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CREATING OPTIMAL MATHEMATICS LEARNING ENVIRONMENTS: COMBINING ARGUMENTATION AND WRITING TO ENHANCE ACHIEVEMENT

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ABSTRACT. The issue of mathematics underachievement among students has been an increasing international concern over the last few decades. Research suggests that academic success can be achieved by focusing on both the individual and social aspects of learning. Within the area of mathematics education, the development of metacognitive skills and the incorporation of discourse in classroom instruction has resulted in students having deeper conceptual understandings of the content and increased mathematical achievement. However, studies in this field tend to focus on the effects of these practices separately, making research that seeks to harness the potential of both quite rare. This paper reports on a study that was aimed at addressing this gap in the literature by examining the effects of writing and argumentation on achievement. Two hundred and eleven students and five teachers participated in this multimethod study that investigated the effects of three treatment conditions on mathematical achievement. These conditions were writing alone, argumentation alone, and writing and argumentation combined. Analysis of covariance revealed significant differences between the groups, and tests of the contrasts showed that students who engaged in both argumentation and writing had greater knowledge gains than students who engaged in argumentation alone or neither activity.

KEY WORDS: learning environments, mathematics achievement, mathematical argumentation, writing

Introduction

Mathematics achievement has been an increasing national concern in the United States, specifically in relation to public education (Schmidt, Wang, & McKnight, 2005). Reports from the Trends in International Mathematics and Science Study (TIMSS) (NCES, 2003) revealed deficiencies in students' understanding and fluency in mathematics and associated these gaps both with curriculum focus and design and also with the quality of instruction, especially in the middle grades (Boe & Shin, 2005). Concern about students' understanding and achievement is not limited to the United States, as approximately 42% of countries that participated in TIMSS 2003 scored below the international average. This lack of achievement in mathematics relative to other nations fosters doubt about the country's future economic competitiveness and its ability to compete

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on the global market, and so countries are becoming more aggressive in their efforts to find a solution.

To address this issue of mathematics underachievement, the United States has conducted several mathematics education reform initiatives geared towards incorporating constructivist-based instructional strategies and departing from more traditional methods of teaching mathematics. In this regard, more attention has been placed on strategies that promote the development of metacognitive and critical thinking skills, and that incorporate discursive practices into the classroom, namely mathematical argumentation (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Garii, 2002; Kramarski, Mevarech, & Arami, 2002; Stein, 2001). Recognition of the importance of these practices has resonated internationally, acknowledging that learning comprises individual and social components both of which are critical to academic success (Cobb, Yackel, Wood, Nicholls, Wheatley, Trigatti 1991; Lesh, Doerr, Carmona, & Hjalmarson, 2003; Pontecorvo, 1993; Schoenfeld, 1992). In light of this, the study combines practices derived from both the cognitive and the socio-cultural domains, focusing specifically on the combined effect of writing and argumentation activities on ninth grade students' mathematical understanding and achievement. In the past, researchers have tended to focus of the effects of practices derived from either domain, making studies that seek to harness the potential of both kinds of strategies rare (Nasir, 2005). This study attempted to address this gap in the literature.

Theoretical Framework

The design of this study is informed by insights from both cognitive and socio-cultural perspectives. Within a strict cognitive tradition, knowledge is seen as constructed by the mind and learning is considered an internal process of assimilating new information or experiences in an effort to understand it (Case, 1996; Prawat, 1996). Alternatively, from a more socio-cultural perspective less focus is placed on these internal processes, and knowledge is seen as constructed through the engagement in the social practices of a particular group (Rogoff, 1995). Although these perspectives are seemingly contradictory in the strict sense, a bidirectionality is assumed in the relationship between the individual and his socio-cultural contexts. This relationship is such that the individual, his thoughts, beliefs, and actions are influenced by his environmental and cultural contexts, and reciprocally these contexts are defined through the individual's position in and contribution to it (Nasir, 2005). In this regard, both socio-cultural and individual factors are considered essential to cognitive change and so an integration of the two should maximize the opportunities for students to learn (Cobb, 1994; Hatano & Inagaki, 2003). Therefore, an environment incorporating strategies aligned with both perspectives should lead to greater achievement than an environment designed around the principles of just one.

This study aimed at investigating this phenomenon and determining if a learning environment specifically designed to support strategies and techniques born out of both cognitive and socio-cultural perspectives would yield higher mathematics achievement than those supporting just one perspective or neither. Specifically, I examined the effects of combining practices that are considered social and cognitive on learning, by incorporating activities that supported the development of metacognitive skills and discourse in the classroom.

Individual Knowledge Construction And Writing

To address the concern that traditional forms of mathematics instruction tends to promote lower-level thinking, researchers in the field of mathematics education have investigated strategies that foster higher-order reasoning. In this regard, numerous studies have been done in the area of cognition and metacognition examining how these processes work together to enhance problem-solving skills and effective strategy use (Brown, 1987; Cornoldi & Lucangeli, 1997; Garii, 2002; Kramarski et al. 2002; Zan, 2000). Researchers have concluded that along with the mastery of basic mathematical skills, both cognitive and metacognitive abilities are crucial to improvement in problem solving ability and developing mathematical expertise (Mayer, 1998; Schoenfeld, 1987; Silver, 1987). These results therefore suggest that if we intend for our students to achieve mathematically, it is important that we engage them in activities that support the acquisition and development of these mental processes.

Writing, although more common in other academic domains, is an activity that helps students generate and connect their thoughts and ideas and consolidate their thinking. As students analyze, compare, and synthesize information they are able to create a clear conceptual picture through written words. Writing in this sense is considered akin to writing as knowledge-transforming, constituting active engagement in knowledge construction and not merely the reporting of information (Bereiter & Scardamalia, 1987). For these purposes it is a valuable learning activity and has enormous potential for promoting metacognitive thinking thereby improving understanding of mathematical concepts.

Social Construction of Knowledge & Argumentation

The idea of mathematics learning being an inherently social and constructive activity has been the focus of several researchers in recent decades (Forman, 1989; Lesh et al. 2003; McClain & Cobb, 2001). This theoretical approach to learning has been primarily influenced by the work of Vygotsky (1978) and proponents of socio-cultural and situated theories who promote the notion of learning beyond individual cognition to the social realm. Essentially, knowledge is considered to be culturally shaped and defined and we develop understandings through our interactions and participation within the 'community of practice' (Case, 1996). Mathematical competence or proficiency is characterized by an individual becoming more expert in the practices of the mathematical community. In this regard, language and various cultural tools are crucial within these communities as they facilitate the individual's increasing ability to effectively engage in the community's practice.

In the mathematics classroom this type of discourse is commonly referred to as mathematical argumentation and is characterized by the sharing, explaining, and justifying of mathematical ideas (Cobb et al., 1991; Leonard, 2000; Stein, 2001). Classroom discussions where students are able to make worthwhile contributions, ask questions, have their ideas evaluated, and receive immediate feedback are considered one of the more effective strategies for knowledge construction (Inagaki, Hatano, & Morita, 1998). This peer collaboration embodies a reciprocal process where each member has opportunities to share his or her thoughts and explore the reasoning of others. The reciprocal process of co-constructing meaning engages the individual in mental processes of both a cognitive and metacognitive nature. Incorporating activities within the classroom that allow students to engage in this form of discourse is essential to the development of students' critical thinking skills and mathematical understandings.

An important factor in the successfully functioning discursive classroom is the role of the teacher. The teacher's role is crucial, not as the repository of knowledge, but as the one who initiates and guides the students in 'community' practices. Maximizing the effectiveness of these classrooms through their transformation into environments of inquiry requires that the teacher take on the role of 'facilitator' and not 'transmitter of knowledge' (Cobb et al., 1991; McClain & Cobb, 2001). In so doing, students' collaborative engagement in argumentation around mathematical ideas and concepts is continuously scaffolded by the teacher, guiding the students towards expertise. Additionally, modeling these practices in whole class settings along with facilitation, characterized by listening and promoting valid but diverse ways of thinking, are crucial teaching strategies for student engagement in meaningful discourse (McClain, McGatha, & Hodge, 2000).

Purpose

This study aimed at examining the effects of engagement in mathematical argumentation and writing activities on the mathematical achievement of ninth-grade Algebra 1[1] students compared to students within a more traditional classroom.[2] The study was guided by three research questions:

1. Will students who participated in mathematical argumentation and writing activities demonstrate greater understanding of mathematical concepts than those students *who only engage in writing activities*?
2. Will students who participated in mathematical argumentation and writing activities demonstrate greater understanding of mathematical concepts than students *who only engage in mathematical argumentation*?
3. Will students who participated in mathematical argumentation and writing activities demonstrate greater understanding of mathematical concepts than students *who have not engaged in mathematical argumentation and writing*?

Additionally, a qualitative component to the study served to examine how both argumentation and writing contributed to students' learning. Specifically, I examined: (a) how students' talk within their groups impacted their knowledge of the concepts being discussed? and (b) how the writing activities consolidated the students' thinking on its own and in addition to engagement in argumentation.

Methodology

Method

A quasi-experimental design was adopted for the quantitative portion of this study. There were four groups; one group served as the control group (these students received no instruction in argumentation or writing), and the remaining three received the following randomly assigned treatments: (a) engagement in activities structured around mathematical argumentation, (b) engagement in activities structured around writing, and (c) engagement in activities structured around mathematical argumentation and writing. Two groups of students (groups of four) were randomly selected from both the argumentation and writing (AW) group and the argumentation only (A-only) group for video-taping in order to provide a more in-depth analysis and to explain the quantitative results.

Participants

The participants were 211 ninth-grade students between the ages 14–15 years and their five teachers. The teachers were distributed across the groups so that each of the four treatment groups had two different teachers.

DATA COLLECTION

Instrumentation

Pre-Post Assessment. A 19-item multiple-choice test was developed to measure the students' learning of the content covered over the 10-week period of the intervention. Each question was aligned to 18 academic achievement standards established by the state in which the study took place[3] (see Figure 1).

Classroom Activities. Students were assessed following instruction over each topic in the curriculum. On the day following the in-class assessment, the students engaged in the argumentation and writing activities. Each activity consisted of two questions that focused on the 'big ideas' surrounding the content; that is, the key ideas for which the students were expected to develop enduring understandings. The students in the three treatment groups received similar questions, but how they were required to engage with the questions was dependent of their group (described below).

Treatment

Argumentation-Only Group. Each student was required to follow a three-step routine: (a) read through the questions individually to gather

1) What is the slope of the line?

a) - 3/2
b) -2/3
c) 2/3
d) 3/2

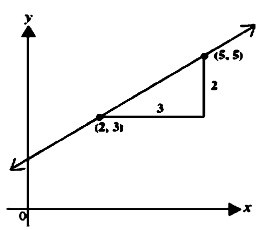


Figure 1. Example of test item. Academic standard: defines slope as rate of change and calculates the slope given the change of the two variables

initial thoughts, (b) each student in the group should state his response and explain why his response is valid, and (c) defend his answers in light of other responses. While each group engaged in the argumentation routine, the teacher was encouraged to facilitate (discussed in the next section). After each activity the students engaged in a whole-class discussion about the questions.

Teacher Facilitation. Teacher facilitation of the treatment conditions was considered important, especially in the initial activities, as students often have difficulty engaging in talk around mathematical activities (Rittenhouse, 1998). The general guidelines for the teachers included ensuring that the students were following the routine and also ensuring that the students were making sense of the questions and developing better understandings of the mathematical content. Specifically, the teachers were directed to go to each group and listen to the students' conversations, ensuring that all students were participating. Additionally, the teacher was to encourage students to justify their responses with mathematically valid statements and when necessary, use questioning (or other appropriate techniques) to push the students towards thinking deeply and more critically about their ideas and statements. In the case where the students were stuck or confused, teachers were encouraged to provide suggestions or hints that served to redirect the students' thinking or guide them toward alternative strategies or solutions. The teachers were told not to use evaluative statements, such as 'that is correct' or 'right answer' when facilitating the group conversations, as in previous research, statements of this type tended to stop the students' discourse (Cross, Taasoobshirazi, Hendricks & Hickey, 2008).

Writing-Only Group. The students were instructed to read and answer the questions individually. Students were encouraged to explain why and how they used the selected formulas and algorithms and why they thought their solution was correct, essentially they were expected to produce a written argument justifying their responses. As the students were writing their responses the teacher was expected to walk around the classroom and read, if possible, the students' statements. By getting an idea of how the students were making sense of the task the teacher could provide initial feedback before collecting the papers that hopefully served to extend the students' thinking or provide needed clarification. The teacher would then collect the papers, review the responses, and provide written feedback.

Argumentation-and-Writing Group. The students in the combined treatment group engaged in both routines. For the first question they engaged in the argumentation routine and then were required to produce a written response for the question. Then, for the second question they engaged in the writing routine.

Control Group. The students and teachers in the control group were excluded from engaging in the argumentation and writing activities. Although the control teachers were aware of the activities that the treatment teachers were incorporating in their classrooms, they were expected to continue with their normal instruction practices. Prior to the start of the implementation, observations of these classes showed that the teachers' primary mode of instruction was more teacher-centered and lecture-oriented, where students sat and listened while the teacher stood at the white board, introduced concepts, and demonstrated step-by-step procedures for solving related mathematics problems. Students were not required or expected to produce written arguments (as described above) for problems nor did they collaboratively engage in problem-solving. Although the students did talk during instructional time, this talk was primarily off-task or checking answers. This type of talk was not regarded as argumentation as it did not constitute the exchange of mathematical ideas or understandings but were basically summative evaluations of final answers.

Observations

During the implementation period, both the teacher and students in all four groups (AW, A-only, W-only and control) were observed twice per week. Detailed field notes were taken during each observation. These notes included rich descriptions of the type of instruction observed and the nature of the teacher-student and student-student interaction.

Results

Quantitative Analysis

The pretest was administered to all four groups prior to the beginning of the treatment and the posttest during the week following the final classroom activity. To determine if the groups were equivalent prior to the treatment, analysis of variance was conducted on the pretest scores showing that the groups were not significantly different prior to receiving the treatment, $F(3,207) = 1.913$, $p = 0.129$. Analysis of covariance was conducted on the data using the pretest scores as the covariate.

Table 1 shows the adjusted means and standard deviations for the posttest scores by treatment. There were significant differences found for the main effects for treatment ($F(3, 206) = 4.284$, $p = 0.006$, $\eta^2 = 0.059$) indicating that there were differences between the performances of the students for the different groups. To specifically identify where the differences in the groups were and to answer the remaining research questions, post-hoc comparisons were done.

Research Question 1. The first research question investigated students' understanding of mathematical concepts for students who engaged in combined mathematical argumentation and writing versus students who engaged in writing only. Post-hoc analyses of pairwise comparison revealed that there were no significant differences between these two groups, AW > W, $p = 0.34$, $\eta^2 = 0.004$.

Research Question 2. Research Question 2 examined the differences in achievement of students who experienced both argumentation and writing activities in comparison to students who only engaged in argumentation. Post hoc analyses of pairwise comparison reflected that there were significant differences ($p < 0.05$) between these two groups, AW > A, $p = 0.019$, $\eta^2 = 0.026$.

Research Question 3. The third research question sought to examine if there would be significant differences in students' achievement for students who engaged in both argumentation and writing activities (AW) over those students who did not engage in either type of activity (C). Post hoc analyses of pairwise comparison revealed that there were significant differences ($p < 0.05$) between these two groups, AW > C, $p = 0.001$, $\eta^2 = 0.050$.

TABLE 1

Means and standard deviations of the four groups

		Pretest		Posttest	
Treatment	N	Means	Standard deviations	Means (adjusted)	Standard deviations
Argumentation only	43	7.60	2.779	9.00	3.169
Writing only	51	7.64	2.629	9.72	3.040
Argumentation and writing	62	8.64	2.797	10.17	2.827
Comparison	55	8.24	2.589	8.66	2.618

Discussion of Quantitative Results

Analysis results suggest that engaging students in activities where they have the opportunity for shared or individual meaning making leads to greater achievement than engaging in neither. From the analyses of the means, we also see that students who engaged in both activities did reflect increased scores over students who worked individually on similar tasks or those who collaboratively discussed these tasks. However, while not all these differences were significant, it does suggest that engaging students in both types of activities provide greater opportunity for learning the content. Further analyses of the pairwise comparisons revealed additional but peculiar information in that, although the means of these individual activity groups (W only and A only) were higher than the control group only the difference between the writing-only group and the control group was significant, A > C, $p = 0.498$, $\eta^2 = 0.002$ and W > C, $p = 0.028$, $\eta^2 = 0.023$.

The lack of significant gains of the argumentation-only group was puzzling because it seems that engaging in mathematical talk should be quite beneficial to mathematical understanding. Having the opportunity to discuss your responses and have these responses assessed and critiqued by one's peers and receive feedback should have provided opportunities for the students to both consolidate their current understandings and extend their knowledge about the concepts. These opportunities to engage in talk did not appear to have enhanced the students' understanding greatly as their gains were minimal. The qualitative analyses shed some light and provided insight into how the discursive engagements and writing activities influenced the assessment scores.

Qualitative Analysis of Transcipts

An in-depth analysis was conducted of the transcripts in an attempt to gain insight into how argumentation may have enhanced the students' understanding of the targeted concepts and the role writing played in consolidating this understanding. Additionally, the papers of the writing-only students were analyzed in order to evaluate their thinking about the concepts and also to provide greater insight and possible explanations for the quantitative results. Specifically, this analysis was designed to provide a clearer picture of why the students in the combined group (AW group) had significant learning gains over the control group and the argumentation only (A-only) group but non-significant gains over the writing only (W-only) group. Additionally, it was expected that this analysis would provide possible explanations for the minimal gains of the A-only group over the control group.

Argumentation. This portion of the analysis focused on the transcripts of the fourth classroom activity (see Appendix A). These transcripts were selected for three reasons: (a) this activity was about midway in the intervention when the students were beginning to engage in argumentation practices (b) it highlighted the importance of teacher facilitation in guiding students' thinking, and (c) the transcripts included several argumentation features that were observed in other activities. The activity required that students determine whether two formulas provided for the slope would result in the same number value and in light of that discuss the validity of the two formulas (see Appendix A). The following excerpt from the transcript of students in the AW group provides an example of this. The students stated their names and then discussed the question:

Kevin*:	What did you say?
Deana:	Abel uses these points with this formula and Cain uses these points with this formula. Are they both correct? Use the points to support your answer
Deana:	Yes, I think it's yeeeess...yeees
Winsome:	No, I put no
Kevin:	I put no
Deana:	I put Abel is correct because that is the only formula I was taught how to use
Winsome:	Is it this one [points to Abel's formula]... I thought they were the same
Deana:	No
Eddie:	No ..it's y_2 - y_1
Deana:	Yeah, that's the right formula
Kevin:	Yeah that one
Eddie:	Yeah that's the real formula
Winsome:	Yeah but when you think about it when you plug in the numbers I'm pretty sure it would work either way.

* For purposes of confidentially, all names are pseudonyms.

After examining the formulas, each student stated his or her response. Three of the students seem to agree that only one formula is correct (Abel's), although Deana initially appears indecisive, because that formula is the one they were taught and the only one they ever used. However, Winsome disagrees with their claim and suggests an alternative viewpoint and a valid means of testing her conjecture. It is interesting to note that although the question required the students to verify their responses by using the ordered pairs, they did not. The students seemed fairly confident in their answers because of previous instruction (inferred from Deana's statement in line 12 and Eddie's in line 19). Having been presented with an opposing view, the students were then encouraged to investigate the idea, thereby creating an opportunity for learning. Having not been presented with this opposing view, the students may not have questioned the validity of their initial response. The discussion continued:

22	Kevin:	I'm gonna try it.
23	Eddie:	Me too.
24	Winsome:	But I put no just because that was the formula [pointing to Abel's formula].
25	Deana:	But it might be yes,
26	Kevin:	What is −2 −3
27	Eddie:	−6.. I mean −5
28	Deana:	−5 yeah
29	Winsome:	You get the same thing except positive.
30	Deana:	So they both can't be correct then.
31	Kevin:	Which one did you get correct though
32	Deana:	This is Cain.
33	Winsome:	I don't know.
34	Kevin:	You get the same numbers but you don't get the…
35	Winsome:	Same sign.

Winsome's suggestion leads the students to use the ordered pairs provided to investigate their initial conjecture. However, due to computation errors they arrive at the incorrect answer and revert to their original conclusion. This interaction between the students demonstrates how discourse provides the opportunity for the distribution and sharing of different perspectives and the potential of this knowledge sharing to encourage conjecturing and further exploration. It forced them to collectively evaluate their previous understanding of how to calculate the slope and to actually try to verify what they had been taught.

Later in the discussion the teacher questioned the students about their discussion and what they concluded:

42	Teacher:	Well, what did you guys put?
43	Eddie:	Well I think we all thought it was yes for the second part.
44	Deana:	Yes and also for the first part.
45	Deana:	Well almost everybody put no and then we came to yes.
46	Kevin:	Well I put no and stuck with it.
47	Deana:	Well all of us really put no.
48	Teacher:	Well you can't just change your answer arbitrarily. You must have a reason.
49	Kevin:	I did, I say no because if you plug in the numbers you will get the same

50		number but not the same sign.
51	Teacher:	Really, let me see [teacher looks at Kevin's paper].
52	Eddie:	Yeah it was like −1 and 1 and...
53	Kevin:	And then you have a −5 and a −5
54	Teacher:	So what is that equal to… if you have a −1 over a −5?
54	Eddie:	Isn't it just 1 over 5?

In this excerpt, the teacher tries to get the students to talk about their answers and to explain why and how they arrived at their conclusions. The teacher identifies the error the students made that lead to the incorrect conclusion (line 51) and guides them towards examining their calculations more closely (line 54). By not providing the correct answer the teacher placed the responsibility on the students for their own thinking, forcing them to reassess their initial ideas and allowing them to create new knowledge for themselves. In this instance, the questioning by the teacher was important for them to take a second look and evaluate the accuracy of their conclusion. The teacher, however, did not stop there but further encouraged the students to explore the reasoning behind this new emergent knowledge that Winsome had alluded to earlier.

69	Teacher:	So what does that mean?
70	Eddie:	Oh yeah..I see...
71	Teacher:	So do you know why that is though?
[about 5 seconds later]		
72	Winsome:	Because it's just the variables that you switch around so it doesn't
73		matter.
74	Teacher:	Can you explain that again and tell everybody?
75	Winsome:	Well because you could have chosen any number to be y_1 either 4 or 5 or
76		you could have labeled any of the points [pointing to the x -values] x_1 or x_2
77		so it would work out to be the same.
78	Deana:	Oh...I see
79	Eddie:	Yeah
80	Winsome:	So it doesn't matter
81	Teacher:	So do you really understand [looking at Deana]? Explain it to me.
82	Deana:	Yeah, I get it because you usually ask us which one do we want to be y1 and
83		y2 and we usually choose the first one but if we switched it around it
84		would be Abel's formula.

The teacher emphasized, by asking them to explain, that understanding why both formulas worked was important. Also, that it was not enough to rely on Winsome's reasoning, but they had to come to an understanding for themselves, making the knowledge their own. Although the students seemed to understand computationally why both formulas would work the teacher could have extended the discussion more conceptually, because while procedural knowledge of this concept may be sufficient to accurately answer most questions that appear on standardized tests at this level, having a more robust understanding of the slope is important for higher-level mathematics. At the end of this conversation, it appeared that all the students had come to a fair understanding of why both formulas will work to produce the value of the slope. The discussion not only allowed them to talk about what they knew about the formula for the slope, but the presentation of an opposing view engaged them in further exploration leading to an extension of their understanding.

Not all the discourse groups engaged in this type of shared knowledge construction; there were differences in the quality of argumentation in the different groups. With this second group of students, all four students are engaged in the discussion, but the onus was placed on one student to provide an answer and defend his position.

1	Toni:	He's wrong...this one [pointing to Abel's formula]…
2	Brad:	Who's wrong?
3	Toni:	You wanna bet.
4	Brad:	What are you talking 'bout? Cain did it different from Abel.
5	Amber:	I know that...this one is right [pointing to Cain's formula].
6	Toni:	You should go y_1 first.
7	Amber:	Prove it.
8	Brad:	Yes you can.
9	Amber:	No you can't.
10	Brad:	Look...you can go up 5 and over 1 or up 1 and over 5. Can't you do it like 11 that both ways?
12	Susie:	Would it be the same thing?
13	Brad:	No.
14	Toni & Amber:	[laughs] ok

Although it is difficult to understand from his discourse, Brad starts out by saying that the formulas will produce the same value although they are different. He attempts to provide evidence for his reasoning by plotting the ordered pairs and determining the value of the slope by estimating the vertical distance between both points and then the horizontal distance. He makes an error with his method and so does not get the same value.

However, he is convinced that they should work and persists following a suggestion from one of the other group members.

15	Brad:	Wait...let me think
16	Susie:	Use the numbers.
17	Amber:	I don't know...let me try it.
18	Brad:	Ok there you go...yes they do, they do equal the same thing
19	Amber:	Don't be yelling!
20	Toni:	You got 1 over 5 and 5 over 1
21	Brad:	But they equal the same thing.
22	Amber:	1 over 5 is not the same as 5 over 1.
23	Susie:	So the answer is yes then.
24	Brad:	Yes! That is what I put, because both formulas can be used to find the point.
25	Susie:	Can you explain it?
26	Amber:	Is it right?
27	Toni:	I quit.
28	Brad:	Ok let me show you...look, look. You take the 3 and then the 5, this is
29		right here and then this is x_1 and then you do this [labeling the next set of
30		points]. Then you subtract this and then you go like this 3 minus −2 which
31		is 3 plus 5, 3 plus 2 which is 5 so...over then you go 5 – 4 is 1 so that's
32		your fraction right there.
33	Susie:	So is it the same thing?
34	Amber:	Aaaaaaaaaaaaaghhh

Brad's previous method did not work for him so he now uses the values from the ordered pairs to prove his hypothesis that the formulas will produce the same answer. He demonstrates to the other group members through calculations that he is correct in his hypothesis, but they do not seem to follow his argument. Although Brad has provided them with an alternative view, they are not encouraged to investigate the possibility on their own but rely on Brad to provide proof. While the first group engaged in knowledge-sharing and developing mutual understanding, these students were satisfied to rely on Brad to produce a valid proof. A few minutes later the teacher tries to have them explain their conclusion:

35	Teacher:	So what do you notice?
36	Amber:	They are the same
37	Teacher:	So this is the one we used in class right ...what's wrong?
38	Amber:	Because it is not supposed to be right.
39	Teacher:	Why not?
40	Toni:	[laughs]
41	Amber:	Ok.

From their responses to the teacher it appears that they are still not sure why both formulas produce the same answer, and it is the cause of some frustration for the students. Following Brad's explanation, the other group members are willing to accept that the formulas produce the same value but are confused because it contradicts what they have been taught in class. The teacher-student interaction is brief, and no guidance is provided to help the students think more critically about the question. The students are in a position of cognitive disequilibrium but are provided with no guidance about how to resolve this issue. Also, it is unclear from Brad's explanation whether he knows why the order of the variables in the formula is not integral to the correct calculation of the slope.

While this conversation may have been beneficial to Brad, it appeared that the other group members did not make sense of the question or come to any better understanding of the underlying concepts. In this regard, the conversation did not provide them with additional opportunities to increase their knowledge beyond the regular classroom instruction. As such, while they had increased exposure to these concepts, the students' opportunities for knowledge building were not significantly different from those students in the control group.

Writing. The writing activity appeared to have helped students organize their thoughts and consolidate their thinking, allowing them to better express their understanding of the concepts. An example of this is demonstrated below in the writings of the AW students for part b of the question following their discussion (Will this (your answer for part a) be true for any 2 points? Why?). The students' written responses to part b are shown because it provides a good example of how the students articulated their reasoning following talk:

Winsome:	Yes, as long as you don't switch around the numbers in the ordered pair. You subtract the *y*'s on top and the *x*'s on the bottom. It works because any of the ordered pairs can be x_{22} and x_{11}.
Kevin:	Yes, if you plug in the numbers in the formulas you will get the same answers because any of the points can be x_{11}.
Eddie:	Yes, it doesn't matter if you switch the y_1 and the y_2 as long as you do the same with x_1 and x_2.
Deana:	Yes, as long as you use the formulas correctly and watch your signs. But as long as you don't do y_1 on top and y_2 on the bottom it wouldn't work. Keep *y*'s together and *x*'s together

From the transcript, both Deana and Winsome had provided explanations of why the different formulas would lead to the same answer. While talking led the students to share their ideas and clarify meanings and understandings, writing allowed the students to focus and to organize their thoughts in a more concise and coherent manner. From these writings one has a better picture of the meaning that the students took from the activity as it allowed then to organize and refine their ideas and produce a more structured response.

For the group that engaged in argumentation and then writing, both activities appeared to have complemented each other, allowing the students to have produced a more cohesive product. However, even without the benefit of generating and sharing ideas with peers, the students in the writing only group seemed to have used the writing tasks to also organize and consolidate their thinking about the two formulas for the slope. Examples of their responses are presented:

James:	Yes, because the x's and the y's are together no matter what ways it goes. They are the same just numbers on different sides.
Amy:	Yes, because the x's and the y's together so you will get the same thing no matter which way it goes it will still have the y's on top and the x's on the bottom.
Eric:	Yes, because the formulas are really the same thing. For Abel you will get 1/5 and for Cain you will get 1/5 just that the numbers start being switched around
Meredith:	Yes, they are the same. Either way you label the points you get 1/5

From the writing samples, both groups of students came to the correct conclusion, however, both sets of writing lacked conceptual depth in that, they both focused on the procedural aspect of the concept and did not reveal much about why they thought this occurred. However, there was a significant difference between the writing samples from the W-only students and the students in the combined activity group. Overall, a larger percentage of the writing-only students used the values provided to support their answer; 87% of the writing-only students used the values (from part a) to support their conclusions, whereas only 54% of the AW students did. This may indicate that after their discussions the AW students began to think more conceptually about the relationship between the two formulas or following part a they saw no reason to provide additional support for this claim. On the other hand, having engaged in the activity individually and not being allowed the opportunity to try out their ideas on others may have forced the W-only students to resolve any confusion or indecision independently. This may have necessitated proving the claim to themselves leading to the use of the ordered pairs as a means of justification. This motivation to justify and explain their responses may have caused these students to activate additional cognitive resources thereby engaging in more intense analysis of the question, ultimately leading to increased understanding of the concepts.

Overall, the writing activities seemed to provide the students with the opportunity to make sense of the question, reflect on and organize their thoughts about the concepts, and structure their ideas to produce a meaningful response. While neither of the groups (AW and W only) came to a more robust understanding of slope, most students seemed to have concluded that both formulas would lead to the same answer. Having to focus so intently on articulating their understanding seemed to have forced them to invest more cognitive resources into making sense of the problem, which appeared to have advanced their thinking somewhat as the two groups that engaged in writing activities had the highest means (AW and W-only groups).

Discussion: Educational Implications and Limitations of Writing and Argumentation Strategies

This study sought to investigate the combined and individual effects of argumentation and writing activities on students' understanding of Algebra 1 concepts. The analyses produced two sets of notable results. First, through examination of the means, the results revealed that engaging in either activity led to greater achievement than none at all and that combined, these activities were more beneficial for students' understanding than individually or none at all. Second, post-hoc analyses revealed significant results for two of the three research questions investigated. Students who engaged in writing activities, either alone or combined with argumentation (AW and W-only), had significantly greater learning gains than the students who engaged in neither activity (C). Additionally, engaging in combined activities (AW) led to significantly greater increases in mathematical achievement than engagement in argumentation only (A-only). However, the AW and W-only groups did not significantly differ in learning gains, nor did the A-only and comparison groups. The qualitative analyses served to both complement and provide explanations for these findings.

The qualitative analyses showed that argumentation was a useful strategy for generating and sharing ideas. This collective sharing allowed students to hear the ideas of others, which helped to both highlight misconceptions and to confirm their own thinking about concepts. In some cases it produced a necessary conflict with the student's current understandings and in attempting to resolve this conflict the student was able to eliminate a misconception and enhance his knowledge of the concept. In some instances, being presented with opposing views led to conjecturing and exploration that provided opportunities for the students to generate new knowledge. This was evident in both discourse groups although the level of knowledge sharing and collaboration differed. The analyses of the group conversations illuminated features of the discourse that appeared to aid knowledge-building and growth as well as features that did not help in developing deeper understandings.

Rivard and Straw (2000) suggested that there are four mechanisms that are important in group discussions for promoting understanding and for mobilizing the conversation: asking questions, conjecturing, formulating ideas together, and explaining. There were differences in the way the two groups actualized these mechanisms. In the first group (AW), all members were actively engaged in working through and talking about each other's responses. With the presentation of a conflicting viewpoint they collectively formulated an approach to investigate this opposing view. Although they arrived at an incorrect answer, the discussion forced the group members to think more deeply and reflect on their initial assumptions and to further investigate the validity of their original ideas. Similar exchanges were present in the second group (A-only), but the knowledge generation and conjecturing was only evident in one group member (Brad). Although the over-reliance of the other group members on their 'prior understandings' seemed to hinder their learning, it served as a point of challenge for Brad to investigate and provide proof of the validity of his own conjecture. Additionally, although the other members of this group appeared not to have come to an understanding of the underlying concept, it did provide some cognitive disequilibrium, which may have encouraged further exploration with the appropriate questioning. An important feature, however, was that although the other A-only group members were confused by Brad's explanation, they did not unquestioningly accept Brad's idea. The propensity of group members to unquestioningly accept claims by seemingly more knowledgeable students is often a problem that decreases the effectiveness of argumentation in groups, as these students rely on the knowledge of their higher achieving peers and do not try to construct their own understandings or make sense of the concepts for themselves (Cross, 2007; Rivard & Straw, 2000).

In light of this, it is imperative for teachers to emphasize that students should not privilege coming to a consensus over their own individual understanding. As such, while students become better at argumentation through increased participation, the teacher should emphasize individual accountability. It is through being encouraged to elaborate one's thinking that thoughts become more refined and synthesized, thereby leading to deeper conceptual understandings. If the practices of negotiating and refining are absent, collective understanding is minimized, thereby decreasing the potency of the learning experiences. These occurrences may explain the minimal learning gains of the A-only group over the control group as the conversations in the A-only group seemed to lack the argumentation features that encourage personal sense-making.

Although the students' argumentation did improve somewhat over the course of the implementation, initially engaging in learning in this way was rather challenging for both the students and the teachers. However, over time, the level of student participation in the conversations improved but their statements still reflected minimal understanding of the mathematical concepts. This appeared to be due to two factors: (a) the students' prior knowledge and understanding of prerequisite concepts appeared to be low (the mean of pretest scores for the discourse groups was 8.3 of a total of 18 items), and (b) classroom social norms for discourse were not sufficiently established in these classes.

Classroom social norms are characterized by explanation, justification, and argumentation (Yackel & Cobb, 1996). Although these norms of rational argument are not specific to mathematics classrooms, mathematical argumentation incorporates these principles and extends them by emphasizing the validity and mathematical sophistication of the statements for them to be considered argumentative. Specifically, it requires the student to be cognizant of what counts as evidence in mathematics and what is regarded as a mathematically valid justification or warrant. However, if the basic social norms are not established, then the difficulty in engaging students in mathematical argumentation is compounded. So although there was an increase in the amount of talk that took place in these groups, many of these conversations still lacked the mathematical rigor that would push the students to extend their thinking.

The writing activities presented a good way for students to consolidate their knowledge about different concepts and how they related, and to synthesize these ideas to produce an organized and coherent response. One notable result from the analyses was that the learning gains for the AW and the W-only group were not significantly different, suggesting that writing is a powerful strategy for promoting learning. A number of researchers who study the generative processes of writing and composition have stated that writing is an extremely challenging and effortful cognitive process (Scardamalia & Bereiter, 1986). The major part of the effort in writing is generating the content, more specifically thinking about what you want to articulate (Flower & Hayes, 1980; Hayes & Flower, 1980). The ability to efficiently generate adequate content so that one has the flexibility to select from what is available and discard what is deemed unnecessary or irrelevant (a skill of more expert writers) appears to be one's knowledge of the subject being written about and the ability to readily access this knowledge. The tasks included in this project required the students to not only recall and state mathematical content but to produce a mathematical argument that was not only valid but also convincing. In order to do this effectively, the student would have to engage in the metacognitively oriented actions of diagnosing (the problem at hand), planning (an effective strategy to obtain a solution), monitoring (one's thought processes as the strategy is being implemented) and evaluating (the reasonableness of the solution in the context of the problem), an approach to writing akin to Bereiter & Scardamalia's (1987) 'writing as knowledge-transformation'. A key function of this type of writing is that students are not simply reporting what they already know, but they are building on prior knowledge and constructing new knowledge through the interaction between the content space (that include acts of recalling, relating, and evaluating mathematical content) and the discourse space (that includes constructing text and appropriate language use) to meet specific goals; in this case, the argumentation goals of the writing task (Bereiter & Scardamalia, 1987; Keys, 2000). Thus, the need to produce a convincing mathematical argument served to place additional cognitive demands on the student than mere knowledge telling would have.

Unlike discourse where the individual can obtain cues from other speakers, writing requires that the individual use their own cognitive resources to generate information. Having engaged in talk about the tasks prior to engaging in the writing activities may have removed some of the cognitive burden of generating the thoughts individually and also allowed for greater meaning-making collectively. It also appeared that engaging in writing in some way forced the students to internalize this argumentation, which may have allowed the students to engage their metacognitive skills when approaching the questions. This opportunity to interact individually with the content and to develop deeper understandings was clearly beneficial to the students and so although the mean scores were lower than the students in the combined activities group, the difference was not significant. From the analyses, writing appears to be a very useful strategy for possibly engaging students in greater meaning-making and helping students to better refine and synthesize their ideas, ultimately leading to greater understandings.

Although individually and combined these activities enhanced student learning, both activities seemed to have their limitations. Both types of activities have the potential to help students develop better understandings and identify and clarify misconceptions, but without the necessary prior knowledge to engage productively in them, learning will be hindered. For example, in the AW group, the students were unable to identify their error without guidance because of they were not competent in computing integers. Similarly, in the A-only group the students seemed to be unable to follow Brad's reasoning. He was aware of alternative methods to calculate the value of the slope but had difficulty explaining it to the students because this method was unfamiliar to them. This inability to follow Brad's reasoning appeared to hinder the effectiveness of the routine and the subsequent understanding of the students. In this case, the presence of the teacher would have been invaluable to help guide the students toward a solution.

This raises the issue of the teacher's role in the facilitation of these activities. Of critical importance to the success of writing and discursive activities is the teacher's role in developing classroom social norms for both group interaction and writing tasks. During the implementation of the tasks teachers had tremendous difficulty in taking on the roles necessary to effectively facilitate the students' conversations. Most of these teachers took a fairly traditional, teacher-centered approach to teaching and so in order to facilitate they had to take a step back and put the responsibility for sense-making on the students. It required that they relinquish some of the control for knowledge building to the students as well as manage a classroom that was active and alive with students' talk and movement. Having to function in this way seemed to threaten the perceptions they had of their teacher role. Additionally, being an active and equal participant in this environment appeared to conflict with the teachers' beliefs about mathematics teaching and how they thought students learned best (Cross, 2007). These factors appeared to impact the teachers' decisions to continue to incorporate similar activities in their classroom. On subsequent conversations with the teachers following the end of the implementation they commented on the usefulness of the activities in revealing the students' misunderstandings and misconceptions, and general lack of conceptual understandings. However, only one teacher stated that he would continue to use the strategies in his classroom and asked permission to share them with other colleagues. The teachers' responses appeared to be more of a reflection of their dispositions and beliefs about mathematics and themselves as teachers than their perception of the impact of the strategies. So although the teachers saw the strategies as useful in enhancing achievement, to continue these practices independently would require some modification of their mathematics-related beliefs, instructional goals, and professional identity.

The results from this research suggest that engaging in activities reflective of both cognitive and socio-cultural views of knowledge and learning does lead to increased understanding and achievement. However, we must be cautious in making claims in this regard since the qualitative analysis does demonstrate that the extent to which these students engaged in the discursive activities as designed was minimal. Having had sufficient time to establish classroom social norms around discourse, students in both discourse groups may have been able to harness the full potential of these activities. It is clear that the teacher's role is extremely important for both modeling these practices and guiding the students during their conversations. While the writing activities may have served as a heuristic for the AW group (as instruments to further encourage them to make meaning of the concepts), for the W-only group they seemed to be a means through which the students were able to transform their current knowledge into deeper conceptual understandings. These activities however, appeared to be more useful for knowledge-building having followed opportunities to collaborate discursively with peers.

Appendix A

1) Points (3,5) and (−2, 4) lie on a line.

Cain uses the formula:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Abel uses the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

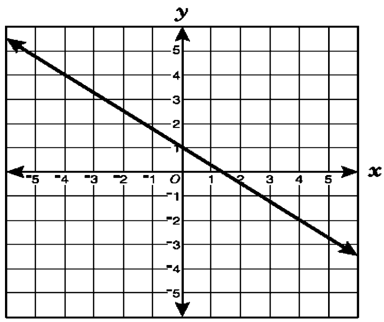
a) Can both Cain and Abel be correct?

Use the points given above to support your answer.

b) Will this (your answer for part a) be true for any 2 points? Why?

2) John calculates the value of the slope of the line says m=1. Sue takes a look at the line and without any calculations says he is incorrect.

a) Explain how Sue knows this without calculating the value of the slope herself? Prove to John (through calculation or otherwise) that he is incorrect.



NOTES

[1] Algebra 1 is a course focused on the study of elementary algebra concepts and skills (including but not limited to relations and functions, functions as rates of change, generalization of patterns, and using symbolic algebra to represent and explain mathematical relationships) typically taken by ninth-grade students in the US. For a full description see *Principles and Standards for School Mathematics*.

[2] The traditional classroom refers to a teacher-centered learning environment characterized by a lecture-style instruction approach, limited student-student and teacher-student interaction, and minimal engagement in conceptually-rich tasks.

[3] In the U.S., each of the 50 states sets its own academic standards. The states also incorporate statewide assessments to determine the extent to which the standards are being attained.

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