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ORIGINAL RESEARCH

Mathematics Apps Under the Magnifying Glass – An Analysis of the Inventory of Math Apps for Primary Schools Using German-Language Apps as A Case Study

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Abstract
The usage of digital media, especially tablet apps, is currently a major concern in educational settings. A controversial discussion is also taking place in relation to mathematics education at primary schools. While on the one hand there are empirical findings on helpful isolated examples and related subject-didactic potentials of digital media, on the other hand numerous apps are criticized. Systematic analyses of the app stores inventory are rare, however. This article therefore deals with the analysis of the app store's inventory of mathematics apps available for primary school use. Using the German app market as an example, 227 apps were analyzed. The results show that those apps are mainly located in the content-related area of *numbers and operations*, while process-related competencies and subject-didactic potentials of digital media are largely neglected. Moreover, apps focus on unstructured forms of practising fluent calculation. All in all, only a fragment of mathematics learning at primary schools is addressed, with frequently no consideration of subject-didactic potentials in mathematics apps, making the need for development of subject-didactically based mathematics apps obvious.

Keywords Apps · Inventory analysis · Mathematics education · Primary school · Digital media

1 Introduction

Since the availability of the first computers in the 1980s, the use of digital media in the educational context has been critically discussed (Freudenthal, 1981; Clements, 2002). Although digital media have not (yet) reached schools to the extent that is widely assumed (Drijvers, 2019), the discussion about the use of digital media is currently a central topic of

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educational policy, scientific and social debate in all areas of education – also in relation to mathematics education in primary schools.

Although there is currently no broad use of digital media in (mathematics) lessons in most countries around the world (Mullis et al., 2020), recent studies point to learning effects (Aspiranti & Larwin, 2021; Drijvers & Sinclair, 2023; Hillmayr et al., 2020; Steenbergen-Hu & Cooper, 2013) and potentials of digital media across all school levels (Bullock et al., 2017; Moyer-Packenham & Westenskow, 2016; Walter, 2018). Additionally, Nikolopoulou. et al. (2021) demonstrated that primary school teachers tend to be more aware of the potentials of digital media than teachers at other educational institutions. In view of such highly promising data, the use of digital media – when embedded appropriately in the classroom – is being encouraged from many different sides. In specific fields of mathematics education, the usage of digital media is highly developed. A substantial body of research has demonstrated how digital media can support both procedural and conceptual understanding processes through digital technologies. For instance, digital technologies have been shown to facilitate the learning of geometry (Turgut et al., 2022; Sinclair & Patterson, 2018) and statistics (Ben-Zvi, 2000; Noll et al., 2022) in secondary education. Yet, in other fields, especially in learning arithmetic at the primary level, the quality of available digital learning opportunities has been and continues to be sharply criticized due to the frequently observed one-sided focus on calculation procedures rather than understanding (Bednorz & Bruhn, 2023; Kim et al., 2021; Krauthausen, 2013; Larkin, 2015; Namukasa et al., 2016).

Since schools (not only) in Germany are currently supported with considerable funds by education policy to improve the digital infrastructure especially by providing tablet computers, tablet apps[1] for teaching mathematics are recently of particular interest in practice and research. Consequently, it is a central goal of mathematics education to design and select mathematics apps on the basis of subject didactic standards (Moyer-Packenham et al., 2015). The literature provides a number of (valuable) contributions dedicated to the analysis of individual apps or a manageable number of apps, highlighting possible uses and limitations (including Larkin et al., 2019). However, there is a lack of a more wide-ranging current analysis that comparatively examines not only single apps but as many potentially available mathematics apps as possible to contribute to a more systematic view of app store inventories. Although such an inventory analysis is postulated by many researchers (Calder, 2015; Trouche et al., 2012; Walter, 2018), there seems to be hardly any current research at present. However, research of this kind seems necessary to provide teachers and researchers an evaluative overview to select apps and to identify gaps that could be addressed by researchers (Larkin & Milford, 2018). In addition, the majority of existing evaluation tools "are not considered adequate to help teachers and parents to evaluate the pedagogical affordances of educational apps correctly and easily" (Papadakis, 2021, p. 18).

Based on the framework for primary school mathematics in Germany derived from the *Standards* of the NCTM (NCTM, 2000), this study provides an example of how the inventory of maths apps for primary schools can be evaluated. Using the German apps market as an example, this study describes the design and results of an inventory analysis of $N=227$ maths apps for primary schools found there. In particular, it is demonstrated to what extent maths apps consider curricular frameworks, learning phases, and subject didactic potentials of digital media. Thoughts on generalization beyond the German example and a critical dis-

[1] This study concentrates on tablet apps, which have become predominant over other types of apps such as web apps or smartphone apps, at least in the context of daily school activities in Germany.

cussion concludes the article, in which consequences for research and practice are derived and limitations are described.

2 Theoretical Background

2.1 Curricular Goals in Mathematics Education - Facilitated by Digital Media?

The central task of mathematics education in primary school is to support learners in acquiring basic competencies for further learning. When it comes to the question of which competencies these are in detail, curricula have been formulated in the vast majority of states. This study gives exemplary insights with regard to the German context, but since tablet apps are accessible in every country and the platforms, on which these apps can be used, are mostly global platforms, the insights from our study have a high international relevance.

For mathematics learning in Germany, the so-called *Bildungsstandards* (KMK, 2022; eng.: *educational standards*) are currently the guiding standards. They differentiate between content-related competencies on the one hand and process-related competencies on the other. While the five content-related competencies (*numbers and operations*; *measurement*; *spatiality and form*; *data and chance*; *patterns*, *structures*, *functional relationship*) define *what* should be learned, the process-related competencies (*reasoning mathematically*, *solving problems mathematically*, *representing mathematically*, *working with mathematical objects and tools*; *communicating mathematically*, *modeling mathematically*) address the question of *how* to learn.

It is postulated that the promotion of content- and process-related competencies should be integrated. Accordingly, the learning should be designed in such a way that the work is not exclusively content-related, for example, by focusing only on the rapid determination of correct solutions. Rather, it entails more than just such surface knowledge, as the understanding of mathematical subjects is provided for by means of different in-depth ways of working, which are linked to the process-related competencies. For example, the comprehension-based teaching of multiplication can be designed in such a way that the focus of the lesson is not solely on the expeditious and accurate execution of tasks. Alternatively, mathematical problem-solving can be encouraged by prompting students to identify as many or all tasks with a specific result (e.g., 24). Prompting students to justify their solutions encourages mathematical reasoning. Similarly, the appropriate use of enactive and iconic representations that show the core of multiplication as thinking in bundled groups (Götze & Baiker, 2021) can facilitate mathematical communication. Figure 1 shows the interplay of content-related and process-related competencies.

Since the *Bildungsstandards* proclaimed by German educational authorities have their roots in the *Standards* of the NCTM (as described in their *Principles and Standards*, National Council of Teachers of Mathematics, 2000), clearly recognizable coherences can be identified. Five *Content Standards* are named there, (*Number & Operations*, *Algebra*, *Geometry*, *Measurement* and *Data Analysis & Probability*), to which the German *Bildungsstandards* are recognizably very closely aligned. Furthermore, five *Process Standards* are described, which are named as follows: *Problem Solving*, *Reasoning & Proof*, *Communication*, *Connections*, *Representation*. Even if the latter terms differ slightly in some cases, there is a great correspondence in terms of content. For example, the NCTM process standard *Con-*

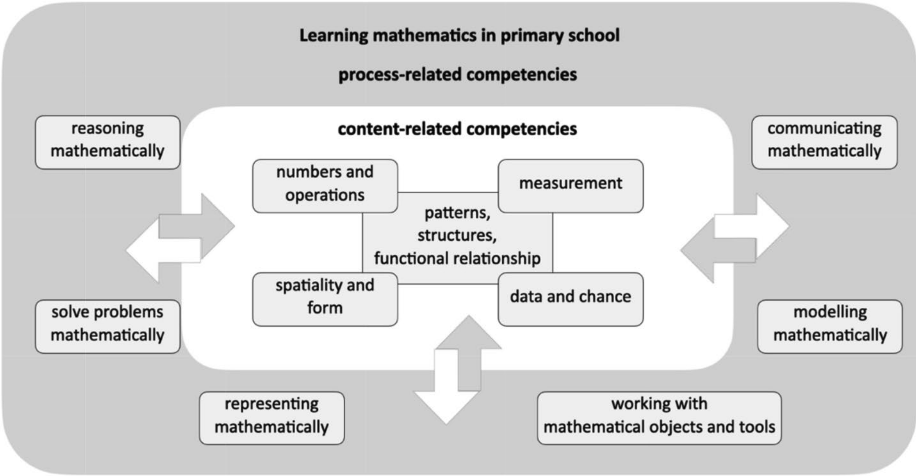

Fig. 1 Areas of competencies of the KMK educational standards (KMK, 2022; translated by the authors)

nections refers to recognition and application of mathematics in contexts outside of mathematics, which is very close to the process-related competence *modeling mathematically* of the *Bildungsstandards*. Although the NCTM standards have been the subject of controversy (Hekimoglu & Sloan, 2005), they have had a considerable impact on educational standards globally, including countries such as Canada (Kilpatrick, 2014) and Germany. It is reasonable to posit that the analyses conducted in this study, based on the German *Bildungsstandards*, are of relevance in international educational contexts.

2.2 Types of Practice

In addition to the consideration of content- and process-related competencies, successful learning processes must also take the phases of the learning process into account. An in Germany established structure for assigning mathematical learning activities to phases is provided by the concept of *productive practicing* (Winter, 1984; Wittmann, 2021). Here, practicing is subdivided into four different types of practice, which result from the *degree of structuring* and from the *degree of abstraction*. This differentiated view, which is discussed in a wide range of German literature, is not as prevalent in the international literature. However, it aligns with the internationally recognized basic belief that students should first develop basic understanding before training their calculation skills (Kilpatrick et al., 2001). For this reason, the evaluation of the possible learning section allocation in this study is summarized here for these two phases.

These forms of practice are necessary in the mathematical learning process and should be addressed in a balanced way in the classroom. In doing so, the goal is to create an appropriate interplay between learning opportunities for understanding and for the automation of what has been understood. International comparative research, such as the most recent TIMSS and PISA cycles (Mullis et al., 2020; OECD, 2019), has repeatedly been able to demonstrate stagnating mathematics competencies among students in numerous countries, which are in need of improvement at the same time. According to this, the competencies described in Fig. 1 do not yet appear to be sufficiently taken into account, especially in the basic understanding phase.

Due to these results, more education policy initiatives have been launched to promote the use of digital media in the classroom. This is linked to the goal of increasing the (subject-related) competencies of students. For example, the strategy *Education in the Digital World* (European Commission, 2022) defined competencies to be acquired in a digital world, which are also to be developed in subject-specific education. In addition to this specification of competencies, the infrastructural equipment, in particular digital devices, at schools in Germany was also promoted by means of the so-called *Digital Pakt Schule* (eng.: *Digital Pact for Schools*) with substantial funding in the amount of five billion euros (BMBF, 2016). The pact consists of schools committing themselves to implementing digital education in the context of adequate concepts in return for the provision of digital infrastructure.

An increasing availability of digital media can be observed in primary schools. Nevertheless, the availability of hardware (here: tablet computers) is no guarantee for the availability of didactically adequate software. The quality of available software for teaching mathematics is repeatedly criticized, among other things, for its non-compliance regarding subject didactic standards and its one-sided focus on calculation procedures instead of understanding (Bednorz & Bruhn, 2023; Kim et al., 2021; Krauthausen, 2013; Larkin, 2015; Namukasa et al., 2016). Related to this, much of the software seems to focus primarily on the – important, but solely insufficient – final phase of the learning process, whereas software that seems suitable for investigating mathematical relationships and structures is hardly reported. In particular, Krauthausen (1991) criticized the German-language inventory of mathematics software for primary schools more than three decades ago, stating that it neither met the curricular requirements nor the findings of modern learning and cognitive psychology and illustrated this with examples. The question therefore arises to what extent this situation has possibly changed.

2.3 Subject Didactic Potentials

In recent years, at least, a variety of studies across school levels have increasingly pointed to learning effects when using digital media (Aspiranti & Larwin, 2021; Drijvers & Sinclair, 2023; Hillmayr et al., 2020; Steenbergen-Hu & Cooper, 2013). These learning effects are closely linked to the subject didactic potentials implemented in digital media. This is understood to mean design features of digital media that open up new ways of acquiring concepts that are not possible with non-digital media (Moyer-Packenham et al., 2016; Walter, 2018). However, these potentials do need to be appropriately framed in the classroom to be fully utilized. Examples of potentials include *feedback with adaptive character* (Reinhold et al., 2020) or *synchronization of representations* (Lisarelli, 2023). The latter potential was analyzed by Walter (2018), for example. He conducted an interview study with $N=19$ children who solved addition problems predominantly through counting strategies to investigate how they use a physical compared to a virtual twenty-frame for number representation up to 20. It was shown that in the case of the virtual twenty frame, which contains multiple representations, children tended to use the possibility of adding five counters simultaneously during the representation process – although this possibility also existed with the physical equivalent. In this way, it was shown how software can be designed and used, so that new ways were revealed not only for automating understood concepts, but also for understanding concepts that are not yet understood.

2.4 Current State of Research – Previous Inventory Analyses

In order to obtain comprehensive overview of the extensive range of software available in the app stores, it is necessary to examine a larger sample. The central findings of frequently cited studies are outlined below:

Dubé et al. (2020) examined 90 apps that are suggested for use after entering "math" in the Apple App Store with focus on basic information (such as the price), the educational content and the rating by users. For this purpose, they analyzed the app descriptions of the suggested apps. The authors conclude that the developers do not provide enough information about the content of their apps to give teachers a well-founded guideline for the selection of apps for their teaching. They analyzed the way the suggestions in the App Store are presented: "Apple needs to be more transparent in how it chooses its top apps and calculates user ratings and rankings. It seems that apps containing complex visuals or a curriculum receive higher ratings from users while apps developed by experts receive higher rankings from Apple" (Dubé et al., 2020, p. 5401).

Highfield and Goodwin (2013) examined $N=360$ popular apps – meaning frequently installed and top-rated apps – in the education category of the English-language Apple App Store. In contrast to Dubé et al. (2020), they did not merely analyze the written descriptions of the app in the app store, but also categorize the content of the apps. The data show that there is more software for pre-school and primary school children with comparatively few apps for secondary school learners. Moreover, only 15% of the apps address curriculum content for mathematics education. Statements on the dominance of certain content are not described in the study. Still, the study can prove a clear dominance of apps with an instructive design (74%). According to the authors, such apps reflect a behavioristic teaching-learning concept, do not appear to be very suitable for the cognitive activation of students and are associated with drill-and-practice elements. Furthermore, apps could be identified that combine instructive design with possibilities to manipulate certain design elements (11%) or contain only manipulable elements without instruction (10%).

Larkin (2014, 2015) analyzed $N=142$ mathematics apps for primary school education to see to what extent content from the Australian primary school mathematics curriculum is addressed. He was able to show that the area of *Number and Algebra* and in particular the associated sub-area *Number and place value* (105 apps) dominates, while the other areas are significantly less frequently included. Thus, *Using units of measurement* (15 apps) and *Data representation and interpretation* (4 apps) are the most frequently represented sub-areas from the other two areas of *Measurement and geometry* and *Statistics and probability*. In a further focus of analysis, it was examined whether the apps are designed in such a way that they can develop declarative, procedural, conceptual or a combination of different types of knowledge. While 44% of the apps address declarative knowledge solely, 30% address procedural knowledge and 10% conceptual knowledge. Apps that address different or all knowledge domains are less represented (9% conceptual and procedural; 1% conceptual and declarative; 5% procedural and declarative; 1% all knowledge domains). These findings are supported by a study by Namukasa et al. (2016) with a focus on the transition between primary and secondary school levels, in which only 4 out of 80 apps were assessed as being potentially suitable for the development of mathematical concepts.

Furthermore, there are some studies whose findings can only be partially applied to the current topic. For instance, Drigas and Pappas (2015) review mobile learning applications for mathematics across all school levels, but they provide only one example from 2010 for primary school, which is no longer available. Similarly, Green et al. (2014) introduce a category system for selecting mobile apps for grades 5 to 12, but they do not analyze the actual inventory. Moreover, their study does not specifically focus on mathematics as a subject or on primary schools, which are the main areas of interest in the present study.

Outhwaite et al. (2023) recently conducted a content analysis and qualitative comparative analysis of math apps for primary school. However, their focus leans more towards assessing the educational value of apps rather than providing an overview of the existing apps, especially since only 23 apps were included in their study. Nonetheless, they uncovered interesting findings regarding the content of these 23 apps: 15 were categorized as *practice apps*, presumably serving as fluency trainers, and 21 out of 23 apps addressed the content standard of *number representation and relationships*. It's worth noting that these 23 apps were hand-picked and do not represent a larger, comprehensive sample, thus limiting the generalizability of their findings, though they do offer valuable insights.

On a related note, Shahjad and Mustafa (2022) conducted a systematic literature review on learning apps covering all subject content and school levels. In terms of mathematics education, they reference the study by Namukasa et al. (2016) mentioned earlier. From their analysis, they conclude that there is a vast number of apps available, with the majority targeting preschoolers, and language being the most frequently addressed content, followed by mathematics. Drawing from the reviewed studies, they outline a category system for app evaluation, which, however, lacks a mathematics didactic perspective.

3 Design of the Study

3.1 Research gap and Research Questions

Results of previous studies thus indicate that mathematics software tends to be instructional in its design, appears to be primarily suitable for the development of declarative and / or procedural knowledge, and has a focus on arithmetic. However, these undoubtedly valuable findings can only be transferred to the current situation for primary school pupils to a limited extent. They were obtained about a decade ago – this is a long time in view of the dynamic development of the use of digital media against the background of scientific and educational policy initiatives (Moyer-Packenham et al., 2015) and requires a current investigation, especially with regard to the existence of subject didactic potentials of digital media. Furthermore, the studies examined only English-language teaching software, that most likely is barely used by German students. Accordingly, no research has been conducted that examines the extent to which a fit between the software design and the German curriculum can be established.

Based on the outlined research gaps, the following research questions are derived for this paper:

RQ1: Which curricular goals are addressed by the currently available mathematics apps in the German-language app stores?

RQ2: To what extent is basic understanding and fluency practice possible with math apps in the German-language app stores?

RQ3: To what extent are subject didactic potentials of digital media implemented in mathematics apps in the German-language app stores?

3.2 Procedure for Sampling and Process for Analyzing the Apps

To answer the above research questions, apps were analyzed that are suggested for tablet computers by the search algorithms of the app stores after entering the search items *Mathe Grundschule* (eng.: *maths primary school*). Initially, these terms appear to be notably vague, particularly in their lack of reference to aspects pertinent to the domain of educational practice, such as the intended purpose of use (e.g., comprehension, automation, etc.). However, they were chosen because they could be used as standard keywords by teachers, students or their parents when searching for apps for primary school mathematics. At the same time, the use of these unspecific keywords allows for the analysis of a larger number of apps. The utilization of more specific keywords may have resulted in a smaller number of apps being analyzed, which would have been contrary to the objective of the study, which was to provide an overview of the (ideally entire) inventory.

It should be noted that other word combinations or word sequences can of course generate different search results. In addition, the devices used by different users seem to lead to varying search results when the same search keywords are entered, as repeated search queries with a relatively short time gap. In the app stores, the actual search algorithm is not made transparent.

In total, a comprehensive list containing $N=227$ apps was generated from numerous searches in August 2022. All apps that appeared after entering the search terms were included in the sample, so no apps were excluded. The apps were analyzed at their stage of development at the time. There is no claim to completeness in the analysis of the apps examined. Instead, the inventory analysis presented here can serve as a foundation for future updates or international adaptations of analyses.

The analysis is explicitly *not* aimed at rating the apps and selecting *the best* app. This does not seem to be feasible, especially since, in addition to the design elements of apps, one must also answer questions about their implementation in the classroom in order to fully benefit from the apps' potential. In the app stores, however, this is precisely what is often suggested by the rankings in the individual sections.

Both authors of this paper initially conducted the analysis of the apps independently according to a previously jointly defined coding manual (Walter & Schwätzer, 2023; example categorization see below in section *Insights into the rating process*). This involved a total of 6.129 criterion decisions per person (27 categories per app). Subsequently, the assessments of the apps were compared. After eliminating obvious input errors (e.g., 0 and 1 mixed up in the coding system), 65 criterion decisions remained for which there was a difference in assessment between the two authors. The resulting Cohen's Kappa of 0.975

indicates almost perfect interrater reliability (Landis & Koch, 1977). Apart from individual cases, the small number of divergent assessments arose in the area of the feasibility of process-related competencies. Following the independent analyses, discussions were held to clarify the differing ratings between the authors with regard to individual terms, such as when an app can be suitable for promoting *reasoning mathematically* and when it cannot. These discussions led to a revised assessment, so that each criterion was ultimately determined by consensus.

3.3 Data Analysis

In order to answer research question 1 (Which curricular goals are addressed by the currently available mathematics apps in the German-language app stores?), the apps were analyzed to determine whether and which of the specific content-related and process-related competencies they address. References to a content-related competence were considered to be present if at least one activity or module was available for it. Similarly, relations to a process-related competence were considered to be given if the tasks to be found in the apps are connected to a process-related competence. If a corresponding task cannot be found in the app, it was also analyzed whether it was conceivable to establish references to process-related competencies through external tasks (e.g., set by the teacher).

To answer research question 2 (To what extent is basic understanding and fluency practice possible with maths apps in the German-language app stores?), the first step was to investigate in which apps tasks are given. As previously described, the observed task types were categorised for German studies into different forms of productive practice (Walter & Schwätzer, 2023). These are summarized here under the categories of *basic understanding* and *fluency practice*.

The answer to research question 3 (To what extent are subject didactic potentials of digital media implemented in mathematics apps in the German-language app stores?) required the analysis of the apps with regard to the consideration of subject didactic potentials (e.g., *synchronization of representations*, *feedback with adaptive character*, etc.) of digital media. Similarly to the two previous focal points of the analysis, a potential was classified as realized if its occurrence was observed at least once.

The evaluations are therefore based on qualitative judgments that merely address the existence of the criteria, but not the subject-related quality of the content implementation in the classroom. As a consequence, the data analysis approach can be characterized as rather *defensive* and can result in the evaluations appearing more optimistic than is actually the case. However, this approach was chosen deliberately, as it goes with the intention of the study to provide an overview of the design of math apps. Furthermore, data analysis requirements aligned with a more *offensive* approach would also have had to address aspects of classroom embedding, which, however, depend heavily on teachers' professional skills in the use of digital technologies in mathematics lessons, are known to vary greatly, and could therefore not be considered in this study.

The results from these codings are processed using methods of descriptive statistics by generating an organizing and summarizing overview of the available data on the individual research questions. In addition to the described areas of analysis (curricular goals, forms of productive practice and subject didactic potentials), further data was also collected, for example on surface features, including reported facts from the app stores on download statistics, ratings, etc. as well as qualitative judgements relating to a design reduced to the essential mathematical core. These data are not discussed in this article due to its focus, but can be retrieved at https://mappsa.de.

3.4 Insights into the Categorization Process

The following information is provided to facilitate transparency in the categorization process, which was conducted using a developed coding manual. This is initially presented and then applied to two apps from the project database, which represent recognizably different approaches in terms of content.

The coding manual is structured into three main divisions: *analysis areas*, *analysis groups*, and *analysis categories*. Each analysis area is divided into analysis groups, which are themselves divided into analysis categories.

Four *analysis areas* were defined. In order to focus on them in this article, we will concentrate primarily on analysis areas 2 to 4, as these are relevant for the *conceptual* design of the apps.

1. surface characteristics.
2. content- und process related competencies.
3. types of practice.
4. subject didactic potentials.

The analysis areas are subdivided into *analysis groups*. For example, the analysis area *content and process-related competencies* is divided into the analysis groups *content-related competencies* and *process-related competencies*.

An analysis group is then subdivided into *analysis categories*. In relation to the analysis group *content-related competencies*, an analysis category is formed for each of the five content-related competencies, for example *numbers and operations*. In this analysis category, it is coded whether the content-related competence of numbers and operations is taken into account in the app being analyzed (code: *1*) or not (code: *0*). Across the entire coding manual, 27 analysis categories were formed in this way, subsumed under the analysis focal points and analysis category groups.

Subsequently, the analysis group is divided into *analysis categories*. With regard to the analysis group *content-related competencies*, an analysis category is established for each of the five content-related competencies, such as *numbers and operations*. In this analysis category, it is coded whether the content-related competence of *numbers and operations* is incorporated into the app under analysis (code: *1*) or not (code: *0*). In total, 27 analysis categories were created in this manner, encompassing the analysis focal points and analysis category groups.

The coding manual provides a detailed description of each category group and instructions on how to utilize them. Potential characteristics are then identified and described in each category, accompanied by a brief explanation in plain text. For each analysis category, an anchor example from the app inventory of the project database is provided, elucidating the rationale behind the selection of this anchor app and its corresponding code. A translated excerpt of this coding manual can be found in the appendix of this article. The complete version can be accessed on the project website at https://mappsa.de.

The use of the coding manual is now illustrated by example through the analysis of two apps. The first app is *Math Land: Basic Arithmetic* (Didactoons Games, 2022), which is gamified and embedded in a pirate adventure. The objective of the game is to navigate islands with a controllable character and collect primarily coins and binoculars. The binoculars are necessary to unlock additional islands. In order to collect binoculars, mental arithmetic tasks must be practiced. In the first exercise level (represented in the game by the first island in the game), addition and subtraction tasks in the number range up to 12 must be calculated mentally. The tasks are presented on coin bags (as shown in Fig. 2), although no direct link between this illustration and the arithmetic task appears to be recognizable. Additionally, various solutions are presented on the bags, and the child must tap the correct one as quickly as possible within a given time in order to receive the binoculars as a reward.

‚Math Land: Basic Arithmetic‘ was coded as *1* in the category group *content-related competencies* (see the relevant section of the coding manual in the appendix) in the category *NO (numbers and operations)*, as arithmetic content as described in the anchor example is recognizably present here. In contrast, the category *SF (spatiality and form)* was coded as *0*, as the app contains only arithmetic tasks and there are no task types that could be assigned to the content-related competence *spatiality and form*.

The coding of *process-related competencies* is slightly more differentiated. For all five analysis categories in this analysis group, it is first analyzed whether a process-related competence is explicitly addressed in the app. This is exemplified by tasks in the app that explicitly require children to demonstrate competence in this area, such as the anchor example for code *2* of category *P (problem solving)* in the coding manual excerpt in the appendix, in which children are instructed to complete a problem-solving task. In the event that such an explicit instruction is not discernible within the app, an analysis is conducted to ascertain whether, from the perspective of the coders, the app could be utilized in a teaching scenario wherein the targeted competence could be imparted (implicit connection, code *1*). In numerous instances, this is indicated by accompanying materials or the coders are able to discern such references based on their expertise in mathematics education. In the event that this is not identifiable, the reference to the examined process-related competence is coded as neither explicitly nor implicitly recognizable (code *0*).

In the example provided, *Math Land: Basic Arithmetic*, this connection could not be identified in category *P (problem solving)* as detailed in the appendix. In this instance, the

Fig. 2 Screenshot of the app *Math Land: Basic Arithmetic* (Didactoons Games, 2022)

children are required to solve basic arithmetic tasks at most. Therefore, the objective is to practice arithmetic skills. There is no task in which they are required to find a creative strategy to solve the problem. They are not asked to do this at any point. Consequently, *Math Land: Basic Arithmetic* was coded in category *P* with *0*.

The *TouchTimes* app (Sinclair, 2021, Fig. 3) provides a different approach to developing an understanding of operations. "Children first create, with the fingers of one hand, some form of unit, and then - through one or more taps of the other hand - create a 'unit of units', 'a product'" (Sinclair, 2021). The aforementioned example, depicted in Fig. 3, illustrates this process. Initially, the child touches the left half of the tablet screen with three fingers, resulting in the appearance of three differently colored dots. Subsequently, the child touches the right half of the screen with two fingers of the other hand. Simultaneously, images of the group of dots created on the left appear under each of the two fingers. This process is dynamic, whereby the app responds immediately to changes in the finger touches on both sides. As a consequence, not only the dot pattern displayed altered, but also the matching symbolic notation that appears at the top centre of the screen.

TouchTimes was coded with *1* in the process-related competence category *P (problem solving)*, in contrast to *Math Land: Basic Arithmetic* (see excerpt from the coding manual). Although the children are not explicitly asked to be creative and solve a mathematical problem, so that code *2* was not used, they can be asked to identify as many different representations and thus factorizations as possible for a given product (e.g., 12). This is also encouraged in the app's accompanying text. In a corresponding teaching scenario, it would be conceivable to address problem solving with the *TouchTimes* app. In this scenario, the coding with *1* would be used.

TouchTimes is also representative of another coding example. In the excerpt of the coding manual in the appendix, the category *SR (Synchronized and linked representations)* is shown for the analysis group *subject didactic potentials*. This is defined as follows in the coding manual: "Enactive, iconic and symbolic representations are related to each other in such a way that changes to one representation automatically result in changes to the other representations." In this instance, the coding options are *0* (not included) or *1* (included). As previously stated, the representations of the individual objects on the left and the grouped objects on the right of the screen react dynamically to each other, as do the symbolically

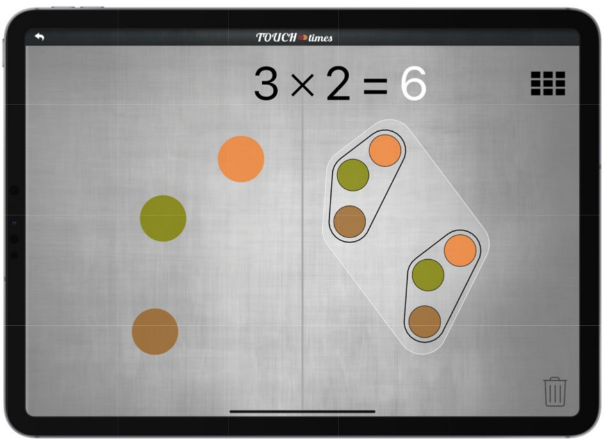

Fig. 3 Screenshot of the app *Touch Times* (Sinclair, 2021)

represented notation at the top center of the screen. Consequently, *TouchTimes* was coded *1* in the *SR* category *(Synchronized and linked representations)*.

4 Results

4.1 Curricular Goals

The findings on the consideration of curricular goals based on the German primary school curriculum goals in mathematics apps in the German-language app stores are presented below. The explanations serve to answer research question 1 by first looking at the content-related and then the process-related competencies.

With regard to the content-related competencies addressed in mathematics apps, 92.5% of the apps have at least one task in the content area of *numbers and operations*. In contrast, this is the case for 21.1% of the apps in the area of *spatiality and form*, 20.3% in the area of *measurement*, and 3.1% in the area of *data and chance*. 7.5% of the apps address the area of *patterns*, *structures and functional relationship* (see Table 1) at least once. Furthermore, 74% of the analyzed apps have a connection to a single content-related competence, 10% to two of these competencies, 16% to three or more content-related competencies.

With regard to the consideration of process-related competencies, the following results can be noted: 223 of the 227 apps analyzed (98.2%) do not have any explicit relationships to the promotion of process-related competencies in the tasks presented. However, 12.3% of the apps seem appropriate to be used by the teacher to address *reasoning* and *communicating mathematically*, 10.6% might be used to *represent mathematically*, 7.9% might be used to *solve problems mathematically* and only one app might be used to *model mathematically* (see Table 2). 28 of the 227 apps analyzed implicitly address more than one process-related competence.

The data thus show two things: On the one hand, as also found in Larkin (2014, 2015) and Highfield and Goodwin (2013), there seems to be a clear dominance of apps with a content-related focus on *numbers and operations*, while the other content-related competencies are noticeably less frequently the subject of mathematics apps. In contrast, addressing process-related competencies is explicitly provided in very few cases through corresponding tasks in the app. With some apps, however, process-related competencies might also be promoted if the teacher provides a suitable framework for the lesson, even if this is not made explicit in the app.

Table 1 Consideration of content-related competencies in mathematics apps (N=227)

N=227 apps		included		not included	
		H(n)	*h(n)*	*H(n)*	*h(n)*
content-related competencies	numbers and operations	210	92.5%	17	7.5%
	spatiality and form	48	21.1%	179	78.9%
	measurement	46	20.3%	181	79.7%
	data and chance	7	3.1%	220	96.9%
	patterns, structures, functional relationship	17	7.5%	210	92.5%

Table 2 Consideration of process-related competencies in mathematics apps (N=227)

N=227 apps		explicitly included		implicitly included		not included	
		H(n)	*h(n)*	*H(n)*	*h(n)*	*H(n)*	*h(n)*
process-related competencies	reasoning mathematically	0	0.0%	28	12.3%	199	87.7%
	communicating mathematically	0	0.0%	28	12.3%	199	87.7%
	solve problems mathematically	1	0.4%	18	7.9%	208	91.6%
	representing mathematically	3	1.3%	24	10.6%	200	88.1%
	modelling mathematically	1	0.4%	0	0.0%	226	99.6%

Table 3 Consideration of different forms of productive practice in mathematics apps

N=210 apps with given tasks		included		not included	
		H(n)	*h(n)*	*H(n)*	*h(n)*
forms of practice	primarily addresses basic understanding	115	54.8%	95	45.2%
	primarily addresses fluency practice	183	87.1%	27	12.9%

4.2 Types of Practice

The data below on the forms of productive practice refer to those N=210 of the total of 227 apps that contain tasks (either completely or in parts), so that an allocation of the apps to different forms of practice can be made. The data serve to answer research question 2.

54.8% of the apps analyzed have at least one material-based task sequence with different levels of structuring. The quality of the visualization was not measured. Nevertheless, they could *possibly* be suitable for stimulating basic understanding. In comparison, formal, purely symbolically notated, structured and unstructured forms of exercises are the most common. These apps, which tend to address fluency practice, are found in 87.1% of the sample (see Table 3). Consequently, it appears that software designed to facilitate the final phase of the learning process, which is typically marked by the *practice of fluency*, is currently the dominant approach.

4.3 Subject Didactic Potentials

The availability of subject didactic potentials of digital media can be seen as an indicator for substantial mathematics apps. The data associated with research question 3 are concerned with the question of the extent to which subject didactic potentials of digital media are included in mathematics apps.

79% of the apps include none of the analyzed subject didactic potentials. These apps can mainly be seen as pure digital duplicates of existing learning materials (e.g., work materials, worksheets) with no further benefits.

Table 4 also shows that in 11.5% of the apps *synchronous and linked representations*, in 14.1% the *structuring of representations*, in 10.1% the potential of *fitting action on material and mental actions*, in 7.0% the *outsourcing of calculations*, and in 6.6% each the *multi-touch technology* and the potential of *informative feedback* can be identified. Accordingly, subject didactic potentials of digital media seem to be taken into account in mathematics apps only in a few cases.

Table 4 Consideration of subject didactic potentials of digital media in mathematics apps

N=227 apps		included		not included	
		H(n)	*h(n)*	*H(n)*	*h(n)*
Subject didactic potentials of digital media	synchronized and linked representations	26	11.5%	201	88.5%
	structuring of representations	32	14.1%	195	85.9%
	fit between action on material and mental action	23	10.1%	204	89.9%
	outsourcing of calculations	16	7.0%	211	93.0%
	multi-touch technology	15	6.6%	212	93.4%
	informative feedback	15	6.6%	212	93.4%

5 Discussion

The presented study analyzed 227 apps that are currently available in the German App Stores for use in primary school mathematics lessons, employing a criteria-based approach. Essentially, the results of the inventory analysis can be summarized as follows:

- *Dominance of the area of numbers and operations*: With regard to content-related competencies, a clear dominance of the area of *numbers and operations* can be observed. For other competence areas, activities or task modules can only rarely be found.
- *Extensive neglect of process-related competencies*: Barely any app contains tasks in the activities or task modules that explicitly include the development of process-related competencies. However, in some cases this might be achieved through adequate external stimuli.
- *Dominance of practice activities, that focus on fluent calculation*: The apps examined have activities or task modules that primarily include formal forms of practice. The investigation of mathematical relationships and basic understanding is only intended in approximately half of the apps.
- *Extensive neglect of subject didactic potentials*: The presence of subject didactic potentials of digital media in apps can be considered as an indicator of an adequate app. However, the implementation of subject didactic potentials could be demonstrated rather sporadically.

Consequently, it can be deduced that the current inventory of mathematics apps for primary school does not adequately encompass all relevant content- and process-related competencies, as well as types of practices. There are too many unaddressed content areas that still require digitally supported learning opportunities. In addition, there seems to be a (too) strong focus on fluency and pure drill, while indispensable offers for the development of basic understanding or structural connections can be expanded. The opportunities provided by digital media are often not taken into account in the design of mathematics apps, especially since subject didactic potentials are rarely realized. Nevertheless, based on the inventory analysis, some apps were identified that incorporate subject didactic potentials of digital media.

Naturally, the 227 apps analyzed were identified through searches for *Mathe Grundschule* (*primary school maths*) in the German app stores. Some of these apps, primarily focusing on subject didactic potentials, were developed by German mathematics education experts in German language and may not be easily found in comparable international search results. However, international studies suggest that similar albeit slightly shifted results are plausible. As demonstrated, German curricular criteria closely align with international standards. These criteria seem applicable not only within the German-speaking context but also internationally, provided they are suitably contextualized.

In this sense, the inventory analysis of the German app stores provides valuable indications for necessary research and development work, not only for Germany It was shown that specific content areas and intended uses dominate, while the stimulation of process-related competencies and subject didactic potentials of digital media are largely neglected. Future app developments should fill these gaps and teachers should consider how to compensate missing process-related aspects. For example, they could combine the app usage with teaching methods, where students compare and discuss their ideas and results. This could be a way to compensate the lack of stimuli for discursive classroom discussions in the majority of the apps (Selter & Walter, 2020).

In order to demonstrate new possibilities of digital media for the understanding of mathematics, subject didactic potentials in particular should be addressed more strongly. Of course, with regard to the implementation of subject didactic potentials of digital media, it is by no means true that 'the more, the better' The authors are not convinced that the presence of several potentials would automatically lead to higher learning success for learners. Numerous apps can be found that only provide some potentials, but can be recommended without reservation from the authors' point of view, if they are used in a way that is appropriate for the learning level. Accordingly, the research-based analysis of existing apps and the development of new apps should not be carried out exclusively with a 'checklist character' along the proposed analysis focal areas.

This article showed how apps are currently designed in the light of curricular goals, for which purposes they could be suitable and which subject didactic potentials are included. However, this approach is also associated with limitations. The existence of certain contents and potentials can be regarded as a necessary condition for successful digitally supported mathematics teaching. However, this alone is not sufficient. The present analysis cannot show how the potential inherent in the apps is framed, whether it is thus effective in the classroom and how it is utilized. That can be analyzed in qualitative approaches as it was done, e.g., in Walter (2018).

From a methodological point of view, it should also be mentioned that this exemplary inventory analysis of the German app stores is based on qualitative judgements by the authors of this paper. Such assessments are certainly subjective and can – even with the same coding manual – lead to different results, although there is almost perfect intercoder reliability here. However, there were several clarification discussions among the authors on individual terms (e.g., when can an app (not) support problem solving?). People working in mathematics education with other background experience might judge differently.

Finally, the apps were examined to see whether the competence areas formulated in the educational standards – reflecting a close affinity between the German standards and those based on the NCTM standards internationally – were taken into account at all. A competency area was considered to be taken into account if at least one activity or task module was present in an app. However, this analysis does not provide any indication of the extent to which the content is addressed in each case. In some apps, for example, it was found that competencies in the content areas of *numbers and operations* as well as *spatiality and form* were addressed, although far more learning opportunities were found for numbers and operations. We assume that an analysis that also records the relative proportion of the consideration of different content areas will find an even stronger dominance of the content area of *numbers and operations*.

Finally, it’s important to mention that comparable results can only be extrapolated from an international analysis of the current scenario. Nonetheless, the international studies referenced do imply a certain level of similarity. It would be intriguing to investigate whether a scrutiny of global app stores, with analysis criteria slightly adjusted for international applicability, could validate or even reinforce these findings.

6 Appendix

Analysis area content- und process related competencies

Analysis group content-related competencies
The app is evaluated for its alignment with the content-related competencies outlined in the 2022 KMK Bildungsstandards (numbers and operations, spatiality and form, measurement, data and chance, as well as patterns, structures, and functional relationships). A detailed description of the five areas of competencies is not provided here. Depending on its design, an app may reference several areas of competencies. For the purposes of coding, it is not necessary for a competence area to be the primary focus of the app; it is sufficient for it to be included at least once.

Category: **NO** – content related competence *numbers und operations*

Code	Name	Description	Anchor examples
0	not included	The app is not related to the competence area *numbers and operations*.	-
1	included	The application contains at least one reference to the competence area *numbers and operations*.	Conni Rechnen 1-100 (*Mappsa-ID A066*) (*App-Store*) The app contains (exclusively) arithmetic content: Numbers 1-100 (number representation in tens and ones), addition and subtraction tasks up to 20, 50 or 100, as well as multiplication tasks. It clearly addresses the competence area *numbers and operations*.

Category: **SF** – content related competence *spatiality and form*.

Code	Name	Description	Anchor examples
0	not included	The app is not related to the competence area *spatiality and form*.	-

Analysis area content- und process related competencies			
Analysis group content-related competencies The app is evaluated for its alignment with the content-related competencies outlined in the 2022 KMK Bildungsstandards (numbers and operations, spatiality and form, measurement, data and chance, as well as patterns, structures, and functional relationships). A detailed description of the five areas of competencies is not provided here. Depending on its design, an app may reference several areas of competencies. For the purposes of coding, it is not necessary for a competence area to be the primary focus of the app; it is sufficient for it to be included at least once.			
1	included	The application contains at least one reference to the competence area *spatiality and form*.	Conni Mathe 2. Klasse (*Mappsa-ID A001*) (*App-Store*) Although the focus is on other content, there is a task module that could be assigned to the competence area of *spatiality and form*: In the Geometry exercise, the number of banana boxes (cube building in oblique view with partially hidden elements) is to be determined using mental geometry.
[…]			

Analysis group process-related competencies

The app is evaluated for its alignment with the process-related areas of competencies outlined in the 2022 KMK Bildungsstandards (solve problems mathematically, communicating mathematically, reasoning mathematically, modeling mathematically, and representing mathematically). A detailed description of the areas of competence is not provided here. Depending on its design, an app may reference several areas of competencies. For the purposes of coding, it is not necessary for a competence area to be the primary focus of the app; it is sufficient for it to be included at least once.

When coding, a distinction is made between three cases. Firstly, it is possible that the app contains explicit prompts or suggestions related to a specific process competence area. For instance, if learners are encouraged or asked to justify their approaches, this could be interpreted as an explicit prompt related to the competence area of ‘reasoning mathematically’. Similarly, if learners are asked to examine the interconnectedness of representations, this could be seen as an explicit prompt related to the competence area of representing mathematically. In addition, these explicit stimuli may be absent, but the manner in which the task is set is arranged in such a way that stimuli or suggestions for a process-related competence area can be implicitly created by the teacher. Finally, an app can also have neither explicitly nor implicitly recognizable references to competence areas.

Category: **P** – **process-related competence***problem solving*.

Code	**Name**	**Description**	**Anchor examples**
0	not included	The app is not related to the competence area *problem solving*.	-

Analysis group process-related competencies
The app is evaluated for its alignment with the process-related areas of competencies outlined in the 2022 KMK Bildungsstandards (solve problems mathematically, communicating mathematically, reasoning mathematically, modeling mathematically, and representing mathematically). A detailed description of the areas of competence is not provided here. Depending on its design, an app may reference several areas of competencies. For the purposes of coding, it is not necessary for a competence area to be the primary focus of the app; it is sufficient for it to be included at least once.
When coding, a distinction is made between three cases. Firstly, it is possible that the app contains explicit prompts or suggestions related to a specific process competence area. For instance, if learners are encouraged or asked to justify their approaches, this could be interpreted as an explicit prompt related to the competence area of 'reasoning mathematically'. Similarly, if learners are asked to examine the interconnectedness of representations, this could be seen as an explicit prompt related to the competence area of representing mathematically. In addition, these explicit stimuli may be absent, but the manner in which the task is set is arranged in such a way that stimuli or suggestions for a process-related competence area can be implicitly created by the teacher. Finally, an app can also have neither explicitly nor implicitly recognizable references to competence areas.

1	implicitly included	The app does not make any references to the competence area of *problem solving*. However, references can be made through the use of impulses and tasks set by a teacher.	Rechendreieck (*Mappsa-ID A010*) (*App-Store*) No tasks are set in the app. There are also no explicit references to process-related competencies formulated within the app. However, the app's accompanying information states that "problem-structured exercises can be worked on properly", which implicitly addresses the process-related competence of *problem solving*. However, the app is also suitable as a digital tool for representing problem-solving processes and talking about them with other children, comparing and evaluating calculation strategies. This also implicitly addresses *representing mathematically*, *communicating mathematically* and *reasoning mathematically* in a suitable classroom setting.
2	explicitly included	The application contains at least one reference to the competence area ‚problem solving'.	Sachrechnen 2.0 (*Mappsa-ID A013*) (*App-Store*) The app presents problem tasks in real-life contexts that are to be solved explicitly in modeling processes. This addresses the competence areas of *problem solving* and *modeling mathematically*.

[…]

Analysis area*potentials of digital media*

Analysis group*subject didactic potentials*

Digital media can have potentials that can support mathematics learning in particular. The following six subject didactic potentials were identified as categories.

[…]

Category: **SR** – subject didcatic potential *synchronized and linked representations*
Enactive, iconic and symbolic representations are related to each other in such a way that changes to one representation automatically result in changes to the other representations.

Code	**Name**	**Description**	**Anchor examples**

Analysis group process-related competencies The app is evaluated for its alignment with the process-related areas of competencies outlined in the 2022 KMK Bildungsstandards (solve problems mathematically, communicating mathematically, reasoning mathematically, modeling mathematically, and representing mathematically). A detailed description of the areas of competence is not provided here. Depending on its design, an app may reference several areas of competencies. For the purposes of coding, it is not necessary for a competence area to be the primary focus of the app; it is sufficient for it to be included at least once. When coding, a distinction is made between three cases. Firstly, it is possible that the app contains explicit prompts or suggestions related to a specific process competence area. For instance, if learners are encouraged or asked to justify their approaches, this could be interpreted as an explicit prompt related to the competence area of 'reasoning mathematically'. Similarly, if learners are asked to examine the interconnectedness of representations, this could be seen as an explicit prompt related to the competence area of representing mathematically. In addition, these explicit stimuli may be absent, but the manner in which the task is set is arranged in such a way that stimuli or suggestions for a process-related competence area can be implicitly created by the teacher. Finally, an app can also have neither explicitly nor implicitly recognizable references to competence areas.			
0	not included	There is no activity/task module in which the potential *synchronized and linked representations* is implemented.	-
1	included	At least one activity/task module is included in which the potential *synchronized and linked representations* is implemented.	Malrechnen (*Mappsa-ID A121*) (*App-Store*) With this app, it is possible to manipulate different representations (point field, number line, symbolic notations) of a multiplication or division task in a linked manner. Changes in one representation are transferred synchronously to the other representations.
[…]			

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