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ORIGINAL RESEARCH

Affordance Access Matters: Preschool Children's Learning Progressions While Interacting with Touch-Screen Mathematics Apps

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Abstract The purpose of this study was to contribute to the research on mathematics app use by very young children, and specifically mathematics apps for touch-screen mobile devices that contain virtual manipulatives. The study used a convergent parallel mixed methods design, in which quantitative and qualitative data were collected in parallel, analyzed separately, and then merged. During the study, 35 children, ages 3–4, interacted with four touch-screen mathematics apps on iPad devices during one-on one clinical interviews while learning seriation and counting. Researchers administered pre and post assessments of learning during the interviews. Each interview was videotaped using a wall-mounted camera and a GoPro camera to provide different views of the interview. Videos were analyzed to examine children's learning progressions, access of affordances, and patterns of behavior while interacting with the mathematics apps. The results suggest that different affordances of the individual apps were perceived in different ways, depending on the age of the child, and that these perceptions were observable in young children's patterns of behavior. Implications are discussed for iPad app use in young children's educational settings.

Keywords Affordance · Virtual manipulative · iPad apps · Touch-screen apps · Preschool children · Mathematics education

1 Introduction

At the earliest stages of development, young children are exposed to the technological world around them. Similarly, early childhood educators are bombarded with new types of virtual learning activities for educational use. The recent emergence of mobile touch-

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screen apps offers a wealth of early mathematics learning tools for young children. In fact, touch-screen devices are being used in today's earliest classrooms creating a need to know how students interact with these devices (Ginsburg et al. 2013) and the effectiveness of this technology for young children.

This study contributes to the research on mathematics app use with very young children, and specifically touch-screen apps that contain virtual manipulatives. A virtual manipulative (VM) is defined as: "An interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge" (Moyer-Packenham and Bolyard 2016, p. 13). We selected virtual manipulative apps for this research that focused on seriation and counting. Through this examination, we hope to contribute greater understanding about young children's learning progressions and suggest implications for children's early use of touch-screen mathematics apps.

2 Literature Review and Theoretical Framework

Current research indicates the existence of consistent relationships between early mathematics learning and later mathematics achievement (Clements and Sarama 2007; Jordan et al. 2010). Seriation and counting are topics commonly found in early childhood mathematics curricula and considered foundational to later mathematics learning. *Seriation*, the ability to sort and order objects according to a defined attribute (often by length or size), is a Piagetian logical reasoning task commonly used in early mathematics assessments. As a pre-number skill, ordering objects by length from longest to shortest entails mathematical reasoning, magnitude comparison, sorting, classifying, and organizing skills. *Counting*, along with number relationships and basic operations, contributes to students' success in later mathematics (Aunola et al. 2004; Jordan et al. 2010). Learning the counting sequence, object counting, cardinal ideas, and the seriation of numbers are important skills tied to the broader construct of counting. These interrelated counting ideas serve as an entry to young children's pathways for developing counting strategies.

As young children learn to seriate and count, their cognitive structures develop along a continuum (Steffe and Cobb 1988; Tzur and Lambert 2011), which many researchers refer to as a *learning progression*. Smith et al. (2006) define learning progressions as "descriptions of successively more sophisticated ways of reasoning within a content domain…" (p. 2). Understanding children's early knowledge base and learning new and innovative ways to access and build on this early mathematical knowledge positively contributes to effective teaching methods for early childhood mathematics. In this study, we used seriation and counting learning progressions to characterize the cognitive restructuring that occurred as children used mathematics apps to learn these concepts.

The use of the iPad in preschools is still relatively new, hence research on its influence and effectiveness on preschoolers' mathematics learning is still in the early stages. Barendregt et al. (2012) study of the iPad game, Fingu, with 5- to 6-year-olds examined how children's behavior with the game developed over three weeks in relation to arithmetic skills. Children developed different skills when playing Fingu. Some improved their conceptual subitizing, some improved their ability to represent numbers on their fingers, and most continued to rely on counting strategies for larger sums. Spencer's (2013) experiment with 5-year-olds using a numeracy app, *Know Number Free*, showed that one week of daily use led to improvements in recognizing, writing, and quantifying numbers 1 to 10. These studies provide emerging evidence of the influence of touch-screen apps on young children's mathematics learning. The studies also encourage researchers to ask more questions about why technological features positively or negatively influence learning and how this occurs during technology experiences in early childhood classrooms.

2.1 Affordances and Learning Mathematics with iPad Apps

Extant research on computer and mouse-based virtual manipulatives provides some basis for hypothesizing that preschool children's interactions with touch-screen devices can foster mathematics learning. Moyer-Packenham and Westenskow's (2013) meta-analysis on virtual manipulatives reported that virtual manipulatives have moderate effects on student achievement when compared to other methods of instruction. A conceptual analysis, including 66 reports, revealed five affordances that promoted mathematics learning: *focused constraint*, *creative variation*, *simultaneous linking*, *efficient precision*, and *motivation*.

Technological affordances influence children's interactions with technology and how they learn mathematics with the technology (Burlamaqui and Dong 2014). Gibson (1986) explained the theory of affordances in terms of the environment and what it affords. He stated, "The *affordances* of the environment are what it *offers* the animal, what it *provides* or *furnishes,* either for good or ill" (p. 127). Gibson's early work helps to distinguish a *feature* of a technological tool from an *affordance* of a technological tool. An instructional designer may create a feature within an app that she hopes will afford learning, however the learner must perceive or attend to the feature and access it in order for it to afford some influence. Osiurak et al. (2010) wrote: "affordances are action-referential properties of the environment that may or may not be perceived" (p. 525). In this study, we examined features of iPad apps to determine how children accessed them as helpful or hindering affordances. Our hypothesis was that affordance access would be more prevalent among those who progressed between pre- and post-assessments, regardless of age. While a feature may have been designed to be helpful, in some cases, the feature afforded confusion instead of learning. Embedded in this idea is the importance of the interaction that takes place between the learner and the app's features.

3 Research Questions

Quantitative Questions:

1. How many children progressed, regressed and stayed the same, in terms of their seriation and counting learning progressions, while using touch-screen mathematics apps?
2. What is the frequency of affordance use, by 3- and 4-year old children, while using touch-screen mathematics apps? What are the changes in frequency of affordance use from pre- to post-assessment?
3. What is the frequency of affordance use disaggregated by learning progression and age?

Qualitative Questions:

4. While interacting with mathematics apps, what do children's patterns of behavior reveal about shifts in the children's learning progressions?

5. What affordances helped young children move forward in their learning progressions and why? What affordances hindered young children in their learning progressions and why?

Mixed Methods:

6. To what extent do the quantitative and qualitative results converge to explain the relationship between young children's affordance use, learning progressions, and age?

4 Study Design

The purpose of this study was to contribute greater understanding about young children's learning progressions by examining their affordance access while interacting with touch-screen mathematics apps. As such, this study used a convergent parallel mixed methods design. Quantitative and qualitative data were collected in parallel, analyzed separately, and then merged (Creswell and Plano Clark 2011). The rationale for this design is that researchers wanted to obtain complementary data on children's progressions in seriation and counting to develop a more complete understanding than just one type of data would provide. The quantitative data analysis focused on the number of children who progressed, regressed, or stayed the same and the frequency of affordance use disaggregated by age. The qualitative data analysis focused on understanding children's patterns of behavior as they interacted with the apps and how affordance access helped or hindered their learning progressions. The results were then merged to assess the extent to which the quantitative and qualitative results converged as a way to triangulate methods using multiple perspectives (Creswell and Plano Clark 2011), thus creating a more complete interpretation of the phenomenon and answering the final research question.

4.1 Participants

The participants were 35 three- or four-year-old preschool children. The participants were identified by their parents as: 20% socioeconomically disadvantaged (low SES) and 94.3% Caucasian. In terms of technology use, parents reported: 91.4% had at least one touch-screen device in the home; 31.4% used a touch-screen device every day; and, 54.3% used a touch-screen device at least once a week.

4.2 Procedures and Data Collection

In preparation for the study, researchers reviewed multiple apps and selected six apps that contained virtual manipulatives that met the Moyer-Packenham and Bolyard (2016) definition and that included features that afforded positive learning effects (Moyer-Packenham and Westenskow 2013). Researchers also developed procedural protocols and tested the iPad devices and video equipment (Schubert 2009). The investigation of preschoolers in this study was part of a larger project that also included kindergarten and second grade students (Bullock et al. 2015; Moyer-Packenham et al. 2015; Moyer-Packenham et al. 2016; Tucker et al. 2014).

The preschool children participated in one 30–40 min structured clinical interview. Each interview followed a prescribed sequence for each learning topic (i.e., seriation or counting) that began with a pre-assessment app, followed by two learning apps, and then a post-assessment app (identical to pre-assessment). All interviews were completed during a 3-month data collection period. Table 1 shows the interview sequence, mathematics topics, and app screenshots.

Researchers conducted interviews in clinical observation rooms with wall-mounted video cameras and an observation booth with a two-way mirror. Researchers videotaped children as they completed seriation and counting tasks with apps on iPads. A wall-mounted camera captured an observer/interview perspective of children's actions on the iPad. A Go-Pro camera (with harness) provided a student view of actions on the iPad. Interviewers were all experienced in conducting clinical interviews with young children. Researchers and parents watched the interviews from the observation booth. One observer made notes of verbal utterances (including terminology), counting and number strategies, purposeful or unhelpful movements, errors and correction of errors, affective responses, and anything unique or noteworthy. Researchers kept a log of each interview with information about the child, interviewer, and observer for each session.

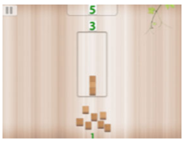
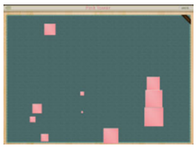
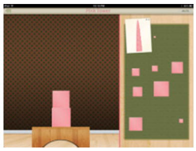
4.3 Data Analysis

The video data were converted into quantitative and qualitative data using four types of data coding and analysis processes (Lesh and Lehrer 2011).

4.3.1 Learning Progression Coding

To answer RQ#1, pairs of researchers developed, independently coded, tested, and refined learning progression coding protocols for the videos in the data set. Over 30% of the videos were coded by two independent researchers.

Table 1 Interview sequence of the counting and seriation activities

Counting	Screenshots of the counting Apps	Seriation	Screenshots of the seriation apps
Pre/post Montessori Numbers (Children use blocks to count quantities from 1 to 9)		Pre/post Pink Tower (Free Moving: Children move pink blocks of different sizes to build a tower)	
Learning app A Montessori Numbers (Children use blocks to count quantities from 1–5)		Learning app A Pink Tower (Card#12) (Children select pink blocks of different sizes to build a tower)	
Learning app B Montessori Numbers (Children use blocks to count quantities from 1–9)		Learning app B Intro to Math (Red Rods) (Children move Red Rods of different lengths from largest to smallest)	

4.3.2 Affordance Coding

To answer RQ#2–3, pairs of researchers developed, independently coded, tested, and refined affordance coding protocols for the videos in the data set. Over 30% of the videos were coded by two independent researchers.

4.3.3 Patterns of Behavior Coding

To answer RQ#4–5, researchers used the notes recorded by each interview observer. This was done for each level on the learning progression to determine patterns of behavior for the levels.

4.3.4 Iterative Videotape Analysis

To answer RQ#6, results from the quantitative and qualitative data sets were converged. Thus, researchers were able to test, refine, and extend their understanding through convergence of the quantitative and qualitative results.

5 Results

The results are presented in two main sections: Seriation Sequence and Counting Sequence. These were the two main topics of the mathematics apps activities and are important in early childhood mathematics curricula and considered foundational to later mathematics learning. Each section begins with quantitative results followed by qualitative results, before the summary of the mixed methods results for readability and to allow for connections between the different types of data. Tables and figures highlight affordance access, learning progressions, and differences between 3-and 4-year old children. The results are disaggregated by learning progression (i.e., progressed, regressed, or stayed the same).

5.1 Seriation Sequence: Quantitative Results

5.1.1 Evidence of Learning Progressions: Seriation

Table 2 displays the results for the seriation learning sequence. Over half of the children showed no shifts in learning progression scores between the pre and post apps (see Table 2, column labeled “Same”). However, more children progressed (28.6%) than regressed

Table 2 Evidence of learning progression change for the seriation sequence

Age	N	Progressed	Regressed	Same
3	16	31.3% (N = 5)	12.5% (N = 2)	56.3% (N = 9)
4	19	26.3 (5)	10.5 (2)	63.2 (12)
Total	35	28.6 (10)	11.4 (4)	60.0 (21)

(11.4%). Interestingly, slightly more 3-year-olds progressed (31.3%) and regressed (12.5%) than 4-year-olds (26.3 and 10.5%, respectively).

5.1.2 Evidence of Affordance Use by Age: Seriation

The seriation sequence apps were: Pink Tower: Free Moving (pre-assessment), Pink Tower Tapping and Red Rods (learning apps), and Pink Tower: Free Moving (post-assessment). Table 3 shows children's affordance access during the seriation sequence. For the Pink Tower: Free Moving app, 4-year-olds were more likely to access a hindering affordance: "difficult to move tiny blocks" (A1: 15.8%) and "difficult to compare tiny blocks" (A2: 10.5%). Both age groups accessed the hindering affordance: "difficult to see block color" (A3). For the Pink Tower: Tapping app, 3-year-olds were more likely to access a helping affordance: "does not allow the child to be wrong" (A4: 81.3%) and "builds the tower for the child in a perfect stack" (A5: 50.0%). Both groups accessed the hindering affordance: "difficult to compare tiny blocks" (A6).

On the Red Rods learning app, 3-year-olds were more likely to access hindering affordances: "sound distraction" (A7: 56.3%) and "difficult to move rods" (A8: 31.6%). However, 4-year-olds were more likely to access the helping affordance "easy visual comparison of length" (A10: 21.1%). Both groups accessed the hindering affordance "rods

Table 3 Evidence of affordance access for each app in the seriation sequence

App	Age	*N*	Affordances		
			A1 % (N)	A2 % (N)	A3 % (N)
Pink Tower: Free moving (pre-assessment)	3	16	6.3 (1)	0 (0)	18.8 (3)
	4	19	15.8 (3)	10.5 (2)	15.8 (3)
	Total	35	11.4 (4)	5.7 (2)	17.1 (6)

			A4 % (N)	A5 % (N)	A6 % (N)
Pink Tower: Tapping (learning app)	3	16	81.3 (13)	50.0 (8)	12.5 (2)
	4	19	68.4 (13)	5.3 (1)	10.5 (2)
	Total	35	74.3 (26)	25.8 (9)	11.4 (4)

			A7 % (N)	A8 % (N)	A9 % (N)	A10 % (N)
Red Rods (learning app)	3	16	56.3 (9)	37.5 (6)	43.8 (7)	0 (0)
	4	19	26.3 (5)	5.3 (1)	42.1 (8)	21.1 (4)
	Total	35	40.0 (14)	20.0 (7)	42.3 (15)	11.4 (4)

			A1 % (N)	A2 % (N)	A3 % (N)
Pink Tower: Free moving (post-assessment)	3	16	12.5 (2)	0 (0)	12.5 (2)
	4	19	26.3 (5)	5.3 (1)	0 (0)
	Total	35	20.0 (7)	2.9 (1)	5.7 (2)

shifted easily" (A9). On the Pink Tower: Free Moving post-assessment app, 4-year-olds were more likely to access the hindering affordances: "difficult to move tiny blocks" (A1: 26.3%) and "difficult to compare tiny blocks" (A2: 5.3%). Only 3-year-olds accessed the hindering affordance: "difficult to see block color" (A3: 12.5%).

5.1.3 Pre-to Post-assessment Changes in Affordance Use: Seriation

During the seriation sequence, 3- and 4-year-olds progressed (pre to post) when they accessed the hindering affordance "difficult to move tiny blocks" (A1) (see Fig. 1). Only 4-year-olds progressed when they accessed the hindering affordance "difficult to compare tiny blocks" (A2). In an interesting contrast, 3-year-olds progressed when they accessed the hindering affordance "difficult to see block color" (A3) on the pre-assessment, but 4-year-olds regressed when they accessed this affordance on the pre-assessment.

5.1.4 Evidence of Affordance Access Disaggregated by Learning Progression and Age: Seriation

Figure 2 shows children's affordance access during the two learning apps (Pink Tower: Tapping and Red Rods). On the Pink Tower: Tapping app, the helping affordance "does not allow the child to be wrong" (A4) was accessed by all groups except 3-year-olds who regressed. The 3-year-olds who progressed were twice as likely to access A5 ("builds the tower for the child in a perfect stack") than the 3- and 4-year-olds who stayed the same and accessed this affordance. The hindering affordance "difficult to compare tiny blocks" (A6) was accessed by 3- and 4-year-olds who progressed or stayed the same, suggesting that something about the difficulty of moving the blocks may have actually been helpful. Also in Fig. 2, all children accessed the hindering affordance "sound distraction" (A7) on the Red Rods app. All 3-year-olds accessed the hindering affordance "difficult to move rods" (A8), while only a small percentage of 4-year-old who stayed the same accessed it. The 3- and 4-year-olds who regressed or stayed the same were the children who most often accessed A9 ("rods shifted easily") on the Red Rods app. Only 4-year-olds who progressed or stayed the same accessed A10 ("easy comparison of visual length").

In summary, the quantitative changes in affordance access in the seriation sequence appear to indicate more frequent access of the affordances among those who progressed in the learning tasks than those who regressed, although this would need further investigation since only two students in each age group regressed. This includes affordances that were helping and hindering. Children's improvement, despite the hindering affordance access, seemed to indicate that some amount of productive struggle was offered by the hindering affordances and this supported children's learning. On the other hand, if the struggle was too much, the hindering affordances prompted children to regress.

5.2 Seriation Sequence: Qualitative Results

5.2.1 Young Children's Patterns of Behavior While Interacting with the Apps

After coding each child's app interactions with a learning progression level during the quantitative phase, in the qualitative phase the videos of children at each learning progression level were examined for patterns of behaviors exhibited at that level to explain differences in children's access of affordances for each app in the sequence. Table 4

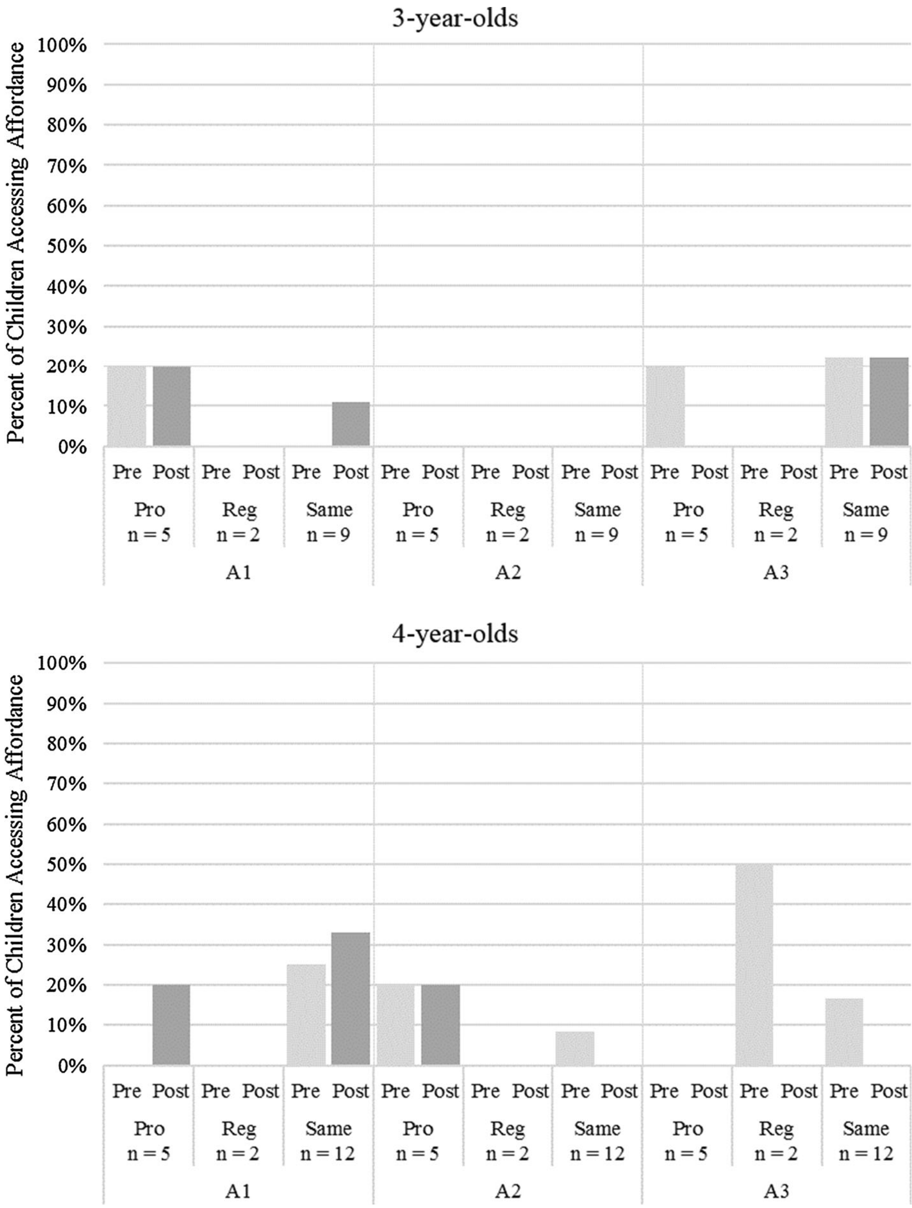


Fig. 1 Seriation Sequence Pink Tower pre- to post-assessment affordance access for age 3 (*top*) and age 4 (*bottom*). Affordances Key: A1 = difficult to move tiny blocks; A2 = difficult to compare tiny blocks; A3 = difficult to see *block color*. *Lighter bars* are children who progressed. *Darker bars* are children who regressed. *No bar* indicates zero children accessed that affordance in that category

displays the results for the Pink Tower Seriation learning sequence. As an example, children coded at Learning Progression level 0 or 1 in the Red Rods learning app made movements such as sliding instead of tapping, or ineffectively sliding, which led to uttered

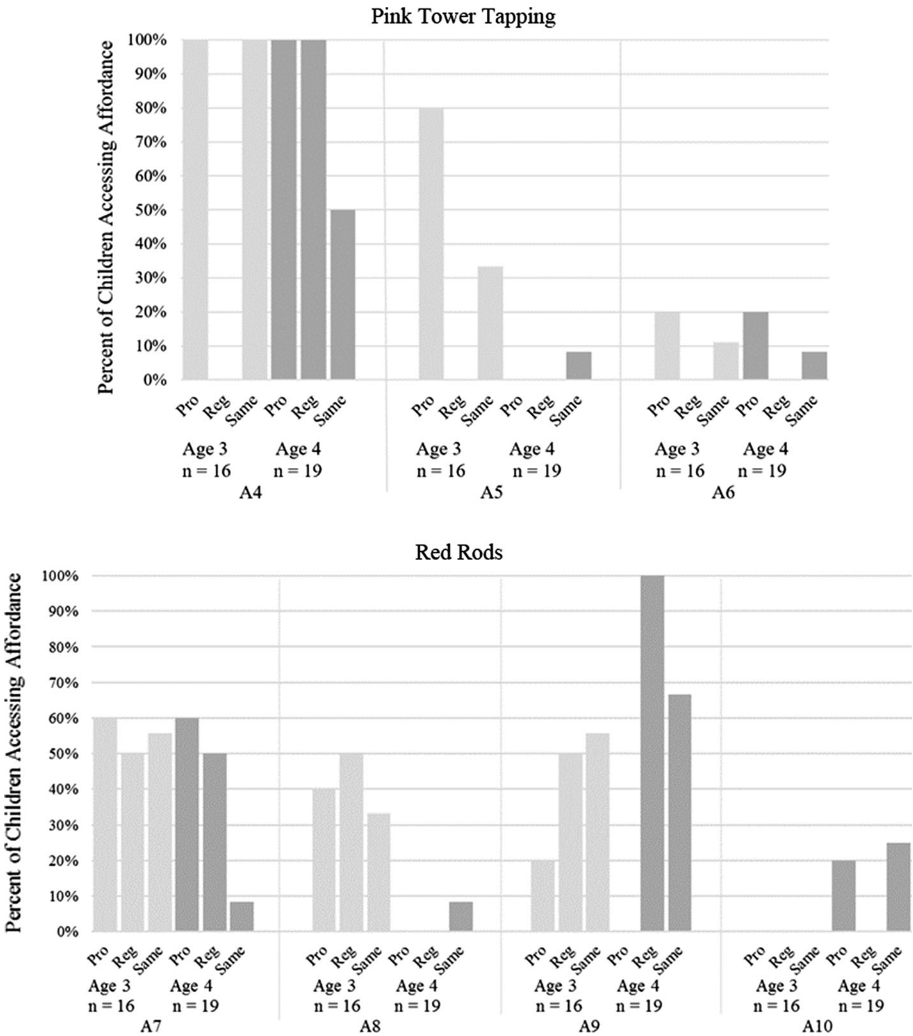


Fig. 2 Seriation sequence learning apps affordance access: Pink Tower Tapping (*top*) and Red Rods (*bottom*). Affordances Key: A4 = does not allow the child to be wrong; A5 = builds tower for the child in perfect stack; A6 = difficult to compare tiny blocks; A7 = sound distraction; A8 = difficult to move rods; A9 = rods shifted easily; A10 = easy visual comparison of length. *Lighter bars* are children 3-year-olds. *Darker bars* are 4-year-olds. *No bar* indicates zero children accessed that affordance in that category

and non-uttered signs of frustration. For example, Tom said, “I don’t like this!” When children repeatedly tried to move the rods and the rods would “jump around” some children gave up and began randomly moving them. At levels 2 or 3 children were more confident and made purposeful statements such as, “I want to do it. There. Up down, up down…This one goes here. Where does this one go?” At Learning Progression level 4, children showed purposeful actions with few utterances as they rapidly and confidently navigated the technological environment to complete the required task(s) with few, if any errors.

Table 4 Evidence of young children's patterns of behavior for the seriation sequence

App	Learning progression level	Patterns of behavior
Pink Tower: Free moving (pre-assessment)	0	Verbal utterances primarily expressing frustration. Technological difficulties in navigating app
	1	No utterances. Correctly built a couple but then randomly guessed
	2	None coded at this level
	3	Utterances primarily purposeful questions that were context focused. A few incorrect choices. Some technical difficulties
	4	Rapid, purposeful movements with no errors. Very few utterances
Pink Tower: Tapping (learning app)	0	Group with most recorded utterances. Affective responses generally positive but more focused on technical aspects rather than seriation. Random guessing. Sliding instead of tapping
	1	Too little coded at this level
	2	Utterances primarily purposeful questions that were context focused. A few incorrect choices
	3	Accurate purposeful movements. Utterances primarily positive. Minor errors
Red Rods (learning app)	0	Majority uttered frustration at inability to move rods effectively. Frustration showed through body language
	1	Some frustration expressed moving the rods effectively. Some could see patterns emerging
	2	Recognized that certain blocks were bigger than others. Utterances more purposeful in context of seriation
	3	Purposeful movements with several errors
	4	Accurate and purposeful movements. Far fewer utterances. Minor errors
Pink Tower: Free moving (post-assessment)	0	Less utterances than pretest. Technological difficulties in navigation
	1	Ample evidence of technical difficulties
	2	None coded at this level
	3	Utterances primarily purposeful questions that were context focused. A few incorrect choices. Some technical difficulties
	4	Rapid, purposeful movements with no errors. Very few utterances

5.2.2 Affordances that Helped and Hindered Young Children's Progressions and Why

Overall, affordances that allowed children to engage in purposeful questioning and movement helped student's progress in learning seriation concepts, with almost one-third of children progressing (29%). An example of a child who progressed from pre to post was 4-year-old Cayden. During the interview, his behaviors included frequent, purposeful verbalizations with statements like, "Does it go here, or here?", "I'm getting it!" and "Nice tower. Do you like my tower?" His movements went from quick movements in the Pink Tower Tap learning app to touching blocks, with pauses for thinking, before moving the blocks. Another child who progressed was 3-year-old Leo. Although he struggled with the Pink Tower Free Moving app on the pre-assessment, the Pink Tower Tapping app allowed him to learn the order of the blocks by size. In the Red Rods app he continued to improve, despite some sound distraction, going in order from the tallest to the shortest rod, while only misplacing a few rods in the middle. On the post-assessment, he was able to put most of the blocks in the correct order. Children's behaviors showed that the apps constraint features (e.g., Pink Tower Tapping app) helped children realize the correct seriation for stacking the blocks from largest to smallest. Because the Pink Tower Tapping app rejected children's incorrect tries, children had opportunities to test their conjectures multiple times as they attempted the correct block sequence.

Some affordances prompted technical difficulties and kept children from meaningfully engaging with the seriation content, causing a small portion of the children to regress from the pre- to post-assessment (11%). For example, 3-year-old Emily regressed. Although she completed the pre-assessment app correctly and quickly, the Red Rods learning app seemed to hinder her due to the technical difficulty she experienced when moving the rods. When she did manage to move the rods, the app shifted the rods contrary to where Emily placed them. Therefore, even though Emily attempted to put the rods in the correct order, the frequent, accidental shifting of the rods was misleading and led to frustration. On the post-assessment app, Emily did not display the same confidence as she had on the pre-assessment. The qualitative results indicate that, for a feature to afford learning, it needs to support the child in question making and answering and in movements towards purposeful actions for sense making. When technical difficulties obscured sense making, the child did not progress, and sometimes, regressed.

5.3 Counting Sequence: Quantitative Results

5.3.1 Evidence of Learning Progressions: Counting

Table 5 shows results for the counting learning sequence. Most children showed no shifts in learning progression scores between the pre- and post-assessment apps (column labeled "Same"). Similar to the seriation sequence, more children progressed than regressed. In fact, more children progressed on the harder task (build 6–9 versus build 2–5). Most of the children who regressed on both tasks were 3-year-olds.

5.3.2 Evidence of Affordance Use by Age: Counting

The counting sequence apps were: Base 10 Blocks: Quantity 1–9 (pre-assessment), Base 10 Blocks: 1–5 and Base 10 Blocks: Numerals (learning apps), and Base 10 Blocks:

Table 5 Evidence of learning progression change for the counting sequence

Task	Age	*N*	Progressed	Regressed	Same
1. Build 2, 3, 4, or 5	3	16	6.3% (N = 1)	12.5% (N = 2)	81.3 (N = 13)
	4	19	15.8 (3)	0 (0)	84.2 (16)
	Total	35	11.4 (4)	5.7 (2)	82.9 (29)
2. Build 6, 7, 8, or 9	3	16	18.8 (3)	12.5 (2)	68.8 (11)
	4	19	15.8 (3)	5.3 (1)	78.9 (15)
	Total	35	17.1 (6)	8.6 (3)	74.3 (26)

Quantity 1–9 (post-assessment). These apps contained blocks that children moved and the audio counted along with each placement of the blocks and celebration sparkles when the correct number of blocks was reached. Table 6 shows children's app affordance access on the pre and post assessments for the counting sequence.

Overall, 3-year-olds accessed affordances more frequently than 4-year-olds. For example, 3-year-olds were more likely to access the following helping affordances during Task 1: "counts as child moves blocks to develop 1–1" (A2) "provides audible feedback through verbal number name or tapping sound" (A3), and "correct response indicated with celebration sparkles" (A4). However, 3-year-olds reduced their access of all three of these helping affordances between the pre and post assessments, perhaps indicating that they were not needed during the post assessment (with the exception of A4, Task 2). The 4-year-olds accessed the helping affordance "links visual/symbolic representations with verbal number name" (A1) on the pre and post-assessments, whereas, 3-year-olds did not access this affordance at all. In addition, 4-year-olds increased their access of A1 from pre to post.

Table 7 shows children's app affordance access when they used the Base-10 Blocks: 1–5 learning app to count quantities from 1 to 5 with the blocks. Similar to the patterns of affordance access on the pre and post assessment, 3-year-olds were more likely than 4-year-olds to access the three app affordances when counting quantities of 3, 4, and 5. Helping affordances of the app were: "links visual/symbolic representations with the verbal number name" (A5), "counts as child moves blocks onto given visual tower of blocks" (A6), and "correct response indicated with celebration sparkles" (A7). It is likely that the 3-year-olds needed much more assistance than the 4-year-olds, and therefore, exhibited greater access to the app affordances.

Table 6 Affordance access on the pre and post apps in the counting sequence

	Age	*N*	A1 % (N)	A2 % (N)	A3 % (N)	A4 % (N)
Pre-assessment						
Task 1 Build 2, 3, 4, or 5	3	16	0 (0)	31.3 (5)	25.0 (4)	56.6 (9)
	4	19	5.3 (1)	0 (0)	5.3 (1)	31.6 (6)
	Total	35	2.9 (1)	14.3 (5)	14.3 (5)	42.9 (15)
Task 2 Build 6, 7, 8, or 9	3	16	0 (0)	37.5 (6)	25.0 (4)	37.5 (6)
	4	19	10.5 (2)	5.3 (1)	15.8 (3)	31.6 (6)
	Total	35	5.7 (2)	20.0 (7)	20.0 (7)	34.3 (12)
	Age	*N*	A1 % (N)	A2 % (N)	A3 % (N)	A4 % (N)
Post-Assessment						
Task 1 Build 2, 3, 4, or 5	3	16	0 (0)	12.5 (2)	18.8 (3)	37.5 (6)
	4	19	10.5 (2)	5.3 (1)	10.5 (2)	15.8 (3)
	Total	35	5.7 (2)	8.6 (3)	14.3 (5)	25.7 (9)
Task 2 Build 6, 7, 8, or 9	3	16	0 (0)	12.5 (2)	18.8 (3)	50.0 (8)
	4	19	21.1 (4)	10.5 (2)	21.1 (4)	26.3 (5)
	Total	35	11.4 (4)	11.4 (4)	20.0 (7)	37.1 (13)

Table 7 Affordance access on the base-10 blocks: 1–5 learning app in the counting sequence

Task	Age	*N*	A5 % (N)	A6 % (N)	A7 % (N)
Count 3	3	16	25.0 (4)	37.5 (6)	31.3 (5)
	4	19	0 (0)	0 (0)	5.3 (1)
	Total	35	11.4 (4)	17.1 (6)	17.1 (6)
Count 4	3	16	25.0 (4)	43.8 (7)	31.3 (5)
	4	19	0 (0)	5.3 (1)	15.8 (3)
	Total	35	11.4 (4)	22.9 (8)	22.9 (8)
Count 5	3	16	25.0 (4)	37.5 (6)	31.3 (5)
	4	19	0 (0)	0 (0)	5.3 (1)
	Total	35	11.4 (4)	17.1 (6)	17.1 (6)

Very little evidence of affordance use was present when both age groups interacted with the Base-10 Blocks: Numerals learning app. In all six tasks where children counted up to 10, the “provides audible feedback as child touches each block to reinforce 1-to-1” (A8) helping affordance was observed an average of 0.95% of the time. The “correct response indicated with celebration sparkles” (A9) helping affordance was observed an average of 4.3% of the time.

5.3.3 Pre-to Post-assessment Changes in Affordance Use: Counting

Figure 3 shows pre to post affordance use by 3-year-olds for the counting sequence. There was no access of the helping affordance “links visual/symbolic representations with verbal number name” (A1) in either task. However, on Task 1, 3-year-olds who progressed accessed the helping affordances: “counts as child moves blocks to develop 1–1” (A2), “provides audible feedback” (A3), and “correct response indicated with celebration sparkles” (A4) during the pre and/or post assessments. Every 3-year-old who progressed accessed these three affordances on Task 1. In comparison, children who regressed did not access A2 and A3 until the post-assessment for Task 1, and perhaps it was then too late to be of help to them. Affordance access on Task 2 seemed to contrast Task 1. For those who progressed and those who regressed, access of the four affordances was more similar than the contrasting access patterns for Task 1.

Figure 4 shows pre to post affordance use by 4-year-olds for the counting sequence. Looking across Task 1, only 4-year-olds who progressed or stayed the same accessed the four helping affordances. These affordances included: “links visual/symbolic representations with verbal number name” (A1), “counts as child moves blocks to develop 1–1” (A2), “provides audible feedback” (A3), and “correct response indicated with celebration sparkles” (A4).

Similar to the patterns for 3-year-olds, the patterns for 4-year-olds on Task 2 seemed to contrast Task 1. The 4-year-olds who regressed were more likely to access A3 and A4 on Task 2, with 100% of those who regressed accessing A3 on the post assessment and A4 on the pre and post assessment. This indicates that the audible number name and the celebration sparkles were providing conflicting counting information to the children.

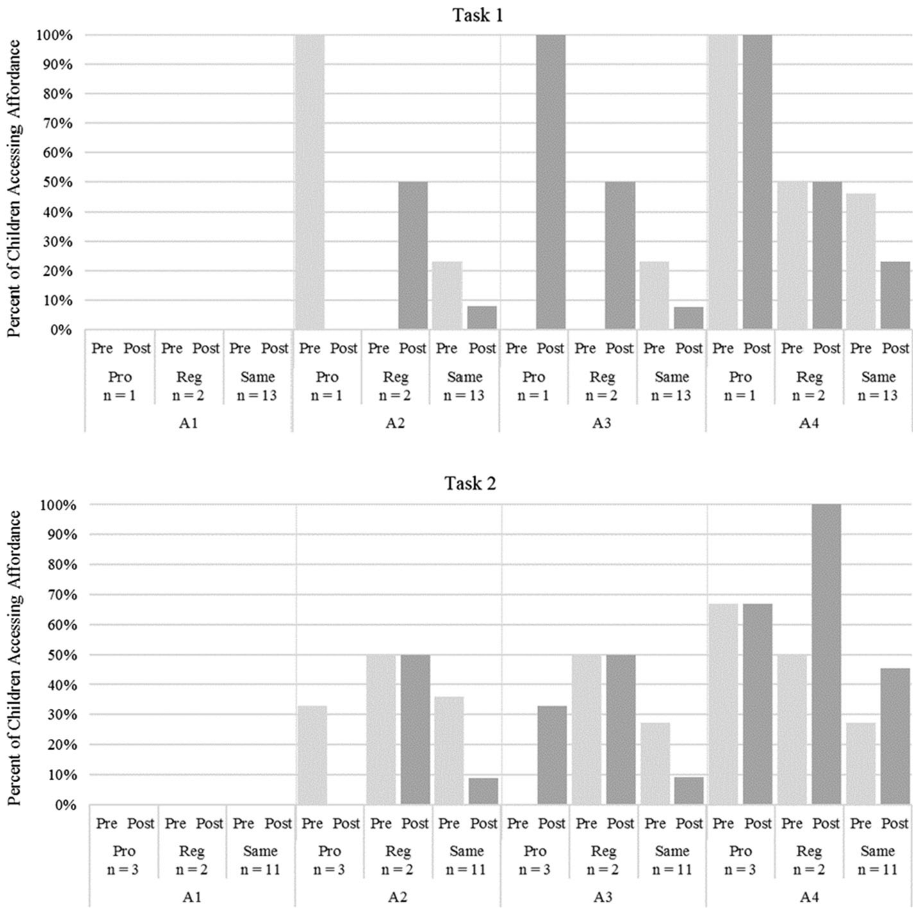


Fig. 3 Counting sequence pre- to post-assessment affordance access age 3: *Task 1* and *Task 2*. Affordances Key: A1 = links visual/symbolic representations with verbal number name; A2 = counts as child moves blocks to develop 1-to-1; A3 = provides audible feedback; A4 = correct response indicated with celebration sparkles. *Lighter bars* are children who progressed. *Darker bars* are children who regressed. *No bar* indicates zero children accessed that affordance in that category

5.3.4 Evidence of Affordance Access Disaggregated by Learning Progression and Age: Counting

Figures 5 and 6 show the affordance results decomposed by interview task on the pre- and post-assessment, learning progression (i.e. progressed, regressed, stayed the same), and age in relation to their interaction with the Base-10 Blocks learning app.

Task 1: Build quantities 2, 3, 4, or 5. In Fig. 5, we see the Learning App where children were asked to count 3, 4, and 5 blocks. For each task the bars show affordance use by children who progressed; age 3 (N = 1) and age 4 (N = 3), regressed; age 3 (N = 2) and age 4 (N = 0), and stayed the same; age 3 (N = 13) and age 4 (N = 16) when building quantities 2, 3, 4, or 5 on the pre- to post. This part of the analysis explains how the children's interactions with counting by 3, 4 and 5 in the learning app may have affected the pre-post outcomes, although we acknowledge that the number of children progressing

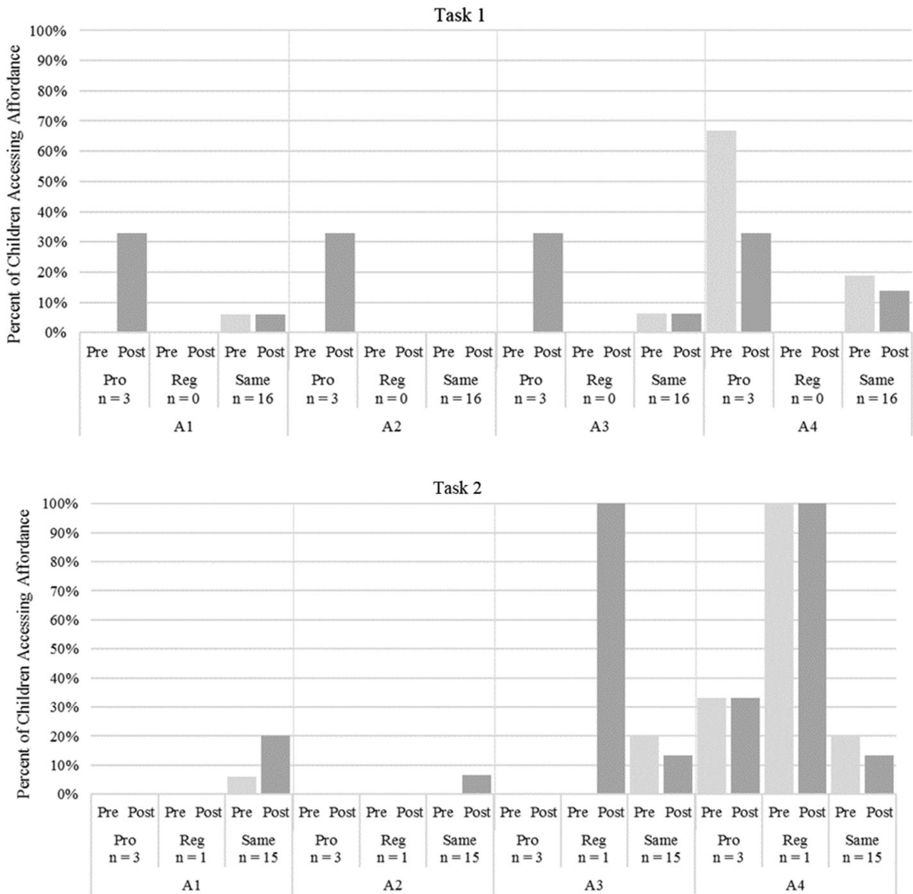


Fig. 4 Counting sequence pre- to post-assessment affordance access age 4: *Task 1* and *Task 2*. Affordances Key: A1 = links visual/symbolic representations with verbal number name; A2 = counts as child moves blocks to develop 1-to-1; A3 = provides audible feedback; A4 = correct response indicated with celebration sparkles. *Lighter bars* are children who progressed. *Darker bars* are children who regressed. *No bar* indicates zero children accessed that affordance in that category

or regressing was very small. The most commonly accessed affordance was by the 3-year-olds who accessed A6, the helping affordance "counts as child moves blocks onto given visual tower of blocks." Only the two 3-year-olds who regressed or stayed the same on all three tasks accessed A5, "links visual/symbolic representations with the verbal number name." In contrast, 4-year-olds did not access A5 or A6 (with the exception of a small number on the counting 4 task). The helping affordance "correct response indicated with celebration sparkles" (A7) was accessed by 3- and 4-year-olds.

Task 2: Build quantities 6, 7, 8, or 9. Figure 6 again shows the Learning App where children were asked to count 3, 4, and 5 blocks. However, in this figure, the bars show affordance use by children who progressed; age 3 (N = 3) and age 4 (N = 3), regressed; age 3 (N = 2) and age 4 (N = 1), and stayed the same; age 3 (N = 11) and age 4 (N = 15) when building quantities 6, 7, 8, or 9 on the pre to post. This part of the analysis explains how the children's interactions with counting by 3, 4 and 5 in the learning app

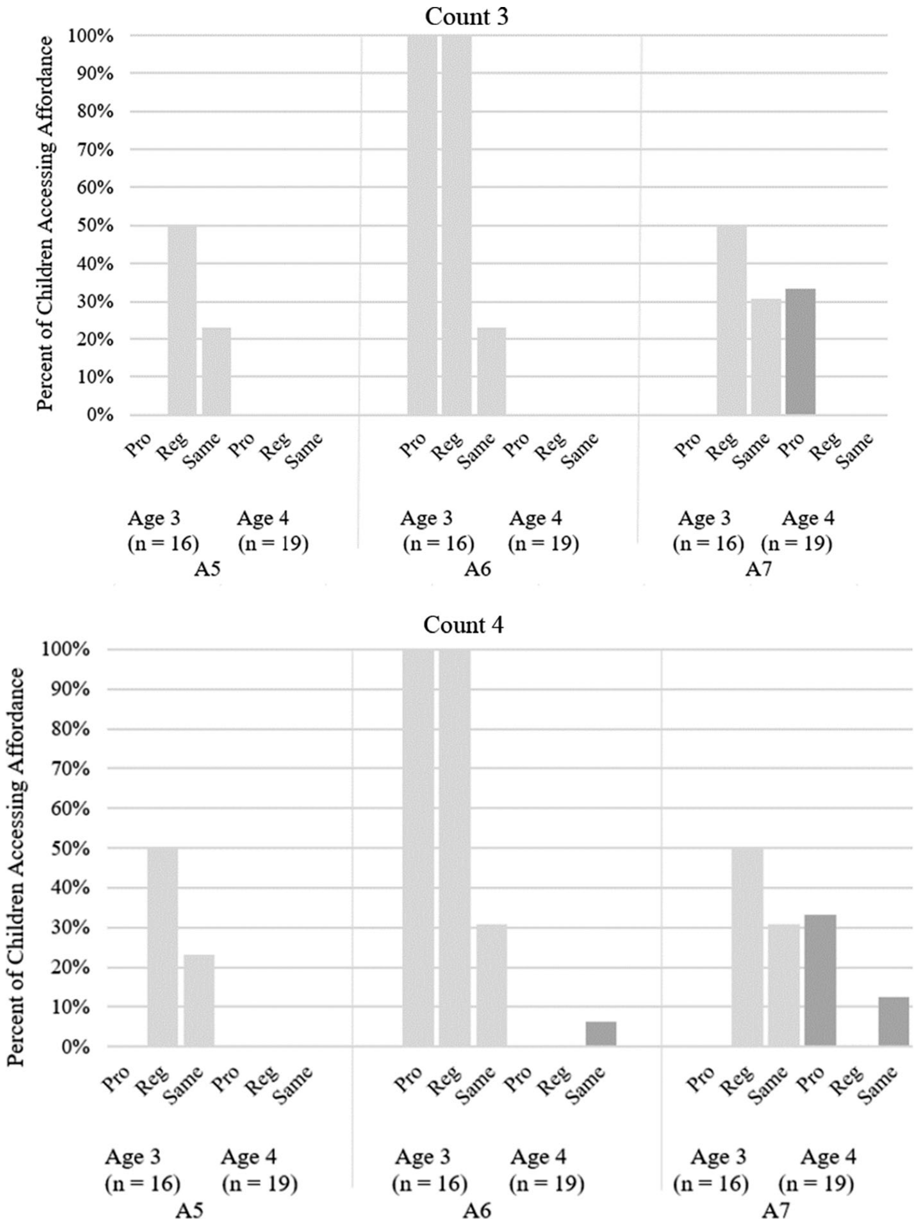


Fig. 5 Counting Sequence learning app Base-10 Blocks 1–5, *count 3*, *count 4*, and *count 5* as it relates to children's progression from the pre-to-post assessment Task 1. Affordances Key: A5 = links visual/symbolic representations with verbal number name; A6 = counts as child moves blocks onto given visual tower of blocks; A7 = correct response indicated with celebration sparkles. *Lighter bars* are children 3-year-olds. *Darker bars* are 4-year-olds. *No bar* indicates zero children accessed that affordance in that category

may have affected the pre-post outcomes, although we acknowledge that the number of children progressing or regressing was very small. Again, the most commonly accessed affordance was by the two 3-year-olds who regressed and accessed A6, the helping

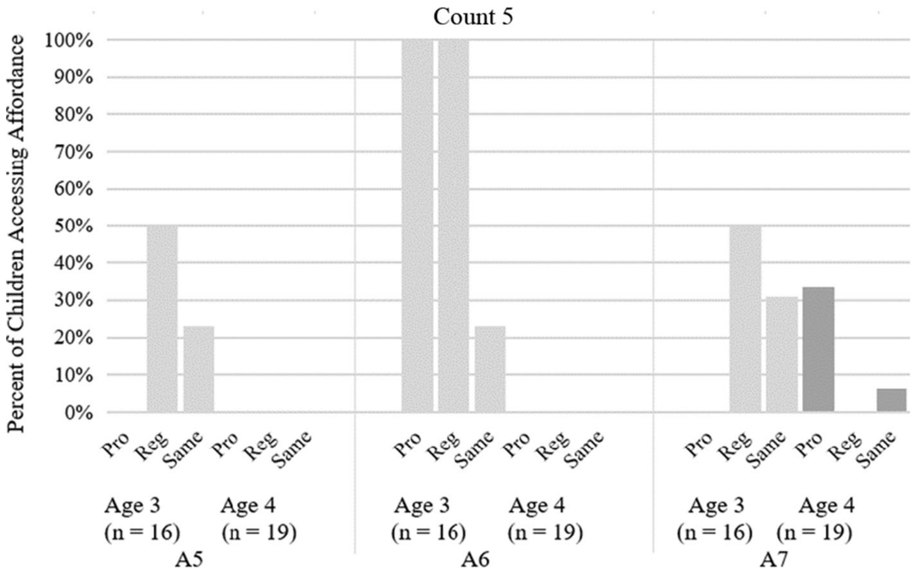


Fig. 5 continued

affordance "counts as child moves blocks onto given visual tower of blocks." Just as they did when counting 3, 4 and 5 in relation to Task 1 from pre-to-post assessment, the 3-year-olds were the only group to access A5 and A6. The 4-year-olds did not access A5 or A6 when counting 3, 4, and 5 (with the exception of a small number when the counting 4) perhaps because they already knew how to build these smaller quantities. Additionally, the helping affordance A7, "correct response indicated with celebration sparkles" was only accessed by the 13 3-year-olds who regressed or stayed the same and 15 4-year-olds who stayed the same.

In conclusion, when considering the changes in frequency of affordance use from throughout the Base-10 Counting learning sequence among all children, there appears to be different perceptions of the affordances by the children who progressed in the learning tasks than those who regressed indicating a need to understand what led to these different perceptions.

5.4 Counting Sequence: Qualitative Results

5.4.1 Young Children's Patterns of Behavior While Interacting with the Apps

Table 8 displays the results for the Base 10 Counting learning sequence. Overall, children coded at a level 0 or 1 displayed an over-reliance on the audio and celebration sparkles feedback across the apps in the learning sequence. Movements were not connected to cardinality. This disconnect explains the differences in perceived affordances among children who progressed and regressed. For example, Brylee's verbal counting utterances did not match up with the audio from the app and only stopped with the celebration sparkles. Because of the delay in the celebration sparkles and audible counting, Brylee counted one too many blocks. Children at level 2 correctly built the required quantity but relied on the celebration sparkles to stop counting. Their verbal utterances matched the

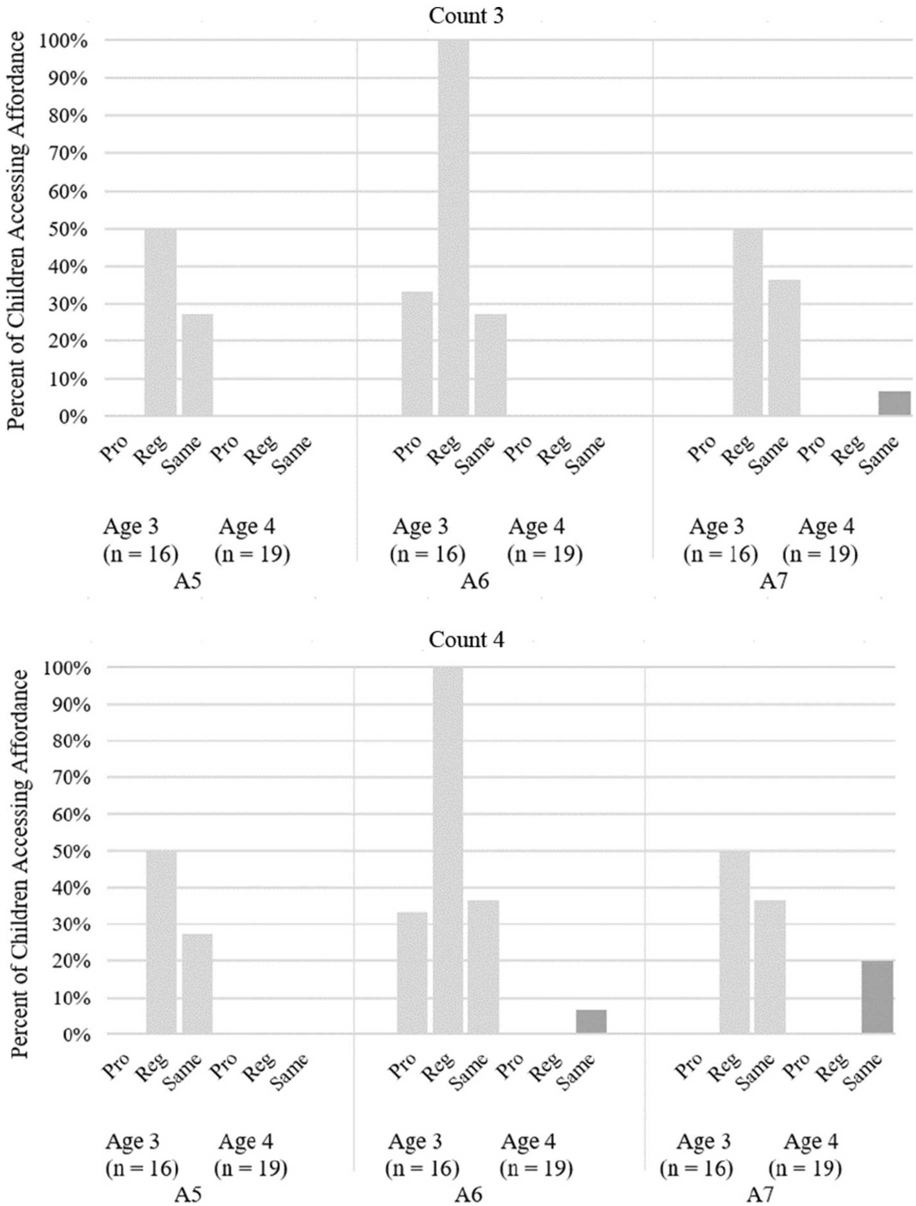


Fig. 6 Counting sequence learning app Base-10 Blocks 1–5, *count 3*, *count 4*, and *count 5* as it relates to children's progression from the pre-to-post assessment Task 2. Affordances Key: A5 = links visual/symbolic representations with verbal number name; A6 = counts as child moves blocks onto given visual tower of blocks; A7 = correct response indicated with celebration sparkles. *Lighter bars* are children 3-year-olds. *Darker bars* are 4-year-olds. *No bar* indicates zero children accessed that affordance in that category

audible counting of the app and, therefore, they achieved much more accuracy than those in levels 0–1. Children coded at level 3 independently built the smaller quantities up to five, but relied on the celebration sparkles for the larger quantities. Like those in level two,

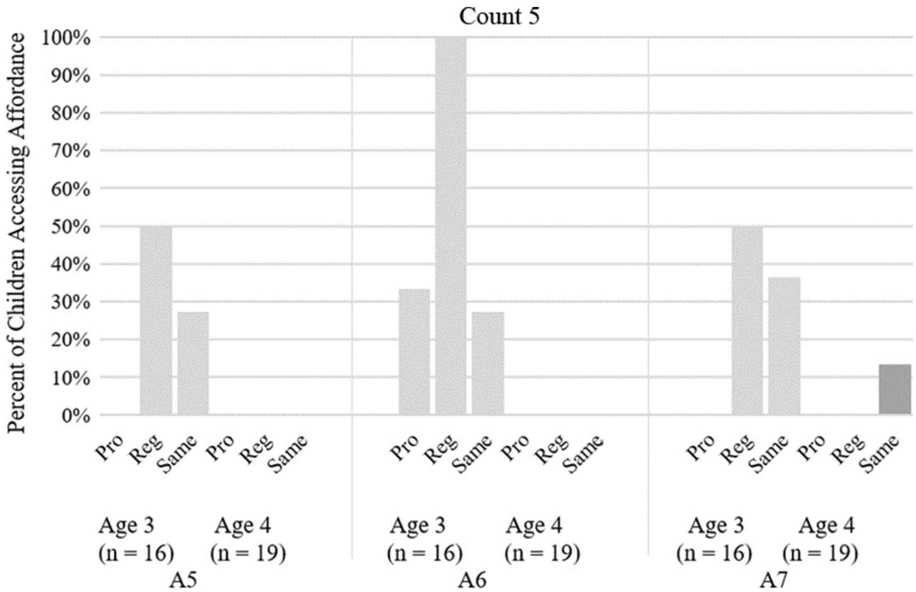


Fig. 6 continued

their verbal utterances matched the audible counting of the app. Children coded at level 4–5 were purposeful and independent in their movements. The app features afforded them error correction or affirmation of accuracy. They counted aloud confidently and their movements aligned with their counting.

5.4.2 Affordances that Helped and Hindered Young Children's Progressions and Why

Overall, affordances that allowed children to engage in purposeful questioning and movement helped them progress in learning counting, with 11% progressing in Task 1 and 17% in Task 2. Affordances that got the child bogged down in technical difficulties, thereby losing focus on the counting concepts and the child's questions, or that failed to lead the child to engage meaningfully in counting, hindered learning. Thus, for an affordance to contribute to counting and seriation learning in positive ways, the child needed to perceive a feature that the app afforded, have their movements with the representation in the app supported, and connect it to the counting and seriation concepts through purposeful actions that contributed to sense making. When children failed to perceive these connections, what the feature afforded was obscured and deterred the sense making process. In these cases, children stayed the same in learning, and sometimes regressed.

5.5 Mixed Methods Results

5.5.1 Affordances, Learning, and Age for the Seriation Sequence

The qualitative analysis showed that when an app feature afforded children's question making and sense making, this supported learning from pre to post, even when a feature seemed to afford a hindering effect. Conversely, children who were not able to manage the

Table 8 Evidence of young children's patterns of behavior for the counting sequence

App	Learning progression level	Patterns of behavior
Base 10 Blocks: Quantity 1–9 (pre-assessment)	0	Relied entirely on sparkles to stop counting. Feedback delay caused inaccurate counting. Inaccurate counting aloud. No sense of cardinality
	1	Correctly built tower without counting aloud. Children relied on sparkles to stop counting and correct answer was due to sparkles, not counting ability
	2	Correctly built tower while also counting aloud. Relied on sparkles to stop counting. Correct answer relied on the sparkles, not counting ability
	3	Able to build smaller quantities under five correctly and purposefully while counting aloud. For quantities over five, correctly built tower while counting aloud, but relied on sparkles, not counting ability
	4	Counted with the audible voice or waiting to say number before moving on. Precise in movements or placement. Focused. Did not rely on sparkles other than as affirmation of correct response
Base 10 Blocks: 1–5 (learning app)	0	Relied entirely on sparkles to stop counting. Feedback delay caused inaccurate counting. No sense of cardinality
	1	Correctly built tower without counting aloud. Children relied on sparkles to stop counting and correct answer was due to sparkles, not counting ability
	2	Relied on sparkles to stop counting and correct answer was due to sparkles, not counting ability
	3	Able to build smaller quantities correctly and purposefully while counting aloud. Used sparkles for affirmation
	4	Many did not want to count aloud even though they could successfully do so. Moved purposefully not waiting for direction to complete activities
Base 10 Blocks: Numerals (learning app)	0	Relied entirely on sparkles to stop counting. Feedback delay caused inaccurate counting. Incorrectly counted aloud. No sense of cardinality
	1	Correctly built tower without counting aloud. Relied on sparkles to stop counting. Correct answer due to sparkles, not counting ability
	2	Correctly built the tower while counting aloud. Relied on sparkles to stop counting, not counting ability
	3	Correctly built tower while counting aloud. Sparkles helped them know when to stop counting, not counting ability
	4	Lots of happy or proud verbal utterances. Laughing or saying "This is easy!" Did not need to physically touch the blocks to count them. Biggest difficult was moving number cards. Quick subitizing in many few cases

Table 8 continued

App	Learning progression level	Patterns of behavior
Base 10 Blocks: Quantity 1–9 (post-assessment)	0	Random movement of blocks. No recognition of feedback given by sparkles
	1	Relied entirely on sparkles to stop counting. Feedback delay caused inaccurate counting. Incorrectly counted aloud. No cardinality
	2	Correctly built tower while counting aloud. Sparkles helped them know when to stop counting, not counting ability
	3	Correctly built smaller quantities under five while counting aloud. For quantities over five, correctly built tower while counting aloud, but relied on sparkles, not counting ability
	4	Several instances of counting with the audible voice or waiting to say a number before moving on. Precise in movements or placement. Focused. Did not rely on sparkles other than as affirmation of correct response or as a cue to fix an error

technological difficulties, or who failed to perceive what an app feature afforded, were hindered in their learning. The results on affordance access and age indicate that some features afforded different experiences based on the age of the child. Children's perceptions of what a feature afforded determined whether the child experienced it as helping or hindering. For example, slightly more 3-year-olds progressed than 4-year-olds from pre-to-post on the seriation sequence. Those 3-year-olds who progressed were far more likely to access affordances A5 and A6 in the Pink Tower Tapping learning app and affordances A8 and A9 in the Red Rods learning app. Since most of these affordances were hypothesized as hindering, this demonstrates that the children who progressed overcame the potentially hindering effects in a productive way. Essentially, children engaged in productive struggle (Hiebert and Grouws 2007, pp. 387–390) that led to their learning.

5.5.2 Affordances, Learning, and Age for the Counting Sequence

Affordances that helped children work through purposeful movements connected to counting concepts helped those who progressed in their learning. On the other hand, children who did not perceive the purpose of a feature and what it might afford, were hindered in their learning, even for affordances hypothesized as helpful. Similar to the seriation sequence, affordances were perceived differently depending on the child's age, and consequently they helped or hindered learning. For example, a greater percentage of 3-year-olds regressed than 4-year-olds from pre-to-post on the counting sequence. Those 3-year-olds who regressed were far more likely to access affordances A5, A6, and A7 in the Base-10 Blocks: 1–5 learning app. Since these affordances were hypothesized as helping, this suggests that 3-year-olds who regressed failed to connect the affordances with the counting concepts. Thus, age seemed to impact children's perceptions of the app features and what they afforded in the learning interaction experience.

6 Discussion

In concurrence with Burlamaqui and Dong (2014), this study found that affordances of children's mathematics education apps influence children's interactions with the technology and how they learn. Our results support the findings of Barendregt et al. (2012), Byers and Hadley (2013), and Tucker (2015), that children develop mathematical concepts differently when interacting with touch-screen apps. These differences depend on the ways young children access the affordances of the apps and also depend on children's age. Children's patterns of behavior in this study showed that specific actions, both in movement and verbal utterances, were unique to those who progressed versus those who regressed in learning. Specifically, young children who displayed purposeful questioning and movement showed higher levels of learning across both sequences. This was true regardless of age. This may explain why some affordances, hypothesized to be hindering, seemed to aid children's learning and some affordances, hypothesized to be helpful, seemed to hinder children's learning.

These findings are consistent with research on the role of external representations in constructing internal representations (Bruner 1964; Piaget 1946/1970). They also complement the results on the affordance-ability relationship reported in a study of second graders (Tucker et al. 2016). However, as Gibson (1986) observed, "some offerings of the environment are beneficial and some are injurious" (p. 137). One implication is that app designers should use evidence from research on their apps to make modifications and adjustments within their apps when there are features found to be hindering student learning. A second implication of these findings is the need for educators and parents to closely observe the interactions of young children with touch-screen apps before allowing unstructured use. Early educators may need to provide clear instructions on technical aspects of the apps, depending on the child's age and developmental level. Some children may need explicit scaffolding to perceive and access features that the apps afford. Without careful observation, the affordances of an app may indeed hinder the child's learning. In some cases, young children may not be cognitively ready for the features in a particular app. These three examples emphasize that children have different needs and that it is important to design instruction that addresses the continuum of these learning needs. Mathematics apps may have the potential to differentiate or customize learning experiences along this continuum of needs. When a child displays inconsistent external representations (as demonstrated by movement and verbal utterances), parents and educators should intercede (as supported by the most recent AAP report) to prevent frustration and a negative mindset toward mathematics experiences. In addition, app designers should solicit feedback about the use of their apps from educators and parents who are utilizing them in real-world contexts.

7 Conclusion

This research demonstrates that affordance access matters for young children when interacting with touch-screen mathematics apps. Pediatricians are now advocating that the use of digital devices among young children should be carefully monitored and limited (Kabali et al. 2015). The patterns of behavior of the 3- and 4-year-old children in this study suggests that children's perceptions of app features and what those features afford has an important influence on what the child learns and how the child learns during app engagement. Children's perceived affordances can be deduced through careful observation of children's patterns of behavior. Specifically, those working with young children can look for purposeful questioning and movement by the child as an indication of accurate perceived features that may afford learning. Children who display evidence of technical difficulty or verbal and bodily expressions of frustration, or a disconnect to the mathematical concepts, may need further scaffolding to effectively support learning using a mathematical app on a touch-screen device. Scaffolding is important because it supports deeper learning by providing sufficient support when concepts and skills are first introduced to children and provides for the careful monitoring and limited use suggested by pediatricians (Berk and Winsler 1995).

Appendix

See Table 9.

Table 9 Affordances KEY

Topic	App name	Affordance code	Affordance description
Seriation sequence	Pink tower: free moving (pre-assessment)	A1	Difficult to move tiny blocks
		A2	Difficult to compare tiny blocks
		A3	Difficult to see block color
	Pink Tower: tapping	A4	Does not allow the child to be wrong
		A5	Builds the tower for the child in perfect stack
		A6	Difficult to compare tiny blocks
	Red Rods	A7	Sound distraction
		A8	Difficult to move rods
		A9	Rods shifted easily
		A10	Easy visual comparison of length
	Pink Tower: free moving (post-assessment)	A1	Difficult to move tiny blocks
		A2	Difficult to compare tiny blocks
		A3	Difficult to see block color
Counting sequence	Base-10 blocks: Quantity 1–9 (pre-assessment) Task 1: 2, 3, 4, 5	A1	Links visual and symbolic representations with verbal number name
		A2	Counts as child moves blocks to develop 1-to-1 understanding
		A3	Provides audible feedback through verbal number name or tapping sound
		A4	Provides feedback indicating correct response through celebration sparkles
	Task 2: 6, 7, 8, 9	A1	Links visual and symbolic representations with verbal number name
		A2	Counts as child moves blocks to develop 1-to-1

Table 9 continued

Topic	App name	Affordance code	Affordance description
		A3	Provides audible feedback through verbal number name or tapping sound
		A4	Provides feedback indicating correct response through celebration sparkles
	Base-10 block: 1–5	A5	Links visual and symbolic representations with verbal number name
		A6	Counts as child moves blocks onto given visual tower of blocks
		A7	Provides feedback indicating correct response through celebration sparkles
	Base-10 block: numerals	A8	Provides audible feedback as child touches each block to reinforce 1-to-1
		A9	Provides feedback indicating correct response through celebration sparkles
	Base-10 blocks (pre-assessment) Task 1: 2, 3, 4, 5	A1	Links visual and symbolic representations with verbal number name
		A2	Counts as child moves blocks to develop 1-to-1 understanding
		A3	Provides audible feedback through verbal number name or tapping sound
		A4	Provides feedback indicating correct response through celebration sparkles
	Task 2: 6, 7, 8, 9	A1	Links visual and symbolic representations with verbal number name
		A2	Counts as child moves blocks to develop 1-to-1
		A3	Provides audible feedback through verbal number name or tapping sound
		A4	Provides feedback indicating correct response through celebration sparkles

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