05$, Exp(B) = 0.40	Wald(1) = 12.02, $p < 0.01$, Exp(B) = 0.28
Achievement in school mathematics	13.09 (1.69)	10.99 (2.52)	Wald(1) = 7.21, $p < 0.01$ Exp(B) = 1.40	Wald(1) = 2.39, $p = 0.12$	
Interest related to school mathematics	2.16 (0.44)	2.14 (0.44)	Wald(1) = 0.61, $p = 0.44$	Wald(1) = 1.77, $p = 0.18$	
Self-concept related to school mathematics	1.95 (0.47)	1.82 (0.45)	Wald(1) = 0.50, $p = 0.48$	Wald(1) = 0.00, $p = 0.97$	
Prior knowledge in scientific mathematics	6.57 (2.21)	4.18 (1.96)		Wald(1) = 15.78, $p < 0.001$, Exp(B) = 1.57	Wald(1) = 19.90, $p < 0.001$, Exp(B) = 1.58
Nagelkerke R^2			0.29	0.40	0.38

Overall school achievement: 1.0 (very good) to 4.0 (sufficient), achievement in school mathematics: 0 (very poor) to 15 (very good), interest and self-concept related to school mathematics: Likert scale from 0 (disagree) to 3 (agree), prior knowledge in scientific mathematics: score 0–11 points

Table 4 Correlations between school-related and university-related learning prerequisites

			Prerequisites related to school mathematics			Prerequisites related to scientific mathematics		
			2	3	4	5	6	7
Prerequisite related to school mathematics	1	Overall school achievement	−.64**	−.05	−.10	−.37**	.13	.04
	2	Achievement in school mathematics	–	.22**	**.29****	**.43****	.14	.25**
	3	Interest in school mathematics		–	.50**	.18*	**.62****	.30**
	4	Self-concept related to school mathematics			–	.26**	.32**	**.59****
Prerequisite related to scientific mathematics	5	Prior knowledge in scientific mathematics				–	.23**	.21*
	6	Interest in scientific mathematics					–	**.54****
	7	Self-concept related to scientific mathematics						–

N = 182 (variables 1–4); N = 139–141 (variables 5–7); $^{**}p < 0.01$, $^{*}p < 0.05$, bold: still significant when considering partial correlations (see text)

The results show small to large correlations between the school-related and university-related learning prerequisites. In particular, the correlation between achievement in school mathematics and prior knowledge in scientific mathematics is only of medium size ($r = 0.43$) and, thus, surprisingly weak (about 18 % explained variance). It is almost of similar size as the correlation between overall school achievement and prior knowledge in scientific mathematics. Further analyses, taking into account partial correlations, revealed that, in fact, there are only substantial correlations between school-related and university-related learning prerequisites among the corresponding cognitive variables as well as affective variables. This, in particular, means that there is no substantial correlation of prior knowledge in scientific mathematics with interest or self-concept related to school mathematics when controlling for interest respectively self-concept related to scientific mathematics.

When considering the affective variables, we found strong correlations between school-related and university-related interest (same for self-concept). These results are based on a slightly positive selected sample because interest and self-concept related to scientific mathematics were measured about 7 weeks after the beginning of the first semester when some of the freshmen already gave up. Despite of this strong correlation and the slightly positive selection it seems that the students experience school mathematics and scientific mathematics as two subjects, which are related but not identical: the absolute values for interest significantly differs depending on whether it is related to school mathematics or to scientific mathematics (related to school mathematics: $M = 2.18$ (0.42) vs. related to scientific mathematics: $M = 1.97$ (0.50), $t(138) = 6.20$, $p < 0.001$, $d = 0.45$). The same result was found for the self-concept (related to school mathematics: $M = 1.90$ (0.45) vs. related to scientific mathematics: $M = 1.57$ (0.55), $t(138) = 8.40$, $p < 0.001$, $d = 0.66$).

For $n = 136$ students, additional information about the learning strategies they have used during self-study phases had been gathered. We considered this information as a learning prerequisite related to scientific mathematics as well: 49 students (36 %) reported that they only reproduce the solution processes of others without using self-explanations (reproducing type), 70 students (51 %) reported that they use self-explanations while reading mathematical solutions of others (self-explanation type), and 17 students (13 %) reported that they solve the mathematical tasks mainly on their own (self-solver type). Unlike in other countries, in many German universities, the weekly exercises are very demanding so that it is not surprising that only a small number of students can solve most of the tasks on their own. To answer the question whether learning prerequisites related to school mathematics are associated with the reported learning prerequisites related to scientific mathematics, we have conducted a MANCOVA with the three different learning types as independent variables and prior knowledge in scientific mathematics as covariate. The results indicated a significant difference between the three learning types concerning their learning prerequisites related to school mathematics (Pillai's trace $V = 0.13$ with $F(8, 260) = 2.18$, $p < 0.05$, $\eta^2 = .06$). However, the post hoc tests (Bonferroni correction) showed only significant differences between the 17 students of the self-solver type and the 119 students of the other two types (see Table 5). This means that the high-performing students of the self-solver type were already at school slightly different concerning their achievement in school mathematics and the related self-concept. The interesting result is that school-related variables do not have an impact on the use of self-explanation strategies in the first semester, which turn out to be influential for study success (see below and Table 6).

To summarize the results for research question 2, it turned out that the individual prerequisites related to school mathematics have only a surprisingly weak connection to the individual prerequisites related to scientific mathematics. Hence, it is the question

Table 5 MANCOVA results concerning the relationship between learning prerequisites related to school mathematics and learning type (covariate: prior knowledge in scientific mathematics)

Estimated marginal means	Reproducing type ($n = 49$) *M* (*SD*)	Self-explanation type ($n = 70$) *M* (*SD*)	Self-solver type ($n = 17$) *M* (*SD*)	ANCOVA
Overall school achievement	2.40 (0.72)	2.24 (0.57)	1.91 (0.47)	$F(2, 135) = 1.27$, $p = 0.28$
Achievement in school mathematics	10.98 (3.00)	11.86 (2.01)	13.82 (0.95)	$F(2, 135) = 3.18$, $p < 0.05$; $\eta^2 = 0.05$, difference between reproducing and self-solver type
Interest in school mathematics	2.12 (0.45)	2.23 (0.40)	2.25 (0.43)	$F(2, 135) = 0.72$, $p = 0.49$
Self-concept related to school mathematics	1.80 (0.46)	1.92 (0.35)	2.28 (0.49)	$F(2, 135) = 5.19$, $p < 0.01$; $\eta^2 = 0.07$, difference between self-solver and the other two types

Overall school achievement: 1.0 (very good) to 4.0 (sufficient), school achievement in math.: 0 (very poor) to 15 (very good), interest and self-concept: Likert scale from 0 (disagree) to 3 (agree) which role school-related variables play for study success in the first semester in comparison to learning prerequisites related to scientific mathematics.

Comparing the Influence of School-Related and University-Related Learning Prerequisites on Study Success

For the third research question, we have examined the influence of school-related learning prerequisites on the study success at the end of the first semester in comparison to that of the individual prerequisites related to scientific mathematics. As for research question 1, a logistic regression analysis with the school-related learning prerequisites and, additionally, prior knowledge in scientific mathematics as independent variables and the dichotomous criterion study success as the dependent variable was conducted. The results are presented in model 2 of Table 3. Only the overall school achievement (Exp(B) = 0.40) and the prior knowledge in scientific mathematics (Exp(B) = 1.57) turned out to be significant predictors of the study success. If these variables are solely included into the regression model (see model 3 of Table 3), then both predictors explain 38 % of the variance (Nagelkerke R^2) for the study success at the end of semester (29.5 % by the prior knowledge in scientific mathematics and only 8.5 % by the overall school achievement).

Again, for the subsample of n = 136 students, we used additional data for interest and self-concept related to scientific mathematics and for their use of self-explanations. We extended the models from Table 3 and also included these variables as independent variables into the logistic regression analysis (the three learning types as dummy variables with reproducing type as reference category). The results show that students' use of self-explanation as learning strategy instead of just reproducing solutions is another significant predictor for study success at the end of the first semester (see model 1 and 2 in Table 6). The few students of the self-solver type also benefit from their specific working style; however, these students belong to the specific group of high-performers which already start with better prerequisites. Introducing the learning type in this model (with the reduced sample), nearly an additional 9 % of the variance (Nagelkerke R^2) in the study success can be explained (see model 2 and 3 in Table 6).

In the previous analyses, study success as a dichotomous variable (successful vs. unsuccessful) was included, though we have more detailed information about the unsuccessful students. In order to analyze the influence of the learning prerequisites particularly for the distinction between failing and dropout students, the dependent variable "study success" was split into three categories (successful students, n = 47, failing students, n = 66, dropout students, n = 69). The results of a multinomial logistic regression with the failing students as reference category showed only a significant influence of the self-concept related to school mathematics (Table 7). At the beginning of their study, the failing students report a higher self-concept related to school mathematics than the dropout students.

Discussion

High dropout rates at universities which are caused by failure in mathematics point to remarkable subject-specific challenges for students in the transition from school to

Table 6 Results of the logistic regression analysis on study success

	Successful (n = 44)	Not successful (n = 92)	Model 1	Model 2	Model 3
Overall school achievement	1.93 (0.50)	2.41 (0.63)	Wald(1) = 7.48, $p < 0.01$, Exp(B) = 0.31	Wald(1) = 8.04, $p < 0.01$, Exp(B) = 0.31	Wald(1) = 8.96, $p < 0.01$, Exp(B) = 0.31
Prior knowledge in scientific mathematics	6.41 (2.16)	4.38 (1.99)	Wald(1) = 7.86, $p < 0.01$, Exp(B) = 1.39	Wald(1) = 8.30, $p < 0.01$, Exp(B) = 1.39	Wald(1) = 14.03, $p < 0.001$, Exp(B) = 1.48
Interest in scientific mathematics	2.15 (0.44)	2.21 (0.41)	Wald(1) = 0.01, $p = 0.92$		
Self-concept related to scientific mathematics	1.95 (0.45)	1.91 (0.42)	Wald(1) = 0.01, $p = 0.92$		
Reproducing type	6	43	Wald(2) = 7.47, $p < 0.05$	Wald(2) = 9.99, $p < 0.01$	
Self-explanation type	25	45	Wald(1) = 5.11, $p < 0.05$, Exp(B) = 3.64	Wald(1) = 5.57, $p < 0.05$, Exp(B) = 3.62	
Self-solver type	13	4	Wald(1) = 6.50, $p < 0.05$, Exp(B) = 10.71	Wald(1) = 9.09, $p < 0.01$, Exp(B) = 10.51	
Nagelkerke R^2			0.41	0.41	0.32

Overall school achievement: 1.0 (very good) to 4.0 (sufficient), prior knowledge in scientific mathematics: score 0–11 points, interest and self-concept related to scientific mathematics from data collection t2: Likert scale from 0 (disagree) to 3 (agree), dummy variables for learning types: frequencies, reproducing type as reference category

Table 7 Mean values (standard deviations) of the learning prerequisites and results of the multinomial logistic regression analysis on study success for drop-out students with failing students as reference category

	Failing students (n = 66)	Dropout students (n = 69)	Regression analysis
Overall school achievement	2.55 (0.59)	2.41 (0.68)	Wald(1) = 1.38, p = 0.24
Achievement in school mathematics	10.97 (2.58)	11.01 (2.48)	Wald(1) = 0.15, p = 0.70
Interest related to school mathematics	2.24 (0.39)	2.04 (0.46)	Wald(1) = 1.38, p = 0.24
Self-concept related to school mathematics	1.95 (0.40)	1.69 (0.47)	Wald(1) = 5.06, $p < 0.05$, Exp(B) = 0.33
Prior knowledge in scientific mathematics	4.39 (1.98)	3.97 (1.94)	Wald(1) = 1.47, p = 0.23

Overall school achievement: 1.0 (very good) to 4.0 (sufficient), school achievement in mathematics: 0 (very poor) to 15 (very good), interest and self-concept: Likert scale from 0 (disagree) to 3 (agree), prior knowledge in scientific mathematics: score 0–11 points

university (Clark & Lovric, 2009). Within the framework of person-environment fit theories, the serious problems of many freshmen can be explained by an inadequate matching between learner characteristics and characteristics of the academic environment in which mathematics learning opportunities are provided. Concerning the environmental characteristics, we focused on the demands of the mathematical learning environments at university and elaborate on the qualitative differences between school and university with respect to the subject mathematics and the way of learning mathematics. Based on these specific differences, we considered students' individual learning prerequisites as learner characteristics. We investigated the relevance of freshmen's learning prerequisites related to school mathematics for a successful first semester in university mathematics which can be considered as an indicator for the fit between characteristics of learners and the environment.

The results of our empirical study indeed reveal that important cognitive and affective students' characteristics related to school mathematics have only a restricted impact on first semester study success in scientific mathematics (model 1 in Table 3). In fact, achievement in school mathematics as well as interest and self-concept related to school mathematics do not have a significant influence on study success of the first mathematics course at university when introducing the prior knowledge in scientific mathematics in the model (model 2 in Table 3; Hailikari et al., 2008). Interestingly, from the school-related variables only the subject-unspecific overall school achievement has a significant impact. These results support the assumption of a qualitative difference between school mathematics and scientific mathematics. This difference becomes apparently relevant for study success when students hardly experience a scientific perspective on mathematics in school. The low scores for the test on prior knowledge in scientific mathematics (M = 4.80, SD = 2.28) indicate that this is the fact in our sample. Moreover, these results are also in line with the assumption that learning mathematics at university requires other learning strategies than learning mathematics at school (Thomas et al., 2010). The overall school achievement can be interpreted as an indicator for a flexible cognitive ability of learning (of different subjects in different contexts) so that a high overall school achievement better allows freshmen to flexibly adapt their learning strategies to the academic learning context.

Following research question 3, we compared the role of freshmen's learning prerequisites related to school mathematics with those related to scientific mathematics (see models 2 and 3 in Table 3 and models 1 and 2 in Table 6). If we restrict our consideration to the factors with a significant influence, then it turns out that the learning prerequisites related to scientific mathematics have a much stronger impact than those related to school mathematics. This result might be considered as trivial because the learning content at university is scientific mathematics and not school mathematics. However, in combination with the findings to research question 2 (connection of school-related and university-related learning prerequisites), the results for research question 3 point to an important aspect. As elaborated for research question 2 (e.g. Tables 4 and 5), the learning prerequisites for scientific mathematics are not strongly correlated with those related to school mathematics. In particular, the cognitive variables *overall school achievement*, *achievement in school mathematics*, and *prior knowledge in scientific mathematics* correlate only on a moderate level. This indicates that students' characteristics related to school mathematics and those related to scientific mathematics are two different things. In particular, it confirms that school mathematics and scientific mathematics require different mathematical knowledge and abilities (Luk, 2005).

Summarizing all these empirical findings, we conclude that the transition from school to university should be considered as a disruption instead of a continuity concerning the learning of mathematics. As elaborated in the theoretical background, there are already theoretical analyses and empirical findings which describe the differences between school and university in mathematics (cf. Clark & Lovric, 2009; Hailikari et al., 2008). Nevertheless, from a mathematics education perspective, continuity in the sense of a spiral learning approach is always the basic assumption when considering learning progressions. Thus, there can be no presumption of simple continuity assumption in this case. The question on which aspects of mathematical knowledge and mathematical abilities students acquired in school are relevant for mathematics learning at university is much more difficult than it appears. Accordingly, the effectiveness of freshmen preparatory courses which are offered at many German universities before the first semester and which often emphasize a repetition of school mathematics can be questioned (Clark & Lovric, 2009). Based on our findings, it seems to be better to follow the idea of a disruption of mathematics learning during the transition from school to university. Hence, from a mathematics education perspective we have to ask which learning prerequisites are relevant for scientific mathematics courses in the first semester. This might provide ideas on how to organize preparatory courses for mathematics freshmen and to reduce the dropout.

Limitations of the Study

The limitations of this empirical study concern the sample and the administered instruments. The sample is restricted to all first semester students majoring in mathematics from a single university. Though this is the only university in the German federal state Schleswig-Holstein which offers a mathematics program and though this university has a good reputation, we cannot assume that the sample is representative for Germany. To explain the study success in the first semester course “Analysis 1”, we restrict our investigation to individual variables only. However, it is obvious that, in addition, factors related to the specific organization of teaching and learning at the considered university as well as the teaching quality of the mathematics professor of the "Analysis 1" course play a role for study success.

Finally, we partly use highly aggregated indicators for our variables (e.g. school leaving examination mark for overall school achievement). These indicators might be too distal so that a more proximal measure might provide results with a higher validity.

Conclusion

Summing up, with this study, we get an insight into the individual learning prerequisites of mathematics students entering university and into factors that predict study success in the first semester. We follow the assumption that an insufficient fit between students' characteristics and characteristics of the academic learning environment lead to unsuccessful learning processes and, in the end, to a dropout. For the subject mathematics, the results indicate that learning prerequisites related to school mathematics have only a minor impact on an adequate student-learning environment fit. Accordingly, specific learning prerequisites related to scientific mathematics should be fostered and developed before or at the beginning of the first semester. We are hopeful that this idea may help to create adequate learning opportunities for first semester students and to reduce dropout.

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