### DOMINIC VOYER

# PERFORMANCE IN MATHEMATICAL PROBLEM SOLVING AS A FUNCTION OF COMPREHENSION AND ARITHMETIC **SKILLS**

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ABSTRACT. Many factors influence a student's performance in word (or textbook) problem solving in class. Among them is the comprehension process the pupils construct during their attempt to solve the problem. The comprehension process may include some less formal representations, based on pupils' real-world knowledge, which support the construction of a 'situation model'. In this study, we examine some factors related to the pupil or to the word problem itself, which may influence the comprehension process, and we assess the effects of the situation model on pupils' problem solving performance. The sample is composed of 750 pupils of grade 6 elementary school. They were selected from 35 classes in 17 Francophone schools located in the province of Quebec, Canada. For this study, 3 arithmetic problems were developed. Each problem was written in 4 different versions, to allow the manipulation of the type of information included in the problem statement. Each pupil was asked to solve 3 problems of the same version and to complete a task that allowed us to evaluate the construction of a situation model. Our results show that pupils with weaker arithmetic skills construct different representations, based on the information presented in the problem. Also, pupils who give greater importance to situational information in a problem have greater success in solving the problem. The situation model influences pupils' problem solving performance, but this influence depends on the type of information included in the problem statement, as well as on the arithmetic skills of each individual pupil.

KEY WORDS: arithmetic, comprehension, mathematics, problem solving, teaching, word problem

#### **INTRODUCTION**

Problem solving has been taught in classrooms for a long time. For instance, it is possible to go back as far as the year 1861 to find, in Quebec, a curriculum where mathematical exercises with significant background information on the problem were presented as text rather than in mathematical notation (Bélanger, Gauthier & Tardif, [1993\)](#page-18-0). In 1945, George Polya published "How to Solve It", an important book on the resolution of word problems and its importance in the learning process (Polya, [1945](#page-19-0)). In 1980, the National Council of Teachers of Mathematics (NCTM) published "An Agenda for Action", a paper that called for the integration of problem

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solving in curricula (National Council of Teachers of Mathematics, [1980](#page-19-0)). Since then, in its reports, the NCTM highlighted the necessity to grant problem solving a major role in curricula (National Council of Teachers of Mathematics, [1989](#page-19-0); [2000](#page-19-0)). The purpose of this research is to study the understanding process at play when pupils are solving an arithmetic word problem. Research suggests that various levels of representations play a role in the process of understanding a word problem. One of these representations is called 'situation model' and is based on pupils' real-world knowledge. The goal of this paper is to assess the effect of a situation model on pupils' performance in solving arithmetic word problems.

## THEORETICAL FRAMEWORK

# Comprehension: an Iterative Process

The process undertaken by pupils to understand problematic situations plays an important role in their problem solving abilities. Research outlines various ways in which arithmetical word problems might be represented by students. Various levels of representation are constructed during the process of solving arithmetic word problems. Kintsch & van Dijk [\(1978\)](#page-19-0) describe the first mental representation as the *text base*, which is strongly based on the actual text. It is a primary analysis of the various notions found within the written text. A second level of representation is described by Kintsch & Greeno ([1985](#page-18-0)) as the problem model. This is a more formal representation in the sense that the problem model is directly related to the question asked and to the mathematical procedures to follow in order to solve the problem. At this level of representation, it is the relationship between the variables that is highlighted, whereas the context surrounding the problem becomes secondary. The problem model allows pupils to construct a significant mathematical representation of the problem. To facilitate the construction of the problem model, one might use the contextual information of the problem and then construct what Reusser [\(1990\)](#page-19-0) identified as the situation model. Reusser ([1990](#page-19-0)) introduced the situation model in the process of problem comprehension in order to fill a gap in the model proposed by Kintsch & Greeno [\(1985\)](#page-18-0), which fails to consider elements that are not essential to solve the problem, but may help pupils to gain a better understanding of this problem. The situation model was described by Reusser [\(1990\)](#page-19-0) as an intermediate and less mathematical representation constructed by using one's real-world knowledge and personal experiences to interpret the information found in the text base. When trying to solve an arithmetic problem, the situation model is constructed when pupils take into consideration information other than the mathematical information required to solve the problem.

# Information Included in the Problem Statement

Considering that all the information included in the statement of a word problem does not play the same role in understanding a problem, this information can be classified in three main categories: solving information, situational information and explanation information. Although many researchers have investigated the analysis of solving information (essential data) in word problems after manipulating the type of problem, the order of presentation of the data or the size of the numbers, few have studied the situational and explanation information found in problem statements, especially in the last 30 years. In the 1970s, American researchers studied what are commonly called 'task variables'. Among these variables were 'context variables' that describe the non-mathematical information found in the problem statement. Context variables encompass both what we previously called 'situational information', as well as explanation information. Context variables include those which make the problem statement more or less relevant to the problem solver's interest, age or experience. They may also help give meaning to the mathematical content (Barnett, [1979](#page-18-0)). Situational information, as we are going to call it, is the information playing a role in the development of a context that anchors the mathematical question in a real life situation. Moreau & Coquin-Viennot ([2003\)](#page-19-0) identified three categories of situational information: initiating events, setting information and temporal information. Their research suggests that situational information contributes to the construction of a situation model. However, not all categories are treated equally by pupils. Explanation information renders the relationship between the various pieces of information found in the text more explicit. Research suggests that these elements also influence the construction of a situation model and may improve comprehension of the problem (Stern & Lehrndorfer, [1992](#page-19-0)). Other information could be considered superfluous, for instance if it cannot help pupils understand the problem in any way.

# The Impact of the Situation Model on the Level of Performance in Solving Word Problems

Though we know that the situation model may play a role in the construction of representations that will facilitate the comprehension of a problem, only a few studies have published data that allow the investigation of the relationship between the construction of a situation model and the performance in the resolution of word problems. In their assessment of the impact of 'context variables' on pupils' performance, Caldwell & Goldin ([1979\)](#page-18-0) assessed the familiarity of students with the context used in a mathematical problem, and results obtained from a sample of 399 grades 4, 5 and 6 students show that realistic problems were solved more easily than abstract problems, when all the other variables are controlled. The authors also found that for realistic problems, there are no significant differences between factual and hypothetical problems. However, for abstract problems, a factual context will make the task harder for pupils. Another research on familiarity, a study performed by El Boudali [\(1984](#page-18-0)), showed that the further the context is from a pupil's real life experience, the harder the problem will be to solve. The author observed, without any real surprise, that when solving word problems in a non-mathematical context, pupils will perform better when they are more familiar with the context.

More recently, Coquin-Viennot & Moreau ([2007\)](#page-18-0) demonstrated that the age of pupils (elementary grade 3 vs. grade 4) influenced the role of the situation model in the resolution of word problems. Authors then analysed whether the situation model played an equal role for students with stronger or weaker problem solving skills, without being able to provide a definitive answer to the question. Stern & Lehrndorfer ([1992\)](#page-19-0) modified the word problems to make the relationship between quantitative data more explicit. This increased pupils' performance in problem solving. One could hypothesize that a representation based on explanation information increases pupils' performance. However, this was not confirmed by other studies that tried to establish a relationship between the construction of a situation model and the level of performance achieved by pupils (Moreau & Coquin-Viennot, [2003](#page-19-0)).

Although certain studies have explored new avenues in this field, the question regarding the influence of constructing a situation model on pupils' performance in solving word problems remains unanswered.

### **OBJECTIVE**

The following question will be the focus of this study: What is the relationship between the situation models and the resolution of arithmetic word problems? To answer that question, we will also consider differences in students' mathematical skills, in order to see if the situation model plays an equal role for students with stronger mathematical skills and for those with less skills in this field.

### **METHOD**

### **Participants**

The sample is composed of 750 pupils of grade 6 elementary school (11 to 12 years old). They were selected from 35 classes in 17 Francophone schools located in the province of Quebec. There are 354 girls and 394 boys in the sample (in two cases, data concerning the gender of the child were not documented).

## **Material**

The problems used for this study are mathematical word problems, with two linear variations having different rates of variation. In each problem, the question is related to the intersection point of these two relations. This type of problem was chosen because it is rarely studied in elementary school, yet can still be solved by using an arithmetical approach. An example is given later in this section.

Three problems were developed, each respecting this mathematical structure. Four versions were built for every problem, in order to be able to manipulate the type of information integrated in the word problem. The first version is similar to the one used to validate the Kintsch & Greeno ([1985\)](#page-18-0) model. It is a simplified version that contains only the information which is essential to solve the problem. The second version, inspired from word problems by Cummins, Kintsch, Reusser & Weimer [\(1988](#page-18-0)), is composed of the simplified version as well as situational information, allowing the creation of a more elaborate context and the insertion of the mathematical question in a situation that is well-known to the pupils (a ski trip, for instance). The third version is also composed of the simplified version, to which an explanatory sentence is added. This allows pupils to better understand the relationship between numerical data, as presented in Moreau & Coquin-Viennot's [\(2003](#page-19-0)) study. The fourth version, also called the complete version, includes all the problem solving information from the simplified version, the situational information from the second version and the explanations from the third version. Table [1](#page-5-0) shows the information categories of Moreau & Coquin-Viennot [\(2003\)](#page-19-0), which we used to develop the statements of our word problems.

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#### TABLE 1

Moreau & Coquin-Viennot's information categories



Here is an example of the complete version of a problem. A number is given before each sentence to better clarify the nature of each section, as described below.

(1) Julie and her brother Antoine have been skiing together for a few years. (2) This year Julie is in grade 6 and her brother is in his first year of high school. (3) Julie's skis are red and Antoine's are yellow. (4) Both wish to save money for their next ski trip. (5) Julie already has \$30 in her bank account saved up for the trip, but Antoine only has \$15. (6) Antoine can save \$5 a week for the trip. (7) Julie can save \$2 per week for the trip. (8) Antoine has less money to start with, but every week his savings increase more rapidly than Julie's. (9) How many weeks will it take for Julie and Antoine to have the same amount of money for their trip?

- 1. Situational information used to introduce the characters (character)
- 2. Situational information used to situate the characters in their school environment (context)
- 3. Superfluous narrative information, unessential for the comprehension of the problem
- 4. Situational information used to set the arithmetic problem in a story (theme)
- 5–7. Solving information, including numerical data, required to solve the problem
- 8. Explanation information explicitly connecting the various numerical data
- 9. Solving information: the question

For every problem in this study, the familiarity with the context and the vocabulary, the number of sentences used and the nature and size of numbers involved were controlled. The familiarity with the context in which the problems were developed was controlled by choosing contexts that pupils could easily relate to. Every situation described involved activities that can be done with friends (swimming, playing chess, skiing).

The three situational elements (sentences 1, 2 and 4) were composed in such a way as to mark a gradation in the contextual information: Sentence 1 introduces the characters, sentence 2 describes the initial situation and sentence 4 introduces the theme. The objective of the contextual information is to allow readers to use their real-world knowledge to build a representation of the context of the word problem.

## **PROCEDURE**

## Task A: Solving Problems

Pupils were randomly assigned one of the four versions of the problems. Each pupil was asked to solve three problems of the same version category: the complete version (type 1), the version containing situational information (type 2), the version containing explanation information (type 3) or the simplified version, which only included the information required to solve the problem (type 4).

# Tasks B: Situation Model

Pupils who were assigned problems from versions 1, 2 or 3 were also assigned task  $B_1$ . After solving each problem, they were asked to identify elements that were not essential, but helped them better understand the problem (see "Appendix [1](#page-16-0)"). The superfluous narrative information added to the problem (sentence #3) becomes a pivotal element in this task. In order to say that a situation model is truly being constructed, it is imperative that pupils do not mark the superfluous information as being important. If they do, we could assume that the pupil gave random answers. A situation model should be constructed using non-essential information from either the situational information or explanation information provided in the problem. This task was inspired from a study performed by Moreau & Coquin-Viennot [\(2003](#page-19-0)).

Students who were randomly assigned simplified versions (type 4) of the problems were then asked to complete task  $B_2$ . The objective of this task was to document data regarding situation models by directly asking them what they believed was helpful in solving the problems. Pupils were told the following: "Here is a word problem that has not yet been solved. Add one or two sentences that would help make the problem easier for other pupils to understand." For this task, pupils were required to develop sentences. This information will contribute to the data collected in task  $B_1$ .

# Teacher Questionnaire

Data were also collected from teachers. Each teacher was invited to provide data related to each child: his/her gender and level of arithmetic skill. Teachers were asked to rate the pupils' performance in arithmetic, based on the overall results obtained throughout the year, as well as on personal observations. Teachers rated each pupil on a three-tiered scale: weak, average and strong. The method of relying on teachers' observations has been used by other researchers (Moreau & Coquin-Viennot, [2003;](#page-19-0) Sovik, Frostrad & Heggberget, [1999](#page-19-0)) and was validated by Sovik et al. [\(1999](#page-19-0)) in using in parallel standardized tests (mathematics and reading tests).

### **RESULTS**

### Results for the Different Versions of the Problems

Each problem was rated on a scale of 1 to 5, based on the solution and the answer of the student (see "Appendix [2](#page-17-0)"). As illustrated in Table 2, when results from all pupils were taken into consideration, two versions had significantly better success rates than the complete versions: the simplified versions and the versions containing explanation information.

<i>Type of problem</i>	Number	Mean	Standard deviation
Complete	267	3.74	1.03
With situation	167	3.82	1.15
With explanation	151	3.99	1.02
Simplified	165	4.01	1.11
Total	750	3.84	1.07

TABLE 2 Pupils' performance, by type of problem

The ANOVA provides statistically significant results  $F(3, 746) = 4.571$ ,  $p = 0.004$ , which means that there is at least one statistically significant difference between the mean achieved by the pupils and the various types of problem statements. To study this difference, we have decided to use Dunnett's test C, even though Levene's test reveals that variances can be considered homogeneous. We made this choice because the distribution of subjects for each problem statement is not uniform. The multiple range test (see also Figure 1) shows significant differences ( $p \le 0.05$ ) between problems with simplified statements and complete problems, as well as between problems with explanations and complete problems.

To study a possible interaction between pupils' mathematical skills and the type of problem statement, a two-way analysis of variance was conducted. The ANOVA did not yield significant results  $F(6, 6)$  $(733) = 1.693$ ,  $p = 0.120$ . However, the diagram and descriptive statistics (see Figure [2](#page-9-0) and Table [3](#page-10-0)) show that weak pupils seem to adopt different



Figure 1. Pupils' mean score, by type of problem statement

behaviours depending on the problem they have to solve. These pupils of our sample obtained higher scores when given a word problem that included an explanatory sentence, as opposed to problems containing situational information. We can think that our unbalanced design did not provide enough statistical power to observe differences between weak pupils and others, in spite of what we can see in the descriptive results.

# Results for Task  $B_1$

For the sample as a whole, explanation information plays a greater role than situational information in the construction of a situation model. Pupils marked 60.7% of the sentences with explanation information as



# Type of problem

Figure 2. Pupils' mean score, by mathematical skills and type of problem statement

<span id="page-9-0"></span>

#### TABLE 3

<span id="page-10-0"></span>

Mathematics skill/problem type		<b>Number</b>	Mean	Standard deviation
Weak	Explanation	19	3.6316	1.38285
	Situational	34	2.8088	1.24325
	Total	53	3.1038	1.34221
Average	Explanation	86	3.9186	1.01129
	Situational	75	3.9356	1.00184
	Total	161	3.9265	1.00379
Strong	Explanation	46	4.2717	0.79377
	Situational	57	4,3333	0.81284
	Total	103	4,3058	0.80104

Pupils' performance, by problem type and mathematics skill

being important, whereas only 16.7% of the sentences with situational information were considered in the same manner. In the statement of word problems, situational information was divided into three different sentences. These three elements were not given equal importance by the students. The theme sentence was chosen by pupils in 44.6% and 64.8% of the time for version types 1 and 2, respectively. The character sentence (4.9% and 17% of the time) and initial situations (context) were chosen in 0.7% and 4.2% of the cases. This analysis leads us to believe that only certain situational details play a role in the construction of a situation model and that other elements do not contribute to this process.

Pupils with weaker arithmetic skills, when constructing a situation model, give greater importance to explanation information than pupils with strong arithmetic skills (81.3% vs. 47.2%). This difference is statistically significant, according to the Brown–Forsythe test  $F(2)$ ,  $60.290$ ) = [4](#page-11-0).076,  $p = 0.022$  (see Table 4).

# Results for Task  $B_2$

Answers given by pupils were classified into three categories: situational, explanation and other. We were particularly interested in knowing whether the pupils' answers were explanatory or situational in nature. Results suggest that added sentences were more explanatory than situational (19.5% vs. 5%). However, a large percentage of sentences were categorized as 'other' (75.5%). Input from this category varied greatly and included giving indications on operations to use and tips to solve the problem, as well as providing the answers.

#### TABLE 4



<span id="page-11-0"></span>Percentage of elements chosen by pupils when given a complete version, by arithmetic skills

In conclusion, the results suggest that, when constructing a situation model, the pupils give greater importance to explanation information, rather than situational information. This is especially true for students with weaker arithmetic skills.

# The Effect of the Situation Model on Pupils' Performance

A correlation analysis was performed for the 95 subjects who had to execute task  $B_2$  as well as solve complete problems. The analysis reveals that the more a pupil uses the theme sentence of the situational information in a complete problem, the better the performance  $r(95) = 0.272$ ,  $p = 0.008$ . For explanation information, at first glance, results seem more surprising: pupils who selected explanation information, when asked which elements of the complete version made the problem easier to understand, did less well when trying to solve the problem  $r(95) = -0.372$ ,  $p < 0.001$ . We will discuss this in the next section.

We then performed a linear regression, with pupils' performance as a dependent variable, and the number of explanation details and theme sentences selected in task A as independent variables. The value of the adjusted  $R^2$  indicates that 22.4% of the variance in the dependent variable is explained by this model. The analysis is statistically significant  $F(2, 92) = 14.585, p \le 0.001.$ 

## DISCUSSION AND CONCLUSION

The objective of this study was to assess how a representation based on real-world knowledge and life experiences (situation model) constructed

during the comprehension process of an arithmetic word problem could influence performance in problem solving. Therefore, we have studied the impact of various types of information included in word problem statements on the comprehension and performance of pupils. We have also considered the level of mathematical skills of these pupils to examine the differences between the comprehension process of strong, average and weak students.

## Roles of Situational vs. Explanation Information in Understanding

The fact that the pupils retained non-essential information, but not the superfluous narrative information that acted as a control for the study, confirms that they construct situation models. These results corroborate those obtained by Moreau & Coquin-Viennot ([2003](#page-19-0)) in a similar task and suggest that pupils use their real-world knowledge and life experiences to solve arithmetic problems. At first glance, the results are not surprising and are in line with those measured in studies on context variables (see Caldwell & Goldin, [1979](#page-18-0); Kulm, [1984;](#page-19-0) Hembree, [1992;](#page-18-0) Nasser & Carifio, [1993](#page-19-0)).

An in-depth analysis of the content of pupils' situation models demonstrates that explanation information is approximately four times more important than situational information for the construction of the situation model. In order to interpret these results, we refer to studies conducted by van Dijk [\(1977\)](#page-19-0), Kintsch & van Dijk [\(1978\)](#page-19-0) and Kintsch [\(1998](#page-18-0)). According to these authors, on a first level, as the pupil reads the text, he or she will construct a representation of each notion. This representation is called the text base. Within this first representation, the various notions of the text will be organized into a coherent structure, the macrostructure, influenced by the readers' objective. The macrostructure allows an elimination of the propositions of the text base which is unrelated to the objective pursued by reading the text, in this case solving the arithmetic problem. The macrostructure is therefore used to transform sequences of propositions into a more general proposition, in order to render the mnemonic organization more efficient (Kintsch, [1998\)](#page-18-0). Hence, by creating more explicit links between the different pieces of information contained in the problem, explanation information could support the development of the macrostructure by helping pupils focus on solving information, making the comprehension of arithmetic problems more accessible.

We wanted to study the differences between the various subtypes of situational information. Results suggest that not all situational information play an equal part in the process of understanding a problem: This was never addressed in previous studies. Within the situational information contained in the problem statement, pupils often described the theme sentence as being helpful in understanding the problem. This information allows pupils to create a context for the arithmetic problem and consequently may allow them to tap into their real-world knowledge to create a better representation of the problem. Although other situational information helped develop a more elaborate story around the arithmetic problem, it did not contribute to a better understanding of the problem.

## Influence of Arithmetic Skills in the Understanding Process

Concerning arithmetic skills, our results suggest that pupils with weaker arithmetic skills do not represent the problem in the same way as other pupils: When constructing situation models, they give greater importance to explanation information than more gifted students. It is interesting to note that pupils with weaker arithmetic skills do not only give greater importance to explanation information during the process of comprehending a problem but they also seem to fare better when given this type of problem, as opposed to problems with situational information. More studies would be necessary to generalize this observation. This could suggest that explanation information can help weaker pupils solve problems. More specifically, this suggests that pupils with weaker arithmetic skills may construct different representations depending on the information presented in the problem. This allows us to take the results obtained by Moreau & Coquin-Viennot ([2003\)](#page-19-0), who studied the differences between pupils with strong and weak arithmetic skills, a step further. By evaluating the relationship between arithmetic skills and the type of information pupils retain in a problem in order to solve it, authors observed that students with weak skills retained 48.1% of the explanation information, whereas strong students retained 42.3% of the same information. Although this difference was not statistically significant, the results obtained in the current study concur with those published by Moreau & Coquin-Viennot ([2003\)](#page-19-0).

The situation model revealed differences based on arithmetic skills. Our results suggest that arithmetic skills have an impact on the representations pupils construct to understand a problem and more specifically on the relationship they have with the various types of information included in a word problem.

To determine whether pupils who construct a situation model fared better in problem solving and to see how specific information contained in a situation model constructed by pupils may have influenced their performance, we analysed situational and explanation information separately. A first correlational analysis, assessing the relationship

between the number of situational elements retained to understand the problem and pupils' performance, allows us to state that pupils who give greater importance to situational information in a problem have greater success in solving the problem. Hence, constructing a situation model using situational information presented in the problem is related to pupils' performance in problem solving.

The results obtained for explanation information reveal differences with situational information. They suggest that pupils who take explanation information into consideration to understand a problem have greater difficulty in solving it. At first sight, these results are surprising, and results from past studies must be taken into consideration in order to interpret them properly. In order to analyse the negative impact of constructing a situation model based on explanation information, it is important to remember that pupils with weak arithmetic skills are those who give the greatest importance to explanatory information to understand a problem. Pupils with weak skills retain 81.3% of the explanation information, whereas pupils with strong arithmetic skills only retain 47.2% of the same information. Pupils who took the most explanatory information into consideration while constructing representations are those with the weakest arithmetic skills. One must also consider that students with weaker arithmetic skills performed better when given problems with explanation information, compared to situational information. The combination of these results tells us that pupils who construct a situation model based on explanation information are primarily pupils with weaker arithmetic skills and would therefore be, at first glance, those who would have the most difficulty solving arithmetic word problems. Moreover, the fact that these students had better performance rates when given problems with explanation information, as opposed to situational information, suggests that pupils who construct situation models based on explanation information may improve their performance in problem solving, even if they do not succeed as well as other students. This will have to be validated by other studies, but it provides a new avenue to explore for teachers working with students with weaker word problem solving skills.

# Situation Model vs. Performance

To date, the literature does not allow us to conclude that constructing a situation model to understand arithmetic word problems has positive effects on performance levels. Coquin-Viennot & Moreau [\(2003\)](#page-18-0) observed that by modifying the information included in the text of a

problem, one may encourage the construction of a situation model. However, the authors were unable to find decisive evidence of the influence of this process on the performance of pupils in problem solving.

The results obtained in the current study allow us to explain the divergence between two studies conducted to evaluate the effect of providing more elaborate contexts in word problems on the performance of pupils in problem solving. First, Cummins et al. [\(1988\)](#page-18-0) were unable to detect a difference in pupils' performance when providing situational information in a word problem. On the other hand, Stern & Lehrndorfer ([1992\)](#page-19-0), who also manipulated word problems, observed an increase in pupils' performance. Stern & Lehrndorfer ([1992](#page-19-0)) suggested that the conflicting results were due to differences in the type of modifications made to the problem. The changes made by Cummins et al. [\(1988](#page-18-0)) were situational in nature, whereas those performed by Stern & Lehrndorfer ([1992\)](#page-19-0) focused more on explanation information. For the current study, neither problems with added situational information nor those with added explanation information were solved with more success than simplified versions. These results, combined with those suggesting that pupils who give greater importance to situational information when trying to understand a problem achieve better performance, lead us to believe that situational information alone is not sufficient to increase pupils' levels of performance.

An important distinction must be made between the different versions of a problem: the situational and explanation information. The mere presence of situational and explanation elements does not imply that pupils will take them into consideration in order to better understand a problem. In other words, including situation information in a problem does not necessarily encourage pupils to construct a situation model. We have noted that the only pupils who increase their performance when given situational information in a problem are those who take these elements into consideration when creating a mental image of the problem. For all other pupils (those who paid no attention to this information), it may be that the presence of this information increases the difficulty of solving the problem, adding 'noise' around information that is essential to solve the problem. What brings us to this conclusion is the fact that results suggest that pupils who take situational information into consideration increase their performance and that problems containing situational information are generally not solved more easily.

In conclusion, we note that the presence of situational information in a problem can only increase a pupil's performance when he or she takes

<span id="page-16-0"></span>this information into consideration during his or her process of understanding the problem and if does so by constructing a mental representation, such as a situation model. Moreover, our analysis demonstrates that not all situational information is integrated the same way into the construction of a situation model. In the current study, only the information that specifically helped relate the arithmetic problem to a context (theme) was retained by pupils when developing a situation model and therefore had a positive influence on the pupil's performance. Finally, the results obtained show that pupils with weaker arithmetic skills do not base their comprehension on the same elements of a word problem statement, when compared with stronger students. These findings could have implications in classrooms. They shed new light on the interpretation of the difficulties experienced by students in solving problems. Teachers could find new ways to help students by insisting on different aspects of a problem, based on their individual characteristics, among other things.

### APPENDIX 1

# Task  $B_1$  (Example)

The problem statement that you just read contains several sentences. In the table below, we have already checked the sentences containing information essential to solving the problem. Among the other sentences of the statement, are there any that helped you understand the problem? Put a check mark  $(\swarrow)$  in the column next to each sentence that helped you understand the problem.



Task B2 (Example)

# Here is a problem that needs not be solved.

David's chess club has 35 members, and Valerie's has only 14. David's club should recruit 2 new members each month. Valerie's club should recruit 5 new members each month. Based on this data, in how many months will Valerie's club have more members than David's?

Add one or more sentences to make the problem easier to understand for a fellow student.

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APPENDIX 2

Rating Scale for Word Problems

5: Complete, relevant approach and correct answer

The student uses a diagram, a table, a list or any other procedure to compare both options and obtain an adequate answer.

4: Complete, relevant approach, but incorrect answer

The student made one or more miscalculations. The student made transcription errors.

3: Misinterpretation of the results obtained OR Coherent, partially relevant approach

The student misinterprets the results obtained to provide his/her answer.

<span id="page-17-0"></span>

<span id="page-18-0"></span>The student uses a coherent strategy (table, list, etc.) but does not thoroughly develop the selected approach. The strategy includes some form of comparison between the two options.

2: Beginning of a relevant approach

As a whole, the approach is not coherent.

The student selects the right operation (repeated addition) to obtain the result, but does not provide a method of comparing both options.

1: The approach selected by the student is not relevant

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