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EFFECT OF THE PRESENCE OF EXTERNAL REPRESENTATIONS ON ACCURACY AND REACTION TIME IN SOLVING MATHEMATICAL DOUBLE-CHOICE PROBLEMS BY STUDENTS OF DIFFERENT LEVELS OF INSTRUCTION

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ABSTRACT. This study explores the effects of the *presence of external representations of a mathematical object* (ERs) on problem solving performance associated with short double-choice problems. The problems were borrowed from secondary school algebra and geometry, and the ERs were either formulas, graphs of functions, or drawings of geometric figures. Performance was evaluated according to the reaction time (RT) required for solving the problem and the accuracy of the answer. Thirty high school students studying at high and regular levels of instruction in mathematics (HL and RL) were asked to solve half of the problems with ERs and half of the problems without ERs. Each task was solved by half of the students with ERs and by half of the students without ERs. We found main effects of the representation mode with particular effect on the RT and the main effects of the level of mathematical instruction and mathematical subject with particular influence on the accuracy of students' responses. We explain our findings using the cognitive load theory and hypothesize that these findings are associated with the different cognitive processes related to geometry and algebra.

KEY WORDS: accuracy, external representations, internal representations, level of instruction, reaction time, solving mathematical problems, split attention effect

INTRODUCTION

Literature review reveals inconsistencies in the role of external representations of mathematical objects (ERs) in problem solving. On the one hand, researchers argue that ERs support problem solving performance and mathematical reasoning by enabling mathematical communication and conveying mathematical thought (Kilpatrick, Swafford & Findell, 2001; Larkin & Simon, 1987; National Council of Teachers of Mathematics (NCTM), 1989, 2000). On the other hand, some studies show that ERs as information sources can damage problem solving performance by increasing cognitive load (Moreno & Mayer, 1999; Sweller, van Merriënboer & Paas, 1998; Tarmizi & Sweller, 1988).

This study aimed to examine this inconsistency by analyzing student's problem solving performance on short double-choice problems that require very quick (correct/incorrect) answers. We examine whether

ERs influence the accuracy of the problem solving performance and the reaction time (RT). Additionally, as part of a larger study, we examine differences between the performance of students who learn mathematics at a high level of instruction and of those who learn mathematics at a regular level of instruction. We apply E-Prime software to implement RT measurement (Schneider, Eschman & Zuccolotto, 2002).

Theoretical Background

External and Internal Representations

Perceptual processes are activated by external representations while cognitive processes are usually activated by internal representations (Zhang & Norman, 1994; p. 118). Commonly, representations can be divided into two interrelated categories: external and internal representations (Goldin, 2003; Goldin & Kaput, 1996; Kaput, 1998; Zhang, 1997).

External representations (ERs) refer to the physically embodied, observable configurations such as words, graphs, pictures, equations, and tables (Goldin & Kaput, 1996). ERs are considered as “acts stimuli on the senses or embodiments of ideas and concepts” (Janvier, Girardon & Morand, 1993, p. 81). ERs, which enable manipulation with mathematical relations and meanings, are involved in mathematical tasks such as manipulations on algebraic expressions, geometry problems, graph understanding, and more (Zhang, 1997).

Internal representations (IRs) are considered to be a mental image corresponding to the internal formulation of what we see around us in external reality. IRs are regarded as “cognitive or mental models, schemas, concepts, conceptions, and mental objects” which are illusive and not directly observed (Janvier et al., 1993; p. 81). IR determines knowledge and memory structures (Zhang, 1997) and refer to possible mental configurations of learners or problem solvers (Goldin & Kaput, 1996).

When communicating and expressing mathematical ideas, students and teachers use ERs (Lesh, Post & Behr, 1987). Lesh, Landau & Hamilton (1983) found five kinds of representations that are useful for mathematical understanding: (a) real life experiences, (b) manipulative models, (c) pictures or diagrams, (d) spoken words, and (e) written symbols. Larkin & Simon (1987) stress that visual representation (pictures, diagrams, graphs, etc.) preserves, in great detail, information about the topological and geometric relations between the components of the mathematical objects more than verbal (spoken words) or symbolic (written symbols such as algebraic equations and formulas) representations do. Another important distinction between different types of ER is the distinction between dynamic ER, for example, shown on a computer monitor and static ER that can be viewed printed on paper (Hegarty, 2004).

Mathematical reasoning requires manipulations with internal representations and translation between representations of different kinds (Janvier, 1987). The nature of external mathematical representations with which students learn concepts influences the nature of internal mathematical representations of the concept (Kaput, 1989, 1998). At the same time, according to Presmeg (2006), visual imagery (internal representation) underlies the creation of a drawing or spatial arrangement (external representation).

In a theory of representation, terms "to mean" or "to signify" are central "as they are used to express the link existing between external representation (signifier) and internal representation (signified)" (Janvier et al., 1993; p. 81). The relationship between internal representations of mathematical ideas that produce networks of knowledge (Hiebert & Carpenter, 1992) corresponding to a mathematical idea is understood only if it is connected to existing networks of previously learned concepts. Thus, mathematics educators agree that the use of different types of representations of mathematical objects in the learning process positively influences development of students' conceptual understanding, their mathematical reasoning, problem solving skills, and mathematical communications (Goldin & Steingold, 2001; Hiebert & Carpenter, 1992; Janvier, 1987; Kaput, 1989; NCTM, 1989, 2000; Pape & Tchoshanov, 2001; Sfard, 1991).

ER and Split Attention Effect

During the learning process, learners need to search for and match different parts of information which are mutually connected. Such parts of information may be a geometric drawing and an associated formula, or a graph of function and its corresponding equation. While different sources of information are crucial for understanding a concept, learners often have to split their attention between different sources of information and then mentally merge them when learning the concept. This mental process is called split attention effect (Sweller et al., 1998; Sweller, Ayres & Kalyuga, 2011).

There are two types of split attention effect. The first is the spatial contiguity effect (Moreno & Mayer, 1999; Sweller et al., 1998; Tarmizi & Sweller, 1988) where the different sources of information are spatially separated. The second is the temporal contiguity effect (Mayer, 2009; Mayer & Anderson, 1991, 1992) where the different sources of information are separated in time. This physical separation (spatial or temporal) can hurt the performance of different types of instructional material due to the necessity of switching from one source of information to another in order to analyze and integrate the material (Bobis, Sweller & Cooper, 1993; Mayer & Anderson, 1991).

On the one hand, it is commonly hypothesized that ERs support working memory and therefore improve people's performance on complex cognitive tasks (Cox, 1999; Zhang & Norman, 1994). However, on the other hand, the presence of several ERs of objects that require switching and connecting them increases cognitive load (Sweller, 1994; Sweller et al., 1998) and thus can impede problem solving performance.

In this study, we explore the influence of the presence of ERs of mathematical objects on students' performance on mathematical tasks by measuring RT and the accuracy of the responses. We assume that a longer RT with a similar accuracy of responses is an indication of higher cognitive load. At the same time, if the presence of ERs increases accuracy of responses, this means that ERs reduce split attention effect.

Reaction Time Methodology

It has been frequently assumed that mental processes are manifested through certain behavioral measures such as a subject's RT and response accuracy (Pachella, 1974; Posner & McCleod, 1982). The subject's RT is measured as the interval between the onset of the presentation of a stimulus to a subject and initiation of the subject's response (Pachella, 1974; p.44).

In the past two decades, a considerable body of research in cognitive psychology has used the RT paradigm in several domains, such as learning disabilities, intelligence, language, reasoning, problem solving, decision making, and movement control (Ashcraft, 1982; Babai, Brecher, Stavy & Tirosh, 2006a; Babai, Levyadun, Stavy & Tirosh, 2006b; Groen & Parkman, 1972; Jensen, 2006; Luce, 1986; Miller & Poll, 2009; Sternberg, 1969).

Sternberg (1969) developed the Stage Model in order to obtain and analyze a subject's RT in the performance of simple perception tasks. According to this model, performance on a task can be divided into a sequence of time-consuming processes such as perception of a stimulus, retrieval of stored information from memory, making decisions based on this information, and preparing a suitable response.

Jensen (2006) analyzes RT on a particular type of task among participants with different IQ levels. These experiments reveal high correlation between IQ and RT; thus, Jensen implies that chronometric tasks can be used as measures of IQ. Some studies in mathematics education analyze students' mathematical thinking using the RT paradigm. For example, the studies by Stavy, Babai, and their colleagues have suggested that the response to tasks with intuitive rules is immediate (Babai et al., 2006a, b; Stavy & Babai, 2008).

In this study, we used RT as a quantitative indicator of cognitive processes associated with solving short double-choice mathematical problems. We supposed that longer RTs indicate a more complex mental process and wondered whether the presence of ERs at the solution stage affects the degree of complexity and, if so, in what way. In particular, we were interested in examining whether ERs of mathematical objects accelerate, decelerate, or have a neutral effect on mental processing.

Neurocognitive Aspects of Representation

A significant number of studies in neuroscience which focus on representations of information in the nervous system emphasize visual, motor, memory, and prefrontal cortical functions of the brain (Funahashi, 2007). In particular, studies that relate to mathematics address the representation of a number in the brain (Nieder & Dehaene, 2009; Cohen Kadosh & Walsh, 2009). The meaning of *representation* in these studies is somehow different from the meaning of representation in our research. While neurocognitive studies analyze topology and processing of external information in the brain, our study analyzes the role of the presence of ERs in mathematical problem solving. To the best of our knowledge, this topic has not been studied, neither in mathematics education nor in cognitive psychology. In addition, our literature review did not identify any systematic study focusing on the differences between students' reasoning in algebra vs. geometry.

Research Goals and Questions

The goal of the current study was to resolve the inconsistency found in the literature review with respect to the role of ERs in thinking and learning processes. We aimed to examine problem solving performance on short mathematical tasks presented to students, either with or without ERs at the solution stage, by measuring RT and accuracy of the responses.

To achieve the study goal, we asked the following questions:

1. What are the differences (if they exist) in the accuracy of problem solving performance on the items that differ in the presence of ER (a) in algebra, (b) in geometry?

2. What are the differences (if they exist) in the RT of problem solving performance on the items that differ in the presence of ER (a) in algebra, (b) in geometry?
3. Are findings in question 1 and 2 related to the level of mathematics that the students study in school?

Additionally, the study presented in this paper was aimed at the validation of items for neurocognitive investigation.

Methodology

Study Participants

Thirty students from a high school in the north of Israel (16 – 17 years old) participated in this study. Twenty seven of the 30 students completed all the tests; thus, we report results related to these 27 students only. Fourteen participants who study at the highest level of mathematical instruction (HL) and 13 who study at the regular level (RL) completed all the assignments. All participants were randomly assigned to one of two groups (group 1 and group 2) with 13 and 14 participants, respectively: group 1 (G1) had six HL and seven RL participants, group 2 (G2) had eight HL and six RL participants. All participants were familiar with the topics included in the experiment's tasks.

Note that mathematics is a compulsory subject in Israeli high schools, and the students can be placed in one of three levels of mathematics: high, regular, and low. The level of instruction is determined by students' mathematical achievements in earlier grades. The differences in instruction at HL differ from that at RL in terms of the depth of the learning material and the complexity of the mathematical problem solving involved. The items we use in our study are basic items for both RL and HL curricula and are learned identically by the students in HL and RL groups. All the students in the study shared a similar socioeconomic background.

Tools and Data Collection

There were four tests: two tests (T1 and T2) in geometry and two tests (T3 and T4) in algebra. Each test contained 60 tasks. All tasks were presented visually at the center of the computer screen, displayed in black characters on a white background. Task presentation and response collection were conducted using the E-Prime software package (Schneider et al., 2002). Each task in each test was presented in two windows with different stimuli (S1—task condition and S2—suggested answer) that appeared consecutively (Fig. 1).

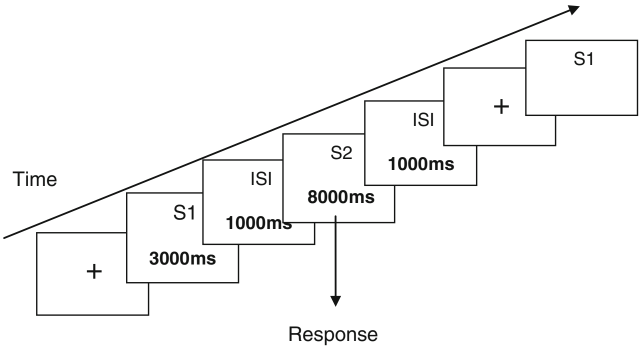


Figure 1. Sequence of events

Each trial started with the fixation cross. The cross was replaced by the problem description (S1) after 500 ms. The problem description was visible for 3,000 ms and separated from the answer (with or without external representation) by a blank time period (ISI) of 1,000 ms. The answer remained visible until the participant responded or for a maximum of 8,000 ms. Time periods were determined by a pretest performed by three participants.

For each task, an answer was presented, and the students were asked to decide whether or not the answer was correct (Fig. 2). In each test, 30 of the 60 tasks depicted a correct answer, while the other 30 tasks depicted an incorrect answer for the task given. We call the task "double choice" since the students had to press "3" (=correct) or "1" (=incorrect) according to their decision. The tasks were presented to participants in random order. The sequence of events is presented in Fig. 1.

Participants were tested individually in a quiet room, seated in front of a computer. They were told that they would be presented with a series of mathematical problems in algebra and geometry from the school curriculum and that they had to make a judgment about the correctness of the result displayed at the end of each problem. Participants were instructed to press one of two keys on the computer keyboard as quickly as possible when the displayed result was correct and the other key when the result was incorrect.

Test T1: Geometry ("Theorems"). Participants received a geometric drawing with the angles marked by Greek letters α and β. The drawing was followed by a statement with α and β referring to this drawing. The participants had to determine the correctness of the statement.

Test T2: Geometry ("Areas"). Participants received a drawing of a geometric object. Part of this drawing was shaded. The participants had to

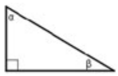
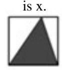
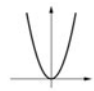
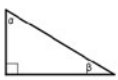
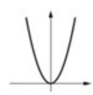
Field	**Geometry** Theorems T1	**Geometry** Areas T2	**Algebra** Graph to formula T3	**Algebra** Formula to graph T4
General instructions for the test	In each task determine correctness of the relationship between angles (α and β)	In each task determine correctness of the algebraic expression provided for the denoted area	In each task determine whether the algebraic formula can represent the function defined by a graph	In each task determine whether the graph can represent the function defined by an algebraic formula
S1 Object displayed in Window 1 common to ER and IR modes	The triangle is a right triangle. 	The area of square is x.  The area of colored triangle is:		$y = \frac{a}{x}$
ISI				
S2 – IR-mode Object displayed in Window 2 in IR mode	$\alpha + \beta = 90^{\circ}$	$\frac{x}{2}$	$y = x^3$	
S2 – ER-mode Object displayed in Window 2 in ER mode	 $\alpha + \beta = 90^{\circ}$	 $\frac{x}{2}$	 $y = x^3$	$y = \frac{a}{x}$ 
After S2 the student was required to press "3" if S2 included a correct answer or "1" if S2 included an incorrect answer				
Correct choice	3	3	1	3
Legend: T1, T2, T3, T4- the study tests, G1-Group1, G2-Group2				

Figure 2. Examples of items in the test

determine what area of the drawing was shaded or what the area of the geometric object was in reference to the shaded part.

Test T3: Algebra ("From Formula to Graph"). Participants received a graph of a mathematical function followed by an equation. They had to determine whether the graph and the equation represented the same function.

Test T4: Algebra ("From Graph to Formula"). Participants received an equation of a mathematical function followed by a graph. The participants had to determine whether the equation and the graph represented the same function.

For each test, two modes were designed. In ER mode, a drawing of a given mathematical object presented in stimuli 1 (S1—window 1—

givens) remained presented in stimuli 2 (S2—window 2—suggested answer). In IR, a mode drawing of a given mathematical object presented in stimuli 1 (S1—window 1—givens) disappeared (was not presented) in stimuli 2 (S2—window 2—suggested answer). Presentation of a suggested answer without presence of the given object (without ER) required activation of a mental image of the given object (activation of IR) in order to examine the answer. Figure 2 presents the task in both modes. Students were asked to cope with four tests (T1, T2, T3, and T4), each one presented in two modes: ER/IR (see Table 1).

Reliability of the Instrument

Internal consistency of each test was evaluated with Cronbach's alpha for accuracy. Table 2 presents the reliability of the instrument for the four tests in ER and IR, respectively (overall, eight tests). Cronbach's alpha was found to be high enough for the implementation of the test.

Data Analysis

Accuracy for each participant was determined by calculating the percentage of correct responses. RT was determined as the mean RT for the answers in all trials of the particular test.

A four-way repeated-measures MANOVA was conducted to investigate the impact on the RT and accuracy of four factors: *group* (G1, G2), *type of representation* (ER, IR), *mathematics subject* (algebra, geometry), and *level of mathematical instruction* (HL, RL) as well as interactions between the factors. An additional repeated-measures MANOVA was

TABLE 1

Distribution of the participants between the different tests and modes of representation

				Geometry		*Algebra*	
	HL	*RL*	*Total*	*T1 (theorems)*	*T2 (areas)*	*T3 (graph to formula)*	*T4 (formula to graph)*
G1	6	7	13	ER	IR	ER	IR
G2	8	6	14	IR	ER	IR	ER

ER external representation, *IR* internal representation, *HL* high level of mathematical instruction, *RL* regular level of mathematical instruction, *G1* group1, *G2* group 2

TABLE 2

Cronbach's alpha for the tests in the study

	ER mode				*IR mode*			
Test	*T1*	*T2*	*T3*	*T4*	*T1*	*T2*	*T3*	*T4*
Cronbach's α	0.662	0.696	0.784	0.719	0.816	0.718	0.809	0.627

conducted to investigate the *tests*' impact on RT and accuracy in an interaction with *representation* and *level of mathematical instruction*. RT and accuracy are two interdependent measures (Pachella, 1974; Jensen, 2006) and therefore were examined as two measures in the same MANOVA. The type of representation and the mathematics subject were within-subject factors, whereas the group and the level of mathematical instruction were between-subject factors.

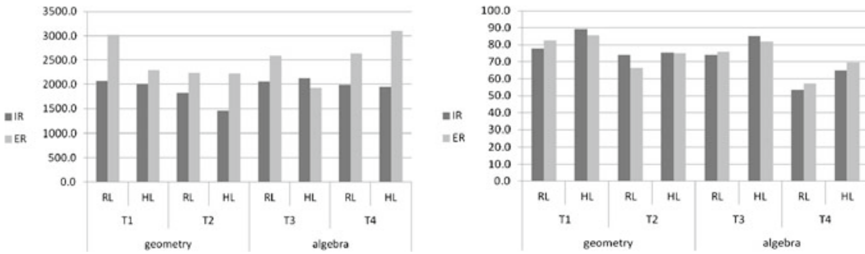
Results

Differences in Accuracy and in Reaction Time

First, we ascertained that the distribution of students among the groups did not affect the study results. Indeed, MANOVA revealed neither a significant main effect of group nor a significant interaction between group and *mathematical topic* (algebra vs. geometry); findings were similar for the interaction between groups and *level of mathematical instruction* (HL vs. RL). This suggests that both groups performed similarly in terms of mathematical topics and levels of mathematical instruction.

Figure 3 demonstrates accuracy (in percent) and RT (in millisecond) revealed in the study for each of the tests in IR and ER modes for HL and RL students.

Table 3 shows that HL participants were more accurate than RL participants on all tests in the two representational modes. On T4, accuracy was lowest for RL and HL participants, and we deduce that among all the tasks used in our research experiment, translation of a formula to a graph was the most difficult. RT was longer for the ER mode than for the IR mode on all tests for both RL and HL participants (with the exception of T3–ER–HL). The accuracy for both modes (ER and IR)



RT Accuracy

Figure 3. Students' results (mean values) on the tests

was similar in both levels of mathematical instruction. Repeated-measures MANOVA revealed three significant main effects (see Table 3).

Data analysis demonstrated a main effect of *representation* (F (2, 22) = 13.264, $p < 0.001$). RT for the IR mode was lower than for the ER mode (1,935.04 vs. 2,503.2, respectively). Accuracy was similar in two representational modes (74.2 vs. 74.2 for IR and ER, respectively). Subsequent ANOVA demonstrated the effect to be significant for RT (F (1, 23) = 27.674, $p < 0.001$), but not for accuracy (F (1, 23) = 0.000, $p > 0.05$).

MANOVA revealed a main effect of *level of mathematical instruction* was found (F (2, 22) = 5.855, $p < 0.01$). The follow-up ANOVA analysis revealed a significant difference in accuracy between HL and RL participants (F (1, 23) = 10.017, $p < 0.01$). HL participants were more accurate than RL participants (78.4 vs. 70.9, respectively). There was no significant difference in RT (F (1, 23) = 0.549, $p > 0.05$, 2303 vs. 2134.5, for RL and HL, respectively).

We found a main effect of *mathematics subject* (F (2, 22) = 12.526, $p < 0.001$) with significant difference in accuracy between tests in geometry and algebra (F (1, 23) = 26.012, $p < 0.001$). The participants performed more accurately on geometric tests than on algebraic tests (78.2 vs. 70.2, respectively). There was no significant difference in RT between tests in algebra and geometry (F (1, 23) = 1.853, $p > 0.05$, 2,299.2 vs. 2,139.1 for algebra and geometry, respectively).

MANOVA also revealed no significant interactions among the following factors: representation, level of mathematical instruction, and mathematics subject. There was no significant interaction between representation, test, and level of mathematical instruction ($p > 0.05$; see Table 3). However, subsequent ANOVA found significant interaction between representation and mathematics subject with respect to the accuracy of the participants' performance (F (1, 23) = 4.638, $p < 0.05$).

TABLE 3

Differences and interactions between the factors' effects

			Accuracy in % *Mean (SD)*		*Reaction time in ms* *Mean (SD)*	
			HL	*RL*	*HL*	*RL*
Means and SD						
Geometry	T1	ER	85.6 (6.7)	82.6 (8.3)	2,289.9 (774.7)	3,014.0 (1039.6)
		IR	89.4 (5.3)	77.8 (11.0)	1,997.3 (291.5)	2,071.9 (619.4)
	T2	ER	75 (8.4)	66.1 (8.9)	2,221.1 (309.7)	2,236.6 (915.7)
		IR	75.3 (11.3)	74 (7.8)	1,459.6 (532.6)	1,822.2 (365.8)
Algebra	T3	ER	82 (5.5)	75.7 (13.3)	1,931.4 (596.6)	2,592.7 (912.1)
		IR	85.2 (7.7)	73.9 (12.0)	2,126.2 (611.6)	2,063.3 (755.9)
	T4	ER	69.6 (9.3)	57.2 (8.7)	3,099.6 (889.8)	2,640.1 (1,420.1)
		IR	65 (9.0)	53.3 (6.6)	1,950.7 (543.1)	1,989.2 (560.4)
Main effects						
Overall			78.4 (6.8)	70.1 (6.8)	2,134.5 (595.2)	2,303.8 (590.9)
Effect of LMI F (2, 22) = 5.855** Wilks' Λ = 0.653			F (1, 23) = 10.017**		F (1, 23) = 0.549	
ER—total mean			74.2 (7.1)		2,503.2 (800)	
IR—total mean			74.2 (8.3)		1,935 (471)	
Effect of R F (2, 22) = 13.264*** Wilks' Λ = 0.453			F (1, 23) = 0.000		F (1, 23) = 27.674***	
Geometry—total mean			78.2 (7.4)		2,139.1 (579.2)	
Algebra—total mean			70.2 (8.4)		2,299.2 (745.2)	
Effect of MS F (2, 22) = 12.526*** Wilks' Λ = 0.468			F (1, 23) = 26.012***		F (1, 23) = 1.853	
Interactions						
ER			78 (7.1)	70.4 (7.1)	2,385.5 (802.4)	2,620.9 (796.5)
IR			78.7 (8.3)	69.8 (8.3)	1,883.5 (472.4)	1,986.6 (469)
R × LMI F (2, 22) = 0.277 Wilks' Λ = 0.975			F (1, 23) = 0.229		F (1, 23) = 0.375	
Geometry			81.3 (7.4)	75.1 (7.4)	1,992 (581)	2,286.2 (576.8)
Algebra			75.4 (8.4)	65 (8.4)	2,277 (747.5)	2,321.3 (742)
MS × LMI F (2, 22) = 1.728 Wilks' Λ = 0.864			F (1, 23) = 1.827		F (1, 23) = 1.129	

TABLE 3

(continued)

		Accuracy in % Mean (SD)		Reaction time in ms Mean (SD)	
		HL	*RL*	*HL*	*RL*
ER	Geometry	77.3 (10.6)		2,445.4 (818.6)	
	Algebra	71.2 (12.8)		2,606.5 (1,020.6)	
IR	Geometry	79.7 (10.6)		1,849 (485.3)	
	Algebra	70 (15)		2,037.7 (587.1)	
R × MS $F(2, 22) = 2.288$ Wilks' $\Lambda = 0.828$		$F(1, 23) = 4.638^{*}$		$F(1, 23) = 0.262$	
ER	Geometry	79.5 (9.2)	75 (11.8)	2,250.6 (532.6)	2,655.2 (1,026)
	Algebra	74.8 (10)	67.1 (14.5)	2,598.9 (960.8)	2,614.6 (1,121)
IR	Geometry	83.3 (10.8)	75.8 (9.2)	1,766.8 (480.7)	1,937.4 (493.5)
	Algebra	76.5 (13)	62.8 (14)	2,051 (568.3)	2,023.4 (629.7)
R × MS × LMI $F(2, 22) = 0.129$ Wilks' $\Lambda = 0.988$		$F(1, 23) = 0.264$		$F(1, 23) = 0.020$	
R × test × LMI $F(2, 22) = 0.374$ Wilks' $\Lambda = 0.899$		$F(3, 69) = 0.463$		$F(3, 69) = 0.175$	

ER external representation, *IR* internal representation, *MS* mathematics subject, *LMI* level of math instruction, *R* representation
$^{*}p < 0.05$; $^{**}p < 0.01$; $^{***}p < 0.001$

Discussion

The present study examined the impact of the ER of a mathematical object (drawing, graph, and formula) on participants' performance in four different mathematical tests. The study aimed to examine the effect produced by the presence of ER of the mathematical object, at the decision making stage of double-choice tasks, on task performance.

The results showed that, in general, students who study mathematics at HL were more accurate on all the tests and performed more quickly than RL students (with the exception of T3 in IR mode). In most cases, RT decreases when students perform tasks in IR mode as compared to their performance in ER mode. Surprisingly, compared to our expectations, students performed more accurate geometry tests than algebra tests.

Research in cognitive science indicates that diagrams used as part of ER can improve decision making or problem solving as compared to texts or tables (e.g. Larkin & Simon, 1987). Larkin & Simon (1987) argued that “the diagrammatic representation preserves explicitly the information about the topological and geometric relations among the components of the problem” (Larkin & Simon, 1987; p. 66). Therefore, diagram use seems to reduce the speed of search and recognition. In addition, external representation is also thought to provide a memory aid and can significantly ease many cognitive tasks (Zhang & Norman, 1994).

Our findings demonstrate that the presence of ER enhances the RT of students’ problem solving performance, but it does not improve accuracy. We explain these findings by the split attention effect that accompanies solving processes with the presence of both the ER of the mathematical object and the corresponding formula. We suggest that split attention effect led to an increase in cognitive load and was not helpful in solving the tasks. However, more careful investigation has to be performed.

The multiple sources of information in our experiment were separated temporally in both modes (since different pieces of information were presented in S1 and S2) and spatially in ER mode (when different pieces of information were displayed simultaneously in S2). The temporal separation occurred in two experimental modes; therefore, we suppose that the differences between RT in the two experimental modes are due to spatial separation. We assume that ER induces extraneous cognitive load by requiring the splitting of attention among several types of information: geometrical object and its property written symbolically or function graph and its formula. This cognitive load requires mental integration of multiple sources of information (Mayer, 2009; Sweller et al., 1998, 2011). Consequently, we assume that maintaining a mental image (IR) of the given mathematical object reduces cognitive load and decreases RT. Moreover, this effect was true for both levels of mathematical instruction. The RT for the ER condition was longer than for the IR condition (except in T3—“graph to formula” for HL students): this extraneous load did not affect the accuracy of responses.

RTs are generally used to determine differences in times of information processing with respect to experiment tasks or experiment conditions: the longer the RT, the more complex the cognitive process (Luce, 1986; Pachella, 1974; Sternberg, 1969). We speculate here that the main effect of the mathematics subject (especially its effect on the accuracy of mathematical performance) and the interaction between representation mode and mathematics subject revealed in this study are associated with different cognitive processes related to geometry and algebra tasks used in our research. We plan to carefully examine the hypotheses raised in this study from both the cognitive and neurocognitive perspectives. We currently conduct a multidimensional examination of mathematical giftedness that examines a population of 200 students with respect to basic cognitive skills and brain activity with ERP methodology (e.g. Waisman, Shaul, Leikin & Leikin, 2012; Shaul, Leikin, Waisman & Leikin, 2012).

We assume that results of this study as well as of the neurocognitive study will contribute to our understanding of underlying principles for design of mathematical tasks with respect to the use of ER or IR. Better explanation of the performance differences associated with ER vs. IR and with algebra vs. geometry tasks (as within-subjects differences) and of the effects of level of mathematical instruction and gender (as between-subjects differences) can be given, and it will inform mathematics educators about the place and the role of ER in instructional design.

Some limitations of our study can be seen in the scope of the research population of the experiment described in this paper. Additionally, we have to note that definitions of ER and IR used in our study were of operational nature for experimental purposes. We are aware that in real mathematics education settings, students may use different ERs and IRs other than those used in this study. For example, in our experiment, we did not address distinctions between static and dynamic ER of mathematical objects (cf., Hegarty, 2004).

NCTM (2000) stressed that "The ways in which mathematical ideas are represented is fundamental to how people can understand and use those ideas" (ibid, p. 67). Based on our findings, we speculate that whereas HL mathematics students can successfully operate with mathematical concepts and solve mathematical problems based on IR, the role of ER becomes especially important in teaching mathematics to students who do not excel in mathematics. We assume that ER of mathematical objects can support learning and solving processes in students who study mathematics at RL and is essential for low achievers in mathematics. Since the former hypothesis was not explored in our study, it opens an interesting direction for future research in mathematics education. Additional direction for future research can be seen in exploring the role of IR and ER in solving more advanced tasks than those that were used in our study.

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