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AN ACTOR-ORIENTED TRANSFER PERSPECTIVE ON HIGH SCHOOL STUDENTS' DEVELOPMENT OF THE USE OF PROCEDURES TO SOLVE PROBLEMS ON RATE OF CHANGE

Received: 29 May 2013; Accepted: 30 November 2013

ABSTRACT. This article reports on a longitudinal observation study about students' development in their use of procedures to calculate instantaneous rate of change. Different procedures for solving tasks on rate of change are taught in mathematics and physics classes, and together they form a repertoire. Our study took an actor-oriented perspective, which we operationalized as a search for students' personal constructions of relationships between (1) learning from mathematics and physics classes and (2) interview tasks. We followed 10 students for 2 years (from grade 10 to 12), during which we administered 4 task-based interviews. We analyzed the breadth and connectedness of students' repertoire of procedures and report on the long-term development thereof. We conclude that often procedures are not part of students' repertoire shortly after the first introduction of this procedure in class. Students need time to acquire single procedures, and much more time to develop a broad and connected repertoire. In the development of their repertoire, there are major differences between students. From an actor-oriented perspective, many personal constructions are visible between learning and interview tasks. Students often use procedures that differ from procedures that are most appropriate from an expert's perspective. We also observed from an actor-oriented perspective that words such as velocity, steepness, or slope act as bridge for creating relationships between situations and procedures.

KEY WORDS: actor-oriented transfer, calculus, longitudinal study, procedures, rate of change, students' development, transfer

Introduction

Transfer of knowledge is a well-known and much discussed issue in the area of learning and instruction (e.g. Anderson, Reder & Simon, 1996, 1997; Greeno, 1997). In research on transfer, it is often reported that students hardly apply assumed knowledge from mathematics to other school subjects, such as physics (Basson, 2002; Cui, 2006; Tuminaro, 2004). These studies on transfer between mathematics and physics are dominated by two perspectives. First, the direction of transfer is considered as unidirectional, that is, from mathematics to physics. Second, studies on transfer often cover a limited period of time because researchers study the effect of a specific mathematics course on a specific physics course. Contrary to these studies, our study focuses on the

Electronic supplementary material The online version of this article (doi:10.1007/s10763-013-9501-1) contains supplementary material, which is available to authorized users.

effect of prior activities that took place in both mathematics and physics classes, on students' choice of procedures to calculate rate of change. Furthermore, we followed students' development over a longer period of time: from grade 10 to grade 12 (approximate ages 16 to 18).

This study aims to contribute to understanding the mechanisms underlying transfer, addressing the question how students develop their use of rate of change procedures learned in mathematics and physics classes and how students, on the long run, construct relationships between procedures learned in these different, but related, school subjects. By studying their performances, we looked for clues on how to foster students' learning of differential calculus. Understanding this process can help mathematics and physics teachers to give students opportunities to develop a more inter-related understanding.

Theoretical framework

There is a rich body of research on transfer and it has a history of over 100 years. Review studies (e.g. Billett, 2013; Lobato, 2006, 2012) describe that in the last decades a shift occurred from a cognitive to a situated view on transfer. In the cognitive view, much attention is given to transfer of knowledge from one situation to another situation, while the situated view emphasizes the role of the learner who constructs similarities between situations. In the next paragraph, we will discuss major differences between these perspectives.

Cognitive and Situated Perspective on Transfer

From a cognitive perspective, transfer is characterized as "how knowledge acquired from one task or situation can be applied to a different one" (Nokes, 2009) or "the ability to apply knowledge learned in one context to a new context" (Mestre, 2005). A feature of these definitions is the role of 'knowledge,' which is subsequently applied to another situation.

The cognitive view investigates if a person transfers knowledge from initial learning to a so-called *transfer task*. For example, Anderson et al. (1996) found evidence that representation and degree of practice are major determinants of the successful transfer from one task to another. The researcher assumes that transfer of knowledge is possible from an initial learning context to transfer tasks. The transfer tasks are designed in such a way that, according to an expert view, specific earlier learned knowledge can be used to solve the task.

Lobato (2003) refers to such forms of transfer as 'traditional transfer'. Traditional transfer suggests that knowledge can be separated from the situation in which it was learned. Lobato considers not only traditional transfer but also its research methodologies as problematic. Lobato & Siebert (2002) object that traditional transfer is based on expert knowledge instead of knowledge of a person who acts in a situation. They state that traditional transfer is the subject's re-application of overt actions in situations that the researcher deems similar.

In contrast with traditional transfer, transfer from a situated perspective highlights the role of the learner. Greeno (1997, p.11) formulates a research question in the situated perspective as: "when someone has become more successful at participating in an activity in one kind of situation, are there other situations in which that person will also be more adept?" Lobato (2003) states that what experts consider as a surface feature in a transfer task may be structurally substantive for a learner. Lobato (2012) describes a situated view on transfer, referred to as actor-oriented transfer, and she defines it as: "the influence of a learner's prior activities on his activity in novel situations" (p.233).

In research on transfer taking an actor-oriented perspective, the focus on the role of the learner has consequences for the design of the transfer situation or transfer task. The goal is to analyze if and how a student is affected by his participation in earlier activities. For example, Greeno, Smith & Moore (1993) were interested in the extent to which participating in an activity in one situation influences the learners' ability to participate in a different situation. Although the transfer task is designed by an expert, the function of the transfer task is to elicit activities by a learner and to investigate if and how students construct similarities between this task and earlier activities.

From the traditional perspective, it is often argued that students cannot apply their mathematical knowledge in physics tasks. In this statement, the direction of transfer is fixed: first students have to learn mathematical principles and knowledge and thereafter apply their knowledge in a different context. A fixed direction of transfer contradicts the basic assumption of the actor-oriented transfer perspective, namely the personal construction of similarities. Also, studies by Zandieh (2000) and Marrongelle (2004) show that transfer is not unidirectional but that knowledge from mathematics and physics mutually interact.

For our research, we choose the actor-oriented transfer perspective as described by Lobato (2003). Lobato states that researchers in the actor-oriented perspective:

- Look for the personal construction of relationships between activities from an actor's perspective;
- Investigate the effect of prior activities on current activities and how actors construe situations as similar;
- Analyze what relationships of similarity are created by actors and how these are supported by the environment.

Studies from the Actor-Oriented Perspective

Some studies report that from a traditional perspective students do not transfer knowledge taught in mathematics lessons to physics tasks (Cui, 2006; Karakok, 2009). However, when data is analyzed from an actor-oriented perspective, students do construct similarities between situations in physics lessons and what is learned in mathematics. To get insight into this statement, we will have a closer look at two studies.

Cui (2006) investigated students' transfer of learning from calculus to physics at college level. The participants in her study were 416 students enrolled in a second semester physics course. For solving tasks in the physics exams, the students could use mathematical procedures taught in two calculus courses. Cui analyzed data both from a traditional and an actor-oriented perspective. The traditional perspective was used by correlating students' calculus course grades and their physics exam problem grade, and additionally by analyzing if variables indicating performance on the physics exams and the calculus exams did cluster. From the traditional perspective, weak evidence was found that students transferred their calculus knowledge to physics exams. From the actor-oriented perspective, the performance of students on physics problems was analyzed by assigning scores for calculus and physics performance for each section of the exam. Cui considers this as an indicator for actor-oriented transfer because the analysis focuses on constructions of similarity between calculus and physics aspects of a given problem. With this analysis, statistically significant correlations were found between students' calculus and physics performance (Cui, 2006, p.85). Cui concludes that there is more evidence for transfer when analyzed from the perspective of the learner (actor-oriented transfer), focusing on students' dynamic constructions of similarities between two aspects of their knowledge.

Karakok (2009) investigated students' transfer of the concept of eigenvalues and eigenvectors learned in physics courses. Seven students participated in three in-depth interviews before, during, and after they had enrolled in these courses. From an actor-oriented perspective, she analyzed what kind of experiences and views students transferred from their courses to the interviews. Her analysis produced evidence that six out of seven participants reconstructed experiences from certain activities, exercises, and examples from the courses. Karakok also analyzed data using a traditional perspective. This means that she analyzed if students did transfer certain knowledge a priori defined by the researcher. This analysis revealed that only one participant seemed to transfer knowledge from the courses to the interview tasks.

These studies (Cui, 2006; Karakok, 2009) concluded that transfer was rare from a traditional transfer perspective; however, from an actor-oriented perspective, it was observed that students constructed relationships between previously learned knowledge and new situations.

Breadth and Connectedness of Rate of Change Procedures in Differential Calculus

In this study, we operationalize actor-oriented transfer as how students construct similarities between prior activities in mathematics and physics lessons and new situations. The new situations are offered through tasks which require the use of rate of change procedures.

Procedures to calculate rate of change are part of the concept of derivative. This concept is multi-faceted, so it is complex to determine to what extent a student understands the concept (Zandieh, 2000). The degree of understanding depends on the number and the strength of connections between facts, representations, procedures, and ideas (Hiebert & Carpenter, 1992). For the concept of derivative, connections can be made between *representations*, such as graphical, numerical, and symbolical representations (Roorda, Vos & Goedhart, 2009; Zandieh, 2000); *procedures* such as those for calculating average or instantaneous rates of change (Kendal & Stacey, 2003); *layers* such as difference and differential quotient (Hähkiöniemi, 2006; Zandieh, 2000); *words* that give meaning to the concept such as slope, increase, and velocity (Zandieh & Knapp, 2006); and *applications* such as marginal costs, velocity, or acceleration (Roorda et al., 2009).

In this article, we will focus on the relationships between procedures learned in mathematics and physics. We use the term *procedures* for methods to solve certain types of problems. We are interested in students' procedural fluency as described by Kilpatrick, Swafford & Findell (2001). Procedural fluency refers to the knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently (Kilpatrick et al., 2001, p.121). In addition, we also study the way students relate chosen procedures. For the construction of relationships between procedures taught in different subjects, one part of procedural fluency is of importance, namely the knowledge of using a procedure appropriately. When solving a problem, students have to choose an appropriate procedure. Sometimes, a physics procedure is most appropriate although a mathematics procedure may also be useable. To make the right choice, an overview over different procedures and connections between procedures is necessary.

The entirety of procedures known by students will be defined in this study as the student's *repertoire*. Students show procedural fluency if they have a broad and connected repertoire of different procedures. These two aspects, breadth and connectedness, are central in this study. *Breadth* of repertoire is defined as the number of procedures a student mentions or uses to solve an interview tasks. *Connectedness* of repertoire is defined as the relationships between procedures that a student construes.

In Dutch physics classes, a number of rate of change procedures are taught in grade 10. Kinematics starts with a graphical approach to demonstrate relationships between distance, velocity and acceleration. At this stage, students are taught a graphical procedure, which is referred to as the tangent method. It is a procedure to calculate instantaneous velocity for nonlinear situations by using a graph and drawing a tangent, from which the *steepness* is calculated (Fig. 1). The word steepness is a reoccurring term in the physics textbooks. Also, students are taught to use formulas for calculating average and instantaneous velocity, such as $v = s/t$ and $v = at$. These formulas are used in physics while relationships with derivatives are not mentioned. So to calculate instantaneous velocity, the emphasis is on the tangent method and on physics formulas.

In mathematics classes, the derivative is introduced in grade 11. It is founded on the transition from graphs to functions and on the transition from a difference quotient to a differential quotient. The rate of change is directly linked to the tangent of the graph. Some

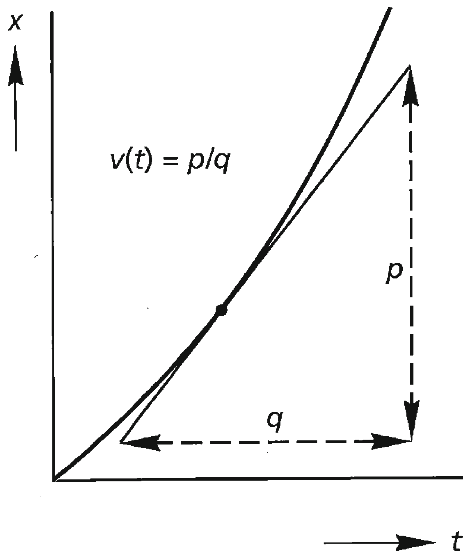

Figure 1. The tangent method as taught in physics classes: to calculate instantaneous velocity, a tangent is drawn and the slope p/q is calculated (Middelink et al., 1998)

exercises in the mathematics textbook use distance–time graphs to illustrate the meaning of instantaneous rate of change. The slope of the line through two points on successively smaller intervals will approximate the slope of the tangent. The physics distance–time situation serves as an example to introduce the mathematical concept of derivative. Note the difference between physics, where the *steepness* of a tangent is calculated by using two points on the tangent which are far apart (see Fig. 1), and mathematics, where the *slope* of a tangent is approximated by using two points on the graph on successively smaller intervals. Later in the school year, the emphasis is on symbolic differentiation rules (power rule, chain rule, product rule, quotient rule) and on the applications of these rules to tasks on calculating extremes and formulas of tangent lines. Studies on calculus (e.g. Kendal & Stacey, 2003; Orton, 1983) indicate that many beginning calculus students master 'symbolic differentiation' without relating it to other procedures. Kendal & Stacey (2003) also indicate that students relate symbolic and graphic procedures, but graphic–numeric relationships and symbolic–numeric relationships are rare. Roorda, Vos & Goedhart (2007) observed that students in think-aloud sessions have difficulty in relating rate of change procedures learned in mathematics classes to rate of change procedures learned in physics classes.

Many studies (e.g. Hähkiöniemi, 2006; Kendal & Stacey, 2003) document students' repertoire after a single calculus course. These studies, however, do not give insight into the long-term process of constructing relationships between procedures learned in different school subjects.

Summary of the Theoretical Framework

The consequences of our choices are now highlighted and visualized in Fig. 2. We will use an actor-oriented transfer perspective by looking at the selection of procedures by students, when working on a rate of change task. Do they use procedures learned in physics or procedures learned in mathematics classes? And which reasons do they give for their choices? Furthermore, we investigate students' breadth and connectedness of repertoire, by looking at the different procedures mentioned or used by the students and the relationships between procedures as indicated by the students. In our research design, we choose a longitudinal approach to follow students' long-term development of their repertoire.

The above is guided by the following research question: *How is students' long term development of breadth and connectedness of their repertoire of rate of change procedures from an actor-oriented perspective*?

Methods

Research Setting and Participants

To gain insight into students' development of their repertoire, we opted for a detailed description and analysis of work by individual students (Roorda, 2012). Their development took place within an educational setting in which procedures are taught in different school subjects without coordination between curricula. According to Yin (2003), case studies can contribute to a better understanding of complex social phenomena. And because of the diversity within and between schools, multiple cases give opportunities to analyze similarities and differences between students (Creswell, 2002). Therefore, we used a longitudinal multiple case study.

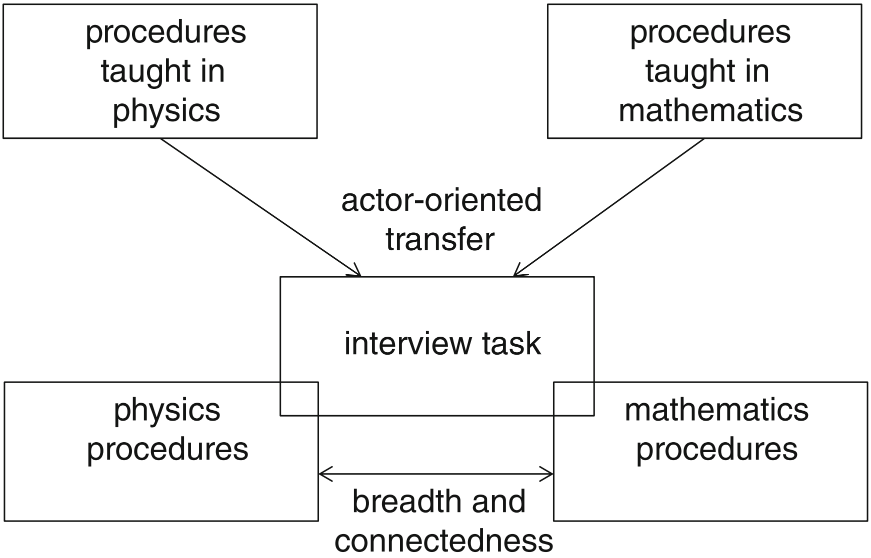

Figure 2. The arrows indicate how we analyze actor-oriented transfer and breadth and connectedness of repertoire

Because of our interest in relationships between mathematics and physics, we selected students following a science track, which meant that they take science and mathematics at an advanced level. We selected students with varying abilities. Based on information of the mathematics teacher in grade 10, ten students (six boys and four girls) were selected from two regular Dutch schools. The teachers indicated one student as weak, four as average, and five as good. In our study, weak students are underrepresented because we looked for students who most likely would move up from grade 10 to grades 11 and 12 without delay. The students are indicated with pseudonyms: Andy, Bob, Casper, Dorien, and Elly from school I, and Karin, Maaike, Nico, Otto, and Piet from school II.

Interviews and Instruments

In many studies on the learning of derivatives, researchers have investigated students' conceptual knowledge by asking them explicitly for the meaning of derivative (e.g. Hähkiöniemi, 2006; Zandieh, 2000). In contrast, in our study we did *not* use words that direct towards derivative. To secure that students choose their own procedures, the procedures to solve the task were not obvious to students, and in the task and during the interviews we avoided using directive words such as derivative, differentiation, rate of change, tangent, or slope. By avoiding these words, we did not lead students to the concept, but they had to make their own choices on procedures.

While the students moved from grade 10 to grade 12, four task-based interviews (Goldin, 2000) were conducted with half year intervals. The tasks for the interviews were designed to provide in-depth information about students' repertoire.

The tasks. In all tasks, situations were described in which the variables had a meaning in real life, such as distance, costs, or volume. In this article, we focus on two tasks, named *Barrel* and *Ball*. These tasks were selected because they offer students ample opportunities to use and relate a variety of rate of change procedures. The task Barrel was used in all four interviews, while the task Ball was only used in interviews 2 and 4 (to avoid recognition of tasks between the four interviews, we did not repeat all tasks in all interviews). In both tasks, the assignment was to calculate velocity at a certain point. However, the Barrel task resembles tasks used in Dutch mathematics textbooks and the Ball task resembles tasks in physics textbooks. Both tasks included a situation description and various representations, such as a graph, a formula, and (in the Ball task) a table. The complete tasks are given in Appendix A and summarized here:

Barrel: A barrel contains a liquid, which runs out through a hole at the bottom. The volume of the liquid in the barrel decreases over time and is expressed as $V = 10\ (2 - {}^{1}/_{60}t)^2$. Also the $V - t$ graph is presented. Students are assigned to calculate the outflow velocity at $t = 40$.

Ball: A ball falls from a height of 90 cm. A table, a graph, and the formula for the height, $h = 0.9 - 4.9t^2$ are presented. Students are assigned to calculate the velocity of the ball at a certain point on the graph, indicated by an arrow.

The interviews: The students were interviewed by the first author in a small conversation room at their respective schools. All interviews were videotaped and transcribed verbatim afterwards. During the interviews, based on think-aloud and stimulated recall techniques, we used a protocol to get as much information as possible on student's knowledge of procedures and the breadth and connectedness of their repertoire. First, a student was asked to solve the task. During the solving of the problem, the interviewer did not interfere. Interventions by the interviewer occurred at following instances:

- When a student thought for over a minute, he was asked for an explication;
- When a student solved a problem, he was asked for clarification of the procedure used;
- A student was asked up to twice if he knew other procedures to check the correctness of the given answer;
- When a student used two or more procedures, he or she was asked to compare these.

The above regulations aimed at encouraging students to mention and use other procedures than the first chosen, and to explain relationships between these procedures.

The first interview was held at a moment at which the concept of derivative had not yet been introduced in mathematics classes. However, at school I, the physics teacher had already introduced kinematics. The second interview was held in the third month in grade 11, a few weeks after the mathematics teacher introduced differential calculus (difference quotient, differential quotient, calculations of derivatives of polynomials). At school II, the chapter on kinematics was introduced in the first weeks of grade 11. Between the second and the last interview, derivatives were a reoccurring topic in mathematics lessons. Table 1 presents the period of the interviews and the tasks used.

Data Analysis

We analyzed the written transcripts of the interviews and the written answers to the problems. The analysis focused on identifying the procedures used and the relationships that the students constructed between the procedures. To determine students' breadth of repertoire, we identified the *adequate* procedures, that is, procedures which lead, if correctly applied, to a correct solution. Next, we analyzed the *accuracy* of the procedures by using three categories: (1) a student only mentions an adequate procedure, (2) a student uses an adequate procedure but makes mistakes in the calculations, and (3) a student uses an adequate procedure correctly.

As an indicator for the connectedness of the repertoire, we analyzed statements, in which students constructed relationships between procedures. For instance, when students explain that an answer could be

TABLE 1

The phasing of the interviews and the tasks used

	Period between grades 10 and 12	*Tasks used for the interview*
Interview 1	Towards the end of grade 10	Task: Barrel
Interview 2	In the third month of grade 11	Task 1: Barrel Task 2: Ball
Interview 3	Towards the end of grade 11	Task: Barrel
Interview 4	In the third month of grade 12	Task 1: Ball Task 2: Barrel

calculated with procedure 1 but just as well with procedure 2, or when students explain that procedures 1 and 2 should deliver the same answer, we indicate this as a relationship between two procedures. The relationship between the two procedures is constructed by their exchangeable applicability.

Results

This section presents the procedures used and the relationships mentioned by students in the interviews based on the tasks Barrel and Ball. First, we illustrate the development of the repertoire by highlighting the work of three students. Second, we describe patterns in the development of the repertoire of the ten students.

The Development of Three Students

We describe the results of three students, Elly, Dorien, and Bob, who vary in the way they worked on the same tasks. We selected them because they provide examples of patterns of actor-oriented transfer and the breadth and connectedness of repertoire. For each of these three students, we first present a table with a description of the procedures used and with details of the problem solving process, and second, we interpret the results with respect to breadth and connectedness of repertoire from an actor-oriented perspective.

To calculate outflow velocity in the Barrel task, the students used or mentioned in total four adequate procedures, indicated as (small)-interval, graphical calculator (GC)-option, tangent method, and symbolic differentiation.

In the Ball task, students used the same four procedures, but also physics formulas. To illustrate these procedures:

1. (Small)-interval method: the calculation of the difference quotient on a (small) interval for example [40; 41] or [40; 40,0001];
2. A graphical calculator-option: the option dy/dx to calculate instantaneous rate of change;
3. The tangent method : drawing an estimated tangent along the graph at $t = 40$;
4. Symbolic differentiation: determining the derivative and substituting $t = 40$.
5. Physics formulas for the Ball task: $v = g \cdot t$, or equating kinetic and potential energy.

Students also used inadequate procedures. These procedures will be described in the results of individual students.

The Development of Elly. According to her mathematics teacher, Elly is a hardworking, but weak student who has to practice many exercises to master a topic; small changes in a task make it difficult for her to solve the task. Table 2 lists procedures used and explanations given by Elly in the four consecutive interviews.

With respect to breadth and connectedness of repertoire, Table 2 shows that Elly mentions few adequate procedures and that she does not relate procedures in the interviews. In the first three interviews with the Barrel task, she mentions inadequate mathematics procedures. In interview 4, Elly asks whether she could use the tangent method (as learned in physics) in this task. In the last interview, she solves the Ball task correctly using a physics formula. Throughout all interviews, she does not use mathematics procedures such as symbolic differentiation, discrete procedures, or graphic calculator options.

From an actor-oriented perspective, we notice that the Ball task, which was designed to resemble tasks in physics textbooks, is at first connected by Elly to a task she remembered from a mathematics test. Yet, she eventually mentions physics formulas, although she does not apply these formulas accurately. However, she cannot reconstruct which formula is the correct one between $s = vt$ and $v = st$. She also mentions $s = ½at^2$ but cannot use either of these formulas to reach a solution. In interview 4, she uses again a physics formula with acceleration of gravity. Although her justification is incorrect (the task is on instantaneous and not on average acceleration), she uses an adequate formula ($v = g \cdot t$) and she reaches a correct answer. Her repertoire in

TABLE 2

Elly's procedures and explanations

Interview 1 (task Barrel only) *Procedures*: No adequate procedures *Explanations*: Elly asks herself if the letter V represents "volume," because in physics the letter v means velocity. She tries to substitute values of t and V incorrectly, for example she inserts $t = 40$ and $V = 40$ simultaneously.
Interview 2 (tasks Barrel and Ball) *Procedures*: No adequate procedures *Explanations*: Elly starts the Ball task with the remark: "I remember something like this from my last mathematics test" [this test was about derivatives, and students had to apply derivative rules in a task on velocity of a moving object]. Then, she writes: "$v = s/t$ or t/s." She says: "I am not good in remembering formulas." She decides to use the physics formula $v = s/t$, because "s over t sounds more familiar to me." She substitutes values for s and t. She remembers also a formula $s = ½at^2$. In the Barrel task, Elly divides the volume in the tank at $t = 40$ by the time passed, 40 min, which is an inadequate procedure.
Interview 3 (task Barrel only) *Procedures*: No adequate procedures *Explanations*: Elly calculates $V(40)$ and says that she now knows that there is exactly $17\ ^7/_9$l in the tank. She says that she could calculate the outflow velocity if the graph was a straight line, but now that it is "curved," she cannot calculate it.
Interview 4 (tasks Barrel and Ball) *Procedures*: Barrel—tangent method, Ball—physics formula *Explanations*: In the Barrel task, Elly calculates the volume at $t = 40$ and notes again that she can find an answer if there was a straight line. Then, she says: "I have to calculate it at a certain time […] but I'm thinking about a tangent, then I have the average velocity." Then, she gives up, sighing: "it certainly is very simple." In the Ball task, Elly asks whether she can look it up in a book with physics formulas. Because the task is about a falling ball, she connects this to a need for a formula that contains acceleration of gravity. She decides to choose the formula $v = g \cdot t$. She says: "I am working on average acceleration, so I think I can use this formula" and then she fills in g and t, which leads to the correct answer. After being asked for ways to check her answer, she does not mention alternatives.

all interviews is narrow and disconnected, and the development of Elly's repertoire is very limited.

The Development of Dorien. According to Dorien's grade 10 mathematics teacher, she has a good mathematical understanding, but when working on a difficult task, she easily gives up. Her mathematics teacher in grade 12 categorizes her as an average student with a reasonable

TABLE 3

Dorien's procedures and explanations

Interview 1 (task Barrel only)
Procedures: Barrel—interval method
Explanations: "In 40 minutes 40 – 17.5 litres flow out of the Barrel, but that will not give velocity at this point." She remarks that if she fills in $t = 40$ into the formula, she will find the coordinates of the point on the graph. She concludes by saying: "I do not know how to calculate velocity at that point, but I can calculate average velocity."

Interview 2 (tasks Barrel and Ball)
Procedures: Ball—tangent method, derivative; Barrel—tangent method, derivative
Explanations: After reading the Ball task, Dorien says: "I think I have to use a tangent [indicates a tangent at the graph]; I think this is about derivatives. But I am very bad in applying mathematics to physics; I have to switch over completely." So, she mentions in her first remark two procedures and two subjects. Then, she writes down: "steepness = velocity" and remarks that she is surest about the tangent method; she draws a tangent and calculates the slope. When asked for other procedures, she calculates the derivative $h'(t) = -9.8t$ and says: "Hey, this is the acceleration of gravity." After filling in the time $t = 0.24$ and delivering the answer, she compares answers and notices that the answers of both procedures match well. In the Barrel task, Dorien says: "Actually this is the same as the Ball task. So I think I will try a derivative." She differentiates the formula without using the chain rule and finds a wrong answer. She decides to check with the tangent method. She is convinced that symbolic differentiation and the tangent method should give the same answer. She explains the different answers to be caused by a miscalculation in the derivative.

Interview 3 (task Barrel only)
Procedures: Barrel—derivative, tangent method
Explanations: Dorien starts by saying: "I will use the derivative [....]. I will get a formula for the velocity and I will fill in $t = 40$." She differentiates the formula correctly and calculates $V'(40)$. She says that she can check this answer by using a tangent. She does not complete the calculation with a tangent because she is convinced that her answer, found through the derivative, is correct.

Interview 4 (tasks Barrel and Ball)
Procedures: Barrel—tangent method, derivative. Ball—tangent, derivative, physics formula
Explanations: Dorien calculates the slope of the tangent. She writes $V = \frac{\Delta y}{\Delta x} = \frac{45}{80} = 0.5625$ and says that the outflow velocity is 563 l per minute [this is incorrect, because of a mistake in reading of Δy]. She checks her answer through the derivative $V'(40)$ and obtains 444 l/min. She compares both answers by saying: "Either this one is inaccurate [points at the tangent] or I made a mistake here [points at the derivative]."
In the Ball task, Dorien first wonders whether this task is about horizontal or vertical velocity. She says that she recently learned theory about a horizontal throw. After a while, she says that, because it is a distance–time graph, it is only about vertical velocity. She recalls a physics formula for vertical velocity of a falling object ($v_y = g \cdot t$), but she does not fill this into the formula. Then she says: "Because this is a distance–time graph, I think I can use the derivative, or I can draw a tangent." She draws a tangent and writes down: $v = \frac{s}{t} = \frac{\Delta y}{\Delta x} = \frac{0.4}{0.16} = 2.5\ \mathrm{m/s}$ and says: "I calculate steepness with a tangent, and I think that everything I do with a tangent can also be calculated with the derivative." Finally, she notes that in the formula of the height, the value 4.9 is half of acceleration of gravity.

insight and a reasonable attitude towards work. Table 3 lists procedures used and explanations given by Dorien in the four consecutive interviews.

With respect to breadth and connectedness of repertoire, Table 3 shows that Dorien does not use adequate procedures in interview 1, but at this stage, she demonstrates an awareness of the difference between linear procedures and nonlinear procedures, of which she indicates that she is only able to calculate the first one. In interviews 2, 3, and 4, Dorien's repertoire centers on two procedures, namely symbolic differentiation (learned in mathematics classes) and the tangent method (learned in physics classes). Dorien is more and more convinced that symbolic differentiation and the tangent method give the same answer. She explains the different answers (in interviews 2 and 4) to be caused by either the inaccuracy of the tangent method or by a miscalculation in the derivative. In interview 4, she explicitly states that answers calculated with a tangent can also be calculated with the derivative. One additional procedure, the physics formula that she uses in interview 4, is not connected to the other two procedures by Dorien.

From an actor-oriented perspective, we notice that from interview 2 onwards (i.e. within 1 year), Dorien selects procedures from mathematics as well as from physic in both tasks. This seems to be based on a deeper insight that both tasks are about velocity at a certain time, which is, according to Dorien, the same as steepness at a point of the graph. The first time she combines both procedures (interview 2), she explicitly states that she is bad in applying mathematics to physics. Additionally, in the Ball task in interview 4, she recalls a physics formula for velocity of a falling object ($v_y = g \cdot t$).

A major step in Dorien's development is made from interview 1 to interview 2. She displays an early uptake of two procedures, both of which she can use appropriately. From interview 2 onwards, her repertoire almost remains the same, but she is surer about connections between symbolic differentiation, learned in mathematics and the tangent method, learned in physics. We qualify her repertoire in the final interview as firmly connected, being based on two procedures which she strongly relates in her explanations.

The Development of Bob. Bob is a boy with a high appreciation of science and mathematics. His mathematics teacher describes him as a clever pupil but sometimes 'sloppy' in his calculations. Table 4 lists procedures used and explanations given by Bob.

With respect to breadth and connectedness of repertoire, Table 4 shows that Bob in interviews 1 and 2, Bob mentions a number of procedures but does not say how they are related. In interview 3, he directly relates the tangent method learned in physics lessons to procedures learned in mathematics classes (symbolic differentiation and small-interval method).

TABLE 4

Bob's procedures and explanations

Interview 1 (task Barrel only)

Procedures: Barrel—tangent method

Explanations: Bob substitutes t = 40 into the given formula. He draws a rectangle under the graph (see Fig. 3). He points at the vertical axis and says: "Velocity is of course the area of this rectangle" and he points at the rectangle under the graph. Bob calculates the area of the rectangle [his choice seems to be based on a procedure learned in physics class where students calculated velocity by estimating the area under the acceleration graph]. After a few minutes, he says: "I think I intend to draw a tangent, I remember it vaguely; I think I will find the average velocity at that point" (see Fig. 3).

After evaluating both procedures, he decides the tangent method is adequate. Again, he connects velocity, this time "in a point," with the tangent method saying: "Yes, with a tangent you can calculate velocity at one point, I am almost sure about that."

Interview 2 (tasks Barrel and Ball)

Procedures: Ball—tangent method, derivative, physics formulas. Barrel—tangent method

Explanations: Bob mentions in the Ball task the tangent method. He proceeds by saying: "Eh, I think, in mathematics you can use a derivative, but how did it work? […] In a distance-time graph you can find velocity with the derivative." Bob is not sure of the derivative and switches back to the tangent method saying: "A tangent will give me the steepness in that point and then the area under the graph is the velocity." At last he mentions also the procedure of equating kinetic and potential energy. [Just as in interview 1, Bob connects velocity with the area under the graph but also with taking the derivative. The derivative is labelled by Bob as a mathematical procedure. The other three used procedures (tangent method, area method, and energy balance) are only learned in physics classes.]

In the Barrel task, he uses the tangent method. When the interviewer asks for other procedures, Bob says he also could calculate the angle of the tangent with the y-axis to find the slope.

Interview 3 (task Barrel only)

Procedures: Barrel—derivative, tangent method, small interval method

Explanations: Bob starts by mentioning the derivative. He says: "This is about the steepness of the line, therefore I have to use derivatives, but, we are not working on derivatives at the moment" [in his explanations the words steepness seems to trigger the use of derivatives].

Bob calculates the derivative inaccurately, without using the chain rule. He plots the graph of his derivative. This derivative graph intersects the graph of the volume exactly in a zero. Bob says: "As the velocity is zero, the volume is also zero. When there is no water in the tank, the water cannot flow with a certain velocity. […] I think it is correct" [he also relates this derivative to "velocity"].

Bob proceeds with other procedures by saying: "I can use a tangent, but now we learned derivatives, tangents are less exact.[…] I can also calculate the volume at t = 40 and t = 40.001 and divide the difference by 0.001. This has to do with limits and is almost the same as taking the derivative."

TABLE 4

(continued)

Interview 4 (tasks Barrel and Ball)
Procedures: Barrel—tangent method, derivative, graphic calculator option. Ball—derivative, tangent method, small interval method, physics formula.
Explanations: In the Barrel and the Ball task, Bob says: "This task is about velocity, therefore I have to calculate the slope in that point, say the tangent, actually taking the derivative. I drew a tangent before, but I think I better use the derivative. Then I have the velocity function and I can substitute t into it", and "the derivative is the velocity graph […] it is about the steepness in this point. I can calculate the tangent with a derivative. […]. This part of the formula [points at $4.9t^2$] is $\frac{1}{2}at^2$ and velocity is g times t; it is logical that the derivative is 9.81; acceleration times time is velocity." In these remarks, Bob uses words such as velocity, slope, tangent, and steepness. In both tasks, he calculates the answer through symbolic differentiation and in the Ball task also through physics formulas.

The relationship between tangent method and derivative is not explicitly stated, but he mentions these procedures in one sentence with an additional remark that tangents are less exact compared to derivatives. In interview 4, Bob relates in both tasks words such as velocity, slope, and steepness and he mentions different procedures to calculate instantaneous rate of change. Relationships between these words seem to promote his understanding of relationships between procedures. At this final stage, the derivative is central in his explanations.

From an actor-oriented perspective, we notice that in interviews 1 and 2 Bob prefers procedures learnt in physics classes, such as the tangent method and physics formulas. The word velocity seems to be pivotal in

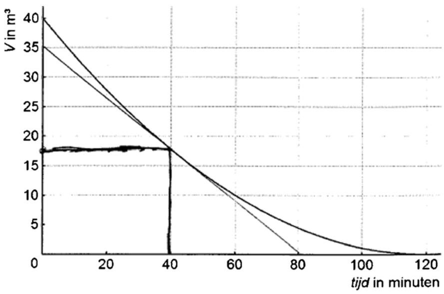

Figure 3. Bob's drawing in interview 1

his explanations. This word also leads to an inadequate procedure because velocity reminds him of the ‘area method,’ a procedure taught in physics classes to calculate distance traveled in a velocity–time graph (Fig. 3). In the Ball task, he also uses the derivative (taught in mathematics classes), but he is unsure about this procedure. In interviews 3 and 4, Bob does no longer label the procedures used in the Barrel task as physics or mathematics procedures, depending on the class where he learned them.

In Bob’s development, we notice a continuously increasing breadth, which at first is still unconnected. In interviews 1 and 2, he mainly uses procedures learned in physics and does not yet use the derivative on all possible occasions (although it was taught). Compared to Dorien, he is later in making connections between procedures, but once he starts making them, the connectedness of his repertoire increases with its breadth. In interviews 3 and 4, he connects different terms (steepness, velocity, slope) and he mentions and uses procedures learned in physics and mathematics to solve tasks.

The Development of All Ten Students

The detailed description of Bob, Dorien, and Elly are now placed in the broader context of the results of all ten students. Table 5 (Barrel task) and Table 6 (Ball task) present all adequate procedures mentioned or used by the students. The tables show for each student which procedures were used in the consecutive interviews, and also if procedures were used accurately, inaccurately, or only mentioned. The ten students are indicated as A, B (Bob), C, D (Dorien), E (Elly), K, M, N, O, and P.

Breadth and connectedness of repertoire. In interviews 1 and 2, most students have a narrow and disconnected repertoire of rate of change procedures. For example, Table 5 shows that most students (seven out of ten) cannot solve the Barrel task in interview 1, in interview 2 the tangent method (learned in physics classes) is the most frequently used procedure (eight students); Table 6 shows that in the Ball task in interview 2 students often use the tangent method (six students), symbolic differentiation (five students), and physics formulas (five students) but relationships between procedures are rare. So, students are unsure about which procedures can be used to calculate an instantaneous rate of change and how these procedures are interrelated. Students who do relate procedures (C, D, and N) explain that velocity or steepness can be calculated with symbolic differentiation and also with the tangent method. The students who display an early uptake of symbolic differentiation and connect this procedure to the tangent method do thereafter hardly increase the breadth of their repertoire.

TABLE 5

Procedures used in the task Barrel

Procedure	Taught in	Interview 1 A	B	C	D	E	K	M	N	O	P
(Small) interval	Math	●		●	○						
GC-option	Math										
Tangent method	Physics	○	●								
Symbolic diff.	Math										

Procedure	Taught in	Interview 2 A	B	C	D	E	K	M	N	O	P
(Small) interval	Math	◍		◍				◍			
GC-option	Math	◍								◍	
Tangent method	Physics	○	●	◍	●		●		●	○	○
Symbolic diff.	Math				◍				◍	◍	●

Procedure	Taught in	Interview 3 A	B	C	D	E	K	M	N	O	P
(Small) interval	Math	○	●								
GC-option	Math	●									
Tangent method	Physics	●	○	●	○		◍		◍	●	
Symbolic diff.	Math		◍	◍	●		●		◍	●	◍

Procedure	Taught in	Interview 4 A	B	C	D	E	K	M	N	O	P
(Small) interval	Math	○		○				●			
GC-option	Math	●	○						●	●	○
Tangent method	Physics	●	○	○	◍	○	●		○	●	●
Symbolic diff.	Math		●	●	●		◍	◍	◍	◍	●

Cells grouped with a borderline indicate that students construct relationships between the procedures

Black circle procedures accurate, *gray circle* procedures not accurate, *white circle* procedures only mentioned

TABLE 6

Procedures used in the task Ball

		Interview 2										Interview 4									
Procedure	Taught in	A	B	C	D	E	K	M	N	O	P	A	B	C	D	E	K	M	N	O	P
(Small) interval	Math						○	○					○					○			
GC-option	Math	●								◌		●									
Tangent method	Physics	◌	●	●	●		◌		●			○	○	●	●						○
Symbolic diff.	Math		◌	●	●		◌		●	◌			●	●	●		●	●	●	●	●
Physics form.	Physics		◌			○		◌	○		○	●	●	○	●	●	●	○	○	○	○

Cells grouped with a borderline indicate that students construct relationships between the procedures
Black circle procedures accurate, *gray circle* procedures not accurate, *white circle* procedures only mentioned

In the later interviews, most students become more pronounced about the relationships between the tangent method and symbolic differentiation (see Tables 5 and 6). Finally, in interview 4, after extension and repetition in mathematics and physics lessons, seven students relate these two procedures correctly in the Barrel task. These students make statements indicating that they understand that the slope of a tangent can be calculated both by the tangent method and the derivative and that both procedures are appropriate to calculate instantaneous velocity. However, making this connection is not self-evident as was observed with some students (e.g. Elly).

Few students express a relationship between symbolic differentiation and physics formulas. In interview 2, four students use physics formulas, but none of them uses them accurately, although physics formulas to calculate velocity of a falling object have been taught at this stage. Also, in mathematics classes at both schools, the students were taught that velocity of an object can be calculated by symbolic differentiation. One year later, in interview 4, two out of ten students (B and P) construct relationships between symbolic differentiation, physical formulas, and other procedures. Bridges between those procedures are the word velocity or the recognition of the acceleration of gravity, 9.8. Piet, for example, states that the derivative of distance is velocity and the derivative of velocity is acceleration. Other students, such as Elly and Andy, use the formula $v = g \cdot t$ correctly, without mentioning a relationship with mathematical procedures.

The actor oriented perspective. We notice the following: When students solve rate of change tasks, they are affected by procedures learned in physics and mathematics classes. The ten students in this study followed nearly the same curriculum; nevertheless, for each student we see different patterns when analyzed from an actor-oriented transfer perspective. Elly uses physics formulas in tasks with a physics appearance (a falling ball) and she does hardly use procedures taught in mathematics. Dorien combines from interview 2 onwards two procedures: one taught in physics and the other taught in mathematics because she knows that velocity and steepness are related and that she can tackle many problems with both procedures. The third student, Bob, prefers procedures learned in physics (tangent method and equating kinetic and potential energy) in the first two interviews. From interview 3 onwards, he uses words like velocity, steepness, and slope more often and he relates tasks more easily to a variety of procedures learned in physics and mathematics.

The observed development of Elly, Dorien, and Bob is person-dependent, but nevertheless we can observe some patterns independent of the individual students. Taking into account that interview 2 took place after the introduction of the tangent method in physics and symbolic differentiation in mathematics, we see in interview 2 that most students prefer the tangent method in the Barrel task. Although this task resembles tasks in mathematics textbooks, students prefer a procedure taught in physics. In later interviews, we see that, in addition to the tangent method, students also use symbolic differentiation. Statements in interviews 1, 2, and 3 by some students show that they see the tangent method as a physics procedure and differentiation as a mathematics procedure. In interview 4, this difference is less visible, and at this stage some students mention the impreciseness of an answer calculated by the tangent method and they prefer the precision of symbolic differentiation. Some students no longer use non-symbolic procedures as an alternative because they consider these as inaccurate.

Discussion

From the perspective that has been referred to as the traditional transfer perspective, one can argue that transfer from mathematics or physics procedures to tasks is disappointing because of the following:

(1) Students often use procedures that differ from procedures that are most appropriate from an expert's perspective. For example, in interview 2, after the introduction of derivatives in mathematics classes, six (out of ten) students do not use symbolic differentiation in a task that resembles tasks used in Dutch mathematics textbooks. Even in interview 4 (November, grade 12) two students do not use symbolic differentiation in the tasks Barrel and Ball.

(2) After introduction of differentiation rules in mathematics and formulas for falling objects in physics, students do not mention relationships between physics formulas and symbolic differentiation. Even in interview 4, these relationships are seldom mentioned.

From an actor-oriented perspective, we see that prior activities in physics or mathematics classes affect students' work in the interview tasks. The direction of relationships is not that students first learn mathematics and subsequently apply this knowledge to physics tasks. Instead, we observe an initial uptake of the tangent method (learned in physics lessons) and a lingering uptake of symbolic differentiation (learned in mathematics lessons), and in the later interviews, students relate terms and procedures of mathematics more and more to terms and procedures of physics reciprocally. This underlines results of Zandieh (2000) and Marrongelle (2004) who observed that some students used physics knowledge to give meaning to mathematics tasks.

In our study, most students construct more and stronger relationships between procedures on the long run. Their constructions of similarities progresses along different routes. The actor-oriented perspective is a good framework to analyze these routes because it is focused on the individual, who is constructing relationships while solving a task. In constructing similarities between situations, words play a crucial role. Some students recognize that in a new situation the same word is central as in a situation that was met before, whether it be in physics or mathematics. For some students, the word velocity is central, while other students use words such as steepness or slope. These students refer to these words when they explain why they use specific procedures. So, such words act as bridge to create relationships between situations and procedures.

This leads to a recommendation for research and education. It seems important that students verbalize relationships between different words that are related to rate of change. Relationships between words such as velocity, derivative, steepness, and slope will give students opportunities to construct relationships between situations. Further research is needed in order to investigate if an educational program in which students are supported to verbalize relationships between words concerning rate of change will help to improve the construction of relationships between situations.

Our study shows that most students do not use procedures after their immediate introduction, but only after repetition and extension of their knowledge on derivatives and kinematics. This leads to the recommendation to teach mathematics as a concentric curriculum—that is, a curriculum in which students repeatedly work with the same concepts and procedures, but in different contexts, at a level of increasing difficulty, with new perspectives and with possibilities for weaker students to catch up. Connectedness of students' repertoire will probably benefit if teachers of mathematics and physics not only mention relationships between procedures learned in mathematics and in physics but also design tasks in which relationships between school subjects are made explicit.

According to Lobato (2003), researchers in the actor-oriented transfer perspective look for the influence of prior activities on current activities and how learners construe situations as similar. In our research, all prior activities were activities in physics and mathematics classes and the current activity was working on tasks about rate of change. This means that we opted for a limited interpretation of the term 'activities.' From the actor-oriented transfer perspective, 'activities' and 'situations' are meant as the broad context, in which students learn. But, it is difficult, or even impossible, for a researcher to describe these situations and activities, and the similarities that students construct, in detail. For example, one student in our research noted that two formulas were related because they were on the same page of the book. Such minor details can easily be overlooked, while they affect the construction of relationships by students.

Our research differs from previous research on the concept of derivative because of the use of a longitudinal design with four interviews in the course of grade 10 to grade 12. The longitudinal design enables the monitoring of students' personal development and the analysis of their repertoire and explanations over time. The longitudinal design causes also methodological complications. Some tasks were used in all interviews and other tasks were used to vary and make the interview unpredictable to students. On this point, we were not fully successful, as a few students recognized tasks from an earlier interview. This may have affected the chosen strategies, although students did not repeat themselves, but instead, demonstrated that their repertoire had expanded. If we had used repetitively the same tasks, this would have enabled the full comparison of students' procedures and explanations between interviews, but then we would have lost on the unpredictability of the consecutive interviews. Therefore, for this type of longitudinal research, we recommend a balanced mix of repeated and new tasks for the sequence of interviews.

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