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Frequency Independent Design of Quasi-optical Systems

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Abstract Beam propagation at millimeter and submillimeter wavelengths is well described by Gaussian beams and quasi-optical theory. Due to the general progress in THz technology, receiver and other quasi-optical systems in the THz range demand increasingly larger bandwidths. In this context, this paper presents a general design methodology for frequency independent quasi-optical systems, based on system matrix analysis. After the presentation of the general ideas, useful design equations are derived for the most common quasi-optical systems. Finally, the derived equations are validated by application to already deployed radio astronomy receivers.

Keywords Submillimeter wave technology · Quasi-optical waveguides · Gaussian beams · Frequency-independent design

1 Introduction

Quasi-optical systems [1] are those which involve the free-space propagation of electromagnetic radiation in the form of beams with lateral size comparable with the wavelength and for which diffraction effects are important. Electromagnetic propagation in this kind of systems can be described well by the paraxial approximation of the wave equation. Gaussian beam modes represent a complete basis for their solutions and are thus of great importance. The paraxial wave equation is especially useful for highly collimated beams at millimeter and submillimeter frequencies. It describes the optical systems of a number of applications, such as radio astronomy [2], material characterization [3], remote sensing of the atmosphere [4], and security [5].

The radiation patterns from corrugated feed horns can be appropriately modeled by a single fundamental Gaussian beam with less than 2% error in power [6]. Therefore, fundamental mode analysis is usually enough for quasi-optical systems with high-quality feed horns. In addition, the propagation of the fundamental mode is well described by simple matrices similar

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to the ray matrices in optical design. This similarity makes it possible to use optical components to modify the beam propagation of radiation in the THz range: mirrors, lens, free-space filters and interferometers, etc. The propagation of fundamental Gaussian beams and their interaction with optical components can be expressed by simple matrix multiplication, instead of the more exact diffraction theory and radiation integrals [7]. This simplifies the design of complex quasi-optical systems. Undesired effects due to beam truncation or distortions in amplitude and phase can be described by the addition of a few higher-order Gaussian modes and could be taken into account in more demanding designs.

In radio astronomy, quasi-optical systems are of great importance to match the beam coming from the telescope large reflector into the receiver feed horns [8] or to transmit the local oscillator signal necessary for heterodyne reception between the place it is generated and the mixer in the receiver [9]. Modern day radio telescopes, such as ALMA [10], try to cover all available millimeter and submillimeter atmospheric windows with as few receivers as possible. In the case of ALMA, the full 35–950 GHz band was divided into 10 frequency bands for practical implementation. Currently, some groups have started working on covering two ALMA bands with a single receiver, which would show extremely wideband performance approaching one octave bandwidth. From the point of view of receiver optics, these must be carefully designed in order to achieve frequency independent performance over such large bandwidths.

There is plenty of literature on quasi-optical systems, and many groups have reported wideband design efforts. However, the problem of frequency independent designs has not been fully covered. The formulas and design ideas in [11] are the most common reference for frequency independent quasi-optical designs and are based on matching the Gaussian beams on the two sides of a focusing element. Recently, similar results have been reported in [12] from Fresnel diffraction integral analysis. However, to the knowledge of the author, there are not any general references for frequency independent quasi-optical design methods. This paper tries to cover this existing gap with an analysis based on ABCD matrices. Firstly, the fundamentals of frequency independent design based on beam matrices will be described. Then, these results will be applied to the most common quasi-optical systems and compared with existing designs.

2 Theory

The basic design ideas for frequency independent quasi-optical designs in this paper have recently been reported in [13] for a particular application. The basic equations and ideas will be repeated here for the sake of completeness.

In quasi-optical theory [1], electromagnetic propagation is well described by Gaussian beams. These can be described by a complex number q, the complex beam parameter, which contains the information of the beam size w and radius of curvature R, as described in (1), where λ is the wavelength.

$$\frac{1}{q} = \frac{1}{R} - j\frac{\lambda}{\pi w^2} \tag{1}$$

The propagation and transformations of a Gaussian beam through the quasi-optical system are described by simple matrix multiplications, similar to ray matrices in geometrical optics.

The relation between the input and output complex beam parameters can be expressed in terms of four ABCD coefficients as presented in (2). These coefficients describe the propagation or transformation of the beam at a quasi-optical component and can be conveniently grouped in a matrix as in (3).

$$\frac{1}{q_{\text{out}}} = \frac{C + \frac{D}{q_{\text{in}}}}{A + \frac{B}{q_{\text{in}}}} \tag{2}$$

$$\text{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{3}$$

Then, the propagation of a Gaussian beam through the optical system is described by simple matrix multiplication. Each element in the optical system is well characterized by simple matrices, and the total system matrix can be obtained by multiplying all the individual beam transformations in the system.

The expression in (1)–(2) can be manipulated to obtain equations (4) and (5) for the output beam size and radius of curvature, respectively.

$$w_{\text{out}} = w_{\text{in}} \sqrt{\frac{\left(A + \frac{B}{R_{\text{in}}}\right)^2 + \left(B \frac{\lambda}{\pi w_{\text{in}}^2}\right)^2}{AD - BC}} \tag{4}$$

$$R_{\text{out}} = \frac{\left(A + \frac{B}{R_{\text{in}}}\right)^2 + \left(B \frac{\lambda}{\pi w_{\text{in}}^2}\right)^2}{\left(A + \frac{B}{R_{\text{in}}}\right)\left(C + \frac{D}{R_{\text{in}}}\right) + BD\left(\frac{\lambda}{\pi w_{\text{in}}^2}\right)^2} \tag{5}$$

One important conclusion which is obtained from (4) and (5) is that if B equals 0, then, the output beam characteristics are frequency independent, provided that the input beam size and radius of curvature do not depend on frequency. This is usually the case for the beams radiated by a horn at the horn aperture plane. In the case that $B=0$, these equations can be simplified as done in (6) and (7).

$$w_{\text{out}} = w_{\text{in}} \sqrt{\frac{A}{D}} \tag{6}$$

$$\frac{1}{R_{\text{out}}} = \frac{C}{A} + \frac{D}{A} \frac{1}{R_{\text{in}}} \tag{7}$$

3 Application to System Design

The condition that the system ABCD matrix has a term B equal to 0, together with equations (6) and (7) can be used to obtain design equations for any quasi-optical system. In general, quasi-optical systems will have a feed horn that radiates (or receives) the initial Gaussian beam, some focusing elements to control the beam divergence along the optical path and, possibly, some other components which modify the beam characteristics.

In the case of radio astronomy, quasi-optical systems are used to match the beam coming from the large reflector antenna to the beam corresponding to the receiver feed horn. In this case, the beam is just focused and propagated along the quasi-optical system. In addition, systems are usually simple with one or, at most, two focusing mirrors. The same matrix describing the matching between two beams can also be applied to the transmission of power, e.g., LO signal, between two horns by means of a quasi-optical waveguide. Since these systems are the most common in quasi-optics, they will be described thoroughly in the next section.

The study of frequency independent conditions for a system with a thick lens, another interesting case which has not been reported in the literature, will be presented next.

3.1 Quasi-optical System with One Focusing Element

A schematic of a quasi-optical system with one focusing element is presented in Fig. 1. The focusing element can be a mirror or a thin lens. A frequency independent beam with beam size w_{in} and radius of curvature R_{in} is transformed into a beam with size w_{out} and radius of curvature R_{out} by means of a focusing element with focal length f. The distances from the focusing element to the input and output planes are d_1 and d_2, respectively.

The system matrix can be readily calculated from individual matrices and its ABCD coefficients are given in (8–11):

$$A = 1 - \frac{d_2}{f} \tag{8}$$

$$B = d_1 d_2 \left(\frac{1}{d_1} + \frac{1}{d_2} - \frac{1}{f} \right) \tag{9}$$

$$C = -\frac{1}{f} \tag{10}$$

$$D = 1 - \frac{d_1}{f} \tag{11}$$

Direct application of the condition $B=0$ yields the design equation (12), whereas (13)–(14) can be easily derived from (6)–(7).

$$\frac{1}{f} = \frac{1}{d_1} + \frac{1}{d_2} \tag{12}$$

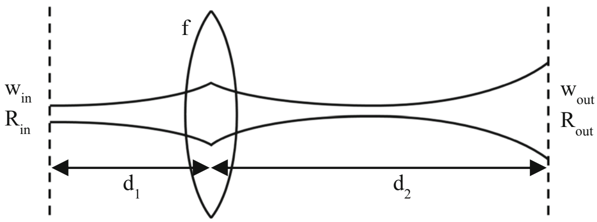

Fig. 1 Schematic of a quasi-optical system with one focusing element

$$w_{\text{out}} = \frac{d_2}{d_1} w_{\text{in}} \tag{13}$$

$$\frac{1}{R_{\text{out}}} = \frac{1}{d_2}\left[1 + \frac{d_1}{d_2}\left(1 + \frac{d_1}{R_{\text{in}}}\right)\right] \tag{14}$$

These design equations are the same which were derived in [11] using beam matching considerations or in [12] using radiation integrals. With the use of simple quasi-optical principles, the same set of equations can be derived quickly without involved mathematics. These equations have been used extensively in the literature as the base for frequency independent designs.

3.2 Quasi-optical System with Two Focusing Elements

A schematic of a quasi-optical system with two focusing elements is presented in Fig. 2. The focusing elements can be mirrors or thin lenses or a combination of both. The different variables are defined in Fig. 2.

The system matrix can be readily calculated from individual matrices and its ABCD coefficients are given in (15–18):

$$A = 1 - \frac{d_2}{f_1} - \frac{d_3}{f_1} - \frac{d_3}{f_2} + \frac{d_2 d_3}{f_1 f_2} \tag{15}$$

$$B = d_1 + d_2 - \frac{d_1 d_2}{f_1} - \frac{d_1 d_3}{f_2} + d_3\left(1 - \frac{d_1}{f_1}\right)\left(1 - \frac{d_2}{f_2}\right) \tag{16}$$

$$C = -\frac{1}{f_1} - \frac{1}{f_2} + \frac{d_2}{f_1 f_2} \tag{17}$$

$$D = \left(1 - \frac{d_1}{f_1}\right)\left(1 - \frac{d_2}{f_2}\right) - \frac{d_1}{f_2} \tag{18}$$

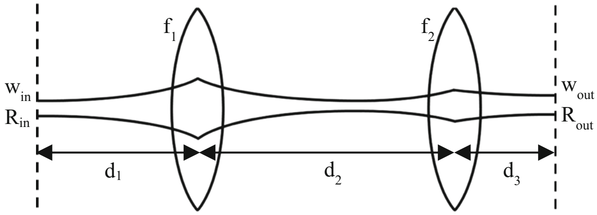

Fig. 2 Schematic of a quasi-optical system with two focusing elements

Under the condition $B=0$, A and D can be rewritten as in (19)–(20).

$$A=-\frac{d_2d_3}{d_1}\left(\frac{1}{d_2}+\frac{1}{d_3}-\frac{1}{f_2}\right) \tag{19}$$

$$D=-\frac{d_1d_2}{d_3}\left(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{f_1}\right) \tag{20}$$

The coefficient B can be factored in a more convenient way after some algebra, as indicated in (21).

$$B=d_1d_2d_3\left[\left(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{f_1}\right)\left(\frac{1}{d_2}+\frac{1}{d_3}-\frac{1}{f_2}\right)-\frac{1}{d_2^2}\right] \tag{21}$$

The condition $B=0$ is stated in (22). It can be separated into two conditions, for f_1 and f_2, by using the parameter k, with $0<|k|\leq 1$, as shown in (23)–(24). The sign of k will depend on the sign of the output radius of curvature R_{out}.

$$\left(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{f_1}\right)\left(\frac{1}{d_2}+\frac{1}{d_3}-\frac{1}{f_2}\right)=\frac{1}{d_2^2} \tag{22}$$

$$\frac{1}{f_1}=\frac{1}{d_1}+\frac{1+k}{d_2} \tag{23}$$

$$\frac{1}{f_2}=\frac{1}{d_3}+\frac{1+1/k}{d_2} \tag{24}$$

Under these conditions, the ACD terms simplify greatly, as presented in (25)–(26):

$$A=\frac{1}{D}=\frac{d_3}{kd_1} \tag{25}$$

$$C=\frac{1}{d_1d_3}\left(kd_1+d_2+\frac{d_3}{k}\right) \tag{26}$$

From equations (6)–(7), two design equations (27)–(28) can be derived, using the parameter k:

$$w_{\text{out}}=\frac{1}{|k|}\frac{d_3}{d_1}w_{\text{in}} \tag{27}$$

$$\frac{1}{R_{\text{out}}}=\frac{1}{d_3}\left[1+\frac{k}{d_3}\left(d_2+kd_1\left(1+\frac{d_1}{R_{\text{in}}}\right)\right)\right] \tag{28}$$

The value of k can be defined in terms of other system parameters. Different expressions for k are provided in (29)–(31). Each expression can be useful for a particular design case.

$$k=-d_2\left(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{f_1}\right) \tag{29}$$

$$k = \frac{d_3}{d_1} \frac{1-\dfrac{d_3}{f_2}}{1-\dfrac{d_1}{f_1}} \left(\frac{w_{in}}{w_{out}}\right)^2 \tag{30}$$

$$k = \pm\sqrt{\frac{\dfrac{1}{d_1}+\dfrac{1}{d_2}-\dfrac{1}{f_1}}{\dfrac{1}{d_2}+\dfrac{1}{d_3}-\dfrac{1}{f_2}}} \tag{31}$$

The design of a frequency independent quasi-optical system with two focusing elements could be directly done using equations (23), (24), (27), and (28) and some mechanical and electrical restrictions. For example, the value of d_1 will be determined from truncation and amplitude distortion considerations apart from mechanical restrictions. Ideally, a long d_1 translates into a larger radius of curvature of the focusing element at the input, which reduces amplitude distortion. However, longer d_1 also means higher beam truncation for a given size of focusing element, which is set by mechanical restrictions and cost. Similar reasoning could be applied to d_2 and d_3. Therefore, when we face the design of a two-focusing-element system, the values of some of the distances, normally d_1 and d_3, will be bounded or even fixed.

The first step in the design would be to use equation (32) for the determination of $|k|$ for certain values of d_1 and d_3. Then, equation (33), derived from (28), will be applied to determine d_2. The sign of k must be the same of the target R_{out}. In other words, k must be positive for a frequency independent beam after the final waist and negative for a frequency independent beam before the final waist of the output beam. The values of focal lengths f_1 and f_2 can then be directly calculated from (23)–(24).

$$|k| = \frac{d_3}{d_1} \frac{w_{in}}{w_{out}} \tag{32}$$

$$d_2 = \frac{d_3}{k}\left(\frac{d_3}{R_{out}} - 1\right) - kd_1\left(1 + \frac{d_1}{R_{in}}\right) \tag{33}$$

In the case d_2 is fixed by mechanical restrictions, equation (33) should not be applied. In this case, the output radius of curvature would not be decided by the designer.

3.3 Quasi-optical System with N Focusing Elements

The generalization of the method presented in this paper for the case of N focusing elements is straightforward but difficult, due to the fact that the size of ABCD matrix elements increases with N and it becomes increasingly difficult to handle. However, some interesting results can be derived from the analysis of the case with N elements.

When a new focusing element is added into the system, this multiplies the matrix M_{N-1} of the system with $N-1$ elements with a matrix for the focusing element with focal length f_N and the additional propagation to the new target plane by a distance d_{N+1}. The matrix for N focusing elements is shown in (34). The ABDC elements of

$M_{N\text{-}1}$ will be noted with the sub-index $N-1$ to distinguish them from the terms of the matrix M_N, with sub-index N.

$$M_N = \begin{pmatrix} 1-\dfrac{d_{N+1}}{f_N} & d_{N+1} \\ -\dfrac{1}{f_N} & 1 \end{pmatrix} M_{N-1} \tag{34}$$

The ABDC terms of matrix M_N in terms of those of M_{N-1} are shown in (35)–(38):

$$A_N = A_{N-1}\left(1-\frac{d_{N+1}}{f_N}\right) + C_{N-1}d_{N+1} \tag{35}$$

$$B_N = B_{N-1}\left(1-\frac{d_{N+1}}{f_N}\right) + D_{N-1}d_{N+1} \tag{36}$$

$$C_N = C_{N-1}-\frac{A_{N-1}}{f_N} \tag{37}$$

$$D_N = D_{N-1}-\frac{B_{N-1}}{f_N} \tag{38}$$

The matrix M_N is created from M_0, with M_0=[1 d1; 1 0], and successive multiplication as in (34). The determinant of M_0 is always 1 as can be seen by simple inspection. By the properties of determinants, the determinant of a product of matrices equals the products of determinants [14], which is 1 in this case. Therefore, the determinant of M_N, and of any intermediate M_i, will always be 1.

In the case of M_N, B_N=0 for frequency independence means that the product $A_N D_N$ is 1 for any value of N. This is stated in (39) for clarity.

$$A_N = \frac{1}{D_N} \tag{39}$$

From the condition B_N=0 and (38), D_N and D_{N-1} can be expressed in terms of B_{N-1} as in (40)–(41):

$$D_{N-1} = -\left(\frac{1}{d_{N+1}}-\frac{1}{f_N}\right)B_{N-1} \tag{40}$$

$$D_N = -\frac{B_{N-1}}{d_{N+1}} \tag{41}$$

The terms C_N and C_{N-1} can be expressed in terms of A_{N-1} and B_{N-1} as in (42)–(43), considering (37) and the fact that the determinant of M_{N-1} equals 1.

$$C_{N-1} = -\left(\frac{1}{d_{N+1}}-\frac{1}{f_N}\right)A_{N-1}-\frac{1}{B_{N-1}} \tag{42}$$

$$C_N = -\frac{A_{N-1}}{d_{N+1}} - \frac{1}{B_{N-1}} \tag{43}$$

The results in (39)–(43) can be put together in three design equations (44)–(46):

$$\frac{1}{f_N} = \frac{1}{d_{N+1}} + \frac{D_{N-1}}{B_{N-1}} \tag{44}$$

$$w_{\text{out}} = \frac{d_{N+1}}{|B_{N-1}|} w_{\text{in}} \tag{45}$$

$$\frac{1}{R_{\text{out}}} = \frac{1}{d_{N+1}} \left[1 + \frac{B_{N-1}}{d_{N+1}} \left(A_{N-1} + \frac{B_{N-1}}{R_{in}} \right) \right] \tag{46}$$

Equations (44)–(46) indicate that design equations with N elements can be expressed in terms of the matrix of the system with N−1 elements. In other words, equations (22), (27), and (28) can be derived from matrix elements (8)–(11) for one focusing element. This can be useful to avoid extra work for the design of systems with more elements.

From the condition of frequency independence, three equations are derived, whereas the number of design variables can be much higher. In conclusion, complex systems with N mirrors allow more flexibility in the design, which can be used to satisfy mechanical restrictions or to impose other conditions to improve the system performance (distortion, cross-polarization, etc.).

Since the complexity of the ABCD matrix increases with the number of elements, the designer may be tempted to concatenate several one or two element designs to achieve an N element design. In fact, in section 3.1, it was shown that a focusing element with a frequency independent beam at distance d_1 from the input will generate a frequency independent beam at distance d_2 given by (12) with the characteristics given by (13)–(14). Then, if we apply the same equations once and again, we will have a frequency independent design in the end. However, it is easy to see that for a given total design length L, all design targets can only be satisfied for the solutions of the N element matrix with $B=0$. For example, in the case of two elements, a first element design f_1 can be done using equations (12)–(14), which will yield a beam with radius w_{aux} and radius of curvature R_{aux} at the frequency independent plane, located at d_2 from the focusing element. Then, for our restriction of a given total optical path length, the distance from that plane to the target plane is L (L=total distance$-d_1-d_2$). For the second focusing element, the input distance to the lens will be $L-d_3$ and the output distance will be d_3. Therefore, for the second element design, we will have three design equations and only two variables d_3 and f_2. Therefore, either w_{out} or R_{out} will differ from our target if the resulting design is frequency independent. All conditions can only be met simultaneously with the dimensions derived from the presented design methodology.

3.4 Quasi-optical System with One Thick Lens

A special case of a focusing element which requires a special treatment is the thick lens. In this case, the thickness is comparable to the other dimensions of the optical design and must be considered. There is no literature about frequency independent quasi-optical designs with a thick lens to the knowledge of the author. Applying the approach in this paper, it is straightforward to get the design equations which apply for this particular focusing element.

In the case of a thick lens, the ABCD matrix depends explicitly on the lens thickness t, the refraction index n, and the input and output radii of curvature R_{in} and R_{out}, through the input and output focal lengths, f_1 and f_2, as stated in (47)–(50). In the usual sign convention, $R_{in}<0$ and $R_{out}>0$ for a usual focusing lens. Therefore, f_1 and f_2 are both positive. Notice that M simplifies to the thin lens matrix when $t=0$.

$$\frac{1}{f_1} = -\frac{n-1}{R_{in}} \tag{47}$$

$$\frac{1}{f_2} = \frac{n-1}{R_{out}} \tag{48}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \tag{49}$$

$$\mathrm{M}_{\text{thick lens}} = \begin{pmatrix} 1-\frac{t}{nf_1} & \frac{t}{n} \\ -\frac{1}{f}+\frac{t}{nf_1f_2} & 1-\frac{t}{nf_2} \end{pmatrix} \tag{50}$$

The ABCD terms of a quasi-optical system matrix for a thick lens and a distance d_1 before it, and d_2 after it, are given in (51)–(54). The terms depending on the lens thickness have been grouped together to appreciate its effect clearly. The term t/n will be replaced hereon by t' to simplify notation.

$$A = 1-\frac{d_2}{f}-\frac{t'}{f_1}\left(1-\frac{d_2}{f_2}\right) \tag{51}$$

$$B = d_1d_2\left[\left(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{f}\right)+t'\left(\frac{1}{d_1}-\frac{1}{f_1}\right)\left(\frac{1}{d_2}-\frac{1}{f_2}\right)\right] \tag{52}$$

$$C = -\frac{1}{f}+\frac{t'}{f_1f_2} \tag{53}$$

$$D = 1-\frac{d_1}{f}-\frac{t'}{f_2}\left(1-\frac{d_1}{f_1}\right) \tag{54}$$

Next, $B=0$ is applied for frequency independence. As done for the case of two focusing elements, f_1 and f_2 can be separated by the use of a dummy variable k, with $0<k<1$. This is obvious after expressing $B=0$ as in (55).

$$\frac{1}{\frac{1}{f_1}-\frac{1}{d_1}} + \frac{1}{\frac{1}{f_2}-\frac{1}{d_2}} = kt' + (1-k)t' \tag{55}$$

After some algebra, some conditions (56)–(58) can be derived from $B=0$,

$$\frac{1}{f_1}=\frac{1}{d_1}+\frac{1}{kt'} \tag{56}$$

$$\frac{1}{f_2}=\frac{1}{d_2}+\frac{1}{(1-k)t'} \tag{57}$$

$$\frac{1}{f}=\frac{1}{d_1}+\frac{1}{d_2}+\frac{1}{k(1-k)t'} \tag{58}$$

This allows to establish the design equations in (59)–(60):

$$w_{\text{out}}=\frac{k}{1-k}\frac{d_2}{d_1}w_{\text{in}} \tag{59}$$

$$\frac{1}{R_{\text{out}}}=\left(\frac{1}{k}-1\right)\frac{d_1}{d_2}\left[-\frac{1}{d_1}-\frac{1}{d_2}+\frac{1}{d_1d_2}\left(t'+\frac{d_1}{k}+\frac{d_2}{1-k}\right)\right]+\left(\frac{1}{k}-1\right)^2\left(\frac{d_1}{d_2}\right)^2\frac{1}{R_{\text{in}}} \tag{60}$$

In a design, there will be four variables to be determined: d_1, d_2, f_1, and f_2. Again, we only have three equations for the design, which means that we have one degree of freedom. This can be used in the case that there are some mechanical restrictions on the distances d_1 or d_2. If not, if can be used to impose some condition on the lens. For example, f_1 and f_2 can be forced to be equal in order to have a symmetrical lens, which simplifies fabrication.

One special case not considered in the derivation above is when $f_1=d_1$ and $d_2=f_2$. In this case, $B=0$ too, and the design is frequency independent. Design equations (12) and (13) are valid and do not depend on the lens thickness and (14) changes into (61):

$$\frac{1}{R_{\text{out}}}=\frac{1}{d_2}\left[1-\frac{t'}{d_2}+\frac{d_1}{d_2}\left(1+\frac{d_1}{R_{\text{in}}}\right)\right] \tag{61}$$

Notice that in the case that $t'\ll d_2$, frequency independent designs are the same for thin and thick lens, for $f_1=d_1$ and $f_2=d_2$.

Another interesting particular case is that of a flat lens surface at input or output. In this case, $f_1\to\infty$ or $f_2\to\infty$, respectively. In this case, f equals the value of f_1 or f_2 which is finite and all equations simplify. Actually, there is no need to use variable k, and the equations are simple. Resulting equations are the same as for a thin lens, (12)–(14), but with a change in d_1 or d_2. If $f_1\to\infty$, d_1+t' must be used instead of d_1. If $f_2\to\infty$, d_2+t' must be used instead of d_2.

4 Application

The formulas and design methodologies with one and two focusing elements will be applied in this section to two receivers for radio astronomy: ALMA band 4 (125-163 GHz) and 10 (787-950 GHz) receivers.

In the case of the ALMA band 4 receiver, described in [15], the optics are composed of a corrugated horn with fields at the aperture modeled by a beam size of 7.722 mm and a radius of curvature of 100.717 mm. A single ellipsoidal mirror is used to match this beam to the beam required for the illumination of the secondary mirror of the ALMA 12-m antenna. The target parameters are w_{out}=319 mm and R_{out}=6000 mm. A relationship between d_1 and d_2 can be easily obtained through (13). If this is substituted in (14), a value of d_2=6367.439 mm can be readily obtained. Using (13), d_1 equals 154.116 mm. The focal length of the mirror f is calculated with (12) and is 150.474 mm in this case. These values are the same as those reported in Table I in [15].

In the case of ALMA band 10 [16], the quasi-optical system is based on two focusing mirrors and it is located completely within the 4K stage of the ALMA cryostat, which imposes strong mechanical restrictions in terms of distances and mirror sizes. The target illumination of the secondary mirror of the ALMA antenna is the same as for band 4. The horn in this case can be modeled by a beam size of 1.931 mm and a radius of curvature of 15.723 mm. In the case of ALMA band 10, the maximum possible d_3 is too short to achieve a radius of curvature of 6 m at the secondary mirror. Therefore, (28) could not be used in this design. Instead, all distances were decided based on mechanical restrictions and only the mirror focal lengths were decided upon frequency independence. The values of distances are d_1=45 mm, d_2=80.044 mm, and d_3=6088 mm. With these restrictions, the value of k is 0.819 from (27). The mirror focal lengths, f_1 and f_2, are 22.25 and 35.821 mm, respectively, from (23) and (24). These values are in good agreement with those presented in [16], which were further refined using physical optics optimization.

5 Conclusions

There are many designs in the literature which are based on frequency independence ideas. Most designs are based on the theory of Gaussian beam matching presented in [11]. With that methodology, one-element equations are applied in the successive sections of a beam waveguide. The beam parameters at the intermediate frequency independent positions are used to connect the equations for connecting sections and equations are solved.

This paper presents a new approach which uses ABCD matrices to find the conditions for frequency independence and is therefore more general, in the sense that it can be applied to any kind of components and beam transformations.

Design equations have been obtained for the cases with one and two focusing elements and for the case of a system with a thick lens. Some interesting results have been derived from the study of the general situation with N focusing elements. Finally, the design equations in this paper have been used to reproduce the results of the quasi-optical designs for the ALMA band 4 and 10 receivers.

It is interesting to point out that some of the formulas in this paper are the same as in [11], notably the case with one or two elements, but they have been derived by applying more general quasi-optical concepts. For example, a quasi-optical system with a thick lens, as analyzed in this paper, cannot be designed directly using the equations in [11].

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