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## INSTRUCTIONAL COHERENCE IN CHINESE MATHEMATICS CLASSROOM—A CASE STUDY OF LESSONS ON FRACTION DIVISION

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**ABSTRACT.** In this study, we examined the instructional coherence in a Chinese mathematics classroom by analyzing a sequence of four videotaped lessons on the topic of fraction division. Our analysis focused on the characteristics of instructional coherence both within and across individual lessons. A framework was developed to focus on lesson instruction in terms of its content and process and the teacher's use of classroom discourse. The analyses of lesson instruction were further supplemented with the analyses of teaching materials and interviews with the teacher. The findings go beyond previous studies that mainly focused on a single lesson to provide further evidence about Chinese teachers' instructional practices and their possible impact on students' learning. In particular, the teacher tried to help students build knowledge connections and coherence through lesson instruction. Results also suggest that coherent curriculum and the teacher's perception of the knowledge coherence facilitated the teacher's construction of coherent classroom instruction.

**KEY WORDS:** Chinese classroom, classroom instruction, fraction division, instructional coherence, lesson structure, mathematics classroom

### INTRODUCTION

Over the past decades, accumulated findings from cross-national comparative studies indicated that students from East Asia performed well in school mathematics (e.g., Mullis, Martin, Gonzalez & Chrostowski, 2004; Robitaille & Garden, 1989). Efforts to improve students' learning of school mathematics worldwide have thus led to the ever-increasing explorations of possible contributing factors in high-achieving education systems in East Asia. It is generally acknowledged that classroom instruction is an important contributing factor. Existing studies have revealed that coherence is an important characteristic of mathematics classroom instruction in Asian countries (e.g., Hiebert, Gallimore, Garnier, Giwin, Hollingsworth Jacobs, Chui, Wearne, Smith, Kersting, Manaster, Tseng, Etterbeek, Manaster, Gonzales & Stigler, 2003; Shimizu, 2007; Stigler & Perry, 1988; Wang & Murphy, 2004) and that coherent mathematics lessons can help lead to students' better mathematics learning

with connected and coherent conceptual understanding (e.g., Baranes, 1990; Fernandez, Yoshida & Stigler, 1992). Yet, much remains to be explored about specific characteristics of mathematics lesson coherence that facilitate students' learning and possible contributing factors in Asian classrooms.

Getting a better understanding of mathematics lesson coherence becomes increasingly important as its benefits for students' learning are recognized. For example, it is specifically emphasized in the United States that mathematics education should provide students with a connected, coherent body of mathematical knowledge and ways of thinking (e.g., National Council of Teachers of Mathematics, 2000). Toward that end, developing and maintaining instructional coherence has gained special attention in the United States over the past few years (e.g., Finley, 2000; Wang & Murphy, 2004). As issues related to instructional coherence development are not restricted to specific regions, an in-depth examination of classroom instructional coherence in China can provide an interesting case for mathematics educators in other education systems. This study was thus designed to focus on a sequence of lessons in one Chinese teacher's mathematics classroom. We aimed to examine instructional coherence from multiple perspectives both within and across lessons. We also tended to identify possible contributing factors that might influence the teacher's lesson instruction. In particular, we focused on the teacher's perception about mathematics and teaching and possible influence of the intended curriculum (i.e., teaching materials).

## RESEARCH BACKGROUND

### *What Do We Know About Instructional Coherence and Possible Contributing Factors?*

Coherence is a concept that has been explored in many disciplines from different perspectives. For example, Thagard & Verbeurgt (1998) provided a computational characterization of coherence as constraint satisfaction that can be applied to many philosophical problems and psychological phenomena. Their characterization of coherence emphasized the dynamic process of reaching the maximum satisfaction given a set of positive and negative constraints. In contrast, the Merriam-Webster online dictionary provides a definition of coherence as “the quality or state of cohering as (a) systematic or logical connection or consistency, (b) integration of diverse elements, relationships, or values.” This definition provides a perspective that focuses on the quality or state of

logically connected or integrated elements, but not the process. Existing studies on instructional coherence tended to explore the quality or state of diverse instruction elements being connected or integrated.

According to Wang & Murphy (2004), instructional coherence can be defined as causally linked activities/events in terms of the structure of instructional content and the meaningful discourse reflecting the connectedness of topics, which benefits students' learning of mathematics (e.g., Baranes, 1990; Fernandez et al., 1992; Stevenson & Stigler, 1992; Trabasso, Secco & van den Brock, 1984). Stein & Glenn (1982) used a metaphor to indicate that a good mathematics lesson is like a story. It is more than just a sequence of events. Each event must be organized and interconnected such that the story has a beginning, middle, and end, as well as a consistent theme that runs throughout the lesson with a clear scheme. Instructional coherence is often observed in terms of how well lessons follow a logical and structured sequence of events and how well lessons focus on one or related topics and proceed from simple to complex situations in the process of concept development. Therefore, instructional coherence should consist of sequences of events that are related to each other and afford a more coherent representation and effectiveness than those that are not (Fernandez et al., 1992).

A well-structured lesson also makes it possible for students to infer relationships among events. It is because "objectives often provide the logical link from one activity to the next, not stating objectives may prevent children from seeing that what they did at one point in the lesson is important for understanding what they are doing at a later point during the lesson" (Fernandez et al., 1992, p. 337). Thus, teachers need not only to structure each activity in a coherent way but also to have a coherent discourse for the clarity of the lesson objectives. In other words, teachers should provide their students with a certain coherent vision of mathematics knowledge through their classroom structure and discourse (e.g., Segiguchi, 2006; Tomlin, Forrest, Pu & Kim, 1997). For example, Chinese teachers specified instructional objectives in their lesson introduction to guide students' learning (e.g., Li & Chen, 2009). By sharing the same objectives, teachers can establish the effective learning environment and make students' activity meaningful.

In existing studies, researchers mainly focused on the structure, activity/event coherence, and teachers' discourse in a single lesson. For example, Stigler & Perry (1988) compared mathematics classrooms in China, Japan, and the United States and found that both Japanese and Chinese lessons were structured more coherently than American lessons. They pointed out that teachers in these Asian countries would spend an

entire mathematics lesson on the solution of only one or two problems for learning a content topic. One problem may also serve to provide topical continuity across different segments of a lesson. Thus, the lesson can be constructed more coherently. Researchers concluded that students could enhance their mathematical reasoning and insight through classroom instructional coherence. The coherence of the lesson's structure and related events can help students build on their former mathematical knowledge, connect with their new knowledge, and comprehend their mathematical knowledge more deeply. Likewise, Hiebert et al. (2003) specified the coherence in terms of the (implicit and explicit) interrelations of all mathematics components of a lesson, and reported that lessons in Hong Kong have a central theme that progresses saliently through the whole instruction process. Leung (2005) also pointed out that lessons in Hong Kong are more coherent than those in other countries based on the TIMSS video study.

It is important to point out that causally linked activities/events are not enough to achieve lesson coherence, either for a single lesson or a sequence of lessons. In order to develop students' mathematics knowledge, it also requires teachers to have coherent mathematics knowledge to implement the coherent curriculum. Classroom instruction coherence found in Asian countries (e.g., Stigler & Perry 1988) may well relate to both the curriculum coherence (Schmidt, Houang & Cogan, 2002) and teachers' knowledge (e.g., Ma, 1999). In particular, Schmidt et al. (2002) identified curricula to be coherent if content standards are articulated over time as a sequential or hierarchical nature. They further indicated that a set of content standards must evolve from particular to deeper structures inherent in the discipline. They argued that the coherent curricula in East Asian countries helped build students' understanding of the big ideas and the particulars of mathematics. Researchers also found that Chinese teachers tended to pay close attention to textbooks when organizing content for teaching (e.g., Li, Chen & Kulm, 2009). However, much remains to be understood about how teachers implement the coherent intended curriculum (e.g., textbooks) in classroom instruction.

In order to implement a coherent curriculum in classroom instruction, another key factor should be teachers' knowledge. Shulman (1986, 1987) raised the idea of pedagogical content knowledge. It required teachers not only to have strong subject matter knowledge, but also to have the ability to use their subject matter knowledge in their teaching practice. Ma (1999) further pointed out that Chinese teachers had specific knowledge packages with regard to particular content topics. These knowledge packages consisted of (1) key ideas that “weigh more” than others in the

package, (2) instructional sequences for developing the ideas, and (3) “concept knots” that link related ideas. Ma coined the term of *Profound Understanding of Fundamental Mathematics* based on the knowledge (a) depth, which referred to large and powerful basic ideas, (b) breadth, which had to do with multiple perspectives, (c) thoroughness, which was essential to weave ideas into a coherent whole, and (d) connectedness, which related to the above three. Yet, few studies focused on teachers' instructional practices to see how they constructed lessons coherently.

Thus, the current study intended to examine and discuss how a Chinese teacher implemented the curriculum in the lesson instruction process and how the teacher created opportunities for students to construct compatible knowledge coherence.

### *Analysis of Instructional Coherence*

Explicit research on instructional coherence is a relatively new endeavor. Existing studies presented various focuses undertaken by different researchers in analyzing instructional coherence.

Using the video data from the TIMSS study, Hiebert et al. (2003) and Leung (2005) pointed out that mathematics classrooms in Hong Kong are thematically coherent. In particular, Leung indicated that “90% of the Hong Kong lessons are judged to be thematically coherent, with the remaining 10% moderately thematically coherent” (p. 207). These studies focused on the coherent instructional scheme and reported that coherence is one characteristic in Asian mathematics classrooms, but with no further explanation.

Wang & Murphy (2004) analyzed the structure coherence of Chinese mathematics lessons and mainly focused on the causally linked activities and related discourse. By focusing on the role of connectedness in the teacher's talk and behavior in the creation of meaningful discourse, their study revealed how mathematical activities within individual lessons relate to each other. Through examining the instructional, psychological, and social dimensions of instructional coherence in the classroom, the researchers indicated that the Chinese classroom instruction could be characterized as coherent from a cultural perspective. Yet, further study is needed to explore how a lesson may help develop students' knowledge coherence. Moreover, although the researchers also tended to analyze the instructional coherence across lessons, they indicated that the results do not present a clear picture of coherence.

Others focused on specific activity components and identified the consistent instructional structure in Asian classrooms (e.g., Shimizu, 2007; Stigler & Hiebert, 1999). For example, Shimizu (2004, 2007)

focused on the feature of “summing up” in Japanese lessons and concluded that summing up can help make the lesson consistent and clear. Summing up, called “matome” in Japanese, is considered an important feature in Japanese mathematics classrooms. According to Shimizu, “matome” can take place not only at the end of a lesson but also in each activity segment of a lesson to pull together students' activities in the lesson. Furthermore, Shimizu pointed out four roles of “matome,” which include (1) highlighting and summarizing the main points of the lesson, (2) promoting students' reflection on their experiences by reviewing what they have done, (3) setting the stage for introducing a new mathematical concept or term based on students' previous experiences and applying it, and (4) making connections between the current topic and previous ones. The results suggested that “matome” helps establish coherent structure in Japanese classrooms. Moreover, “matome” also can be considered important to bring the whole lesson together to a central theme.

Shimizu's analysis (2007) further revealed that Japanese teachers use explicit classroom discourse to accomplish the role of “matome.” In Japanese classrooms, lesson activities are often devoted to solving a single problem. Thus, teachers used explicit discourse in “matome” to make all activities interrelated to each other so that a lesson structure can be coherent. Compared with Asian classrooms, Shimizu pointed out that “summing up” in US classrooms does not look like a consistent structural feature of a lesson in the same way it appears in Asian (e.g., Japan, China) classrooms. Therefore, “matome” has its unique characteristic for coherent teaching in Asian classrooms.

Stigler & Perry (1988) also pointed out the importance of the explicit discourse to make a lesson structure coherent. They argued that, although some American lessons were well structured, students could not perceive the structure because the teacher failed to discuss the needed connections. Separate activities were not automatically put together to make sense in relation to each other. Teachers must explicitly point out the relationship between two activities or events. Therefore, classroom discourse, or the language that teachers use, is very important for students to understand the relationship between two activities.

Taken together, existing studies suggest that a consistent topic or solving a single problem through multiple activity segments and teachers' language are very important for having a coherent instruction in mathematics classrooms. However, how multiple activity segments are constructed in a single lesson, how the central theme is conducted through a sequence of lessons, and how the class discourse complements instructional coherency need to be further explored.

## RESEARCH QUESTIONS

In this study, we aimed to examine instructional coherence of lessons on the content topic of fraction division. Studies of students' and teachers' understanding showed that fraction division is one of the least understood concepts and algorithms in elementary and middle school mathematics (e.g., Li, 2008; Sinicrop, Mick & Kolb, 2002). The content topic of fraction division is procedurally straightforward and is often taught and learned as “invert and multiply.” The difficulty is partially due to the fact that fraction division requires conceptual understanding of both division and fraction concepts (e.g., Armstrong & Bezuk, 1995). Fraction division has many different interpretations and the use of fractions in division makes this concept even more complicated for the students and teachers (e.g., Borko, Eisenhart, Brown, Underhill, Jones & Agard, 1992; Ma, 1999; Sowder, 1995). Thus, the topic is conceptually rich and difficult, as its meaning can be explained through its connections with other mathematical knowledge, various representations, or real-world contexts (e.g., Li, 2008). By focusing on a sequence of four lessons on fraction division, we aimed to develop a deeper understanding of possible instructional coherence embedded in one Chinese mathematics classroom in terms of the teacher's discourse and the lesson structure. In particular, we tended to address the following research questions:

1. What is the nature of instructional coherence in individual lessons that a Chinese teacher constructed from the curriculum in teaching fraction division?
2. What is the nature of instructional coherence in a sequence of lessons that a Chinese teacher constructed from the curriculum in teaching fraction division?

## METHOD

*Participant and Data Source*

A case study approach and qualitative analysis methods were employed in this study to address these research questions. The case study examined the features of instructional coherence in one Chinese teacher's, Ms. X, classroom both within individual lessons and across a sequence of four lessons on fraction division. The school where Ms. X worked was a “key” elementary school<sup>1</sup> in the eastern part of China.<sup>2</sup> Ms. X had over 10 years

of teaching experience and actively participated in teaching research within the school and in the school district. Ms. X continually taught at the sixth grade level and was familiar with the textbook. She was also familiar with the content topics that she taught and the connections and relationships among them. On the other hand, through the interview, we found that, although she had not taught these students until the sixth grade, she knew about her students' learning very well. The data of Ms. X's classroom instruction for this study came from a larger data set of a research project. Although Ms. X shared a similar professional background as many other Chinese elementary school mathematics teachers in the project, Ms. X can be taken as an above-average mathematics teacher.

The data we used in this study included the videotaped lessons, the curriculum materials, and the interview. Videotaped lessons were used as primary data and the lesson analysis was supplemented with the data of curriculum materials and our interviews in order to triangulate our findings. Videotaped lessons included four lessons of fraction division (i.e., a fraction divided by a whole number; a whole number divided by a fraction; a fraction divided by a fraction; and application of fraction division). The curriculum materials included a textbook and teachers' instructional guidebook. Semistructured interviews were carried out to study the teacher's perceptions of teaching and learning mathematics.

There were over 50 students in the class. Each lesson lasted about 40 min. With this rich set of data, we examined this Chinese teacher's instructional coherence in depth, within individual lessons and across a sequence of four lessons.

### *Data Analysis*

The videotaped lessons were transcribed and analyzed in the original Chinese first and then translated into English if needed. The curriculum materials and the interview data, which were also analyzed and then translated into English, supplemented the analysis of videotaped lessons.

To examine the nature of instructional coherence, existing studies suggested the importance of both the lesson content and its instructional process (e.g., Li & Li, 2009; Stigler & Hiebert, 1999; Wang & Murphy, 2004). We thus developed a two-dimensional framework that includes both content and process aspects for analyzing instructional coherence embedded in individual lessons and a sequence of lessons. Moreover, instructional coherence was further revealed through examining the teacher's discourse.



*Analyzing a Single Lesson.* In analyzing a single lesson, we focused on the first lesson of a fraction divided by a whole number. We first examined the lesson's content aspect. That is, what was the content covered and how was it organized in this lesson? The analysis aimed to reveal how the knowledge or content theme was developed and interconnected for students' learning. We focused on the relationship between the instructional objectives and important and difficult points of the lesson.

We further examined the lesson from the process aspect in terms of causally linked activities (i.e., Stigler & Hiebert, 1999; Wang & Murphy, 2004) and the pedagogical strategies the teacher used during the lesson. Following the method used in the TIMSS video study (Stigler & Hiebert, 1999), the lesson was first partitioned in terms of its activity segments. The relationships among activity segments were then coded. In particular, possible relationships between segments in these four lessons were classified into three categories: (1) two segments were similar with respect to the basic mathematical idea; (2) the second segment was dependent on the first segment procedurally, that is, students could apply the methods they used in the first segment to begin creating or solving problems; (3) the second segment extended the first one procedurally and conceptually as the complexity of the problem increased.

By analyzing possible relationships between segments in the first lesson, the lesson's structure coherence was examined. The lesson's structure coherence was further revealed in terms of the way that the whole lesson is devoted to a clear objective or theme and connections of relevant concepts, facts, and procedures.

*Analyzing a Sequence of Four Lessons.* In analyzing a sequence of lessons, we also focused on the content and process aspects. For the content part, we focused on the lesson connections between adjacent lesson and the flow of mathematical ideas between them. We further discussed possible links in the sequence of these four lessons as the process aspect. In particular, we examined reviewing and closure parts in each lesson and focused on knowledge interconnections across lessons. Besides the knowledge connections used to create coherence, we also focused on the hierarchical structure of the content topic across the sequence of lessons.

*Analyzing the Teacher's Discourse.* In general, the teacher's discourse is very important to enhance a coherent structure both within individual lessons and across a sequence of lessons. In order to analyze discourse

coherence in the instructional process, we coded the teacher's discourse as guided by Shimizu's work (2004, 2007). In particular, our coding focused on the teacher's discourse in enhancing the lesson's structure and theme coherence. Relevant codes include: (1) making explicit transitions from one activity to another, which occurred in a single lesson; (2) making causal links to previous knowledge both within a single lesson and across the sequence of lessons; (3) promoting students' reflections on their experiences by reviewing what they have done both within a single lesson and across the sequence of lessons; (4) highlighting or summarizing the main points of the lesson; (5) setting the stage for introducing a new mathematical concept or term based on students' previous experiences; and (6) making explicit statements about the lesson's objectives or goals. Here, the explicit transition and causal link were identified if an explicit verbal reference was made by the teacher to ideas or events from another lesson or part of the lesson. The reference must be concrete (referring to a particular time or topic, not to some general ideas) and should relate to the current activity or topic.

*Analyzing the Teaching Materials and Interviews with the Teacher.* In order to explore what may contribute to the instructional coherence, we went beyond the teacher's classroom behavior and discourse. We supplemented the lesson analyses with the teaching materials to see how the content topic was structured in the textbook and what relationship may be presented among different content topics. Besides the teaching materials, the semistructured interviews with the teacher complemented our understanding of the teacher's perceptions about mathematics and teaching. Because the teacher's perception might strongly influence the way she prepared and constructed her classroom instruction, the data allowed us to examine how this teacher thought about instructional coherence and why she explored her teaching in a coherent way.

## RESULTS

Results from lesson analyses revealed the characteristics of the teacher's classroom instruction from both within individual lessons and across a sequence of four lessons. In general, all four lessons used worked-out examples. Each individual lesson mainly focused on one example, which was also provided in the textbook. Moreover, the teacher consistently used a similar approach in teaching fraction division in all four lessons, which can be generally characterized as "conjecturing and justifying." In

the following sections, we first report the features of instructional coherence revealed from individual lessons. Furthermore, we examine the features across the sequence of these four lessons.

### *Main Features within Individual Lessons*

In general, four lessons were structured in a similar way. They all contained the same seven activity segments, which are reviewing previous knowledge (RPK), presenting one or two problems related to previous knowledge (PP), group discussion (GD), students solving individually (SSI), discussing methods or solutions (DS), summarizing and highlighting (S/H), and application and homework (A/H). All these segments can be grouped into three big categories, including reviewing, introducing new content, and closure (see Figure 1).

In the following subsections, we will first present the content features of the first lesson's instructional coherence and then focus on the way that each of those activity segments was structured in order to make the thematic coherence.

*Content Features of Instructional Coherence in the First Lesson.* The content topic in the first lesson is “a fraction divided by a whole number.” According to the teachers' guidebook (Jiangsu Province Research Group for Elementary and Middle School Mathematics Teaching, 2001b), the instructional objectives are the concept of fraction division and the computational rule of a fraction divided by a whole number and the difficult point of this lesson is the conceptual understanding of the algorithm.

Overall, the whole lesson was devoted to a clear instructional scheme of understanding the concept of fraction division and mastering the algorithm of a fraction divided by a whole number. When the teacher introduced this new content, she indicated clearly that the concept and the algorithm are the two important aspects of fraction division learning. The concept of fraction division was defined as the computation of finding a factor with the given product and another factor. The teacher devoted over 30% of the time for reviewing previous knowledge and introducing

Reviewing	Introducing new content				Closure	
RPK	PP	GD	SSI	DS	S/H	A/H

Figure 1. Transition between activity segments

the concept of fraction division. From this concept, the teacher focused on the idea that fraction division is the inverse operation of fraction multiplication, which is the same idea between whole number division and whole number multiplication.

In introducing the algorithm of a fraction divided by a whole number, the teacher devoted 36% of the time to conjecture, justify, and discuss their answers. The teacher provided a worked-out example that was presented in the textbook (Jiangsu Province Research Group for Elementary and Middle School Mathematics Teaching, 2001a), and discussed two ways for solving it with students (see Figure 2). By discussing multiple strategies of carrying out the algorithm, the teacher intended to overcome the difficult point of students' learning in this lesson, which is the conceptual understanding of the algorithm.

The teacher compared these two computational ways to help students discover that the first computational strategy has the limitation and the second one is a general way for the computation of a fraction divided by a whole number. The teacher then extended the example to the case when the numerator is not a multiplier of the divisor in order to generalize the computational rule of a fraction divided by a whole number.

At last, the teacher provided multiple types of problems in order to achieve two instructional objectives for this lesson. The problems helped students not only learn the concept, but also understand the quantity relationship involved in fraction division. The teacher then provided several more problems for students mastering the algorithm.

*Process Features of Instructional Coherence in the First Lesson.* In general, the lesson was structured with three activity parts from reviewing

<p>Problem: A <math>\frac{4}{5}</math>-meter-long rope was divided into two equal pieces, what is the length of each piece?</p> <p>1) The first way is to divide the <math>\frac{4}{5}</math>-meter-long rope into two equal pieces, which means to divide 4 of <math>\frac{1}{5}</math>-meter-long pieces into two equal groups, that is, <math>\frac{4 \div 2}{5}</math>. Thus, each group contains 2 of <math>\frac{1}{5}</math>-meter-long pieces, which is <math>\frac{2}{5}</math>-meter.</p> <p>2) The second way of solving is that dividing <math>\frac{4}{5}</math>-meter rope into two pieces is the same of finding <math>\frac{1}{2}</math> of <math>\frac{4}{5}</math>-meter. Thus, it would be <math>\frac{4}{5} \times \frac{1}{2}</math>.</p>
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Figure 2. Two ways of solving a problem used in content introduction

to closure (see Figure 3). Each part included multiple activity segments. The pedagogical flow proceeds from reviewing, teaching new content, to closure (see Figure 1). The relationship among different activity segments was coded in terms of three categories, as specified in the “Data Analysis” section. All the activities in the lesson were carried out through problem solving.

Figure 3 reveals that each activity segment is connected one to another. The PP segment extends RPK procedurally in terms of the instructional process and conceptually as the problem's complexity increased. The GD segment can be considered as procedurally dependent on PP. Students used their previous knowledge to conjecture the answer. SSI depends on GD procedurally. DS expands SSI procedurally and also conceptually in terms of the content. By discussing problem solving throughout the whole lesson, the teacher explored the problem from the specific (i.e., the

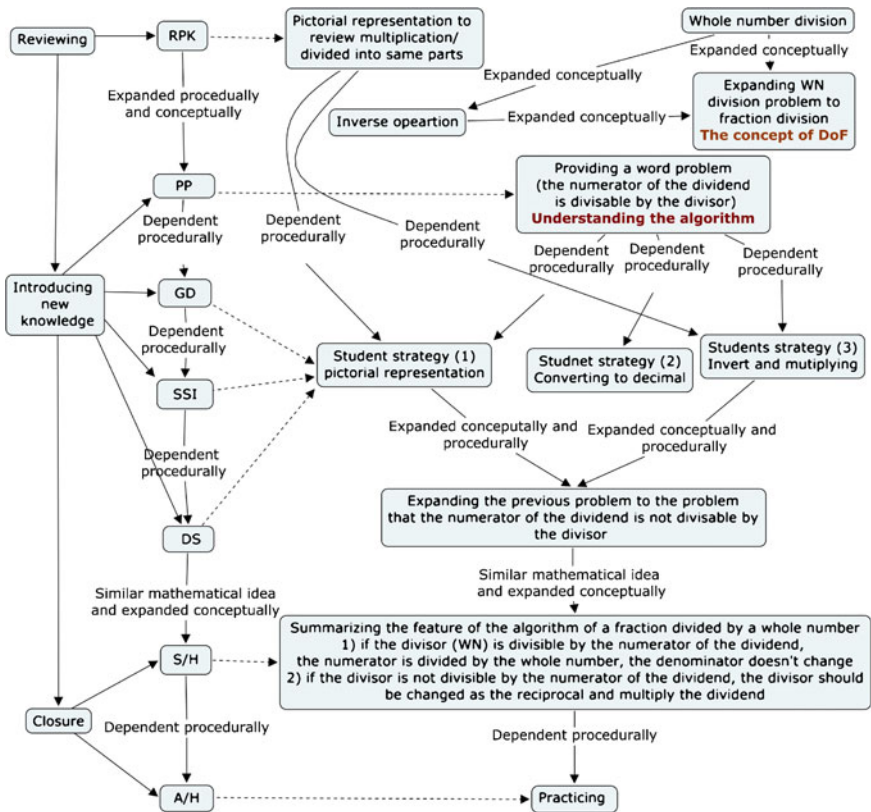


Figure 3. Relationships among the lesson's activity segments and its content

numerator of the dividend is divisible by the divisor) to the general case. Through their discussions, the teacher and students provided multiple representations for understanding. S/H is summarizing and can be considered as addressing similar mathematical ideas and making the generalization. A/H can be considered as extending S/H because the complexity of the problems increased.

Along the process of classroom instruction, the lesson content was developed within and across activity segments (see Figure 3). For example, in the activity segment of RPK, Ms. X used a pictorial representation to review the relationship between fraction multiplication and fraction division. She drew a pictorial representation to show " $\frac{2}{3}$ " (Figure 4-(1)). Ms. X continued the drawing (Figure 4-(2)) and asked the students what it means. A student answered, "dividing two thirds into two same parts and taking one part of it." Ms. X asked what it means in other words. "Finding how much is  $\frac{1}{2}$  of  $\frac{2}{3}$ ?" students answered. Ms. X confirmed that these two expressions (e.g.,  $\frac{2}{3} \times \frac{1}{2} = \frac{2}{3} \div 2$ ) have the same answer. In this way, Ms. X made a connection among students' previous knowledge, in particular, the fraction multiplication and even division.

Through this reviewing, Ms. X introduced the new mathematical content topic, fraction division. Furthermore, Ms. X used real-world examples to review the concept of whole number division. The examples indicated the relationship between the concept of whole number division and whole number multiplication. Based on these examples, Ms. X asked students to convert the whole number (e.g., 750 g) into a fraction (e.g.,  $\frac{3}{4}$  kg) by changing the unit, without changing the quantity of the equation. Ms. X indicated that the concept of fraction division is the same as the concept of whole number division and is also the inverse operation of multiplication.

The whole lesson focused on solving one worked-out example. Among all seven activity segments, four activity segments (i.e., PP, GD, SSI, and DS) were used in presenting and discussing the worked-out example,

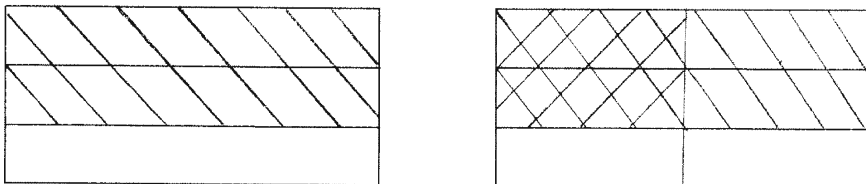


Figure 4. Pictorial representations of (1) and (2)

while all seven segments were devoted to the same objectives or goals in each lesson. Furthermore, exploring the verbal explanation in the textbook, the teacher asked the students to conjecture their solutions and to prove their conjectures. The teacher also used line segment representation and mathematical reasoning for understanding. Here, the teacher combined the goal in the teacher's guidebook and the textbook's worked-out example to help students make sense of the algorithm.

In order to make each activity segment and knowledge coherent, this Chinese teacher provided explicit explanations. For example, after explaining fraction multiplication and fraction division, the teacher clearly addressed the relationship between them. The following example shows that Ms. X used explicit explanation to make a connection between two pieces of knowledge.

Dividing two third into two parts and taking one part of it, it is (One student:  $\frac{1}{2}$  of  $\frac{2}{3}$ ). If we want to find  $\frac{1}{2}$  of  $\frac{2}{3}$ , what do we need? (Students: Multiplication). (Teacher wrote the expression  $\frac{2}{3} \times \frac{1}{2}$ ). Good, when I drew the picture, someone told me this is dividing  $\frac{2}{3}$  into 2 parts. If we use the knowledge of even division to rewrite this expression, what is it? (One student: would be  $\frac{2}{3} \div 2$ ). Agree with him? (Students together: Yes). (Teacher wrote the expression of  $\frac{2}{3} \div 2$ ). It means evenly divided  $\frac{2}{3}$  into 2 parts, and to find how much in one part, right? ... If the two expressions described the same answer, we can use (equal sign) to connect them. It means the answer of these two different expressions is same.

Moreover, the teacher provided explicit discourse in each segment to make a transition from one activity to another. Each transition lasted at least 10 seconds. The explicit transition helped students to reflect on what they had already done in the previous activity. Here, the explicit transition played an important role to constitute a coherent system of all the activities. For example, from Figure 1, we can notice that the teacher intended to connect with prior knowledge and experiences (i.e., the knowledge of fraction multiplication learned from the previous lesson) as follows.

T: We used two pictures and came out with two multiplication expressions. We came out with two division expressions based on our knowledge of "evenly dividing." We have learned multiplication of fractions, but we did not learn division of fractions. We can learn division of fractions based on our previous knowledge. This is important for mathematics learning in the future. ... We will learn division of fractions today. Can you guys recall what we learned about multiplication of fractions?

Taken together, the lesson's thematic coherence and its structure coherence were enhanced with the teacher's explicit explanation in classroom discourse.

*Main Features Across Lessons*

Following the content structured in the textbook, the teacher constructed four consecutive lessons as: (1) learning the concept of fraction division and the algorithm of a fraction divided by a whole number; (2) learning the algorithm of a whole number divided by a fraction; (3) learning the algorithm of a fraction divided by a fraction and generalizing the algorithm of fraction division; (4) using fraction division to solve word problems. Except for the last lesson, the other three lessons focused on students' conceptual understanding and mastering of the computational rules. Furthermore, these four subtopics were devoted to a consistent theme, which is to develop students' conceptual and procedural understanding of fraction division.

*Content Features of Instructional Coherence Across Lessons.* The first three lessons were devoted to developing students' conceptual understanding of the algorithm from specific cases (e.g., a fraction divided by a whole number) to a general case (i.e., a fraction divided by a fraction). In the first lesson, the teacher helped students conclude the first two strategies of doing the algorithm of a fraction divided by a whole number based on students' conjecturing and justifying their answers. The first way is that the numerator is divided by the whole number directly and the denominator does not change if the numerator is divisible by the whole number. The second way is to multiply the reciprocal of the divisor. Using multiple strategies, the teacher tried to develop students' conceptual understanding of why the computational rule works. Furthermore, the teacher guided students to realize that the second strategy is the general way of doing the algorithm of a fraction divided by a whole number.

Thus, in the second lesson (a whole number divided by a fraction), some students conjectured that the general way that they learned previously could also be used in this case. Like the first lesson, the teacher first provided students a real-world problem (i.e., a car runs 18 km in  $\frac{3}{10} h$ , what is the speed of this car?). The teacher further gave a pictorial representation for the quantity relationship (i.e., a line segment was divided into ten equal parts, three parts were used to represent that the car runs 18 km in  $\frac{3}{10} h$ ). Based on the pictorial representation and quantity relationship, students indicated that the number sentence for this problem is  $18 \div \frac{3}{10}$  (i.e., speed [kilometers per hour]=distance $\div$ the number of hours). In order to solve the problem, the teacher guided students to think in two steps. (1) A  $\frac{3}{10} h$  means three of  $\frac{1}{10} h$ . Students



first needed to know how far the car goes in  $\frac{1}{10} h$ , thus, it should be  $18 \div 3$ . (2) One hour means ten of  $\frac{1}{10} h$ . Students should multiply ten to get the answer of how far the car goes in 1 h. Thus, it should be  $18 \div \frac{3}{10} = (18 \div 3) \times 10 = 18 \times \frac{10}{3}$ . These two steps showed why the reciprocal should be multiplied.

The third lesson used the same example but changed the quantity to a fraction (as the case of a fraction divided by a fraction). The students were also encouraged to conjure and justify their answer based on the previous knowledge. Most students could justify their answer in the way that the teacher did in their previous lessons. The examples in the three lessons devoted to a scheme that helped students conceptually understand why the algorithm should be “invert and multiply.” The same activity segment and method for conjecturing and justifying across the three lessons can help students make the connection among these lessons. The results showed that students were given only 18% of lesson time for their individual work (conjecturing and justification) in the first lesson, while they spent 43% of lesson time in the third lesson.

The last lesson was for the application. It required students to not only do the algorithm clearly, but also be able to write a number sentence (or an algebraic equation) based on their understanding of the concept of fraction division and related quantity relationship.

Taken together, the first three lessons can be coded as procedurally developed based on the previous lesson. The last lesson reflected the algorithm and tried to help students master the algorithm through problem solving. The first three lessons were used to focus on relevant concepts and procedures of fraction division. Each lesson focused on one aspect of the algorithm. In the fourth lesson, the teacher reviewed the conceptual aspect of fraction division, the computational rules of fraction division, and then focused on the application of fraction division. The application focused on solving word problems that involve fraction division. These four lessons were structured as evolving from dividing a fraction with a whole number to solving word problems, which made the whole content unit coherent as structured in the textbook. All four lessons emphasized what fraction division is, how to calculate it, and why the computation rule works.

Moreover, the teacher used the experience that the students had in the previous lesson and the knowledge they had already learned in lesson instruction. The teacher mainly directed students to justify their conjectures in the first two lessons, helping students to become familiar with the process of mathematical thinking. In the third lesson, the teacher gave students more opportunities to think and explore possible solutions

by themselves. The consistency of events across the lessons (i.e., students guessing the answer, justifying their answers, and reporting their strategies) allowed the teacher to enhance the instructional coherence in the teaching process.

*Process Features of Instructional Coherence Across Lessons.* In order to examine the structure of the lesson sequence, we focused on the reviewing and summing up (or called “matome” in Japanese lessons; Shimizu, 2007) in each lesson (Figure 5). The teacher frequently used explicit verbal references to point out the relationship with the prior knowledge at the beginning of a lesson and to make connections for further learning at the end of a lesson.

Figure 5 shows that the summarizing in the previous lesson was reviewed at the beginning of the follow-up lesson. The beginning of the first lesson reviewed whole number division and fraction multiplication in order to introduce the content topic, fraction division. All four lessons were structured coherently for the goal of developing students' understanding and mastering the algorithm of fraction division. At the end of the fourth lesson for this content topic, the teacher worked with students to summarize the features of the algorithm.

Moreover, the teacher's explicit explanation enhanced the coherent structure of a lesson. The explicit explanation can be found in both the reviewing (beginning) and summarizing activity segments with a certain amount of time (see Table 1). The teacher's explicit explanation also helped build connections across lessons as for teaching and learning of a coherent content topic.

Interestingly, the teacher made explicit links not just at the beginning or the end of a lesson. In the middle of a lesson, the teacher often made clear connections by asking her students to check the pattern of relevant

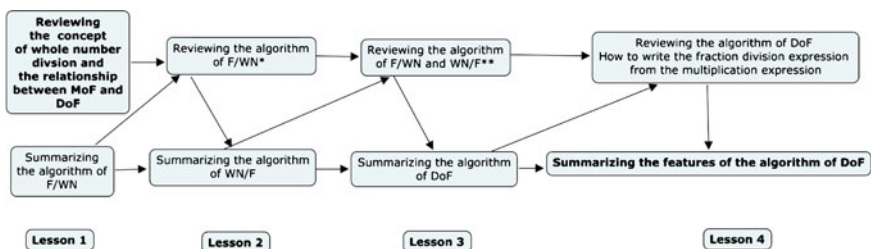


Figure 5. The relationship between reviewing and summarizing across lessons. F/WN means a fraction divided by a whole number (indicated by *one asterisk*). WN/F means a whole number divided by a fraction (indicated by *two asterisks*)

TABLE 1  
Time used for explicit explanation

<i>Explicit explanation during</i>	<i>Lesson 1</i>	<i>Lesson 2</i>	<i>Lesson 3</i>	<i>Lesson 4</i>
Review (beginning)	13:30 <sup>a</sup>	1:55	9:36	5:49
Summarizing/highlighting	1:08	4:38	2:24	8:04

<sup>a</sup>For example, 13:30 means 13 min and 30 s

computations or by encouraging students to use the same method when solving a different problem. Especially when individual students worked to examine the formula of fraction division in the latter lessons, the teacher encouraged students to recall and use the same method that they had used in the first lesson. Through encouraging students' use of similar methods in solving problems, the teacher helped provide a coherent picture for students of what they need to learn and why.

Another feature is that the teacher used a similar approach and presentation to make those four lessons consistent and coherent. For example, Ms. X used the approach of conjecture and justification to help students understand the algorithm both conceptually and procedurally. Moreover, she consistently used the same pictorial representation (i.e., line segments) in those four lessons.

### *The Teacher's Perception*

Based on the interview data, we found that the teacher considered the connectedness in the mathematical content topic is very important. For instance, the teacher considers that the concept of inverse is one important idea in mathematics learning (i.e., multiplication is the inverse of division). Thus, using the previous knowledge to solve the problem that students have not known is an important strategy to learn mathematics. Thus, she intended to construct the lesson in a way of "learning new knowledge based on students' previous knowledge and learning." The teacher provided an example during the interview. The students have already learned fraction multiplication. A number ( $\frac{1}{2}$ ) multiplying another ( $\frac{3}{4}$ ) means to find how much is  $\frac{1}{2}$  of  $\frac{3}{4}$ . It also can be considered that dividing  $\frac{3}{4}$  into two parts and how much is each part. This is the concept of evenly dividing. The students already learned multiplication and can easily find the connectedness between these two expressions. Thus, students can easily conjecture the answer of a fraction divided by the whole number.

## DISCUSSION AND CONCLUSIONS

*What Can We Learn About Instructional Coherence in Chinese Classroom from this Case Study?*

This study aimed to examine the characteristics of instructional coherence in a Chinese teacher's classroom. Based on the results, we analyzed the teacher's lessons from both their content and process aspects.

For the individual lesson, all activity segments were devoted to achieve the lesson's instructional objectives. The content features showed that the teacher used and explored one example for helping students understand the algorithm of a fraction divided by a whole number. The example explored the case from specific (i.e., the numerator of the dividend is divisible by the divisor) to general (i.e., the numerator of the dividend is not divisible by the divisor). All activities that were fit into three categories mainly concentrated on solving this one worked-out example, which was provided in the textbook. A clear central theme went throughout the lesson with a single worked-out example. This instructional approach let the teacher construct the lesson more coherently.

Moreover, the teacher structured the lesson with a coherent pedagogical flow that proceeds from reviewing, teaching new content, to closure. This structure made each lesson like a story, having a beginning, middle, and end (Stein & Glenn, 1982). Like Stein and Glenn mentioned, each activity segment was organized as more than just a sequence of events. It was organized and interconnected such that there is a consistent theme, understanding the concept of fraction division and mastering the algorithm of a fraction divided by a whole number, which runs throughout the whole lesson. Thus, activity segments had a close relationship with each other and they were put together to explore the mathematical idea.

Besides the activity segments or categories, the teacher used explicit and clear explanation to complement her coherent teaching. From the results, we know that the teacher consistently used transitions between activity segments. The explicit explanation was provided not only in the transition parts, but also when the teacher made a connection with previous knowledge. The explicit explanation also helped students understand the purpose of an activity, how to do it, and why to do it. Since "objectives often provide the logical link from one activity to the next, not stating objectives may prevent children from seeing that what they did at one point in the lesson is important for understanding what they are doing at a later point during the lesson" (Fernandez et al., 1992, p. 337).

Across the sequence of these four lessons, we noticed that the four lessons were constructed from easy to difficult and from specific to general. Moreover, the sequence of the four lessons was devoted to a single content topic, fraction division, through relevant concepts, computations, and applications. Thus, these four lessons constituted for teaching a coherent content topic.

The results showed that the teacher used a consistent approach in teaching these four lessons, which is to let students conjecture and justify their own answers. The students' guessing of the solution, their individual work, and the discussion were the most important parts, and thus occupied over half of the time in each lesson. The length of students' individual work segment also increased over the lessons (used 13 min in the third lesson). Besides the approach, the teacher used the same representations with a similar order in each lesson. The teacher first used the pictorial representation (mainly used line segment) to help students understand the problem. Moreover, the pictorial representation also helped students conjecture the answer. Then, the teacher provided further verbal explanation of the pictorial representation. Finally, the numerical representation of a possible solution was provided. The consistent use of representations may help students infer relationships among representations. The consistent approach and use of representations also helped students know what they needed to do next and how to do it. From the video, we found that, in the third and fourth lessons, the majority of students were able to use at least one way to present and justify the algorithm of fraction division.

To make the instructional coherence possible, our analyses revealed that the textbook's coherent content structure helped the teacher to construct classroom instruction coherently both within individual lessons and across those lessons. As Schmidt et al. (2002) revealed in their study that the textbooks in the top achieving systems often presented a coherent curriculum, it is also revealed in this Chinese textbook. Before presenting the topic of fraction division, the textbook introduced the definition of the reciprocal. Students had an opportunity to learn some background knowledge about the reciprocal and its properties. Fraction division is the content topic following fraction multiplication and it is divided into four pieces from a fraction divided by a whole number to application. Each piece of knowledge is constructed as connected to each other. The content construction of fraction division, as presented in the textbook, shows a meaningful and hierarchical structure from easy to complex, from specific to general, from computation to application. This Chinese teacher followed the textbook and the teacher's guidebook with fidelity to implement the

coherent curriculum both within individual lessons and across a sequence of those lessons. The content unit was introduced based on students' prior knowledge, which should be connected with the new knowledge.

Through the interview data, we found that the teacher viewed mathematical knowledge as a knowledge package, which is coherently connected to each other. From the preinstruction interview, the teacher emphasized that she would spend the most time connecting students' previous knowledge with the new knowledge in order to help them understand mathematics as a "package of knowledge." For each lesson, she had clear instruction objectives. In order to achieve instructional objectives, her lesson plans were clearly structured as containing reviewing, introducing new content, and application parts (Li et al., 2009). The post-lesson interview confirmed the teacher's idea of connectedness in mathematics teaching and learning. She explicitly indicated that coherent instruction could help students form a good habit of learning mathematics, which include conjecturing, justifying, and application. Moreover, the sequence of four lessons consistently emphasized one important mathematical idea, transformation, as she specified during the interview. The teacher also explored the idea of transformation through teaching fraction division.

Taken together, the results obtained from this study revealed instructional coherence in a Chinese mathematics classroom through not only individual lessons, but also the instructional flow across lessons. The teacher explored and structured her coherent teaching based on the teaching materials and her perceptions of mathematics, teaching, and her students.

### *Conclusions, Limitations, and Future Directions*

Teaching is a cultural activity (Stigler & Hiebert, 1999). China has a cultural view and practice about teaching that is different from the West (Li & Li, 2009). This study provided detailed information about one particular yet valuable aspect of the Chinese teaching culture: instructional coherence. In a way, the study helped reveal specific characteristics of instructional coherence practiced in Chinese classrooms. Indeed, the analysis made it possible for us to learn beyond the teacher's instruction activity itself. As we indicated at the beginning of this article, an in-depth examination of classroom instructional coherence in China can provide an interesting case for mathematics educators in other education systems to reflect on their own practices.

At the same time, it is important to point out that the findings should not be simply applied to (or even used to evaluate) teachers' practices in

other education systems. In fact, instructional coherence may have different appearances and can be explored from different perspectives. If teachers design and carry out their lesson instruction using different instructional approaches, it is very likely that their instructional coherence, if existed, can show different states of being coherent. While this study revealed the characteristics of instructional coherence practiced in one Chinese classroom, it would be important to investigate instructional coherences practiced in other education systems and related features in the future.

As with many other studies, this study is limited in analyzing only one Chinese teacher's lesson instruction. Although Chinese teachers share many similarities in their lesson planning (e.g., Li et al., 2009) and classroom instruction (e.g., Li & Chen, 2009), it remains unclear whether their lessons share similar features of instructional coherence. Moreover, students and their learning, as part of the lesson instruction, can presumably benefit from coherent lesson instruction. Although it is generally documented that Chinese students performed well in school mathematics, specific connections between features of coherent instruction and what students may learn remain as an important question. As a result, future studies will be needed to investigate possible other features of Chinese teachers' instructional coherence and students' learning from coherent instruction for a comprehensive understanding of the issues related to instructional coherence.

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#### NOTES

<sup>1</sup> Key schools are those schools that are perceived by the public to offer a high-quality education and to be more selective in admission.

<sup>2</sup> China refers to the Chinese Mainland hereafter in this article.

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