VINCE GEIGER, MERRILYN GOOS and SHELLEY DOLE

THE ROLE OF DIGITAL TECHNOLOGIES IN NUMERACY TEACHING AND LEARNING

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ABSTRACT. This paper presents a model of numeracy that integrates the use of digital technologies among other elements of teaching and learning mathematics. Drawing on data from a school-based project, which includes records of classroom observations, semi-structured teacher interviews and artefacts such as student work samples, a classroom-based vignette is presented, which illustrates possibilities for technology integration into classroom numeracy practice. This vignette provides evidence of the influence of digital tools on students' development of skills, mathematical knowledge, dispositions and orientation towards using mathematics critically.

KEY WORDS: applications, digital tools, mathematical literacy, numeracy, technology

Introduction

The notion of numeracy (which in some international contexts is also known as mathematical literacy) as the capacity to make use of mathematics to accommodate the demands of the lived worlds of private and public life has been an issue of discussion within mathematics education from at least the time of the Crowther Report (Ministry of Education, 1959). Subsequent government reports and research literature have emphasised the importance of numeracy as a focus for schooling (see for example, Cockcroft, 1982; Steen, 1999). Within the Australian context, this has meant the initiation by government of many programs and policies (see for example, Council of Australian Governments, 2008) aimed at addressing the perceived deficiencies in the capacities of young people to apply mathematical knowledge in post school environments (Zevenbergen & Zevenbergen, 2009).

While such initiatives may be interpreted as a call for vocationally oriented mathematics instruction, Straesser (2007) warns against narrow approaches to mathematics education and training. He views mathematics as a strategic tool that can be adapted for a range of contexts and settings. In particular, he signals a concern for the "black box" view of mathematics in the workplace where the underpinning features and functions of mathematics are subsumed into simple routinised practice. Straesser argues that what is needed is as new type of knowledge that bridges the divide between mathematics and the rest of the world. Traditionally school curricula have placed little emphasis on the use of mathematics in the beyond school world (Damlamian & Straesser, 2009), but there are developing areas of research and practice that focus on both the teaching and learning of mathematical knowledge and on the utilisation of this knowledge in real world contexts.

Increasingly, the use of mathematics in the real world involves the use of digital tools, having such an impact on nearly all aspects of life that Steen (1999) describes the world in which young people are growing up as "data drenched". How the numeracy demands of a task are changed through the availability and use of digital tools and the way in which this change impacts on the skills, knowledge and dispositions of individuals within and out of school is an undertheorised area of research (Zevenbergen & Zevenbergen, 2009). Regardless of our understanding of such processes, young people in the workplace have begun to accommodate their information saturated environments through the development of more holistic approaches to solving problems by making use of all available tools—especially digital technologies (Jorgensen Zevenbergen, 2011). This phenomenon raises the question of how we might adapt school teaching and learning practices in order to facilitate more holistic approaches to the application of mathematics in preparation for beyond school environments.

This paper explores how numeracy learning and teaching in schools can be supported through the use of digital tools and in so doing enable students to develop technology-integrated mathematical capacities that will prepare them for the beyond school worlds of work and active citizenship. To guide this exploration a theoretical framework, underpinned by relevant research literature, is presented. This theoretical framework was used to direct the selection of data collection procedures and as a tool of analysis. The use of digital tools to support effective numeracy practice is illustrated through a vignette constructed from observational and interview data. Through the analysis of this vignette, we seek to address the research question:

How can digital tools be used to support effective numeracy teaching and learning practice?

Digital Tools and New Numeracies

While the balance between teaching mathematics for its own value as a discipline and for its usefulness in addressing problems in the real world is still a matter of debate, there appears to be international acceptance that applications of mathematics have a valuable place in school mathematics programs. To enhance teaching and learning, however, application tasks must make connection to current practices in working, private or civic life and provide opportunity to take advantage of the capabilities offered by digital technologies (Noss, 1998).

The importance of digital tools in supporting the use of mathematics in the workplace is noted by Zevenbergen (2004) who argues there is an intergenerational difference in the way numeracy skills are manifest in contemporary retail industries. Drawing on data from a large scale survey of young people working in emerging industries (e.g. leisure, hospitality and information technology) within a major regional centre, she observed that younger workers were happy to defer cognitive labour (e.g. mental arithmetic) to digital tools. This approach enabled them to take on the more strategic aspects (e.g. planning, problem solving) of their work more effectively. Zevenbergen concludes, from this and subsequent studies (e.g. Jorgensen Zevenbergen, 2011), that the influence of technology in schools and the workplace, and by implication other aspects of the lived in world, has shaped the habitus of young people who, as a result, are reshaping the various structuring practices that serve to recognise and validate particular dispositions and skills within their workplaces. This new generation of workers also make use of their personal mathematical knowledge and their capabilities with Information and Communication Technologies (ICTs) to solve on the job problems in more inventive ways than their experienced co-workers.

While the use of digital tools has an increasing impact on the way we live and work, and the potential of digital technologies to enhance the learning and teaching of applications of mathematics is widely acknowledged (Niss, Blum & Galbraith, 2007), research into the role of technology when solving problems within the beyond school world is limited. As Zevenbergen (2004) observes:

> While such innovations [ICTs] have been useful in enhancing understandings of school mathematics, less is known about the transfer of such knowledge, skills and dispositions to the world beyond schools. Given the high tech world that students will enter once they leave schools, there needs to be recognition of the new demands of these changed workplaces. (p. 99)

The demands of the workplace require individuals to think adaptably, to have a positive disposition towards learning new approaches to solving problems as they arise and to make effective use of digital tools. While these are capabilities and dispositions that appear to address the demands of Straesser's "in between worlds", it is less clear how these capabilities can be promoted through instructional practices in school mathematics classrooms. In the next section of this paper, two complementary research perspectives are presented on the affordances offered by digital tools when individuals engage with problems set in real world contexts.

Theorising the Affordances of Digital Tools for Applications of Mathematics

An area where there is developing interest in the use of digital tools when teaching students how to apply mathematics is the field of mathematical modelling. We draw on two perspectives from this field as a way of illustrating the state of research in this area.

Confrey & Maloney (2007) identify four approaches to using technology in mathematics instruction: (1) teach concepts and skills without computers, and provide these technological tools as resources after mastery; (2) introduce technology to make patterns visible more readily, and to support mathematical concepts; (3) teach new content necessitated by technologically enhanced environments (estimation, checking, interactive methods); and (4) focus on applications, problem solving and modelling, and use the technology as a tool for their solution (p. 57).

While acknowledging that each of these approaches has a role in the teaching and learning of mathematics, Confrey & Maloney (2007) argue that mathematical inquiry related to the application of mathematics to real world must play a more central role in mathematics instruction. They draw on Dewyian principles to develop a framework in which technology is assigned a vital role in coordinating inquiry, reasoning, and systematising processes. In their view, the process of mathematical modelling, and applications of mathematics more broadly, is founded on two activities: inquiry and reasoning. Inquiry is a means of gaining insight into an indeterminate situation—such as a loosely bound problem in the real world. Reasoning is the process that draws on bodies of knowledge to transform the indeterminate situation into a determinant outcome—a model. In their view:

> Mathematical modelling is the process of encountering an indeterminate situation, problematizing it, and bringing inquiry, reasoning, and mathematical structures to bear to transform the situation. The modelling produces an outcome—a model—which is a description or representation of the situation, drawn from the mathematical disciplines, in relation to the person's experience, which itself has changed through the modelling process. (p. 60)

The process of inquiry gives rise to observations, responses, measurements, interactions, indicators and methods of sampling and data collection that are typically mediated by various forms of technology. According to Confrey & Maloney (2007), it is through the coordination of these artefacts and the processes of inquiry, reasoning and experiment, that an indeterminate situation is transformed into a determinate situation. In their view digital tools play a central role in this coordination.

An alternative perspective is offered by Geiger, Faragher & Goos (2010) in a study of the use of Computer Algebra Systems (CAS) as a tool to support mathematical modelling in senior secondary mathematics classrooms. In contrast to the role attributed to digital tools within mathematical modelling in earlier work (e.g. Galbraith, Renshaw, Goos & Geiger, 2003) where technology was seen as an enabler at the point where a final solutions was being produced, instances were reported in Geiger, Faragher and Goos' study where the electronic outputs available through CAS forced students to re-evaluate fundamental assumptions they had made in relation to the context in which problems were situated. As a result, students chose to reformulate, solve, interpret, and evaluate the problem in the light of an adapted assumption set. This finding is consistent with the position of Confrey & Maloney (2007) in that technology can have a role at every level of the inquiry process including the coordination of the inquiry and the reasoning and systemising processes that lead to an outcome.

While both of these studies provide insight into an understanding of how digital tools can support the use of mathematics to solve problems in real world contexts, neither attempts to address the broader issue of how this potential can be harnessed in concert with other important aspects of teaching and learning mathematics such as students' dispositions towards the use of mathematics to solve problems in real world contexts or the use of mathematics to inform a critical view of real world events and phenomena. Further, neither model is concerned with how to teach the application of mathematics to the real world or the place of applications in school education programs.

Theoretical Framework

In response to changes to the demands of the workplace and consequent criticisms of the mathematical capabilities of school leavers in Britain through the 1970s, Cockcroft (1982) led a government inquiry into teaching mathematics. In the resulting report, he redefined numeracy as an "at-homeness" with numbers that enabled an individual to accommodate the mathematical demands of everyday life. More recently, the importance of numeracy as an enabler of informed and participatory citizenship has been recognised in the OECD's Program for International Student Assessment (PISA). According to PISA's definition mathematical literacy is:

> …an individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2004, p. 15)

While these definitions recognise that being numerate requires the capacity to use mathematics in a critical way, they do not fully accommodate other aspects of critical thinking such as challenging positions or arguments through evidence based reasoning or the role of digital tools in solving problems in the lived-in world. In considering the concept of numeracy in relation to the changing nature of knowledge, work and technology, Goos (2007) designed a model that captures the richness of current definitions of numeracy while introducing a greater emphasis on tools as mediators of mathematical understanding, reasoning and action. The model incorporates attention to real-life contexts, the deployment of mathematical knowledge, the use of physical and digital tools and consideration of students' dispositions towards the use of mathematics. Developing a critical orientation was also emphasised in relation to numeracy practice, for example, the capacity to evaluate mathematical information used to support claims made in the media (Fig. 1).

The model was constructed as an accessible instrument for the purpose of teachers' planning and reflection and has been validated in earlier work when used as a framework for auditing mathematics curriculum designs (Goos, Geiger & Dole, 2010), for the analysis of teachers' attempts to design for the teaching of numeracy across the curriculum (Goos, Geiger & Dole, 2011), and for mapping teachers' learning trajectories in effective numeracy pedagogy (Geiger, Goos & Dole, 2011). A description of the model is presented below along with relevant research literature that justifies its structure.

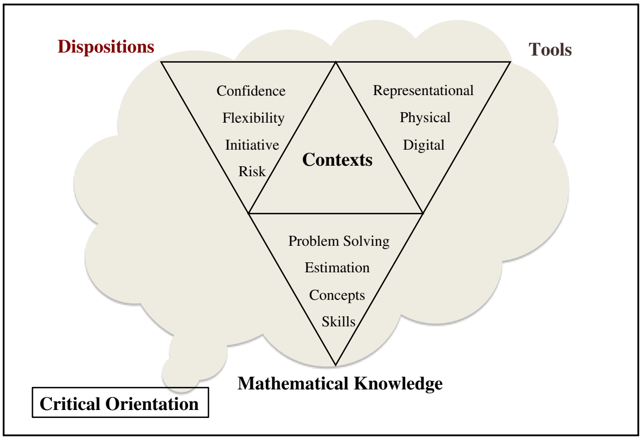

Figure 1. A model for numeracy in the twenty-first century (Goos, 2007)

According to Steen (2001), numeracy is about the use of mathematics to act in and on the world; thus, *context* is at the centre of the concept of numeracy. This position is supported through numerous studies that examine the role of mathematics in the workplace where context determines which mathematics is used, how it is used and when it is used (see for example Hoyles, Wolf, Molyneux-Hodgson & Kent, 2002). Typically, when mathematics is used in a context it is applied in a way different from how it is traditionally taught in school (Noss, Hoyles & Pozzi, 2000; Straesser, 2007) and so to learn to be numerate individuals must be exposed to using mathematics in a range of contexts (Steen, 2001).

Appropriate mathematical knowledge is required to act on problems within a given context. The capacity to understand and do mathematics provides access to powerful mathematical ideas that "significantly enhance opportunities and options" for participation in society and for pursuing aspirations in students' imagined futures (National Council of Teachers of Mathematics, 2000, pp. 3–4). In a numeracy context, mathematical knowledge includes not only concepts and skills, but also higher order thinking such as problem solving strategies and the ability to make sensible estimations (Zevenbergen, 2004). How to interpret a problem from outside of mathematics in a mathematical way, and then how to choose which mathematical knowledge is needed to engage with the mathematised problems is a challenge that lies at the interface of *contexts* and *mathematical knowledge*.

The desire and confidence to apply mathematics in real world contexts is related to the *disposition* of an individual in relation the use of mathematics. Affective issues have long been held to play a central role in mathematics learning and teaching (McLeod, 1992), and the importance of developing positive attitudes towards mathematics is emphasised in national and international curriculum documents (e.g. Australian and Reporting Authority, 2011; National Council of Teachers of Mathematics, 2000; OECD, 2004). Kloosterman (2002) emphasises the connection between beliefs and motivation in learning mathematics and concludes, from a review of literature based on a range of psychological perspectives (e.g. attribution theory, self efficacy theory, goal orientation theory, etc.), that individuals will only invest effort in an activity when they have an expectation of succeeding at a task and see value in attempting to engage in a task. Perkins, Tishman, Ritchhart, Donis & Andrade (2000) have argued that learners must believe in their ability to solve problems as otherwise they will not persevere with challenging tasks for long enough to succeed. Similarly, Greeno (1991) found that persistence and the capacity to continue to work determinedly towards a solution to a problem solving task are vital attributes to success. These attributes are particularly important when students work on ill-defined problem solving tasks (Valanides & Angeli, 2008). Consequently, it is not sufficient to focus on the mathematical skills and capacities we want students to learn alone, but that teaching must take place with students' dispositions in mind if they are to develop an affinity with a discipline (Gresalfi & Cobb, 2006). This affinity is vital for students to be disposed to making use of mathematics in their current lived in worlds and in their future lives (Boaler & Greeno, 2000). These dispositions include not just confidence with mathematics but a willingness to think flexibly, to show initiative and to take risks.

An increasing number of studies identify tools, and especially digital tools, as mediators of meaning making, reasoning and action in relation to mathematical learning (e.g. Artigue, 2002; Drijvers & Weigand, 2010; Geiger, 2005; Goos, Galbraith, Renshaw & Geiger, 2000, 2003; Guin, Ruthven & Trouche, 2005; Pea, 2004). In school and workplace contexts, tools may be representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and physical (models, measuring instruments), but increasingly tools are digital (computers, software, calculators, internet) (Noss, Hoyles & Pozzi, 2000; Zevenbergen, 2004).

Within the workplace, mathematical skills are becoming increasingly inter-related with information technology skills (Hoyles et al., 2002). This inter-relatedness results in changes to the nature of mathematical skill required. The changed nature of mathematical skills, in turn, changes the manner in which digital tools are used. Thus, the use of mathematics and digital tools in real world contexts generates a cycle of shaping and reshaping of the use of both mathematics and digital tools in a way where it is difficult to separate one from the other.

Other researchers have sought to define more clearly specific ways in which the use of digital tools can enhance the study of mathematics in context within school mathematics classrooms. In a report on research into uses of ICTs in mathematical modelling in Brazilian schools Villarreal, Esteley & Mina (2010) conclude that ICTs were vital elements in the construction and validation of mathematical models for both secondary school and university students. In addition, ICTs allowed teachers to offer learning experiences that were beyond the scope of the official curriculum. Digital tools can also promote students' effective engagement with real world problems by providing a means of accommodating gaps in requisite mathematical knowledge. In a case study of students using CAS-enabled calculators, Geiger (2011) reported on a teacher's observation that the algebraic facility of the calculator had facilitated some students' development of mathematical models for real world problems even though these students had previously demonstrated limited competence with algebraic manipulation.

All elements of the model are embedded in a critical orientation, as the fundamental purpose of numeracy in practice is that it empowers individuals with the capacities to evaluate and to make judgements and decisions about their options and opportunities in the lived in world. Thus, we view this critical orientation as a vital capacity for informed and participatory citizenship and for exercising effective and socially conscious decision making in an individual's personal life. This includes, for example, the capacity to evaluate quantitative, spatial or probabilistic information used to support claims made in the media or other contexts. Ernest (2002) views social empowerment as an important reason for teaching mathematics. This social empowerment can range from the purely utilitarian mathematical skills that are needed to function, in the simplest sense, in work and society, through to the critical skills that enable individuals to: make decisions and judgements; add support to arguments; and challenge an argument or position. As Ernest points out:

> The empowered learner will not only be able to pose and solve mathematical questions (mathematical empowerment), but also will be able to understand and begin to answer important questions relating to a broad range of social uses and abuses of mathematics (social empowerment). Many of the issues involved will not seem primarily to be about mathematics, just as keeping up to date about current affairs from reading broadsheet

newspapers is not primarily about literacy. Once mathematics becomes a '*thinking tool*' for viewing the world critically, it will be contributing to both the political and social empowerment of the learner, and hopefully to the promotion of social justice and a better life for all. (p. 6)

From Zevenbergen's (1995) perspective, attention to the critical aspect of numeracy in school classrooms will assist students to understand their social, cultural, political and environmental worlds, and empower them to make decisions, in their future lives about maintaining or challenging a status quo. In developing this view, she draws on Habermas (1972) tripartite theory of knowledge to distinguish between three types of numeracy: technical, which is related to basic skills and the capability to perform traditional, context free mathematical tasks; practical, which is the capacity to apply technical mathematics skills appropriately within life related contexts; and critical or emancipatory, which involves the use of mathematics for social or ideological critique. We argue that in the past, this critical aspect of numeracy has received limited attention within school numeracy teaching and learning practices.

The capacity to be critical assumes greater importance in a world that is increasingly data driven. Steen (2001) argues that in our data drenched world, where information is increasingly freely available, quantitatively literate citizens need to be able to do more than calculate and apply algorithms to problems set in familiar contexts; they need to be capable of thinking quantitatively about common-place issues. Because so many decisions in society are now supported by arguments based on numerical data, Steen believes that to thrive in these new times, individuals need the critical tools available to those who are quantitatively literate in order to support or confront authority confidently. This position is also consistent with that of Frankenstein (2001) and Jablonka (2003) who argue for the need to recognise how mathematical information and practices can be used to persuade, manipulate, disadvantage or shape opinions about social or political issues.

This model brings together clearly different but interrelated dimensions of numeracy. While the purpose of this paper is to identify and outline the use of digital tools in supporting numeracy development, a discussion of the use of digital tools cannot be conducted in isolation from these other dimensions as they are enacted in authentic classroom settings.

Research Design

The project was conducted across one Australian state, South Australia, over a period of 1 year within a state based educational system. In

Australia, education is a state responsibility and, as a result, state-based education jurisdictions function independently. The aim in this project was to empower teachers to work with numeracy across all curriculum areas. Consequently, participation in this project was sought from generalist primary and middle school teachers and specialist secondary teachers. This resulted in participants possessing a range of subject specialisations (secondary) or self-nominated strengths (middle years and primary) including, for example, mathematics, English, science, social education, health and physical education and design studies. The experience of teachers ranged from novice to very experienced. Schools nominated pairs of teachers to project managers within the school system who then made selections from the pool of applicants. The nomination of two teachers per school allowed for collaboration and support within a specific school setting while, at the same time, providing opportunity for pairs of teachers to compare and contrast their experiences from within the context of their own classrooms. Through this process, 10 pairs of middle school (Grades 6–9) or secondary teachers (Grades 8–12) were selected from schools across South Australia. Selected schools were inclusive of those situated in metropolitan, rural and remote settings.

A design-based research approach was used for the project as the methodology: involved iterative interventions; was initiated through specific theoretical intent; and developed and tested theory about how teaching practice and student learning might change, and how these changes can be identified as they emerge through the study. Design based research evolved out of the need to examine the potential of educational innovation within the reality and messiness of authentic classroom settings. In such contexts, experimental and quasi-experimental methodologies, where the environment and associated variables require strict control, cannot accommodate the complex, interactive and reflexive nature of classroom interventions that focus on the 'systematic generation and examination of data and refinement of theory' (Schoenfeld, 2006, p. 193). Cobb et al. (2003) argue that design based research is both theory focused and pragmatic in nature, as it involves iterative interventions that take place in practical educational settings with an aim to generate theory about improved educational practice. In keeping with the contextualised, pragmatic nature of design-based research, our approach wove together a number of effective models of professional learning such as action research, immersion experiences, curriculum implementation and collaborative partnerships between teachers and university researchers (Loucks-Horsley et al., 2003).

The framework of Loucks-Horsley et al. (2003) for professional development underpinned the design of the intervention/development component of both projects. From their perspective, all learning is contextual and so professional development needs to occur in school-based contexts so teachers can try out and validate ideas in their own classrooms. Teachers also need time and opportunities to discuss pedagogical and curricular issues with supportive colleagues as they attempt to implement new practices. These opportunities were provided on a regular basis. At the beginning of the project, teachers came together for an initial meeting to become familiar with the ideas embedded in the numeracy model and to work through investigations that allowed for the elaboration and clarification of the ideas embedded in the model. After this initial meeting, teachers were asked to adapt activities presented in the workshop to their own classroom contexts, or to develop new ideas based around the elements of the numeracy model and trial these in their classrooms. After a number of months, teachers were brought together again to present examples of activities they had trialled and to engage in further curriculum planning while being supported by teachers from other schools. After this meeting, a second round of trialling activities and whole project group meetings where teachers were asked to evaluate the outcomes of the just completed action learning cycle. In a final whole group session at the end of the project, teachers were also asked to consider ways in which pedagogical change could be sustained over time. Between each of the whole project meetings, a research team consisting of the authors of this paper and representatives of the sponsoring system authority visited teachers to discuss the success of the activities they were trialling and to provide further input and support as was necessary.

The data used in this paper are drawn from field notes of classroom observations, audio recordings of semi-structured interviews with teachers and students, which took place when the research team visited teachers, and artefacts such as teacher planning documents, student work samples and computer files collected during school visits. Field notes from lesson observations documented teacher and student activity and the extent to which this incorporated elements of the numeracy model presented in Fig. 1. Pre- and post-lesson interviews captured teachers' intended approaches to numeracy focused instruction and then their evaluation of the effectiveness of the tasks they trialled. Post-lesson interviews with students sought their perceptions of the connection between the lesson they had just experienced and the elements of the numeracy model—mathematical knowledge, contexts, dispositions and tools. Artefacts such as planning documents and student work samples provided additional evidence of the implementation of numeracy practices that were consistent with the model.

Audio-recorded interviews were transcribed and coded through a process of constant comparison with the categories of the numeracy model. While not all conversations could be categorised against the elements of the model, all noteworthy episodes were documented. Teacher and student activity, as recorded in lesson observation records, and the use of artefacts, were similarly coded. These aspects of analysis were then combined to present a holistic representation of classroom learning experiences, as defined by the numeracy model, in the form of vignettes. The vignette presented for this paper was selected because of the axial role digital tools played in promoting student learning.

Vignette

As part of the project, one teacher developed an activity within her Grade 8 (12–13 years old) Physical Education (PE) program where students investigated the level of their physical activity through the use of a pedometer they were asked to wear during all waking hours over a period of 1 week. The number of paces walked or run were recorded daily and entered into a shared Excel spreadsheet. Students were asked to analyse their own data using facilities within Excel, for example, the graphing tool, and then to compare their results with those of other students. This activity also provided the teacher with the opportunity to discuss the appropriateness of the graphs chosen for comparison between students. Thus, for example, in the case of the graphs presented in Fig. 2, there was a conversation between the teacher and the student about the difficulties of comparing the weekly totals between males and females when graphical representations of the performance of the two groups were of different types. After some discussion, the student made changes so that both sets of data were presented as bar graphs.

As an additional part of their analysis, students were asked to convert their total daily and total weekly paces into kilometres to gain a sense of how far they typically walked in the course of a day or a week. The task was also designed to help students realise that the number of paces they walked alone did not determine the distance covered but that an individual's pace length was also a factor. In order to make this conversion, students were required to design a process for determining the length of their own pace. After a discussion guided by the teacher, students negotiated an approach that was acceptable to all members of the

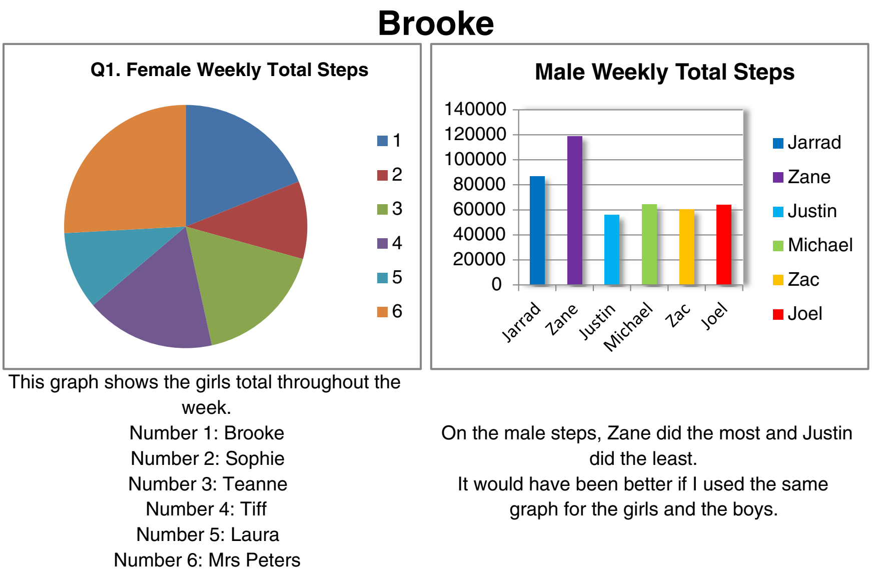

Figure 2. A comparison of males' and females' weekly total steps

class. This involved marking out a distance of 100 m along the footpath that bordered the school against which students counted the number of paces they each took to walk this distance. After demonstrating the procedure for obtaining the length of her pace and the converting paces in a day to kilometres from her own personal data, the teacher asked students to complete conversions of their own pace totals to kilometres by writing the formula for the conversion into the spreadsheet. She also suggested that students compare their kilometric distances with each other and to discuss why they were different. Totals varied considerably and, generally, according to the level of regular, organised activity, such as participation in sporting teams or walking to school (for a more detailed account of the approach to transforming "steps" to distance, please see Peters, Geiger, Goos & Dole, 2012). At the conclusion of the lesson, the teacher indicated the next session would include a further investigation related to the number of paces Usain Bolt (the world 100 m record holder) takes during a 100-m sprint.

Tools were used throughout the lesson. Physical tools such as tape measures were used to measure distances. *Digital tools* included pedometers, electronic calculators and Excel spreadsheets and provided the capability in this investigation to collect data (pedometer), perform initial calculations (electronic calculators) and record, manipulate, analyse and represent data (Excel spreadsheet).

Digital tools connected with other dimensions of the model in a number of ways. The number of steps per day was collected via the use of a pedometer and then converted (measurement and ratio) using both electronic calculators and through formulas in the Excel spreadsheet. Students were expected to select appropriate graphical representations for the comparison of the results of different groups. Thus, the aspects of *mathematical knowledge* required to engage with the problem incorporated the use of *digital tools*.

The use of pedometers provided the opportunity to introduce a personal, life-related problem *context*. Data gathered via the use of the pedometer provided a measure of students' personal levels of physical activity through the duration of the task. Thus, the use of the pedometer as a *digital tool* allowed students to raise their awareness of their own levels of physical activity, which the teacher eventually connected to the levels of activity needed in order to maintain good health in their current and future lives. This can be seen in the following transcript (R1, R2 refers to Researcher 1, Researcher 2; and S1, S2 refers to Student 1, Student 2 etc.)

R1: *We saw you earlier today and you were using pedometers…so what was going on there? What was that all about?*
S1: *We were measuring how many steps we took over a period of 1 week, so from Saturday to Friday.*
R2: *So did you have to wear the pedometers the whole time?*
S2: *Yep. And we were told that when we measured our steps it had to be around the same time every day. So we have the even* (meaning consistently measured) *amount of steps every weekday.*
R1: *So you weren't wearing them all day?*
S2: *Yeah, you wear it most of the day.*
S1: *Whenever we walked.*

The capacity to display data in different formats via the use of an Excel spreadsheet challenged students to think flexibly about the representation of their personal data so that they could compare their levels of activity on a daily basis and also compare their activity levels to others in the class. Students' *dispositions* towards this activity were also enhanced because they felt the opportunity to use digital means of gathering and representing data was more engaging and effective than performing the same activity using pen and paper methods across a range of subject areas.

R3: *So do you think it (Excel) is a program you could use in other learning areas?*
S1: *Definitely! We had to do in Science, recently, a prac and it required amounts and*

percentages. I haven't done it yet, but I'm going to get percentages and use Excel to make a graph.
R3: *And that will be OK with your teacher?*
S1: *Yeah, the idea was to create a graph—so on the computer is fine. And it's easier than drawing up one as well and having to count some of the more advance kind of graphs.*
R1: *It is a good way of doing calculations on lots of numbers—I was thinking of the pedometer—so rather than doing divisions for every line ... So I was looking at that in todays lesson. Do you know how you were converting the number of steps to the number of kilometres and you were just doing it for your own data—do you know how to set up formulae in Excel?*
S2: *In the new one you can choose which formula you want and it does it itself.*
S1: *Like the average formula or the total formula.*
S2: *Last year we learnt how to write them in.*

Students were more readily able to take a *critical orientation* to their personal levels of physical activity as the use of Excel permitted the ready comparison of their own results with others and thus for individual speculation on the reasons for differences between themselves and others. The visual displays students were able to generate with Excel also mediated discussion between students in relation to the differences they observed which, in turn, allowed for reflection on where individual students stood in relation to peers.

R1: *It was interesting looking at that table. One thing that stood out to me from the totals were differences for all of you—but did you notice that looking at that on different days of the week each of you were walking different numbers of steps?*
S2: *Yeah, Sunday was smallest.*
R1: *I noticed!*
S2: *Thursday and Saturday probably would have been probably been biggest for a lot of us because we play sport (on these days).*
S3: *We had to do graphs on the computer showing 2 days. And I did Saturday and Sunday using a like graph—there was a major difference! Saturday was like this (gesticulating with her hands to indicate a high level of activity was recorded on the graph) and Sunday was like this (indicating with her hands that the graph showed a low level of activity).*

Digital tools also provided the opportunity for the teacher to modify her teaching in order to challenge students to take a more *critical* approach to their own learning. She indicated that this was one of the goals she had set herself for the year and believed that the way to achieve this was to take a less "direct" approach to her teaching. She commented on her attempt to change this aspect of her teaching when discussing how she worked with students while they were exploring the type of graph that would best represent their data (T1 refers to Teacher 1).

R1: *So what sort of things were you and the students doing with Excel?*
T1: *Putting data into a table and using AutoSum—just little things like that. And then*

looking at different types of graphs—just the discussion around—well, what is that graph showing. And because I'm used to being so directive, I'd say that's not a good graph for this situation—for this reason. So I've really tried to stop doing that.
R1: *So what did you do instead?*
T1: *Well I'll give you an example. Say the kids had to compare their results on a Tuesday to their friends' results on a Tuesday. I really wanted them to have the same style of graph for both but of course some didn't because they liked the prettiness and the difference. So I said—if you got some else to look at that what do you think they might find a bit difficult to understand. Still a leading question from me—I know—but at least it made them think. And when some line graphs that were just inappropriate, I'd say, OK, if we go halfway along that line what is it showing?*

The teacher indicated that she had found it difficult to stop herself addressing errors or misconceptions in students' work directly but commented that she thought it was important to change her approach in order to promote students' capacities to take a more critical stance when engaging with information they were expected to understand and interpret.

Discussion and Conclusion

In order to prepare students for the types of data drenched and technology integrated worlds and workplaces Steen (1999) and Jorgensen Zevenbergen (2011) have described, the teaching and learning of numeracy must receive high priority. Simplistic approaches to numeracy instruction, such as those that emphasise basic skills to the near exclusion of other aspects of numeracy, however, will not empower students with the skills and capacities needed to function effectively and productively in the out of school world (Straesser, 2007). This means that approaches to the teaching and learning of numeracy must accommodate for social, contextual and critical aspects of the use of mathematics in action. For teachers to implement such approaches they must have access to models of teaching practice in which different aspects of numeracy are addressed in a balanced and holistic manner (Goos, 2007).

We have argued in this paper that for students to function effectively in their "data drenched" present and future worlds approaches to numeracy teaching must also incorporate the use of digital tools. The case presented here as a vignette demonstrates that the integration of digital tools into classroom teaching practice is possible and that the use of digital technologies can enhance the way in which other elements of numeracy are addressed.

In the vignette digital tools afforded the teacher the opportunity to promote students' mathematical knowledge. Students made use of their knowledge of number, measurement, ratio, graphical representations and algebraic concepts. The use of this knowledge was supported via digital tools in the following ways: (1) pedometers were use to collect data (number of paces per day); (2) Excel was use to store, display and present data in graphical formats; and (3) Excel formulas and hand held calculators were used to convert the number of paces into kilometres travelled.

The evidence presented here is consistent with previous research in support of the use of digital tools to enhance learning within content domains such as number (e.g. Kieran & Guzma'n, 2005), geometry (e.g. Laborde, Kynigos, Hollebrands & Straesser, 2006), algebra and calculus (e.g. Ferrara, Pratt & Robutta, 2006). Where our findings differ, however, is that digital tools have been used to assist the learning of mathematical knowledge within cross-curricular life-related contexts.

In Noss (1998) view, the use of digital tools is inseparable from the application of mathematics to individuals' lived-in-worlds. The activity presented in the vignette is bound to a life-related context that is explored through the use of digital tools. These were used to collect, display and analyse data related to the number of steps taken by students as part of a physical health education lesson. The teacher's focus was on developing students' knowledge of their levels of activity and then on how their level of activity compared to other members of their class. Thus, digital tools were integral to raising the awareness of students about the amount of exercise they performed on a regular basis in an absolute sense and then in comparison with others. The sharing of these data and the graphical representation of the data acted as a segue into a discussion about the level of activity required to maintain good health.

The capacity to be persistent and to continue to work through challenges is necessary when engaging with any ill-defined task (Gresalfi & Cobb, 2006) such as the use of mathematics to solve probelms in the real world. Persistence is only likely, however, when individuals believe they have a reasonable chance of success on a task, and they see value in what they have been asked to do (Kloosterman, 2002). During interviews, students indicated that they found the use of technology encouraged them to investigate their data through different types of graphs because it provided an effective and efficient means to do so. It is a reasonable speculation that if students had been hamstrung by a requirement to complete graphs and the analysis of data by hand then far less exploration would have taken place.

If social empowerment is an important reason for teaching mathematics in schools (Ernest, 2002), students must develop a critical orientation to the way they use mathematics to engage with and work in the world. A critical orientation allows students to recognise how mathematics can inform, persuade and shape opinions (Frankenstein, 2001; Jablonka, 2003) and to assist them to make decisions and judgement of their own based on mathematical reasoning. The various displays and data analysis capabilities, provided by the available digital tools, allowed students to critically examine the situation they were investigating and to speculate on what measures were necessary to change outcomes in their favour. In the case presented here, students were able to identify how their levels of daily activity change over a 1-week period and how these compared to other class members. This provided them with information about where their best opportunities lay in relation to increasing their amount of exercise. Students' opportunity to take a critical orientation to learning and doing mathematics was enhanced by the teacher adopting a less directed approach to instruction. During the interview that followed the lesson, she revealed that her decision to allow students to choose the types of graphs they used to represent data related to classmates' steps was deliberate. This strategy opened up the prospect of students making their own decisions and then reflecting on the appropriateness and effectiveness of the resulting representations in forming and justifying their opinions about different aspects of their classmates' levels of activity.

While it is acknowledged that the data drawn from this study represents an outstanding example of numeracy teaching and learning, it serves to illustrate the important role of digital tools in fostering rich numeracy practices. In order to find ways that extend quality practice in numeracy beyond outstanding but isolated cases further research is necessary into how to assist teachers to develop ways of thinking about their practice that disposes them towards recognising, and taking advantage of, opportunities to create tasks relevant their students' present lived-in-worlds and beyond.

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School of Education
PO Box 456, Virginia, 4014 QLD, Australia
e-mail: vincent.geiger@acu.edu.au