International Journal of Infrared and Millimeter Waves, Vol. 27, No. 3, March 2006 (© 2006)
DOI: 10.1007/s10762-006-9054-2

STUDY ON EXCESS CONDUCTION LOSS IN NORMAL METALS AT/BELOW SUB-MILLIMETER WAVELENGTHS

Wei Pan [a], Xiaoxia Zhang [b]

[a]*School of Information Science and Technology,*
Southwest Jiaotong University,
Chengdu, 610031, China
[b]*School of Optic-electronic Information,*
University of Electronic Science and Technology of China,
Chengdu, 610054, China.;

Received 12 December 2005

Abstract,

We have used the spatial dispersion theory to develop a rigorous model to investigate excess conduction loss in normal metals. We have used the model to account excess conduction loss and dissect the discrepancies between excess conduction loss measurements and classical theoretical predictions in normal metals at/below sub-millimeter wavelengths. Moreover, we have compared the results of this model with the results of the classical skin-effect model and classical relaxation- effect model. Our analysis shows that the conductivity is not only frequency but also wave vector dependent. The results of the calculations indicate a good quantitative agreement with the published experimental data for the room temperature excess conduction loss of normal metals at/below sub-millimeter wavelengths.

Keywords: spatial dispersion, sub-millimeter wavelengths, excess conduction loss, normal metals, wave vector.

1. Introduction

Excess conduction loss depends on the surface resistance. Surface resistance is a classical topic in electrodynamics and there is an extensive theoretical and experimental literature on the subject [1–3]. Starting with the pioneering work by Ament and Rado, theoretical studies of the surface resistance in metallic ferromagnets have been based on solving Maxwell's equations for electromagnetic fields, with appropriate boundary conditions at sample's surfaces [4-9].

Measurements of the surface resistance provide an accurate tool for the investigation of the Fermi wavevector. Accurate values of the surface resistance of conductors are very important to the efficient utilization of the sub-millimeter-wave region which is under development for applications in communications, radar, and radio astronomy at present.

However, a number of researchers have reported discrepancies between surface resistance measurements and classical theoretical predictions at/below sub-millimeter wavelengths [10-11].

This paper presents a rigorous model for analyzing excess conduction loss of normal metals. The model is used to evaluate the excess conduction loss in normal metals at/below sub-millimeter wavelengths. The model includes spatial dispersion within the metal.

2. The model

It is well know that the temporal dispersion leads to a frequency dependence of the dielectric function and the spatial dispersion often requires dielectric function that depend on the wave vector. In this section we provide the quantum expression for the dielectric function for normal metals within the spatial dispersion theory and use perturbation theory to find the change in the exciton wavefunction caused by an electric field with frequency ω and wave vector q. We consider the simplest model of a metal like the free-electron gas. The dynamical dielectric function $\varepsilon(q,\omega)$ in the MKS units can be calculated from the quantum expression of the free-electron gas as given by [6].

$$\varepsilon(q,\omega) = 1 + \frac{2e^2}{q^2\varepsilon_0}\frac{1}{V}\sum_k \frac{f(k) - f(k+q)}{E(k+q) - E(k) - \hbar\omega + i\eta} \tag{1}$$

Where f (k) is Fermi-Dirac distribution function, E (k) is the energy, V is volume. All other variables have their usual meaning. We have started from this basic equation performing the replacement k+q→-k' in the term containing f (k+q) (than k' is relabeled as k). We also use the dynamical dielectric function in the long wavelength limit. We notice that $\frac{1}{V}\sum_k f(k) = \frac{n}{2}$, $\frac{1}{V}\sum_k (\frac{\hbar k}{m})^2 f(k) = \frac{3}{5}v_F^2\frac{n}{2}$ and the DC conductivity $\sigma_0 = \frac{ne^2}{m}\tau$ where the Fermi velocity $v_F = \hbar k_F / m$, $2\eta/\hbar = 1/\tau$, τ is the electron collision time and n is the electron density. We obtain

$$\varepsilon(q,\omega) = 1 + \frac{\sigma_0}{i\omega\varepsilon_0(1 + i\omega\tau(1 - \frac{3v_F^2q^2}{5\omega^2}))} \tag{2}$$

Because of including spatial dispersion, the dynamical dielectric function $\varepsilon(q,\omega)$ is not only frequency but also wave vector dependent.

The surface impedance Z is defined as the ratio of the (complex) electric and magnetic field components in the direction parallel to the surface of incidence, evaluated just inside the surface. This concept gives a convenient description of the optical properties of a material half-space. The real part of Z, which is called the "surface resistance", is proportional to the energy absorbed from the wave and it is useful in determining the shapes of the Fermi surfaces of metals. The imaginary part of Z, called the "surface reactance", determines the frequency shift of a resonant cavity bounded by the conductor. Our definition of the surface impedance in the MKS Units is

$$Z = E(0)/B(0) = \mu c/n = \mu c/\sqrt{\varepsilon} \tag{3}$$

The propagation of electromagnetic waves through matter is determined by solving the Maxwell equations, in which the dielectric function ε, equal to the square of the index of refraction , carries information about the material properties. From Equ. (2) and Equ. (3), we obtain,

$$Z = \sqrt{\frac{i\omega\mu\mu_0}{\sigma_s + i\omega\varepsilon_0}} \tag{4}$$

So surface resistance is

$$R_{sd} = \mathrm{Re}\left[\sqrt{\frac{i\omega\mu_0\mu_r}{\sigma_{sd} + i\omega\varepsilon_0}}\right] \qquad \sigma_{sd} = \frac{\sigma_0}{1 + i\omega\tau\left(1 - \frac{3v_F^2 q^2}{5\omega^2}\right)} \tag{5}$$

We have developed a rigorous phenomenological model which is called the "spatial dispersion model".

The excess conduction loss $\Delta\alpha$ are defined as

The spatial dispersion model $$\Delta\alpha_{sd} = \frac{R_s}{R_{sd}} \tag{6}$$

The classical skin-effect model [10] $$\Delta\alpha_0 = \frac{R_s}{R_0} \tag{7}$$

The classical relaxation-effect model [10] $$\Delta\alpha_R = \frac{R_s}{R_R} \tag{8}$$

Where, R_s is measured surface resistance, R_0 is normalizing classical skin-effect surface resistance, R_R is normalizing classical relaxation-effect surface resistance. Their define are following.

$$R_0 = \mathrm{Re}\left\{\sqrt{\frac{i\omega\mu_0\mu_r}{\sigma_0 + i\omega\varepsilon_0}}\right\}, \quad \sigma_0 = \frac{ne^2}{m}\tau,$$

$$R_R = \mathrm{Re}\left[\sqrt{\frac{i\omega\mu_0\mu_r}{\sigma_R + i\omega\varepsilon_0}}\right], \quad \sigma_R = \frac{\sigma_0}{1 + i\omega\tau}.$$

In Fresnel optics spatial dispersion is neglected. This simple free-electron form of $\varepsilon(\omega)$ is, however, often not sufficient for real metals. In the "spatial dispersion model", we include spatial dispersion within the metal in the approximation. The following analysis shows that using this model demonstrate good quantitative agreement with the published experimental data for the room temperature excess conduction loss of normal metals at/below sub-millimeter wavelengths range from the literature.

3. Results and discussion

In this section we have used the spatial dispersion model to predict the excess conduction loss in normal metals at/below sub-millimeter wavelengths range. We have compared the results of the model with the results of the classical skin-effect model and classical relaxation- effect model.

Table 1 Solid-state parameters for different metals[9, 12]

Parameter / Metal	Element Density g/cm^3	Atomic Weight g/mole	τ fs	σ_0 $\times 10^7$ S/m
Silver	10.49	107.868	38.182	6.301
Gold	19.32	196.967	27.135	4.517
Aluminium	2.70	26.982	7.407	3.774

Because of a lot of experimental data for the room temperature surface resistance of high electrical conductivity metals: silver, gold, aluminium, we calculate of the excess conduction loss in this work takes only silver, gold, aluminium, it should be equally applicable to other normal metals. Calculation is based on the values of q given in [5], other values are given in table 1 and other sources [10-11]. The results for silver, gold and aluminium are shown in Table 2- Table 5, including the results from the classical skin-effect model and the classical relaxation-effect model [11, 14].

Table 2 Excess conduction loss for silver

$\lambda_0\,(\mu m)$	$R_s(\Omega)$	$R_0(\Omega)$	$R_R(\Omega)$	$R_{sd}(\Omega)$	α_0	α_R	α_{sd}
32	0.3679	0.7661	0.3531	0.3615	0.480	1.042	1.018
30	0.3695	0.7913	0.354	0.3658	0.467	1.044	1.01
28	0.3701	0.8190	0.3549	0.3701	0.452	1.043	1
26	0.3731	0.8499	0.3558	0.3740	0.439	1.049	0.998

Table 2 shows excess conduction loss factor for silver below sub-millimeter wavelengths range (32 μm ～26 μm). An extracted measurement value of surface resistance was given by Bennett et al [13]. An excess conduction loss factor (α_0 and α_R) for both classical excess conduction loss models was reported by [10]. Using the spatial dispersion model, we obtained a value for the excess conduction loss factor of α_{sd}. It is clearly seen that the measured were: 56.1%, 54.8%, 53.3%, 52% with the classical skin-effect model; 4.9%, 4.3%, 4.4%, 4.2% with the classical relaxation-effect model; 0.2%, 0%, 1%, 1.8% with the spatial dispersion model in the wavelengths range 32 μm, 30 μm, 28 μm and 26 μm, respectively.

It is easy to see that there are very large deviations from the classical skin-effect model below sub-millimeter wavelengths range. The classical relaxation-effect model can be approximated to measurement value. The spatial dispersion model is sufficiently accurate to describe the excess conduction loss of silver below sub-millimeter wavelengths range at room temperature.

Table 3 Excess conduction loss for gold

$\lambda_0\,(\mu m)$	$R_s(\Omega)$	$R_0(\Omega)$	$R_R(\Omega)$	$R_{sd}(\Omega)$	α_0	α_R	α_{sd}
32	0.5287	0.9079	0.4852	0.4956	0.582	1.090	1.067
30	0.5322	0.9309	0.4870	0.5023	0.572	1.093	1.060
28	0.5333	0.9643	0.4892	0.5089	0.553	1.090	1.048
26	0.5380	1.0072	0.4917	0.5152	0.534	1.094	1.044

Table 3 shows excess conduction loss factor for gold below sub-millimeter wavelengths range (32 μm ～26 μm). An extracted measurement value of surface resistance was given by Bennett et al [13]. An excess conduction loss factor (α_0 and α_R) for both classical excess conduction loss models was reported by [10]. Using the spatial dispersion model, we obtained a value for the excess conduction loss factor of α_{sd} . The measured were: 46.6%, 44.7%, 42.8%, 41.8% with the classical skin-effect model; 9.4%, 9%, 9.3%, 9% with the classical relaxation-effect model; 6.7%, 6%, 4.8%, 4.4%with the spatial dispersion model in the wavelengths range 32 μm, 30 μm, 28 μm and 26 μm, respectively.

Table 4 Excess conduction loss for gold

$\lambda_0\,(\mu m)$	$R_s(\Omega)$	$R_0(\Omega)$	$R_R(\Omega)$	$R_{sd}(\Omega)$	α_0	α_R	α_{sd}
318	0.3120	0.2870	0.2650	0.3016	1.087	1.178	1.034
269	0.3308	0.3121	0.2840	0.3259	1.060	1.165	1.015
236	0.3416	0.3332	0.2993	0.3474	1.025	1.142	0.983
200	0.3664	0.3620	0.3190	0.3666	1.012	1.149	0.999
165	0.3813	0.3985	0.3421	0.3887	0.957	1.115	0.981
143	0.4016	0.4281	0.3593	0.4075	0.938	1.118	0.986
125	0.4082	0.4578	0.3752	0.4237	0.892	1.088	0.987
111	0.4155	0.4859	0.3888	0.4377	0.855	1.069	0.949
100	0.4086	0.5119	0.4004	0.4555	0.798	1.020	0.897
90.9	0.4136	0.5369	0.4106	0.4659	0.770	1.007	0.888
83.3	0.4085	0.5609	0.4196	0.4750	0.728	0.973	0.860
76.9	0.4186	0.5837	0.4274	0.4830	0.717	0.979	0.867
71.4	0.4100	0.6058	0.4343	0.4990	0.677	0.944	0.821
66.7	0.4360	0.6268	0.4403	0.4961	0.696	0.990	0.879

In addition to the work of Bennett et al. Brandli et al. reported measurement for gold that extended longer wavelengths range (318 μm ～

66.7 μm).[14]. Table 4 is comparison of Brandli's experiment data and three models predictions excess conduction loss factor for gold. The classical relaxation-effect model can be approximated to the classical skin-effect model at these relatively long wavelengths. There are some deviations from both classical model. The spatial dispersion model is sufficiently accurate to describe the excess conduction loss factor for gold at/below sub-millimeter wavelengths range at room temperature.

Table 5 Excess conduction loss for aluminium

$\lambda_0(\mu m)$	$R_s(\Omega)$	$R_0(\Omega)$	$R_R(\Omega)$	$R_{sd}(\Omega)$	α_0	α_R	α_{sd}
32	0.8165	0.9900	0.8012	0.8100	0.825	1.019	1.008
31	0.8226	1.0059	0.8088	0.8194	0.818	1.017	1.004
30	0.8286	1.0225	0.8166	0.8287	0.810	1.015	1
29	0.8343	1.0400	0.8245	0.8378	0.802	1.018	0.996
28	0.8460	1.0584	0.8327	0.8488	0.799	1.016	0.997
27	0.8511	1.0778	0.8410	0.8594	0.790	1.012	0.990
26	0.8627	1.0983	0.8495	0.8679	0.785	1.015	0.994
25	0.8671	1.1201	0.8582	0.8807	0.774	1.010	0.985

Table 5 is comparison of experiment data and three models predictions excess conduction loss factor for aluminium. A measurement value of surface resistance was given by Bennett et al [13]. An excess conduction loss factor (α_0 and α_R) for both classical excess conduction loss models was reported by[11]. Using the spatial dispersion model, we obtained a value for the excess conduction loss factor of α_{sd}. The analysis for aluminium is also similar to silver in same wavelengths range as mentioned in the discussed Table 2.

4. Conclusion

In this paper, we have developed and demonstrated a rigorous model applied to the excess conduction loss. We have used the model to predict the excess conduction loss of normal metals at/below sub-millimeter wavelengths. We have compared the experiment data with three models predictions the excess conduction loss for silver, gold, aluminium. Our analysis shows that conductivity is not only frequency but also wave vector dependent. We have seen that the spatial dispersion model is very useful for the description of excess conduction loss in normal metals. With normal metals at room temperature, the classical relaxation-effect model can be approximated to the classical skin-effect model up 300 μm. The classical relaxation-effect model is more complicated and accurate than the classical skin-effect model at shorter wavelengths range. It can be clearly seen in Table 4 that there are some deviations from both classical model with longer wavelengths measurement setups. It is show that there are very large deviations from the classical skin-effect model in the shorter wavelengths range (32 μm ～26 μm). The classical relaxation-effect model can be approximated to measurement value. In this work we developed model, the spatial dispersion model, is sufficiently accurate to describe the excess conduction loss behavior at/below sub-millimeter wavelengths range.

Acknowledgment

This work is supported by the National Natural Science Foundation of China (Grant Nos 10174057 and 90201011) and the key project of Chinese Ministry of Education (Grant No. 2005-105148).

References

[1] Bozorth R 1956 Ferromagnetism (Princeton: Nostrand Comp).

[2] Landau L D and Lifshitz E M 1960 Electrodynamics of Continuous Media (New York: Pergamon).

[3] Akhiezer A I, Baryakhtar V G and Peletminskii S V 1968 Spin Waves (Amsterdam: North-Holland).

[4] Ordal M A, and Long L L et al 1983 Optical properties of the metals Al,

Co, Cu, Au, Fe, Pd, Ni, Pd, Pt, Ag, Ti, and W, Appl. Opt. Vol. **22,** 1099 – 1119.

[5] Halevi P. 1992. Spatial Dispersion in Solids and Plasmas. (Amsterdam: North-Holland).

[6] Giuseppe G. 2000. Solid State Physics (Cambridge: Academic Press).

[7] Abrikosov A A 1972 Introduction to the Theory of Normal Metals (New York: Academic Press).

[8] Michael P. M. 2000 Condensed Matter Physics (New York: John Wiley & Sons, Inc).

[9] Lide D R 2003 83rd edn, CRC Handbook of Chemistry and Physics(CRC Press).

[10] Zhang X X, Pan W and Lucyszyn S, 2004, A new theoretical modelling of surface resistance in normal metals at terahertz frequencies, International Journal of Infrared and Millimeter Waves, vol. 25, 16211-1620

[11] Lucyszyn S, 2004 Investigation of anomalous room temperature conduction losses in normal metals at terahertz frequencies, IEE Proceedings Microwave, Antennas and Propagation, vol. 151, 321-329

[12] Fluck and Heumann 1991, Periodic Properties of Metals (UK: VCH Publishers).

[13] Bennett H E and Bennett J M 1966, Validity of the Drude theory for silver, gold and alumina in the infrared, in Abeles, F. (ed): Optical Properties and Electronic Structure of Metals and Alloys (Amsterdam: North - Holland).

[14] Brandli G and Sievers A J 1972 Phys., Absolute measurement of the far infrared surface resistance of Pb, Rev.B vol. **5** 3550.