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'Boys Press All the Buttons and Hope It Will Help': Upper Secondary School Teachers' Gendered Conceptions About Students' Mathematical Reasoning

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Abstract Previous results show that Swedish upper secondary school teachers attribute gender to cases describing different types of mathematical reasoning. The purpose of this study was to investigate how these teachers gender stereotype aspects of students' mathematical reasoning by studying the symbols that were attributed to boys and girls, respectively, in a written questionnaire. The results from the content analysis showed that girls were attributed gender symbols including insecurity, use of standard methods and imitative reasoning, and boys were assigned symbols such as multiple strategies especially on the calculator, guessing and chance-taking.

Keywords Gender · Gender symbols · Mathematical reasoning · Teachers' conceptions · Upper secondary school

Introduction

In Sweden at upper secondary level, the differences between boys' and girls' performances in various mathematical courses are marginal. However, there is still segregation especially in undergraduate and graduate education, among academics and as a professional field (Brandell, Leder, & Nyström, 2007). This segregation starts already at upper secondary level where fewer females than males study the more advanced mathematics courses. Similar patterns can be found in other countries, e.g. the USA (Piatek-Jimenez, 2015). In addition, it seems that a view of mathematics as a male domain exists among Swedish students at secondary level (Brandell et al., 2007; Brandell & Staberg, 2008; Sumpter, 2012). Boys are attributed motivational beliefs such as mathematics being enjoyable and a subject you will need for the future. They

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are also thought of as successful in mathematics and therefore logical and clever. Girls are considered diligent and hardworking, but since they have to work more and harder than boys, they are not as clever (Brandell & Staberg, 2008). The view of the successful woman because of hard work (e.g. Hermione Granger) and the male genius (e.g. Sherlock Holmes) is considered one of the main reasons for the gender imbalance at university level (Leslie, Cimpian, Meyer, & Freeland, 2015).

Such conceptions create a norm that follows a traditional view of gender, which has been observed both in students' behaviour (Ben-Shakhar & Sinai, 1991; Carr & Jessup, 1997; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Gallagher & DeLisi, 1994) and in teachers' conceptions (Tiedemann, 2000a, b, 2002; Walkerdine, 1998). Research indicates that there exist differences between girls' strategy choices compared to boys when solving mathematical tasks (Carr & Davis, 2001; Carr & Jessup, 1997; Fennema et al., 1998), although these differences seem to develop over time. Although there is no significant difference between the sexes in the first grade three years later, the girls tend to choose the strategy their teachers had shown them, whereas the boys indicated a more abstract and creative mathematical thinking (Fennema et al. 1998). Gender differences can be explicit and measureable; when Öhrn (1990) studied interaction between Swedish lower secondary school students and their teachers, boys used 2/3 of the common talking space. The norm, including conceptions, can affect interactions and self-perceptions (Kahlin 2008).

Furthermore, a research review has shown that teachers from different countries share similar conceptions: that mathematics in one way or another is a male domain (Philipp, 2007). For instance, Tiedemann (2000a, b, 2002) studied German teachers at primary level and found that boys were thought more capable of mathematical reasoning based on logic, and girls were thought to find mathematics more difficult (compared to boys). According to sixth-grade teachers from the USA, if girls are successful, it is due to hard work, whereas boys succeed because they are bright (Jussim & Eccles, 1992). In a study focusing on gender symbols and Israeli pre-school children, mathematical success was by teachers attributed to different things depending if you talked about a girl or a boy (Klein, Adi-Japha, & Hakak-Benizri, 2010). If a boy was prosperous, the success was related to, according to these teachers, spatial ability, but if a girl was thriving, it was thought to be dependent on the girl's verbal ability. These different views then affected how the teachers interacted with the children and as a consequence shaped different conceptions of mathematics and mathematics education. Different terms and conditions were created for children's participation in the subject based on the social relations that exist within the context and culture of the subject. Yet, a more recent study has shown that Swedish teachers do not think that gender issues are relevant in their teaching (Gannerud, 2009).

Here, I focus on one part of the norm; what in students' reasoning do upper secondary school teachers consider typically male or female? This study aims to provide insights on upper secondary school teachers' conceptions about potential gender differences in students' mathematical reasoning. A first study was made to see if different types of reasoning were considered gendered (Sumpter, 2015). The results indicated that boys were connected to reasoning types when you use several strategies and/or chance-taking, and girls were indicated to use familiar strategies. But to find out what it is in the different types of reasoning that is thought of as male or female (or neutral), we need to analyse the teachers' arguments. The research question posed is 'Which gender symbols are attributed by the teachers to students' reasoning?'.

Gender Perspective

The starting point is the definition of gender as an 'analytic category which humans think about and organize their social activity rather than as a natural consequence of sex difference' (Harding, 1986, p. 17). This means that gender is a social construct and what is thought of as male or female is not a static phenomenon but changes through time and place (Damarin & Erchick, 2010). The construct is explicit for instance when people assign gender to non-human entities (Harding, 1986). You can attribute a gender to an object, characteristics or an action (e.g. a ship is female), or you can attribute an object, characteristic or an action to a gender (e.g. boys are more likely to use the graphic calculator). In either situation, an element is identified and picked out as typical with the assignment to a specific gender. This assignment is called gender stereotyping. Gender as a social construct is asymmetrical which means that human thought, social organisation and individual identity and behaviour are categorised in an order with some things thought of as more female and others more male. This part of gender is called symbolic gender and includes symbols and discourses attributed to a specific gender (Bjerrum Nielsen, 2003). So even though gender as a construct does not need to be a binary concept, gender symbols often are since they are based on gigantic system of beliefs, values, etc. that have been accumulated during a long period of time (Connell, 2009): elements are identified and picked out as typical with the assignment to a specific gender. This assignment is called gender stereotyping. As stated earlier, students at Swedish upper secondary school perceive mathematics as a male domain, which means that the structure, the symbols and the identity are all more likely to be pro-male. What you expect of a person seems to have an impact on their performance (c.f. Cadinu, Maass, Rosabianca, & Kiesner, 2005), a phenomenon that is part of the idea of norms. But also, how you gender stereotype seems to influence your conception of a student's competency (Tiedemann, 2002).

Mathematical Reasoning

This paper is based on a framework that allows different types of mathematical reasoning (Lithner, 2008). Reasoning is defined as the line of thought that starts with a task (e.g. exercises, tests) and ends with an answer. It includes more than just deductive mathematical reasoning. In this framework, there are two main types of reasoning: creative and imitative mathematical reasoning. *Creative mathematically founded reasoning* (CR) fulfils the following conditions: (1) novelty, (2) plausibility and (3) mathematical foundation. Even though most teachers at upper secondary school think that creative reasoning is only for high-ability students (Boesen, 2006), this type of reasoning is not restricted to people with an exceptional ability in mathematics.

Imitative reasoning is a group of different types of reasoning (Lithner, 2008). They share the property that they are all, from a mathematical point of view, superficial reasoning (Lithner, 2008). The different types of reasoning are (1) *memorised* *reasoning* (MR) where the strategy choice is founded on recalling an answer and the strategy implementation consists of writing this answer down with no other consideration and (2) *algorithmic reasoning* (AR) where the strategy choice is to recall a certain algorithm (set of rules) that will probably solve the problematic situation, and the implementation is straightforward once the rules are given. Two of the subcategories to AR are familiar AR (where strategy choice is founded on identifying the task as being familiar, and the solution is based on a corresponding algorithm) and delimiting AR (where the strategy choice is to repeatedly choose an algorithm from a set, and the set is delimited through surface properties). In imitative reasoning, the verifying argumentation is made on surface considerations related to expectations on the solution or the answer.

Conception and Gender Symbol

Conceptions can be seen as 'conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences' (Thompson, 1992). I follow this description, and conceptions are defined as abstract or general ideas that may have both affective and cognitive dimensions, inferred or derived from specific instances. Teachers' conceptions will be studied by looking at how they gender stereotype aspects of students' reasoning when solving school tasks. Gender stereotyping is identified with use of symbols. Gendered symbol is here defined as the actions, objects and characteristics that teachers pick out from students' work and attributed as having a particular gender. Actions include behaviours such as 'follow rules' or 'guess'. Objects include particular mathematical tools, such as calculators, or more abstract entities like a particular kind of solutions. Characteristics include inclinations like 'being careful' or 'having a long memory'. These symbols, and in a more prolonged term norms, are binary (Allard, 2004). Either something is considered male or female (or neutral), not both.

Methods of Data Collection

This study consists of two parts: (1) a questionnaire with the aim to see how teachers gendered different cases of reasoning and to use as stimuli to provide with arguments containing gender symbols and (2) interviews with the aim to clarify what elements within the reasoning were identified and assigned to a specific gender. The questionnaire describes eight cases (fictive and real, based on observations, marked A–H) with different aspects of reasoning. The following reasoning types were included: CR, MR, familiar algorithmic reasoning (FAR) and delimiting algorithmic reasoning (DAR). Two main aspects of each case were chosen. For each case, the teachers were asked to select one of the following responses to the question: 'Who is more likely to behave like this?': *BD* boys definitely more likely than girls, *BP* boys probably more likely than girls, *ND* no difference between boys and girls, *GP* girls probably more likely than boys and *GD* girls definitely more likely than boys. There was a chance to comment for the option selected. Two pilot studies were made in order to test the instrument. (For more information of the development of the instrument, see Sumpter, 2015).

The questionnaire was handed out to six public upper secondary schools, chosen to have students participating in all levels of mathematics courses in four different towns with different locations and sizes: rural north, rural south, two mid-Swedish larger towns and two city schools. Because of the long distances, the questionnaire was sometimes handed out by the head teacher instead of the researcher. The same information about the study was given to the teachers in both cases, and since there were no clear differences in the data from these two situations, they are treated as one case. The questionnaire was handed out to 62 teachers and they all responded; however, there were more teachers teaching mathematics at these schools, but they were not present when the questionnaire was answered. With the questionnaire as a background, six semi-structured interviews (face to face or via telephone) were made with experienced teachers (+10 years of teaching) from four different schools in three different towns. They volunteered to further explain what symbols they could see in the cases. The aim of the interviews, as well as the comments given in the questionnaire, was to clarify *why* certain reasoning was considered gendered. Their arguments were based on gendered symbols such as ‘This is a boy/girl since…’. Data carrying information about these symbols were identified and marked and classified in order find explanatory factors. Two additional interviews were also made with two of the 12 teachers that answered neutral on all the cases. They are presented as short interviews since the only question was ‘Why did you choose neutral?’.

As a triangulation, a post-questionnaire was made. It was sent out to the six of the teachers who participated in the interview and who used gendered symbols. They were asked to comment on the result of the gender symbols (see Table 1).

Method of Analysis

In a previous paper, the data was analysed with statistical analysis, investigating which cases of reasoning that were significant (Sumpter, 2015). The goal of the analysis in this paper is to present and highlight teachers’ gendered conceptions, and more specifically gendered symbols, about students’ reasoning. The interviews were transcribed. The data comes from the interviews and the comments from the questionnaire. In the teachers’ arguments, attributions are made about male and female characteristics. These attributions will be called symbols, as part of symbolic gender (Bjerrum Nielsen, 2003)

Table 1 Results of the written comments and interviews as symbols

Boys	Neutral	Girls
(Use the) calculator	Good student	Safety
Multiple strategies	Standard solution	Use the standard method
Chance-taking, to guess	Guess within context	Imitative reasoning
Make a mess, not careful		Insecurity
Quick solution		Long reflection time
Graphic solution		Wants *the* correct answer
Explore		Think

since they are attributed when the teacher has picked out elements, parts of the reasoning, as 'typical' with a specific association to gender (this feels like a 'boy thing' or a 'girl thing') or considered neutral. This means that some elements can be considered more gender-specific than others by the individual teacher making them stand out more than others. The same element can also be assigned by two different teachers to different genders (or thought of as neutral).

In order to analyse the data, I use content analysis. This is a technique used to 'extract desired information from a body of material (usually verbal) by systematically and objectively identifying characteristics of the material' (Smith, 2000, p. 314). Here, the body of material is written and verbal data. Aiming to do a systematically and as objectively analysis as possible, a coding and identification process were created. It consisted of three steps. First, passages carrying information about the teacher's conceptions were marked. Then, there was the identification of symbols. Symbols were indicated by points in the passages, written comments or verbal data, where teachers identified actions, objects and/or characteristics as part of the reasoning in the students' solutions and attributed these to a specific gender (or was considered as neutral). This identification of gender symbols worked as the coding scheme. The analysis was inductive since there were no previous categories to use. The symbols were gathered case by case to analyse if there were some attributions more common for boys or girls, respectively. Then, the focus was on commonalities between the cases. The last step was to analyse the arguments where the focus laid somewhere else than reasoning, for instance teachers choosing a specific gender only because of the language used in the description of the cases. This worked as an evaluation of the questionnaire. The teachers' comments from the post-questionnaire were summarised.

Results

Questionnaire

To order the data in the presentation, the interview responses are marked with 'Int' and a number and the questionnaire responses with 'Q' and a number or, if response from the post-questionnaire, with [Post]. The cases from the questionnaire are translated from Swedish and presented with reference if real. It covers the same substance, but it is slightly compressed in its presentation. All comments are presented. […] stands for omitted passages not relevant for the study.

The number of respondents who answered 'no difference' to all cases was 19 % ($n=12$). As presented in the first paper (Sumpter, 2015), when analysing the comments given and combining these with a short interview with two of the teachers, two rather different reasons emerged. The first reason was about recognition based on actual observations illustrated by the following teachers that were interviewed:

> I can't answer [this questionnaire]. I don't get a picture of any student. These are not our students' [Short Int teacher 1]; 'I don't recognise my students in these descriptions. When I read, I don't think 'This is Charlie'. But I do have my prejudices. And according to them boys just continue and 'bubbles on' even

> though they are wrong, and girls hesitate even though what they are doing is correct. But they just stop. They are 'stoppers' [Short Int teacher 2].

Adding to this view, in the questionnaire, one teacher wrote, "Have never observed any 'structural' differences" [Q 5]. Altogether, it seems likely that the reason why they decided to go for the middle option is because these teachers have not observed any differences in their particular students instead of an absence of stereotyped conceptions.

The second reason presents a moral and/or political view. This view is illustrated by the following reply: 'From an educational perspective based on equality, I don't see any gender differences' [Q60]. Another teacher wrote, 'Can't see anything gender specific in [mathematical] reasoning' [Q62]. This is more of an explicit decision to not to see any differences in the cases than recognition.

Written Comments and Interviews

The responses from the 50 teachers (81 %) who marked a gender differences in one or several of the cases constitute the database for investigating which gender symbols that have been attributed.

FAR. Let us start with FAR, cases A and C. These two cases describe two aspects of choosing the standard method for a routine task. The results from the statistical analysis were that both these cases had the majority of the responses in one of the girl categories but only one (case A) was statistically significant (Sumpter, 2015). In case A, the algorithm does not behave in an expected way, and the student therefore concludes that the reasoning is incorrect despite a correct supporting mathematical argumentation.

> *Case A* (Sumpter, 2013): Student A is trying to solve following task: Find the largest and smallest values of the function $y=7+3x-x^2$ on the interval [−1, 5]. The student differentiates the function and puts that equals to zero.
>
> A: This shouldn't be correct since it should be like two roots. When I say that the derivative is zero, then I mean that the slope is zero. Wait a minute… you should be able to calculate $3-2x=0$. That one should have two solutions. One positive and one negative, since it should be a maximum and a minimum. If you write it… [writes $3-2x=0$]
>
> A: But wait a minute. [silence 40 s]
>
> A: No. When I think that this should be zero, then it is just one way and that is $x=1.5$, and if you take it negative it becomes another number.
>
> A concludes that $x=1.5$ is completely wrong.

Gender Symbols. Most of the written comments for A are about girls and 'reflecting' and 'waiting' referring to that A takes time to reflect over the strategy choice [Q10, Q16, Q44, Q45, Q48]. One of the gender symbols attributed is insecurity: 'girlish dithering' [Q17] and 'female insecurity' [Q12]. One teacher sums it up by writing 'It is probably a girl since A knows how to solve it ($y'=0$), solves it correctly but then is afraid to trust her own solution' [Q34]. Analysing the interviews, similar attributions are made by four of the teachers:

> [Girls] assume they are going to fail, and if they succeed it is only because they were lucky. Boys assume they are going to succeed, and if they fail it is only because they are unlucky. [The student] remembers something about what it means mathematically, but is unsure. […] 40 s. If it was a boy it would be 5 s and then he would say 'What the heck' and write something down [Int 1]; Mathematics is often like that… confidence: 'This is what it is and this is just how it is'. [If] Hesitation, then it is a girl [Int 3].

Two teachers picked instead out messiness with the attribution to boys [Int 2, Int 4]. In case C, there is no supporting mathematical argumentation made by the student:

> *Case C* (Bergqvist, Lithner & Sumpter, 2007): Student C is trying to solve following task: Solve the equation
>
> $$4 - x = 3x + 14.$$
>
> C writes:
>
> $$\begin{aligned} &4-4-x = 3x + 14-4 \\ &x \;=\; 3\;x \;+\; 14 \\ &x + x = 3x + x + 14 \\ &4x = 14 \quad 14/4 = 3.5 \\ &x \;=\; 3.5 \end{aligned}$$
>
> C tries to control the solution by using the calculator. C computes 4–3.5, but stops. C is silent and looks at the equation and the calculator.
>
> C: I don't know if I should take plus or minus $3x$. [stops the solution attempt]

Gender Symbols. The majority of the written comments for case C are about the student being unsure and stops [Q12, Q17, Q44, Q45, Q56]. The safety aspect is emphasised with the attributions 'careful' [Q10, Q56] and 'ambitious' [Q35]. One teacher picked out the way of working as being typical female saying 'She wants to be completely sure before she answers. Controls her line of thought' [Q1]. Just as in case A, insecurity or to do something because it is safe appears to be a female characterisation, sometimes linked to imitative reasoning:

> [A boy] would do the short version', [he] would use a method that is faster in front of one that he understand [Int 1]; To write like that, with a minus sign in

> front, girls let it go so much later [than boys] [Int 3]. My perception is that they are much more… [into] copying so to speak. They don't think so much… the weaker ones. Maybe have not released their ability to think. [They] Copy the teachers' explanations model. Girls are often very industrious and then it is often rote learning. They follow rules. They are rule monitored so to speak [Int 3]; Girls often want to please and follow the [given] structure more than boys [Int 4].

A few comments were about boys. 'Fast solution' is one attribution [Int 2] and two aspects of control are raised: the lack of control [Int 2] and to control on the calculator [Int 4].

Summary FAR. Combining the two cases and their symbols, FAR could be considered as a safe strategy and more likely to be performed by a girl. There is a link to rote learning and to follow the teacher. A girl is hardworking, ambitious and careful. When she does not know, she thinks it is better to stop.

CR. This type of reasoning has novelty, plausibility and mathematical foundation as conditions. Neither of the two cases describing CR, B and E were statistically significant (Sumpter, 2015).

Case B describes a solution using a conventional strategy; however, not following the order, most textbooks would present it:

> *Case B*: Student B is trying to solve following task: Find the largest and smallest values of the function $y=7+3x-x^2$ on the interval [−1, 5].
>
> B: Hold on. Second degree… it only turns once. Then either the smallest or the largest must be at the border of the interval.
>
> B calculates the function values for $x=-1$ ($y=3$) and $x=5$ ($y=-3$).
>
> B: And it only changes direction once… so it is either one of these [draws with the hand in the air ∪] or one of these [draws with the hand in the air ∩]. Ok, it is one of these [∩] since it is minus in front of x^2. [differentiate and puts it equal to zero, arriving with $y=9.25$ when $x=1.5$]
>
> B: Ok, largest value must be 9.25 and smallest −3.

Gender Symbols. The largest proportion of the responses to case B is 'no difference', and the situation is similar with the written comments. Half of them are about 'no difference' saying that this is a standard solution, and it is produced by a good student [Q10, Q34, Q35, Q46]. One teacher says that this is a girl since she 'Follows a strict solution method' [Q30]. Analysing the arguments given in the interview, the responses are equally nuanced. One teacher chose neutral; this is a correct conclusion and it is carefully performed and it could be a good boy or a good girl [Int 1]. One teacher says it is a boy, a good boy with the argument that clever students more often are boys [Int 3]. Standard algorithm is assigned to girls in two different ways:

> They use more formal mathematics. [Girls] follow the standard algorithm, the book steers them. Boys are not affected in the same way [Int 2]; It is more typical that a boy is reasoning. A girl would have followed a more traditional strategy [Int 4].

These two teachers use the same property assigned to the same gender for arguing for two different results: to follow a traditional strategy is a female behaviour. In this case, the same attribution is made. One teacher picks out the graphical element of the reasoning:

> It is a guy… it is too 'swooshy'. To draw with your hand, I've seen several of my guys doing that. […] Their verbal language is not… They feel comfortable to illustrate [Int 5].

According to this teacher, girls are more verbal and boys more graphical including using tools (here drawing with your hands).

Case E presents a solution using an unconventional strategy:

> *Case E*: Student E is trying to solve following task: Solve the equation $4-x=3x+14$
>
> E: Solve the equation… well, then I solve it by looking at the graphs. These are two straight lines, and where they intersect is the answer [to the equation]. Because that's where it [x] is the same. [sketches a coordinate system] Ok… 4 $-x$, it is 4 when x is zero and then it is a negative slope. There we go… . And then there is $3x+14$. [sketches $3x+14$] Then x has to be negative. Like minus 2. Let see… if x is minus 2, the left part is 6 and the right part 8. And minus 3 is not right as well… so x has to be −2.5. Then the left hand is 6.4 and the right hand is 14–7.5 that also is 6.5.

Gender Symbols. The written comments do not really provide for any good explanations why most teachers, 48 % (Sumpter, 2015), would think that student E probably or definitely is a boy. Compared to case B, which had the largest proportions of the responses in the 'no difference' category, E stresses the factor of novelty. The comments arguing for 'boy' bring up a number of different aspects. One teacher refers to the factor of novelty when saying that the student 'is not sticking to a conventional method in order to solve the problem and reason himself, however still unsure, to a solution' and must therefore be a boy [Q30]. Another one writes that 'boys solve [problems] more often with graphical solutions than algebraic' [Q45], and a third writes 'dare to try' [Q17]. Some of the teachers who argue that this is a girl pick out various aspects of the strategy choice: 'More girls than boys go for a long-winded method during all circumstances' [Q34]; 'Girls loves structure' [Q12]; 'If this clumsy method was made by a boy, he would've used the graph function [on the calculator]' [Q35].

The interview data goes in two directions. Two teachers argue for case E being neutral behaviour where one says that this is not a specific gender because it is 'a creative solution by someone who dares to use an unconventional method' [Int 1], and the other talks about personal preference [Int 4]. Two teachers say that this is a boy because of the graphic solution:

> Girls. I don't perceive them as sketchers. They do it more meticulously. More carefully [Int 2]; Boys […] do it more [in a] graphical [way]. A girl would use a standard method. They […] wouldn't even think that there is a graphic solution. It is more 'this is how we did it on the black board and this is how we should solve it' [Int 3].

Summary CR. Combining the two cases, most comments say that this is a standard solution produced by a good student, but case E differs slightly with more responses in one of the boys categories compared to case B. Altogether, neither the comments nor the interviews give a unanimous picture.

DAR. Delimiting algorithmic reasoning is represented by two cases involving multiple strategies. The results from the statistical analysis show that both cases D and G were statistically significant ($p<0.001$) for 'boys' (Sumpter, 2012). The student pictured in case D used a smaller set of strategies.

> *Case D* (Bergqvist et al., 2007): Student D is trying to solve following task: Find the largest and smallest values of the function $y=7+3x-x^2$ on the interval [−1, 5].
>
> D: Here you are expected to differentiate so you can find maximum and minimum. Or at least I think so. [differentiates, puts the derivative equal to zero and arrives with $y=9.25$ when $x=1.5$]
>
> D: Uhm… I wonder why. I thought I would arrive with two values and I don't know what I did wrong.
>
> D looks at the graph on the calculator and tries to use the minimum function value tool. When D can't find anything, D moves on to the table function tool on the calculator and looks at the y-values for $x=-1,0,\ldots,5$.
>
> D: Between −1 and 5 [referring to the interval for x]… then the smallest value should be −3 and the largest value 9. Here [points at the differentiation] I've got 9.25. That's… clever.
>
> D then solves $y=7+3x-x^2=0$ and gets $x_1\approx4.54$ and $x_2\approx-1.54$, and gives them as answers to the task.

Gender Symbols. Half of the written comments for case D refer to the use of the calculator [Q25, Q31, Q45]. Two more teachers argue for D being a boy; one writes

'Considers himself as clever' [Q17] and refers to the characteristics of being confident, and another 'takes a chance' [Q10] focusing on the action. One teacher writes that this must be a girl since 'Incorrect solution, but ambitious' [Q35]. Analysing the interviews, most of the teachers choose 'boy' except for one teacher that argues for this is a girl because of insecurity [Int 6]. The first symbol attributed to boys is multiple strategies: '[He] Doesn't exactly have a clue what he is supposed to do, and then [uses] several strategies [Int 1]'; 'Typical calculator behaviour. Several strategies on the calculator [Int 4]'. The other symbol is the calculator itself and the confidence to use it:

> [He] Throws himself over the calculator and uses it with pleasure. Has informed himself in all the possibilities [on what you can do on the calculator] [Int 1]; 'Yes, I think so anyway': he is confident when he is unsure what to do. He knows what to do—to bring out the calculator. The calculator […] is considered to be reliable [Int 2]. Don't care about being diligent with details such as bad settings on the calculator [Int 1]; The way of speaking is more like a girl, but the way of using the calculator portrays more a lad. [Int 5]

Compared to case D, the second case (G) encompasses a larger number of different strategies:

> *Case G* (Sumpter, 2013): Student G is trying to solve following task: Find the largest and smallest values of the function $y=7+3x-x^2$ on the interval [−1, 5]. The student first tries to calculate $7+3x-x^2=0$, but cannot remember the formula for a second degree equation. Then, G calculates the function values y for the border of the interval, $x=5$ and $x=-1$, and says they are the largest and the smallest value. After that, G looks at the graph on the calculator. G can see where the largest value is on the graph and tries to use the minimum function value tool on the calculator to get the smallest value. When this is not successful, G looks at further 14 different tools on the calculator with no result before saying:
>
> G: The answers are these ones [pointing at the borders of the interval].

Gender Symbols. The most frequently used argument for case G is multiple strategies on the calculator [Q16, Q30, Q31, Q34, Q35, Q45, Q46, Q57] especially as a tool for searching or exploring. This is illustrated by this expression: 'Boys press all the buttons [on the calculator] and hope it will help' [Q34]. Some teachers use reversed argumentation [Q30, Q35, Q46] illustrated by 'None of my female students have ever tried 14 tools' [Q35]. One concludes that boys 'rely on the calculator, do *not* think beyond that' [Q25] emphasising the lack of consideration of the intrinsic mathematical properties. One teacher states that 'this behaviour seems to frequently recur independent of the math course' [Q59] saying this is an overall male tendency. Other male symbols are the calculator [Q10] and braveness [Q44]. Analysing the interviews, the situation is similar. All teachers conclude that case G describes male behaviour and how it differs to girls':

> The typical boy. Absolute no clue but the calculator is an extended index finger [Int 1].; Tries with the calculator a lot. Testing strategies so to speak. A guy do

> that: look for tools, look for tools. Look for all the different tricks [Int 3].; [laughter] Well.. a bloke. The behaviour in itself [counts up all the different strategy choices] [Int 5]. He is unbelievable. He searches. He is fixated that the calculator is the thing that is going to solve this for him. Girls don't do that. They ask 'How do you solve this?'. Boys don't ask the teacher. They searches […] or ask each other [Int 2].; It is with no doubts more common that boys searches through the calculator, that boys explores the calculator. Girls […] do what they are suppose to do. They don't have to go through the whole lot [Int 4].

Summary DAR. Combing the symbols for cases D and G, most teachers refer to the calculator and/or multiple strategies. Most often is a combination of the two. Some teachers use binary reasoning saying that this can't be a girl since girls do not use multiple strategies on the calculator. The very few references to girls are insecurity and being ambitious.

MR. This type of reasoning is represented by two aspects of a guessing strategy; one (case H) more within the context of the task than the other (case F). Both these cases were statistically significant ($p<.001$) for 'boys' (Sumpter, 2015). Case F describes a guess with reference to not remembering a specific answer.

> *Case F*: Student F is trying to solve following task: Rewrite $100cm^3$ in litres.
>
> F: Well. Not easy to remember. But I guess it is 1 l.

Gender Symbols. The most common argument in the written comments for case F is chance-taking [Q10, Q12, Q45, Q48]. One teacher writes that student F, a boy for sure, 'must and wants to make a decision quickly' [Q30]. There is also a comparison between girls and boys: 'Girls rarely dare to guess unless they are certain' [Q48]. Analysing the interviews, to take a chance and/or to guess are also the main arguments where some teachers point out the asymmetry:

> Because… well, he doesn't even bother to think. 'What the heck'—it is that type of principle. 'I'll take a litre. That will do'. [Int 1]; Sounds like a bloke. The tiredness… a guess, get it done quickly: 'I don't know, I can't be bothered to think.' [Int 5]; He just grabs something. There is no deduction, just a guess. Girls, strangely enough, have learnt to remember. They have better memory than boys. They would try to deduce [Int 2]; A guess. […] It is done so fast. It is more like this 'Oh well, just take something'. [..] Girls, they think a bit more [Int 3]; No clue, but a guess. […] Here it is crystal clear 'I don't know, I guess. A bit random. Girls want to have the correct answer [Int 6]'.

When boys are thought of as random guess-makers, girls are here assigned symbols such as 'good memory', 'deduction' and 'wanting *the* correct answer'. One teacher says that this is a neutral behaviour since 'units are difficult for everyone. If you don't know, you guess' [Int 4]. Case H illustrates a guessing strategy also based on lack of memory, here a procedure.

Case H: Student H is trying to solve following task: Simplify $(x+y)^2-z^2$.

H: That is a sneaky one. And I can't remember the trick. [silence]

H: I answer $(x+y-z)^2$.

Gender Symbols. There are several variations of different aspects in case H. The teachers who choose 'boy' seem to pick out 'chance-taking' as one key element [Q10, Q12, Q45, Int 2]. One teacher describes boys as 'jugglers' as in using tricks more than girls [Q56], and another says this must be a girl since student H 'admits ignorance' [Q16]. The teacher choosing neutral explains that this type of mistake is 'gender indifferent' [Q35]. The interview material gives some more information. Two teachers say this is neutral behaviour with the argument that the guess described in case H is based on some mathematical properties. The guess is therefore within the context of the area and not a random one [Int 1, Int 4]. A third teacher refers to this element but makes the conclusion that this is instead a familiar activity, and therefore, it is more likely to be a girl who performs this guess [Int 3]. One teacher argues for this is a boy simply because of the language: 'Girls don't talk like that in our acquaintance' [Int 5].

Summary MR. Combining the gender symbols from the two cases, the majority of symbols are about guessing, and it is attributed to boys. A few comments are about neutral behaviour based on the mathematics, and very few say that this is a girl based on language or this is familiar algorithmic reasoning.

Summary and Triangulation

Summarising the results, these are the gender symbols most frequently used in the arguments made by the teachers for the categories boys, girls and neutral (see Table 1).

Most of the symbols in the girls' column describe someone who is insecure and wants safety. Girls use the standard method and produces imitative reasoning. In order to arrive with 'The correct answer', they would spend more time on thinking, but if not successful, then guessing would not be an option. The symbols linked to boys portray a different personality. Boys are rather messy, and they take chances and make quick solutions. They would use multiple strategies on the calculator to search for an answer, strategies most likely to be chosen based on the surface properties in relation to the task. A standard solution performed by a good student, and guessing based on mathematical properties is considered neutral.

As a post-questionnaire, Table 1 was sent out to the teachers participating in the interviews. They were asked to comment on the results. Two teachers declined the offer to comment with the explanation that it is not a question about gender differences; it is all about different personalities [Post 1, Post 4]. The other teachers agreed to the description, parts or the whole table [Post 2, Post 3, Post 5, Post 6] with a few exceptions. One of the exceptions was 'careful' compared to 'security': this teacher wants to put the emphasis from 'insecurity' to 'carefulness' [Post 2]. The column describing neutral behaviour was also in focus in the teachers' replies:

> I think you have summarised male/female in mathematics in a good way. Then there are always some boys that remind of girls' methods and vice versa. The only thing that I found strange is neutral, 'standard method' I find more female than male [Post 6]; I am a bit doubtful about the column describing neutral. […] One cutting comment I've read is: If necessity is the mother of inventions then play is the father. And that maybe says a lot about our traditional putative behaviour [Post 5].

The aim of this study was to present a set of examples that differed with respect to the reasoning types used and see what kind of attributions of symbols the teachers made. The results in this study showed few symbols that were directly connected to the reasoning types but that the different reasoning examples brought forward several attributions that were related to other aspects of reasoning and/or task-solving. This is most likely due to that the fact that the different types of reasoning are a research framework aiming to describe and analyse different types of reasoning and arguments respondents give, more than it is a common knowledge among teachers in the field. It is probably not explicit to the teachers that the cases are describing different cases of reasoning so therefore their arguments are not focused on that. On the other hand, the framework was used in order to give a spectrum of different types of reasoning, and since the framework is derived from empirical studies, this is behaviour that has been observed previously. The cases are also real or based on real task-solving. Having the types of reasoning in focus a few points can be raised. Delimiting algorithmic reasoning is by most of the respondents a male way of reasoning, and the main argument is the use of multiple algorithms especially on the graphic calculator. MR is also a male way of reasoning with symbols such as risk-taking and gambling. Here, it is a case of 'throwing in something' just for the sake of giving an answer at all. However, this seems to be related to how much the answer is considered (by the teachers) to be within the context of the task. Analysing the questionnaire, the generality is not so strong except for the very few choosing any of the girl responses. The conclusion is that MR has aspects that are male and not female behaviour.

The cases describing aspects of familiar algorithmic reasoning were the only ones with the largest proportions of the responses in 'girl'. This type of reasoning has aspects connected to girls such as the strategy choice, to use the standard method, and the conclusion to stop and not continue with other strategies. Creative reasoning is the type of reasoning where the cases have the least gender stereotyping and produce the fewest numbers of attributions. The most common response is that this is a good student producing a standard solution. Comparing the two cases B and E, the latter one has aspects considered male more than case B. Whether the dissimilarities between the responses to these two cases are due to the different emphasis on the factor of novelty or the use of graphic solution, I cannot say from this data.

When analysing the arguments where the focus is laid somewhere else than reasoning, one teacher [Int 5 case H] explicitly states that he or she has based the choice on the language used in the case description: 'Girls don't talk like that in our acquaintance'

[Int 5]. Even though trying to avoid this situation (that choices are made expressly based on the language) by making different pilot studies, it seems almost impossible to do so most probably because reasoning and language are so closely connected to each other.

Discussion

According to most of the teachers participating in this study, there is a difference between girls and boys task-solving. It looks like gender plays a role in how teachers think about student's reasoning. Gender symbols and the association made between gender and different reasoning are interesting since they add to describe the norm upper secondary school students operate within. Although it might be tempting to think that the relationship between teachers' beliefs and/or conceptions and their way of instructing a class is straightforward, the situation is far more complex (Fennema et al. 1998). There is no coherency in the field (Thompson, 1992). Nevertheless, the questions about the effect of these views in the classrooms or if these conceptions mirrors the reality are still interesting and worth some consideration.

According to upper secondary school teachers, creative reasoning is only for high-ability students (Boesen, 2006). This study adds a gender perspective to this result. There is no significant gender difference between high-ability students (Sumpter, 2015); a standard solution produced by a good student is often thought of as neutral behaviour. This is similar to Tiedemann (2002) where the findings of gender stereotyping when considering the performance level was held for average and low-achieving students and not for high-achieving students. However, there is a difference in the symbols attributed to the students. Previous result has indicated that girls are diligent and successful because of their hard work (Brandell & Staberg, 2008) and not because of their creative mathematical thinking. In this study, girls were linked to imitative reasoning and the use of standard methods, a way of working that if you are diligent and careful you are most likely to be successful. This is line with previous research saying that girls are more likely to choose the standard method and stick to it (Fennema et al., 1998). Familiar algorithmic reasoning is effective if you want to be sure that you solve certain tasks but highly restrictive when facing a genuine problem. It is here thought of as a safe way of reasoning.

The high-ability boys, on the other hand, are successful because of them being bright, which is within the same norm as indicated in Brandell and Staberg (2008) and Jussim and Eccles (1992). In this study, only one of the cases describing creative reasoning generated attributions that could be interpreted as an indication of creativity. Case E stresses the factor of novelty when the student, instead of using the conventional method, solves the task with a graphic solution. Graphic solution is one of the symbols attributed to boys. It is not clear if it is the graphic solution in itself that brings forward this symbol or if it is the use of it instead of a standard method or the combination of the two. In any of these cases, standard method is assigned to girls and by asymmetry not considered male. Combining these two cases, FAR and CR, and the attributions, we arrive with a possible variation of the norm described by Leslie et al. (2015): the industrious girl learns the standard algorithm and the boy with the innate capacity is more creative.

Some of the gender symbols more frequently attributed to boys were multiple strategies (especially on the calculator) and to guess/to take a chance. This would fit in with the view of boys being gamblers and risk-takers (Ben-Shakhar & Sinai, 1991; Carr & Jessup, 1997) in the sense that they try different methods to increase the probability to stumble across something that might be right. This is not a strategy choice based on intrinsic mathematical properties of the task: in delimiting algorithmic reasoning, the choices and the argumentation for these choices are made on surface reasons, and in memorised reasoning, the strategy choice is simply trying to recall an answer and writing it down without any other consideration. This sort of gambling is part of the effortless strategy that we can see, for instance, in Walkerdine's (1998) description of the English gentlemen: you do as little as possible, but you should gain as much as possible. The use of calculator was reoccurring as a symbol, in particular in relation to multiple strategies. This is probably due to that a calculator can offer lots of different strategies with little effort. Here, it is illustrated by the following quote: 'Boys press all the buttons [on the calculator] and hope it will help' [Q34].

If teachers' conceptions mirror the reality, what does it mean to do girls' mathematics relative to boys' mathematics? Piatek-Jimenez (2015) concluded that:

> It appears that individual encouragement and recognition by others continue to be instrumental in some women's interest and persistence in mathematics. Therefore, it is important for parents, teachers, professors, and advisors to continue to encourage and support women in their interest in mathematics. Ideally, this encouragement would begin at a young age, but ought to be continued throughout their educational careers. (p. 45)

Given this and the fact that the women disappear between upper secondary school and university level (Brandell et al. 2007) leaves us with the important role mathematics teachers play especially at upper secondary school level. Girls choose *not* to do mathematics at university level. The view of mathematics as a male domain at university level might be one reason, but a more likely explanation is the context with its norms and trajectories including teachers' expectations of expected behaviour. It seems plausible to conclude that mathematical reasoning and task-solving are different for boys and girls in mathematics education. But, if so, when do these differences start and why?

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