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PROFESSIONAL NOTICING PRACTICES OF NOVICE MATHEMATICS TEACHER EDUCATORS

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ABSTRACT. The focus on professional noticing in mathematics education has recently gained increased interest as researchers work to understand how and what is noticed and how this translates into practice. Much of this work has focused on the professional noticing practices of inservice teachers and preservice teachers, with less attention focused on those educating teachers. This research explores how novice mathematics teacher educators professionally notice as they engage in teaching experiments and create models of student's mathematical thinking. Findings indicate the novice teacher educators are including some evaluative comments in their professional noticing practices but lack in-depth interpretive analysis about student thinking and rarely make connections between student's thinking and the broader principles of teaching and learning. These findings provide evidence for the importance of supporting teacher educators with developing their abilities to professionally notice.

KEYWORDS: mathematics, model building, professional noticing, teacher educators, teaching experiments

As educators engage in teaching, experts are distinguished based on what they professionally notice as well as what they do not notice; those who excel emulate an observer of practice as they engage in professional opportunities (Miller, 2011). As highlighted in the work of Jacobs, Lamb, & Philipp (2010), professional noticing demands that teachers attend to students' thinking, interpret their thinking, and make decisions about how to respond based on their own assertions. Noticing student thinking is an intentional act requiring active engagement from the educator (Mason, 2011; Sherin, Jacobs, & Philipp, 2011). The assumption is that educators who come to notice student thinking and make decisions based on what is noticed will implement lessons focused on what students know and need to learn.

The focus on professional noticing in mathematics education has recently gained increased interest as researchers work to understand how and what is noticed and how this translates into practice (Star & Strickland, 2008; van Es, 2011). Much of this work has focused on the professional noticing practices of inservice teachers, finding that the development of professional noticing is dependent on extended opportunities to focus on students' thinking and make connections between the teaching that occurs and the learning taking place (Jacobs, Lamb, Philipp & Schappelle, 2011; van Es, 2011). At the same time, the emphasis on professional noticing in the context of teacher education is also heightened, with research findings indicating that preservice teachers can develop some ability to professionally notice in as little as one semester of coursework (Star & Strickland, 2008).

From studying preservice teachers, Star & Strickland (2008) assert that the ability to learn from teaching is dependent on the ability to notice; however, preservice teachers' prior experiences as learners of mathematics, as opposed to teachers of mathematics, influenced how and what was noticed. At the onset of teacher education courses, preservice teachers lack the critical skill of professional noticing. "Teacher educators likely need to carefully consider the ways by which preservice teachers can improve their observation skills" (Star & Strickland, 2008, p. 124). As teacher educators consider how to best prepare preservice teachers to professionally notice, teacher educators' own professional noticing, and the means through which they convey this noticing to preservice teachers, is called into question. Current research has focused on how preservice teachers notice and how inservice teachers notice, but little attention has focused on how teacher educators notice. Understanding more about how teacher educators notice is important for knowing how to support the development of professional noticing as they work with inservice and preservice teachers.

The need to understand how professional noticing is perceived from teacher educators is exacerbated by the notion that not all teacher educators have terminal degrees and may be developing their own ability to notice, raising into question the means through which they are teaching preservice teachers to notice. As a result, there is a need to understand how teacher educators professionally notice and conceptualize their own abilities to professionally notice. This study answers the following research questions: (1) How do mathematics teacher educators shift in their professional noticing practice? (2) How do mathematics teacher educators conceptualize their ability to professionally notice? This paper focuses on novice teacher educators, defined as those who are relatively new to educating inservice and preservice teachers and are in the process of seeking terminal degrees to pursue teacher education as a career. Understanding the professional noticing practices of these teacher educators provides insight for knowing how to support these individuals as they advance in their careers as mathematics teacher educators. For the purposes of this paper, the term *noticing* is used to describe *professional noticing* (Jacobs et al., 2010; van Es & Sherin, 2008) that occurs when teachers attend to and interpret student thinking.

Theoretical Framework

This study is framed with the theoretical construct of noticing. Derived from the concept of situational awareness, noticing is essential for effective teaching (Miller, 2011). At origin, Dirkin (1983) studied cognitive tunneling, focusing on teachers' cognitive and perceptual work through what is noticed. This work was complimented by Goodwin's (1994) focus on professional vision, described as ways of making sense of events that are of interest to specific groups. Extending the work on professional vision (Goodwin, 1994), the construct of noticing has evolved to encompass the ability to notice significant events and decide how to respond based on what is noticed (Jacobs et al., 2010). In this process, educators maintain an "awareness of awareness," meaning that teachers are cognizant about the extent to which they are conscious about classroom happenings (Mason, 2011, p. 43). This necessitates metacognition focused specifically on the consideration of active instances of observation. Teachers must be aware of the noticing taking place before they can analyze and construct tasks focused on students' reasoning. When noticing, educators draw attention to students' thinking in instances that are most pertinent for improving instruction.

The process of engaging in noticing is an active and intentional act (Sherin et al., 2011) void of "theorizing, emotional content, justification, and explanation" with the intent of providing an accurate account of specific instances (Mason, 2011, p. 39). The purpose is to emphasize teachers' "in-the moment instructional decision making" with a focus on children's thinking to help teachers make sense of children's reasoning within the complex environment in which they teach (Jacobs et al., 2011, p. 97). Van Es & Sherin (2008) define this type of noticing as the ability to identify noteworthy aspects of a classroom situation, use knowledge to reason about classroom interactions, and make connections between classroom events and principles of teaching and learning. In a more recent study, Jacobs et al. (2010) modified the definition of noticing to include attending, interpreting, and deciding how to respond on the basis of children's thinking, thus extending prior work of van Es & Sherin (2008) to include the decision-making component for addressing children's needs.

Noticing And Teaching

Noticing is an important skill for focusing on students' thinking. Teachers who notice are cognizant about student understanding and misconceptions occurring in the classroom, while enacting a lesson (Miller, 2011). This is complicated in a classroom where multiple events are happening simultaneously and there is a need to focus on multiple intricacies of the environment. Having the expertise to focus attention where needed, to improve student understanding, is a key component of teaching mathematics (An & Wu 2012). "An effective teacher needs to …construct tasks that bring them to learners' awareness and in order to draw attention to them when they are relevant" (Mason, 2011, p. 44). Teaching experiments, which involve work with one or two students, provide a context for learning to notice because of the emphasis on students' thinking and the reduction of classroom interruptions (Weilandet al., 2014; Norton & McCloskey, 2008; Steffe & Thompson, 2000).

Teaching Experiments

Teaching experiments provide a context for studying students' thinking while learning to notice (Norton & McCloskey, 2008; Steffe & Thompson, 2000). "In mathematics education, a particular type of in-the moment instructional decision making has been emphasized—decision making in which children's thinking is central" (Jacobs et al., 2011, p. 97). Teaching experiments provide this context and are described as extending clinical interviews to encompass the scientific process of building explanatory and predictive models of students' mathematical understanding (Norton & McCloskey, 2008; Steffe & Thomposon, 2000). In a teaching experiment, educators meet individually with a student and interview the student to construct models of the student's understanding, which are the result of the degree of noticing taking place. Research on children's mathematical learning highlights the efficacy of building models of students' knowledge through teaching experiments (Norton & McCloskey, 2008; Steffe & Thompson, 2000), which involves "ongoing assessment through careful observation, hypothesizing about a student's current knowledge and strategies, and selecting learning activities closely attuned to the child's current reasoning" (US Math, 2005, p. 6).

Within the context of teaching experiments, "One needs to perceive what is important in a given situation and to infer what it portends with respect to the goals of that situation" (Miller, 2011, p. 51). For someone learning to notice, understanding the relationship between the context of teaching experiments and students' thinking, and knowing the mathematical content to emphasize while conducting a teaching experiment can be difficult; however, preparing teachers to understand the relationship between their noticing and analysis of student thinking through teaching experiments is important for improving student understanding in mathematics.

Focusing on the relationship between noticing and the analysis of student thinking can be difficult for American teachers because their lesson design is often not conductive for studying the development of student thinking or understanding (Stigler & Hiebert, 1999). This type of interaction is dependent on an interpretive discourse structure that focuses the teachers' thinking on making sense of student thinking based on evidence to reason through teaching situations (van Es, 2011). Cultivating this type of interpretation requires extended opportunities focused on learning to notice, with an emphasis on students' thinking and the relationship to learning. Jacobs et al. (2011) purport, "expertise in attending to children's strategies is foundational to deciding how to respond …our cross-sectional findings showed that neither form of expertise is something that adults routinely possess but is something they can gain with support" (p. 111).

Teacher Educators

Recent research on noticing has focused on the noticing capabilities of both inservice and preservice teachers (Jacobs et al., 2010; Star & Strickland, 2008). Much of this research has emphasized the need to explicitly teach preservice teachers to notice because they are initially quite weak at observing classroom events and interpreting student understanding (Star & Strickland, 2008). At the same time, research has called for the support of inservice teachers as they further their noticing capabilities (Jacobs et al., 2010). What is less clarified is the noticing practices and interpretations from those who are supporting the development of noticing in preservice and inservice teachers. Kazemi et al. (2011) studied the noticing practices of expert mathematics teacher educators during professional development with inservice teacher leaders. In this process, the teacher educators utilized the noticing structure of Jacobs et al. (2010) and focused on attending, analyzing, and deciding how to respond on the basis of the teacher leaders' needs during professional development on mathematical tasks. Findings indicated that the noticing practices of the expert teacher educators were instrumental in drawing conclusions about how to restructure the professional development for increased benefits to the teacher leaders; specifically, Kazemi et al. (2011) wrote, "through our noticing of leaders' thinking about videocases, we have reframed our work with leaders" (p. 201). While this article focused on the noticing practices of teacher educators, the method through which the noticing was analyzed was not explicit but rather provided a more holistic examination of how noticing can influence practice. In essence, this work demonstrated the influence of noticing on the practices of teacher educators but highlighted the need for continued research on noticing among teacher educators. As teacher educators work to develop the noticing practices, it is important to understand how they notice and how they conceptualize noticing, so they can be supported as they work with teachers.

Method

An exploratory study was used to examine novice mathematics teacher educators' abilities to notice as they engaged in teaching experiments. The purpose was to provide an in-depth examination of how teacher educators think and reason with respect to noticing.

Participants and Context

The current educational structure in the USA includes a spectrum of educators spanning from preservice to inservice teachers to teacher leaders (i.e. curriculum experts and coaches) to novice teacher educators and finally expert teacher educators. The noticing practices of preservice, inservice, and teacher leaders have been studied (i.e. Jacobs et al., 2010; van Es & Sherin, 2008); however, there is a need for additional studies examining the noticing practices of both novice and expert teacher educators. This study focuses on the novice teacher educator population to provide insight about those who are not yet experts in teacher education.

The four participants were novice teacher educators and comprised all students in one doctoral education course titled Mathematics Education at a medium-sized university in the western portion of the USA. The course was optional as part of the doctoral program, so only those interested in improving in mathematics teacher education enrolled. The small sample size was purposely selected to allow for the in-depth examination of noticing practices of these novice mathematics teacher educators. The intent of this study was not to make broad generalizations about novice mathematics teacher educators but to understand the noticing practices of a few to know how to support developing teacher educators. The term novice is used in this paper to describe individuals who have experience educating teachers and who are working toward careers in teacher education but do not currently hold academic appointments in the field. Novice is used because they are new to the milieu of teacher education.

Sean was a community college biology instructor who taught future teachers, had a strong mathematics background, and planned to be a STEM teacher educator. Justin was a former civil engineer, taught mathematics methods courses to future elementary and secondary teachers, and planned to be a mathematics teacher educator. Cassidy was certified as a mathematics teacher, and Victoria had taught high school mathematics for 2 years. Both Cassidy and Victoria regularly conducted professional development for inservice teacher leaders across two states, and both planned to be teacher educators following completion of their terminal degrees. All participants regularly taught university-level courses to preservice teachers or engaged in intensive professional development activities with K-12 inservice teachers of mathematics, but none of them were in academic positions for this purpose. During the first week of class, all four participants indicated they were focused on improving their mathematics teaching of both K-12 students, preservice teachers, and inservice teachers to increase their competence as teacher educators. Pseudonyms are used in the study for all participants and the students with whom they worked.

Course Components

As a part of the course, the participants were asked to select one student, in grades 9–16 to work with for the duration of the semester. The novice teacher educators were purposely asked to select a student of mathematics, as opposed to a preservice teacher, so that the novice teacher educators could engage in the experience of noticing students' mathematical thinking. The idea is that if the novice teacher educators are teaching preservice or inservice teachers to notice how students think about mathematics, they themselves need to have experience noticing how students think about mathematics. The term *students* will be used in this paper to refer to these individuals in grades 9–16.

Working with the students, participants conducted two teaching experiments (Weiland, Hudson & Amador, 2014; Steffe & Thompson, 2000), built models on the student's thinking (Norton & McCloskey, 2008), wrote reflections on these interactions, and discussed them collectively with other class members through five live video feeds. To conduct the teaching experiments, the participants each selected a mathematical topic they considered relevant for the learning of their selected student. They then designed interview protocols constructed of five main tasks or questions with subquestions focused on their specific topic (see example in Fig. 1).

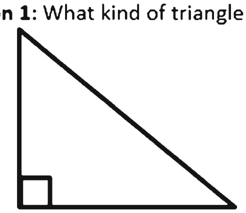
Each participant arranged a time to interview his or her selected student using the self-created interview protocol. Table 1 shows the mathematical content of each teaching experiments.

These interviews were audio recorded by the participants. After each teaching experiment, the participant listened to the audio recording and wrote a reflection based on the interview. Reflections were prompted with the following questions modeled after Weiland, Hudson & Amador (2014):

> Give at least two specific examples of things your student said or did in the interview that help you understand more about how that student thinks. For each of your examples, describe the context of the task given to the student.

It is important to note that this prompting encouraged participants to focus specifically on student's mathematical thinking and provided a scaffolding structure for them to learn to notice. Following the written reflection, the participants identified and transcribed a salient point from the teaching experiment focused on mathematical thinking and built a model that was both predictive and explanatory about the student's thinking (Norton & McCloskey, 2008). They were specifically prompted to write a statement capturing the essence of a student's mathematical

Question 1: What kind of triangle is this?



Subquestions: What do you notice? How do you know it is a right triangle? What is your definition of a right triangle?

Figure 1. Example of a portion of a teaching experiment interview protocol

TABLE 1

Mathematical content of each teaching experiment for each participant

Participant	*Teaching experiment 1*	*Teaching experiment 2*
Cassidy	Properties of right triangles	Algebraic patterns
Victoria	Solving quadratic equations	Graphs and equations
Justin	Slope as a rate of change	Factoring and graphing polynomials
Sean	Hardy-Weinberg theorem	Measures of central tendency

reasoning. As an example, based on Fig. 1, Cassidy wrote the following model:

The student can state and use the Pythagorean Theorem to solve for a side of a right triangle given two other sides. The student can also state another method to solve for a side of a right triangle given two other sides. If given another right triangle with two different side lengths, the student could find the third side using Pythagorean Theorem. The student could try and find the sides using Cosine, Sine, and Tangent, but the student might not accurately find the third side without using the Pythagorean Theorem.

After analyzing their first teaching experiment and building a model of student thinking, all participants met via live video feed and discussed both their teaching experiment and their corresponding model of student thinking. Following the first teaching experiment, the participants began a 6-week unit in the course focused on noticing. During these six modules, participants analyzed their teaching using various frameworks for noticing (i.e. Jacobs et al., 2010; van Es, 2011), analyzed videocases, compared frameworks, analyzed actual teaching episodes of preservice and inservice teachers, used the van Es (2011) framework to analyze videos of lesson study meetings, and evaluated their own noticing practices. Emphasis was placed on attending to students' thinking, interpreting students' thinking, and deciding how to respond on the basis of students' thinking (Jacobs et al., 2010). Participants were oriented with the van Es (2011) framework at the beginning of the modules on noticing, regularly evaluated teaching using the framework, and discussed analysis results together by justifying their decisions via written and live interaction. This included analysis of the same teaching episodes or excerpts, so that the participants could negotiate application of the framework to arrive at a consensus. Near the end of this unit, the participants repeated the same cycle of conducting a teaching experiment and model building. At the end of the semester, participants considered all of their experiences with their student, reflected on the teaching experiments and model building, and wrote a synthesis paper describing their student's mathematical thinking. In the same document, they reviewed the audio recordings and written documentation from their work and analyzed their own ability to notice as they engaged in the first and second teaching experiments. The purpose of the self-analysis was for the participants to evaluate and justify their ability to notice during the teaching experiments.

As a result, data for this study included (a) teaching experiment questions and reflection one (occurred at the beginning of the semester), (b) teaching experiment questions and reflection two (occurred later in the semester), (c) models of student thinking one (corresponding to the first teaching experiment), (d) models of student thinking two (corresponding to the second teaching experiment), (e) transcripts from the most salient points of the teaching experiments (as defined by the participants), and (f) written final synthesis about the student's thinking from each participant. Figure 2 shows how the data items correspond to the research questions.

Data Analysis

The purpose of conducting an exploratory study was to understand participants' noticing practices and understand how participants conceptualized their ability to notice through an in-depth examination of practices. To first determine the extent to which the participants were noticing, their teaching experiment questions, model building descriptions, and reflections were analyzed following a framework on how teachers notice (van Es, 2011). Within this framework, van Es (2011) provides four categories for explaining how one may learn to notice: level 1-baseline, level 2-mixed, level 3-focused, and level 4-extended. This framework spans from forming general impressions, providing descriptive, and evaluating comments and little or no evidence (level 1-baseline) to highlighting noteworthy events, providing interpretive comments, referring to specific events, elaborating on events, making connections, and proposing alternative pedagogical solutions (level 4-extended).

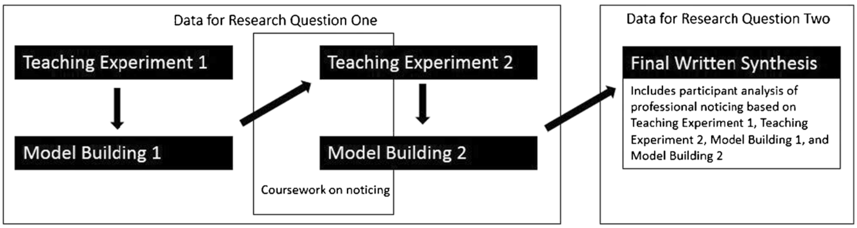


Figure 2. Process of teaching experiments and model building during semester and data collection in relation to the research questions

Two researchers each independently analyzed each data item by assigning one of the four categories (baseline, mixed, focused, or extended) to each participant's first and second teaching experiment questions and reflection. Similarly, each model of student thinking was analyzed based on the same framework, by both researchers independently. This resulted in four coded levels of noticing for each participant (teaching experiment 1, teaching experiment 2, model 1, and model 2) with analysis focused on changes in noticing from the first to second teaching experiments and model building for each of the participants. The researchers only varied once on the difference in levels from the first to second instances of the activity; this occurred between the first and second teaching experiments for one participant. The researchers discussed this instance and reconciled the disagreement based on evidence from the teaching experiment write up. As a component of the course, the participants had each been asked to evaluate their own teaching experiments and reflections using the same framework.

Results

For the teaching experiments and corresponding reflection, the participants' formed general impressions of what occurred, provided descriptive, and evaluated comments with little or no evidence or began to provide some interpretive comments with minimal reference to specific events and interactions. Table 2 shows noticing levels for the teaching experiments.

After participants finished their teaching experiments and corresponding reflections, they engaged in model building based on each of the teaching experiments. The process including scaffolding that encouraged participants to notice student thinking. Specifically, participants were given a template and asked to select and transcribe a given

TABLE 2

Researcher assessed levels of noticing during the first and second teaching experiments

Participant	*Teaching experiment 1*	*Teaching experiment 2*
Cassidy	Level 1-baseline	Level 2-mixed
Victoria	Level 2-mixed	Level 2-mixed
Justin	Level 1-baseline	Level 2-mixed
Sean	Level 2-mixed	Level 1-baseline

section of their teaching experiment that they considered important for understanding student thinking. They were prompted to consider what the student knew mathematically, what the student did not know, and what additional evidence they would need to better understand the student's reasoning. Table 3 shows the noticing levels for model building.

As the participants engaged in, and learned about, the construct of noticing, they analyzed their own ability to notice during both the first and second teaching experiments. Table 4 provides the participants' assessment of their noticing on the teaching experiments.

Analyzing their own noticing levels was important for participants because it provided scaffolding for them to reflect on their professional practices and described what more-advanced noticing may include; in essence, it gave them a tool to guide reflection that supported them in thinking about how they notice and how they could improve their noticing.

The aforementioned noticing levels, as determined by both the researchers and the participants, are included in the findings to provide an understanding of how noticing was conceptualized. The intent is not to compare similarities or differences in the determined noticing levels, but rather to understand how the participants were thinking about their ability to notice when they used the van Es (2011) framework. The following considers the shift in noticing practices of each of the participants and describes their individual conceptualizations of their noticing practice.

Cassidy: Pedagogical Perspective

For the teaching experiment and model building, Cassidy demonstrated growth in noticing from the first attempt to the second. Specifically, with the teaching experiment, she went from level 1-baseline to level 2-mixed and for the model building, she went from level-2 mixed to level 3-focused. As Cassidy reflected on her first teaching experiment and worked to describe specific examples of her student's thinking and

TABLE 3

Research assessed levels of noticing during the first and second model building

Participant	*Model building 1*	*Model building 2*
Cassidy	Level 2-mixed	Level 3-focused
Victoria	Level 1-baseline	Level 2-mixed
Justin	Level 1-baseline	Level 2-mixed
Sean	Level 1-baseline	Level 1-baseline

TABLE 4

Participant assessed levels of noticing during the first and second teaching experiments

Participant	*Teaching experiment 1*	*Teaching experiment 2*
Cassidy	Level 3-focused	Level 3-focused
Victoria	Level 3-focused	Level 4-extended
Justin	Level 1-baseline	Level 3-focused
Sean	Level 1-baseline	Level 1-baseline

reasoning, she focused more on her own actions during the interview than on student thinking. For example, she wrote:

> The main questions that was asked was what are the properties of right triangles. Then after asking that main question, I asked the student if they knew anything about the sum of the two adjacent sides versus the hypotenuse. In this question, I was thinking of triangle inequality and wanted to see if the student knew this property. However, the student came up with 'the sum of the two squared equals the hypotenuse squared.'

Cassidy's reflection focused on her pedagogical practices, with little emphasis on how the student was actually thinking. She provided a quote from the student but did not elaborate on the student's understanding. Instead, she continued to reflect on what she had thought and asked during the interview by forming general impressions of what occurred, as opposed to interpreting what the student's understanding meant.

By the point of the second teaching experiment, Cassidy was forming more general impressions and providing evaluative comments while beginning to refer to specific events:

> I could tell that the student knew what perimeter and area mean. Because he correctly found the perimeter and area of each figure he was asked to, I can infer that he has a good understanding of what they mean and how to find them in a square.

In this example, Cassidy focused on what the student could do and understood and included some initial evidence by mentioning that the student could find perimeter and area, demonstrating her ability to progress beyond descriptive comments. Similar to the example with the teaching experiment, Cassidy's noticing developed similarly during the model building.

Cassidy's own evaluation of her noticing practices revealed that she considered herself to deeply analyze her student's mathematical thinking and she considered herself to make clear connections between student thinking and evidence. When reflecting on her interview, Cassidy commented:

> In this question (see Fig. 3), I was thinking of triangle inequality and wanted to see if the student knew this property. However, the student came up with "the sum of the two squared equals the hypotenuse squared." Then asked later what the property was called, the student responded with the Pythagorean Theorem. This surprised me. I expected the student to just know the variable version of the Pythagorean Theorem and not to recognize it in words.

Cassidy used this example as evidence of an instance in the teaching experiment when she focused on the student's thinking and was able to reason and make connections to the broader implications of his mathematical learning. When she analyzed this teaching experiment 2 months later and reflected on her own reflection by analyzing her level of noticing, she referred to her noticing as a level 3-focused. She wrote:

> Level 3, I referred to a specific student (the interviewee) and what he said and did throughout the interview that helped me get a better understanding of how he thinks. For example, I presented an example of the student stated the Pythagorean Theorem in words and explained how this surprised me while interviewing. I later used this example to infer what I thought the student knows and does not know. I inferred that he had a good understanding of the Pythagorean Theorem.

In this excerpt, Cassidy considered herself to have made assertions about what the student knew and did not know to come to the conclusion that her student had a "good understanding" of the Pythagorean Theorem. Her argument that her noticing was at a level 3-focused was based on the notion that she attended to a "particular students' mathematical thinking." To further substantiate her claim, Cassidy commented, "I was able to see that using the method of Pythagorean Theorem stays with students better than using trigonometric functions to solve for a side of a right triangle."

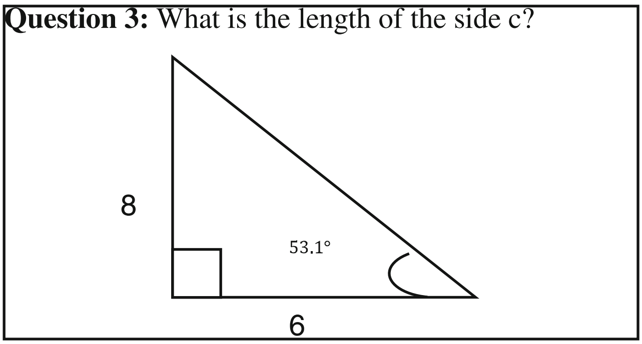


Fig. 3. Triangle graphic referenced during teaching experiment

She equated this with making connections between teaching strategies and mathematical thinking, which is a hallmark of level 3-focused, extended noticing.

Pedagogical Emphasis. Cassidy's noticing practices were characterized with a focus on her own pedagogical practices. Over time, she began to form general impressions about students' thinking, but she considered students from the perspective of her own teaching actions. At times, she made general statements about students' thinking, but these were not clearly connected to evidence from the teaching experiments or model building, but rather focused on what she had done pedagogically. In essence, she assumed a pedagogical perspective that focused on her own actions and occasionally gave way to general statements about students' thinking as she attempted to professionally notice. Nevertheless, she demonstrated growth with her teaching experiments and model building, which indicates growth in her awareness to student thinking.

Victoria: Interpretive Initiation

Victoria noticed at a level 2-mixed for her first and second teaching experiments and shifted from a level 1-baseline to a level 2-mixed for the model building. The level 2-mixed for the teaching experiments was distinguished by the extent that she mentioned specific events and interactions and incorporated interpretive comments to further describe what the student's thinking meant in the larger mathematical realm. She commented:

> The student solved an equation that is very easy to factor by using the quadratic formula. So, I originally asked him how he solved this equation and then why he chose to solve it that way, 'Why did you choose to use the quadratic formula?' Student: 'Because that is the way I was taught; and that is the way I was told is the easiest to solve these.' I gathered that the student has been told that the easiest way to solve quadratic equations is by using the quadratic formula. Also, it does seem that even though he was taught other ways, he does not remember how to use them.

In this example, Victoria made a specific reference to an instance in the interview when she was working to understand why her student used the quadratic formula instead of factoring to solve the problem. She began to interpret his response by mentioning that he likely forgot other ways to solve the problems. While Victoria did not elaborate on the events, or come to deep interpretations, she began to make meaning and draw some conclusions about her student's use the quadratic formula and reasons for his use of the particular method for solving the problem.

With model building, Victoria initially noticed at a level 1-baseline and noticed at a level 2-mixed during the second attempt. During the second model building, she began to make some interpretations about the student's thinking. She notes, "The student knows the features of slope-intercept form and how they apply to the graph." She goes on to describe how the student graphs linear and quadratic equations. The inclusion of evidence and interpretation distinguished the noticing level on her second model building from the first.

When assessing her own noticing Victoria was cognizant of the noticing framework and even cited van Es (2011) herself when she wrote, "I believe that I was able to 'attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking' (van Es, 2011, p. 139)." She referenced a description about quadratic graphs and how her student was thinking and began to refer to specific events. She considered instances of pointing to specific comments students had made, in this case about graphs, as students' mathematical thinking, even in instances when she simply recalled what the student had said or done when graphing. She considered instances that drew connections between something a student said or did and her response to be examples of attending to relationships.

Interpretive Initiation. Victoria's noticing is characterized by repeated instances of initial interpretations. With the teaching experiments and model building, she begins to incorporate interpretive comments about students' thinking but does not elaborate on these events. As a result, she considers her noticing to include elaborations, but she does not make connections to broader principles of teaching and learning and her interpretations are budding without the inclusion of deep analysis. As a result, she is tending toward more advanced levels of noticing which would occur with increased depth when interpreting students' thinking. While her first and second teaching experiments were both at the same level of noticing, Victoria demonstrates some growth with the model building, shifting to a higher level of noticing.

Justin: Conscious Awareness of Focus

Justin's noticing for the teaching experiment and model building went from a level 1-baseline to a level 2-mixed type of noticing in both instances, indicating an increase in the inclusion of evidence and interpretation. Justin demonstrated his ability to notice by beginning to make interpretive comments and reference specific events in his second teaching experiment, in which he asked his student to factor polynomials. He remarks:

> I can infer that Allison can factor polynomials with numbers using her rule of finding numbers added together and numbers multiplied together to find the factored values … However, when Allison was given a polynomial with letters instead of numbers she had a tough time fitting her strategy to factor a polynomial like, $x^2-ax-2a^2$. Plus, the negative signs seemed to give her some problems when factoring …For example, at one point she explained that $-2ax$ would be zero since she added $-2+1+1=0$.

In this example, Justin began by making interpretive comments about Allison's understanding of factoring polynomials. He then supported these claims by providing a specific mathematical example and explaining the misconceptions that Allison had. In this instance, Justin highlighted noteworthy events and began to refer to specific events and interactions as evidence, resulting in level 2-mixed noticing. As with the other participants, the level 2-mixed noticing that took place during the second teaching experiment was distinguished from level 1-baseline noticing because Justin went beyond recalling events and formulated initial interpretations of events that occurred during the teaching experiment. Likewise, these instances of noticing did not extend to a level 3-focused because he did not elaborate on events and interactions while making interpretive comments. A similar pattern was seen with his model building.

In Justin's first teaching experiment, he considered his noticing to be at a level 1-baseline. He commented, "What I noticed was not specific. I noticed more general moves by the student …I was aware of the whole environment and I did not have a specific awareness to the goal." Justin went on to further describe how noticing the whole environment influenced his focus on student thinking, "By noticing the total environment of the interview, I most likely lost information that was pertinent to the student's thinking …I needed to choose where to focus my attention on student thinking instead of focusing on aspects that were not important in the interview." Here, Justin made a connection to the framework by describing how he was forming general impressions of what occurred (van Es, 2011), without a specific focus on student thinking. In this instance, Justin conceptualized his noticing by realizing that his focus on extraneous components of the interview resulted in his neglect to consider student thinking.

Conscious Awareness. Justin's noticing was characterized by an understanding of the focus of his noticing, meaning that he was aware of the subjects or content upon which he was noticing and made decisions to shift this awareness for professional growth purposes. Initially, Justin was focused on the contextual features of the teaching experiment and when model building and realized that these external foci kept him from concentrating on students' thinking. As a result, he was able to internalize the focus of his attention and made appropriate shift to consider students' thinking as opposed to environmental features. His conscious awareness of the object of his noticing was important for his own understanding about professional noticing. Justin demonstrated a shift toward higher levels of noticing with both the teaching experiments and the model building.

Sean: Introspective Refection About Content

Sean presented a unique case in the dataset because he was the only participant to notice at a lower level in the second teaching experiment as compared with the first, going from level 2-mixed to level 1-baseline. His noticing was at a level 1-baseline for both model buildings.

During Sean's first teaching experiment, he included evidence about his student's understanding regarding the Hardy-Weinberg Theorem. Specifically, he commented, "Lucie knew what the variables were but wasn't sure what they summed to." He went on, "I gave Lucie a value for *p* and asked her if she could then solve for *q*. She responded immediately, 'I don't know I'd have to … ' And then she paused." Following this description, Sean reasoned that "she couldn't approach the math before she had conceptualized it." In essence, Sean was beginning to make interpretive comments about Lucie's understanding by referring to specific events and interactions related to her biological and mathematical understanding.

By contrast, during his second teaching experiment, Sean made broad statements about Lucie's thinking, without supporting claims with evidence. He generalized, "The mathematics of this problem was quite simple, and she solved it easily, but she had difficulty with terminology and she really struggled to put what she told me in words into a mathematical formula." He neglected to refer to specific events and maintained a general description of what occurred.

When Sean assessed his own noticing practices, he was extremely methodical and introspective. This level of metacognition with reflection distinguished his assessment of his noticing practices from the other participants. In his synthesis paper, he wrote:

> With regard to my own ability to notice in an educational setting, I'll suggest that I'm somewhere between an apprentice and a journeyman. I feel that I notice events during instruction and that I have the ability to reflect on those events and make sense of them. However I had never given any serious consideration to the idea of noticing before beginning the current exploration of this construct.

As Sean considered his own limitation to notice, he provided a rationale for his low levels of noticing. He remarked, “It’s possible that my failure to notice effectively is due, in part, to my lack of understanding regarding the complexities of mathematical thinking and the places students commonly run into trouble.” It should be noted that Sean was the only participant to select a university level student for the teaching experiments. Of all of the participants, this was the only instance in which the participant deeply analyzed reasons for the level of noticing that was occurring and made a connection to larger constructs of teaching and learning, such as the teacher’s knowledge, as rationale.

Content Reflection. Sean’s noticing is characterized with an emphasis on reflection about content. The focus of his noticing is on the mathematics content the student is encountering as well as his own mathematical content knowledge and the relationship between his knowledge and his delivery of the teaching experiments. Essentially, he considers interactions between the student and the content and between his own content knowledge and his ability to deliver the teaching experiment. In this way, he is reflective about the role of content and notices how the content mediates the teaching experiment process as well as how noticing influences his interpretation of students’ thinking. Of the participants, Sean was the most introspective, but was also the only one to shift noticing to a lower level on his teaching experiments; his model building noticing was stagnant.

Discussion

The emphasis on noticing for the class structure highlighted what it means to focus deeply on students’ thinking as opposed to noticing other features of a classroom. “The range of what we think and do is limited by what we fail to notice” (Goleman, 1985, p. 24). As such, orientation toward noticing is likely to provoke participants to increase urgency for trying to notice. In this study, during the first teaching experiment and model building, the participants were not yet oriented with the construct of noticing. During the second teaching experiment and model building, they were immersed in learning about noticing and were aware of what it meant to notice, thus influencing their awareness of what they had previously been failing to notice (Mason, 2011). Despite the awareness with respect to noticing during the second teaching experiment and model building and the emphasis on noticing in the course, the participants noticing levels were not drastically increased from the first to second attempts and decreased in one example.

Over and Under Evaluation

While the intent of the study was not to compare the noticing levels as determined by the researchers with the noticing levels as determined by the participants, differences were evident, which may provide some insight into the participants' noticing practices. A trend across participants indicated that they assessed their noticing practices at higher levels than the levels assigned by the researchers. Analysis of the noticing of Cassidy and Victoria as compared with Justin and Sean reveals a difference in the degree of deep self-reflection that occurred throughout the course and with their self-assessments of noticing. Cassidy and Victoria did not describe their own conscious awareness of noticing to the extent that Justin and Sean wrote about their metacognition related to noticing. Recall that Justin's noticing was characterized as conscious awareness of focus and Sean's was characterized as introspective reflection about content. By contrast, Cassidy took a pedagogical stance with her noticing and commonly detailed her own actions. Victoria made initial interpretations but lacked elaboration; neither Cassidy nor Victoria engaged in the introspection that was evident with Justin and Sean. Perhaps this difference in the extent to which participants considered their own noticing resulted in differences between the researchers' levels of noticing and Cassidy and Victoria's determined levels of noticing. The two participants who were more reflective were more closely aligned with the researchers when it came to evaluating levels of noticing.

Examining Shifts

During the first teaching experiment and model building, the participants were not oriented toward noticing and had yet to encounter the framework for assessing how and what was noticed (van Es, 2011). At this point, their noticing levels were also relatively low. Some may speculate that as novice teacher educators, levels of noticing would already be beyond level 1-baseline at any point in teaching. However, this was not the case; all participants were either level 1-baseline or level 2-mixed. One possible explanation could be the contrived structure of the teaching experiment and model building. The participants all indicated they had not taken part in a similar practice, so the process was new to them. Likewise, they were in the process of learning about effective questioning as a part of the course, so their emphasis during the teaching experiment and model building could have been on other factors, such as the questions they asked. Admittedly, all participants indicated that the teaching experiment was not as straightforward as one may assume because of the necessity to pose questions based on students' understanding, as opposed to asking a checklist of questions (Franke et al., 2009).

One explanation for the limited shift in noticing levels could be the short period of time in between the two cycles. The framework used in this study (van Es, 2011) was generated from a professional development project spanning ten sessions and 1 year with the intention of being a tool to describe the development of noticing. The span of ten sessions in the work of van Es (2011) provided greater opportunity for participants to develop ways of noticing at the more advanced levels on the framework. By contrast, in this work participants had the opportunity to engage in a repeated cycle, but they may not have yet developed adequate competence and were trying to understand the notions of noticing, teaching experiments, and model building simultaneously, while also attempting to take part in these professional practices. Therefore, the instances of limited shifts with noticing, or no growth in the example with Sean, may be the result of limited engagements with opportunities to professionally notice and reflect on the experience. This raises questions about how novice teacher educators should be supported to develop the ability to professionally notice.

When considering the noticing of these participants, Jacobs et al. (2010) note that shifts in noticing may be minimal at first and "professional developers need to be patient and initially expect limited, rather than robust, evidence of shifts" (p. 196). While Jacobs et al. (2010) work was with inservice teachers, the participants in this study are novice teacher educators, and thus similar findings regarding the rate of growth could be expected. As a result, these findings, when compared with the work of van Es (2011) and considerate of Jacobs et al. (2010), suggest that professional development on noticing for novice teacher educators should include multiple repeated experiences for engaging in noticing and reflecting on the experience. As evidenced in these data, the process initiated growth with noticing, suggesting that continued cycles may be beneficial for novice teacher educators. Specifically, the course could be restructured to include additional repeated cycles of the teaching experiments and model building.

Noticing Trends

These four individuals represent unique examples to aid in understanding how to support novice teacher educators. The incorporation of the teaching experiments and model building provided a context in which the participants could assume a self-reflective stance to consider their own professional noticing. Cassidy demonstrated a pedagogical perspective that slightly shifted over the course toward thinking about students' thinking. Similarly, Victoria was unique in that she formulated initial interpretations of students' thinking and developed in her noticing, but lacked clear connections between evidence and generalizations. Both Cassidy and Victoria would benefit from increased opportunities to focus deeply on how students' think about mathematics, continued opportunities to analyze what they noticed, and time to consider how they internalize those findings to make future teaching decisions. Of the four, Justin and Sean both represented an increased awareness of their awareness (Mason, 2011). This did not necessarily indicate that they noticed at more advanced levels, but they demonstrated a more in-depth understanding of recognizing how they noticed and possessed a greater cognizance of their ability or inability to notice particular aspects of students' thinking. This level of in-depth reflection is promising for experienced teacher educators because it provides evidence that some novice teacher educators are introspective about their pedagogical and content delivery practices and recognize how these practices may influence student learning. Based on these findings, the following section describes how experienced professional developers can orchestrate opportunities to further develop noticing practices in novice teacher educators.

Future Professional Development

In the present study, the novice teacher educators conducted their teaching experiments with high school or undergraduate students; in these instances, they assumed the role of teacher as opposed to teacher educator. Again, this was purposeful, so they would have opportunities to learn to notice students' mathematical thinking before teaching others to learn to notice students' thinking. The emphasis here was on how they understood how students' thought about and interacted with mathematics content. According to Kazemi et al. (2011), the next step would be to provide opportunities for these novice teacher educators to take part in professional development focused on understanding how preservice and inservice teachers engage with mathematics. This suggests that once the novice teacher educators have developed the ability to professionally notice students' thinking through repeated cycles they should shift to focusing on how teachers engage with mathematics. When experienced teacher educators work with novice teacher educators to develop professional noticing, it is important they convey differences in how teachers hold knowledge as compared with how students consider mathematics, so that the novice teacher educators are able to notice teachers' mathematical thinking (Kazemi et al., 2011).

Limitations

Studying the four novice teacher educators provided opportunity for careful analysis into the practices of these individuals; however, these findings cannot be extrapolated to the larger population. It should be emphasized that this exploratory study focused on four novice mathematics teacher educators and the findings are representative of their experiences professionally noticing. These findings are not generalizable to all novice teacher educators, but they do provide an in-depth understanding of how these four participants professionally noticed, how they shifted in their noticing, and how they conceptualized their noticing. Furthermore, these cases may provide insight for knowing how to support novice teacher educators as they work on developing their own noticing, but these findings are not generalizable. Likewise, the data collected were all written accounts of noticing; this study did not examine the thinking process of these participants, so it could be possible that varying degrees of noticing occurred among the participants, but they did not record these thoughts. Despite these limitations, these findings add to the research literature on noticing by focusing on an understudied population and providing an in-depth analysis of four novice mathematics teacher educators' practices.

Future Research

The findings from this study call to question the noticing practice of both novice and experienced teacher educators. Knowing more about how these populations notice would provide insight for knowing how to support their development of noticing and their teaching of noticing. Likewise, researching the implications of restructuring this course to include additional cycles of the teaching experiments and model building would provide understanding about the use of the van Es (2011) framework and the development of noticing over time. Since noticing is no trivial task (Jacobs et al., 2010) and experienced teachers often have a difficult time noticing, there is a need to further understand how teacher educators notice and how this noticing can be developed to know how to support this population's work.

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