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ORIGINAL RESEARCH

The Complexity of the Affordance–Ability Relationship When Second-Grade Children Interact with Mathematics Virtual Manipulative Apps

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Abstract The purpose of this study was to explore relationships between app affordances and user abilities in second graders' interactions with mathematics virtual manipulative touchscreen tablet apps. The research questions focused on varying manifestations of affordance–ability relationships during children's interactions with mathematics virtual manipulative touchscreen tablet apps. Researchers qualitatively analyzed video recordings and ethograms from clinical interviews of 33 second-grade children. Each 45-min clinical interview involved one child interacting with two sequences of mathematics virtual manipulative touchscreen tablet apps: one sequence focusing on place value concepts and the other sequence focusing on skip counting concepts. Results provided evidence of Moyer-Packenham and Westenskow's (Int J Virtual Pers Learn Environ 4(3):35–50, 2013) five affordance categories of virtual manipulatives. Approach to and degree of affordance access varied depending on a child's corresponding ability, and some children modified their affordance access as their ability changed. Results also indicated that outcomes of accessing an affordance also related to a child's ability. Context also influenced affordance access. These results imply that affordance–ability relationships are multifaceted. Overall, these results imply that is important to consider affordance–ability relationships in relation to mathematics education technology.

Keywords Virtual manipulatives · Affordances · Mathematics · Mobile devices

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1 Introduction

Technology is an important element of mathematics teaching and learning (National Council of Teachers of Mathematics 2000), with a range of tools offering access to an array of digital representations of mathematical content. Human cognition is grounded in perception of and interaction with the physical environment (Alibali and Nathan 2012). Therefore, to learn mathematics, children interact with representations of mathematics in the physical environment. These interactions are evidence of mathematical thinking, and changes in these interactions are evidence of mathematical learning (Nemirovsky et al. 2013). The physical environment affords possibilities for interacting with representations of mathematics. However, children interact with representations of mathematics in many ways, leading to a broad array of outcomes.

The purpose of this paper is to explore relationships between app affordances and user abilities in second graders' interactions with mathematics virtual manipulative touchscreen tablet apps. This study draws on research on affordances of technology, specifically those related to the chosen tools. Data analyzed in this paper were gathered as part of a larger study of children's use of mathematics virtual manipulative touchscreen tablet apps. In the larger study, second-grade children showed statistically significant improvements in performance and efficiency for skip counting tasks, but not for place value tasks (see Moyer-Packenham et al. 2015). Affordance–ability relationships contribute to children's interactions with mathematics while using technology and thus are important considerations for those who work with mathematics education technology.

1.1 Theoretical Background

1.1.1 Affordances and Abilities

Based on Gibson's work (1986), Greeno (1994) defined an affordance as something that "relates attributes of something in the environment to an interactive activity by an agent who has some ability" based on its own attributes, which are characteristics of the environment or agent (p. 383). One can qualify affordances in many ways, as Gibson (1986) claimed that affordances can be objectively beneficial, neutral, or injurious. Greeno (1994) considered affordances to be graded properties, with each affordance existing only in relation to a corresponding ability, and vice versa. Chemero (2003) extended this, positing that affordances and abilities are paired in continuous systems. Two main views of affordances in technology have emerged. Gaver (1991) began with Gibson's conception of affordances and expanded it to how design suggests affordances. However, Norman (1988, 1999) interpreted application of affordances as one of perceived possibilities. In each case, affordances refer to "action possibilities relative to the agent" (Burlamaqui and Dong 2014, p. 13) and affordance access depends on a corresponding ability. However users do not always perceive and engage with affordances as designers intend, though there are ways to prime users to increase the likelihood that they will perceive the intended affordances (Burlamaqui and Dong 2015). Thus, affordance–ability relationships vary based on the attributes of the environment (technology tool) and the agent (user), and may have a range of iterations and outcomes.

1.1.2 Affordances of Virtual Manipulatives

Virtual manipulatives are "an interactive… visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al. 2002,

p. 373). The effectiveness of virtual manipulatives is well established, with a meta-analysis of studies comparing virtual manipulatives with other instructional treatments yielding a moderate effect in favor of virtual manipulatives (Moyer-Packenham and Westenskow 2013). Instruction with virtual manipulatives may also contribute to the equalization of achievement across demographic groups (Moyer-Packenham et al. 2014). However, interaction modality may also influence results, as long-term interaction with touchscreen virtual manipulatives may be more effective than long-term interaction with mouse-controlled virtual manipulatives (Paek 2012). Furthermore, researchers found that children adjusted their external representations of mathematical content by refining their input gestures when interacting with touchscreen virtual manipulatives (Barendregt et al. 2012; Ladel and Kortenkamp 2012). Emerging research suggests that interactions with multi-touch virtual manipulatives may have unique potential to support development of specific mathematical skills and strategies (Baccaglini-Frank and Maracci 2015). Yet such interactions influence children's performance and efficiency on mathematics tasks differently across grade levels and content areas within mathematics (Moyer-Packenham et al. 2015), and student characteristics may influence access to features of virtual manipulatives (Moyer-Packenham and Suh 2012).

In their meta-analysis, Moyer-Packenham and Westenskow (2013) identified five categories of affordances of virtual manipulatives that contribute to learning mathematics:

> *focused constraint* (i.e., VMs focus and constrain student attention on mathematical objects and processes), *creative variation* (i.e., VMs encourage creativity and increase the variety of students' solutions), *simultaneous linking* (i.e., VMs simultaneously link representations with each other and with students' actions), *efficient precision* (i.e., VMs contain precise representations allowing accurate and efficient use), and *motivation* (i.e., VMs motivate students to persist at mathematical tasks). (p. 35)

These affordances also relate to scaffolding, which involves control of task components that may initially be too difficult for a learner (Wood et al. 1976). Technology tools use scaffolding for various purposes (Quintana et al. 2004) and can change scaffolding as children progressively master instructional objectives (Murray and Arroyo 2002). Children may also ignore scaffolding offered by virtual manipulatives as task proficiency increases, such as decreasing use of audio feedback (Paek 2012). Research on virtual manipulatives also suggests links between scaffolding and affordance categories such as simultaneous linking (McLeod et al. 2012) and focused constraint (Tucker 2015).

Emergent research has focused on affordances of touchscreen virtual manipulatives. Tucker and colleagues (e.g., Tucker 2015, in press; Tucker and Moyer-Packenham 2014; Tucker et al. in press) concluded that children's affordance access varied depending on their ability. For example, when interacting with Motion Math: Zoom, efficient precision was evident when children with greater technological proficiency chose specific places to use the app's features to zoom in within a given range. Other children, zooming inaccurately or inefficiently, or even avoiding zooming altogether, were neither as precise nor efficient as children who could effectively zoom. Affordance access could also vary by approach and degree, could change over time, and could lead to different outcomes (Tucker 2015, in press). Together, this research suggests that virtual manipulative touchscreen tablet apps offer an array of affordances that may influence learning, and that children access these affordances in different ways. However, these studies involved interactions with fewer apps, and the fine-grained analyses (e.g., Tucker 2015, in press) used smaller sample sizes and different mathematical content. Therefore, additional research was required to examine affordances and affordance–ability relationships when children interact with touchscreen virtual manipulatives to determine if the emergent findings apply beyond the initial contexts.

1.2 Research Questions

The research questions that guided this study were:

1. What evidence of affordances is present when children interact with mathematics virtual manipulative touchscreen tablet apps? Our hypothesis was that evidence of affordances would be present and could be structured using Moyer-Packenham and Westenskow's (2013) categories of affordances of virtual manipulatives.
2. What variations in affordance–ability relationships occur when children interact with mathematics virtual manipulative touchscreen tablet apps? Our hypothesis was that affordance–ability relationships would vary by approach and degree, could change over time, and could lead to different outcomes, based on previous results, including those by Tucker (2015, in press) and Moyer-Packenham and Suh (2012).

2 Methods

2.1 Design

This study used a qualitative research design, which is appropriate for testing hypotheses and illuminating related quantitative data (Miles et al. 2013). As part of the larger study, researchers examined children's performance and efficiency and found statistically significant improvements for skip counting tasks, but not for place value tasks (see Moyer-Packenham et al. 2015). In this study, we focused on a qualitative analysis of the children's videos by examining app affordances and the affordance–ability relationship as a way to interpret and explain the performance and efficiency results in the larger study. Our analysis focused on an in-depth examination of second grade children's interactions with the virtual manipulatives for the presence of app affordances and affordance–ability relationships. Greene (2007) suggests that in-depth qualitative analysis following quantitative analysis can provide complementarity for convergent or divergent results by examining different facets of the same phenomenon. Thus, this study led to complementary findings that shed light on quantitative results in the larger study (Moyer-Packenham et al. 2015).

2.2 Participants

Researchers used letters and brochures to recruit thirty-three second-grade children, ages 7-8, from local schools. Children's parents reported on the use of personal touchscreen devices (PTDs) in the home with 15 % ($n = 5$) having more than five PTDs, 76 % ($n = 25$) with between one and four PTDs, and 9 % ($n = 3$) with none. Four of the children had their own personal device at home. Parents reported that the children used the PTDs every day ($n = 16$, or 48 %), 4–6 days per week ($n = 2$, or 6 %), 1–3 days per week ($n = 12$, or 36 %), and never ($n = 2$, or 6 %). Parents reported that the children used mathematics apps on the PTDs every day ($n = 3$, or 9 %), 1–3 days per week ($n = 10$, or 30 %), and never ($n = 15$, or 45 %).

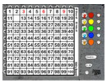
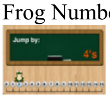
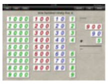
2.3 Procedures

As part of the larger study, researchers used a multi-step piloting process to test and select appropriate apps, draft research protocols, pilot the protocols with children in local schools, and pilot the research protocols and procedures in a clinical interview setting. Criteria for initial app selection included presence of virtual manipulatives, relevant mathematical content, and developmentally appropriate representations. These decisions were based on research on how young children learn mathematics (e.g., Sarama and Clements 2009) and the research team's combined 70+ years of K-8 teaching experience. App piloting involved children interacting with the apps as researchers noted learning opportunities, affordance access, ease of use, and other elements of the interaction experience. (Additional details, including piloting of procedures and protocols, can be found in Moyer-Packenham et al. 2014.)

During the larger study, children participated in video-recorded clinical interviews featuring tasks designed to reveal their mathematical understandings as they interacted with mathematics virtual manipulative touchscreen tablet apps (Ginsburg and Pappas 2004). The interviews took place in a research building on a university campus, in a room equipped with wall-mounted video cameras and two-way mirrors connecting to an adjacent observation room. At the beginning of each interview, parents or guardians completed informational surveys and consent forms. Each child participated in a 30–45 min one-to-one clinical interview. Clinical interviews feature tasks intended to reveal a child's underlying knowledge, allowing opportunities for the child to demonstrate understanding and describe his or her thinking while researchers observe the process (Ginsburg and Pappas 2004). A researcher conducted the interview with the participant while a researcher in the observation room recorded observations while listening via headphones.

During the interviews, each participant interacted with six mathematics virtual manipulative touchscreen tablet apps presented on an iPad and organized in two three-app sequences. Each sequence consisted of pre-assessment tasks, two learning activities, and post-assessment tasks. Each post-assessment was identical to the corresponding pre-assessment, but within each sequence, the order of learning apps varied. The interviewer

Table 1 Apps used in iPad interviews

	Skip Counting	Place Value
Pre/Post	100s Board 	Base-10 Blocks 
Activity A	Frog Number Line 	Zoom Number Line 
Activity B	Counting Beads 	Place Value Cards 

Researchers did not update the apps during the study, and subsequent iterations of the apps may differ from the study version. See "Appendix" for commercial app names provided a brief demonstration of how to use an app before the participant began the corresponding tasks. The first sequence focused on skip counting concepts and the second sequence focused on place value concepts (see Table 1). The following sections describe the apps and tasks.

2.4 Skip Counting Sequence Apps and Tasks

2.4.1 100s Board

The pre-assessment and post-assessments on the 100s Board app consisted of three tasks using the 100 board: count by 4–28, 6–18, and 9–36, in each instance explaining how many times they counted by the given number to reach the target (e.g., "I counted by four seven times to get to 28"). The 100s Board app was a single-touch virtual replica of a physical 100 board. For these tasks, the app displayed the numbers 1–100 in rows of 10, increasing from left to right and top to bottom. Tapping a space on the board once made the number change color, a second tap caused the number to disappear, and a third tap caused the number to reappear, allowing participants to track skip counting patterns on the board.

2.4.2 Frog Number Line

In the Frog Number Line learning app participants practiced skip counting in response to app-generated prompts by using single-touch input to move a frog across a labeled number line ranging from 0 to 15. A display above the number line indicated randomly sequenced instructions to skip count by 2's, 3's or 4's, while the frog appeared on the number line at the position of 0, 1, 2 or 3. Participants dragged the frog along the number line to skip count. Each label turned yellow when the frog was directly above it. To indicate a response, the participant let go of the frog over a number. The label turned green if the input was correct. When the participant correctly skip counted, the frog made a "ribbit" sound and the display provided a positive message (e.g., "Outstanding!"), and a new task appeared.

2.4.3 Counting Beads

The Counting Beads learning app presented a virtual mat and a tray containing linked beads in strings of one to ten. For these researcher-generated tasks, the interviewer asked each participant to use the beads to count by 4–28, 6–18, and 9–36. To indicate skip counting, the participant used single-touch input to drag a string of beads to the mat, joining each subsequent string to the previous string. The app allowed the participant to place only one type of string at a time (e.g., only strings of three beads). It was not possible to remove connected beads except by clearing the entire sequence. Each time the participant placed a string of beads, a number card showing the total number of beads on the mat appeared in a pile in the last section of the tray. At any time, the participant could move cards from the tray to label the skip counting sequence. The app provided feedback in the form of a checkmark if the participant correctly labeled the entire sequence represented on the mat.

2.5 Place Value Sequence Apps and Tasks

2.5.1 Base-10 Blocks

For the pre-assessment and post-assessment on the Base-10 Blocks app, the interviewer asked each participant to use the blocks to complete the same six researcher-generated tasks, modeling: (a) a number between 12 and 30, (b) a number between 54 and 62, (c) 181, (d) a number between 181 and 200, (e) 267, and (f) 20 less than 267. The app presented participants with a place value chart and blocks (ones, tens, and hundreds) to form target numbers less than 1000. Each block fit only within the assigned section (e.g., ones in the ones area), where the app organized them into a vertical stack. When a participant changed the number of blocks, the numeral tiles changed accordingly. The app bundled groups of ten blocks, placing the new block in the next available place value. The narration function was disabled because the voice was slower than participants' input. The single-touch app required tapping, dragging, or flicking motions for interactions.

2.5.2 Zoom Number Line

In the Zoom Number Line learning app, participants navigated a number line to place app-generated target numbers using multi-touch input. Animals of varying sizes demarcated numbered intervals (e.g., rhinos for hundreds, dogs for tens, frogs for ones). To place the number, participants swiped to move the number line left or right, or zoomed in or out to change interval size (e.g., ones, tens, hundreds) by bringing together or pulling apart two fingers ("pinching"), finally tapping to pop the bubble containing the number. The app responded to correct answers by allowing the number to fall into place on the number line and providing sounds and animations. Incorrect answers caused the bubble to shudder instead of pop. Scaffolds, such as visual and auditory cues to navigate in the correct direction, appeared when participants' progress slowed. In this study, most participants encountered ranges from 0 to 20 through 0 to 1000. All participants reached levels where changing intervals could be more efficient than swiping alone (e.g., finding 198 from 723), but few participants reached levels where they were forced to change intervals (e.g., finding 13 from 200, from intervals of 100).

2.5.3 Place Value Cards

In the Place Value Cards learning app, participants used single-touch input to drag Place Value Cards to form app-generated target numbers that appeared in word form and were read aloud by the app. Participants only encountered tasks featuring three-digit numbers without zeros. Columns of cards were arranged from left to right in order of hundreds, tens, and ones. Within each column, cards were arranged from least being furthest from the child to greatest being closest to the child (see image in Table 1). Participants could represent the number using the addition frame (e.g., 900 + 40 + 3) before dragging the cards into the "sum" box, or drag the cards directly into the sum box. A check appeared on the screen to indicate a correct answer. Participants could touch the word form of the number to repeat the audio prompt.

2.6 Data Sources

Instruments used to collect data for this study included two video recordings, an observation ethogram, and a demographic survey. The interview room's wall-mounted camera and the GoPro camera worn by the participant provided multiple angles of video recordings of the interviews. Researchers documented the interviews using observation ethograms, which allow researchers to describe the frequency and duration of observed behaviors without evaluation (MacNulty et al. 2007). This technique is often used in animal behavior researcher and has been adapted for use in educational settings (e.g., Moyer-Packenham et al. 2014). The ethograms supplemented the videos and noted strategies used, errors and error correction, affective responses, interviewer actions, and unique or noteworthy occurrences. Demographic surveys provided information on the child's age, the availability and use of PTDs at home and school, and attitudes toward mathematics.

2.7 Data Analysis

Researchers analyzed data using a two-cycle coding process, with the first cycle used to summarize the data and the second cycle used to identify patterns across the codes and categories (Miles et al. 2013; Saldaña 2013). Videos of each child's interview were examined and coded. In each cycle, researchers revisited the videos multiple times as codes evolved and exemplary cases became apparent. Researchers used provisional coding during the first coding cycle, beginning with a predetermined flexible start list of codes or categories informed by relevant literature, the research questions, and previous research findings (Miles et al. 2013; Saldaña 2013). Moyer-Packenham and Westenskow's (2013) affordance categories served as the provisional coding categories, but codes emerged to describe evidence of specific affordances and manifestations of the corresponding affordance–ability relationships. In the second coding cycle, researchers used pattern coding, which involves the identification of themes or explanations for first-cycle codes (Miles et al. 2013; Saldaña 2013). This coding cycle focused on themes and explanations that connected first-cycle codes and coding categories.

3 Results

The results in this section appear by research question. The first section presents evidence of the affordance categories identified in the literature. The second section presents evidence of how children's interactions demonstrated the affordance–ability relationships, including themes of variation by approach and degree, change over time, variation of outcomes, and the influence of context. Performance and efficiency results from the pre-assessments and post-assessments indicated statistically significant improvements in performance and efficiency from pre-assessment to post-assessment on the skip counting tasks but not on the place value tasks (see Moyer-Packenham et al. 2015). A related analysis examined helping and hindering affordances in relation to subgroups of children based on their progression or regression in performance and efficiency (Moyer-Packenham et al. 2016). The results below, that answer our two research questions, help to provide some interpretation and explanation for the performance and efficiency results that were reported in the larger study.

Table 2 Examples of affordance categories from children's interactions with virtual manipulative mathematics apps

	Skip counting			Place value		
	100s Board	Frog Number Line	Counting Beads	Base-10 Blocks	Zoom Number Line	Place Value Cards
Simultaneous linking	Color change indicating the child's input	Action of "skipping" connected to movement along number line	Number tile appears when bead group is added	Block arrangement, numeral tiles, and voice linked to user block placement	Increasing or decreasing magnitude along number line connected to left or right navigation	Combining cards for hundreds, tens, and ones to make a number
Focused constraint	Focus on set range (e.g., 1-100)	Focus on number line with range of 1-15	One bead type per construction	Construction limited to 999	Limited navigation to maintain focus range	Allows only one card from each place value to build number
Efficient precision	To indicate number, must tap exactly in box	Requires exact placement and pause for decision	Bead strings cannot be split	Blocks can only fit in specific corresponding place value	Labeled intervals contain specific range	Cards organized by place value and increasing magnitude
Motivation	Task-related (from researcher)	Sounds, animations, and messages (e.g., "Winning!")	Checkmark	Task-related (from researcher)	Sounds and animations	Checkmark
Creative variation	Limited by research tasks	Multiple possible strategies	Limited by research tasks	Multiple possible strategies	Multiple possible strategies; app supports efficiency	Order of building solution

3.1 Presence of Affordance Categories

The first research question focused on what evidence of affordances was present when children interacted with mathematics virtual manipulative touchscreen tablet apps. Data analysis of each child's video interview revealed evidence of affordances that aligned with Moyer-Packenham and Westenskow's (2013) affordance categories. Four affordance categories—simultaneous linking, focused constraint, efficient precision, and motivation—were explicitly supported throughout the data, while creative variation was less common. This is consistent with Moyer-Packenham and Westenskow's (2016) most recent meta-analysis in which creative variation was a less-frequently identified affordance category. Examples of each affordance category are provided for each app (see Table 2). In each category, the description refers to an affordance–ability relationship involving access to the specific example of the affordance, rather than all possible examples of the affordance or affordance category, which is beyond the scope of this study. There were no differences by learning app order or demographics.

3.1.1 *Simultaneous Linking*

Simultaneous linking refers to the concurrent connection of multiple representations or actions (Moyer-Packenham and Westenskow 2013) and was prevalent throughout the interactions. The Base-10 Blocks app simultaneously linked changing block arrangement and numeral tiles with participant input of block placement to indicate changing quantities on the place value chart. When a child placed a block, a numeral tile corresponding to the newly modeled quantity replaced the numeral tile in the place value chart label and slid over to update the sum. Although children's input was synched with the pictorial representation of the blocks, the symbolic place value chart representation did not update until the participant paused input briefly, and it took nearly one second for the sliding numeral tile to finish indicating the new sum. Thus, children could place many blocks before the symbolic and pictorial representations aligned (see Figs. 1, 2, 3). Other examples of simultaneous linking included change in magnitude on the number line connected to directional navigation in Zoom Number Line and a number tile showing the total number of beads present appearing when a set of beads was added in Counting Beads.

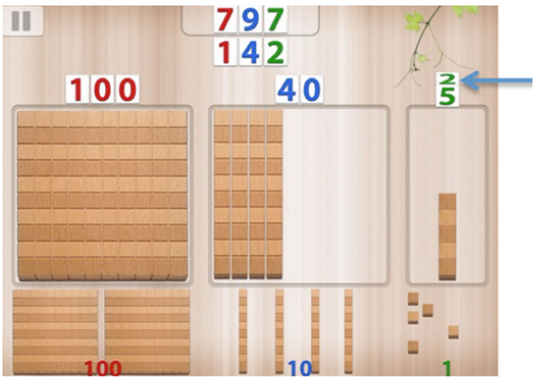


Fig. 1 Annotated screenshot of symbolic place value chart representation changing to reflect pictorial representation

3.1.2 Focused Constraint

Focused constraint refers to the direction and maintenance of attention on specific mathematical content (Moyer-Packenham and Westenskow 2013) and was prevalent throughout the interactions. The Zoom Number Line app restricted navigation to focus children's attention on magnitude and comparison within a given range. The app limited navigation by stopping users from moving outside a given range (e.g., 0–20) or accessing certain intervals (e.g., allowing intervals of 1 and 10, but not 0.1 or less, nor 100 or greater), and these restrictions changed by level and task within the app. Other examples of focused constraint included limited ranges for exploration in the assigned tasks, such as 100–999 in base-10 blocks, 111–999 in Place Value Cards, 1–15 in Frog Number Line, and variable ranges in 100s Chart.

3.1.3 Efficient Precision

Efficient precision refers to accurate, fluent representation of mathematics content (Moyer-Packenham and Westenskow 2013) and was prevalent throughout the interactions. The Frog Number Line app afforded efficient precision by requiring exact placement and a pause in input to indicate a decision made for identification of the target number during skip counting. Furthermore, upon completion of a task, the app instantly cleared the used number line and presented a new task. Other examples of efficient precision included the requirement of tapping exactly within a numbered square to indicate number choice in 100s Chart, the layout of the cards in Place Value Cards, and the restriction of cube or bundle placement to specific regions of the place value chart in Base-10 Blocks (e.g., ones cubes only in ones place).

3.1.4 Motivation

Motivation refers to the desire to engage in mathematical activity (Moyer-Packenham and Westenskow 2013). Each app had combinations of attributes that afforded motivation and demotivation. Mathematical and non-mathematical attributes could contribute to affordances. Mathematical attributes included the potential for using negative numbers in Zoom Number Line. Non-mathematical attributes included feedback of a check for a correctly

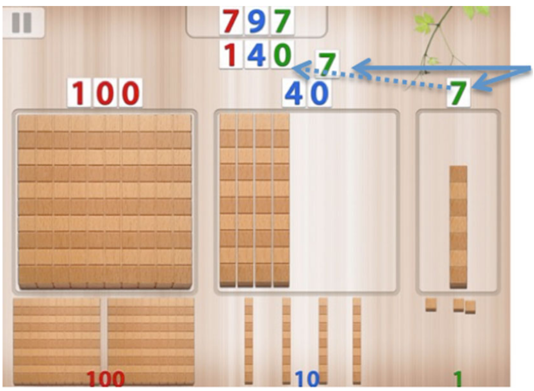


Fig. 2 Annotated screenshot of symbolic place value chart representation sliding to change symbolic sum representation

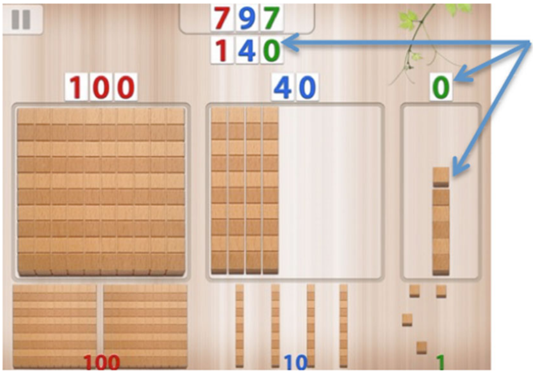


Fig. 3 Annotated screenshot of symbolic representations out of sync with pictorial representation

labeled sequence in Counting Beads. Repetition in the form of rapidly providing similar tasks could also influence motivation in Place Value Cards and Frog Number Line.

3.1.5 Creative Variation

Creative variation refers to the proliferation of varied mathematical strategies and solutions (Moyer-Packenham and Westenskow 2013) and was less common in these interactions. In Place Value Cards, Zoom Number Line, and Frog Number Line, one could take multiple, usually similar paths to generate one correct outcome for each task. Researcher-generated tasks limited possible creative variation in interactions with the 100s Board and Counting Beads. The only tasks with multiple correct solutions occurred in Base-10 Blocks, where one could take various paths to find a correct solution and some researcher-generated tasks had more than one correct solution.

3.2 Affordance–Ability Relationships

The second research question focused on what variations in affordance–ability relationships occurred when children interacted with mathematics virtual manipulative touch-screen tablet apps. Data analysis of each child's video interview revealed variations in affordance–ability relationships during children's interactions. Five main themes were evident: (a) affordance access could vary by approach (b) affordance access could vary degree, (c) affordance–ability relationships changed as ability changed, (d) outcomes of affordance access varied, and (e) context influenced affordance access. The following section presents the themes with select examples of each.

3.2.1 Affordance Access Could Vary by Approach

Affordance access could vary by approach, aligning with findings from Tucker (2015, in press) and implications from Moyer-Packenham and Suh (2012). Varying by approach means that it was possible to access the same affordance in different ways. For example, children accessed simultaneous linking in Base-10 Blocks in various ways. Most children slowly placed the blocks and watched the numerals change to indicate the new quantity during at least one task. Some children accessed this affordance when challenged, such as the child who exclaimed, "I know how to do it!" when assigned the first task, yet slowed considerably to wait for the numeral tiles to match the input when presented with more challenging tasks. Other children largely ignored the simultaneous linking, focusing on the blocks rather than the numeral tiles for most tasks. However, 32 of the children modeled quantities so quickly during at least one task that they appeared not to attend to the changing numeral tiles, and two children repeatedly called out the answer before modeling. Mistakes were rare during quick modeling, suggesting that children may have been confident enough in their modeling ability to ignore the symbolic representation. Two children consistently counted aloud to track their modeling, which may have permitted them to ignore the changing symbolic representation.

Another affordance approach variation was evident in when children accessed focused constraint in the form of restricted navigation scaffolds when interacting with Zoom Number Line. For example, a child trying to reach 73 from the range of 70–80 might be limited to zooming into the ones intervals, rather than accidentally zooming into the tenths intervals. Similarly, children encountered navigation restrictions after passing the target number while swiping along the number line, as the app stopped eventually stopped further travel in the incorrect direction. All 33 children encountered at least one of these restrictions, but one child frequently used the navigation restriction to aid planning for the next task by zooming out as far as the app permitted between inputting a correct answer and the presentation of the next task. Thus, differences in children's abilities led to variations in approaches to accessing the corresponding affordances.

3.2.2 Affordance Access Could Vary by Degree

Affordance access could vary by degree, which also aligned with findings from Tucker (2015, in press) and implications from Moyer-Packenham and Suh (2012). Varying by degree means that it was possible to access the same affordance to different extents. Access to motivation often varied by degree, often relating to particular mathematical or non-mathematical attributes of the app or task. One child spoke in a higher pitch voice as the target numbers in the Zoom Number Line app increased, while other children specifically requested to work with negative numbers. However, these reactions were less common. Children's access to motivation also connected to non-mathematical attributes, including feedback and repetition. Few children clearly showed motivational responses related to feedback, and those that responded were evenly split between enjoying and disliking the style of feedback. For example, sounds in Zoom Number Line induced mild laughter and occasional complaints, while feedback of a check for a correctly labeled sequence in Counting Beads elicited little evident motivational response.

However, many children reacted to repetition in Place Value Cards and Frog Number Line, which both presented children with very similar tasks in quick succession. Frog Number Line had few potential tasks, and many children commented on the repetition, including two children who explicitly noted identical tasks. Children who were developing their skip counting skills may have benefited from a second chance at the same task, but some children with strong skip counting skills became demotivated and asked to move to another app. Place Value Cards offered many possible combinations of numbers to create using the same process, thereby changing content without changing presentation. Only two children made uncorrected mistakes on at least 10 % of their tasks, but whereas some children expressed boredom and asked to change apps, many children settled into a routine and may have benefitted from the practice the apps afforded. Mathematical and non-mathematical attributes contributed to affordance–ability relationships concerning motivation, and children's access to the affordance of motivation (and demotivation) varied by degree, depending on their corresponding abilities.

3.2.3 Affordance–Ability Relationships Changed Over Time

Affordance–ability relationships changed over time as children's abilities changed, as discussed by Tucker (2015, in press) and implied by Paek (2012). For example, when asked to model a quantity using Base-10 Blocks, many children purposefully accessed simultaneous linking by slowly placing the blocks and watching the numerals change to indicate the new quantity. As their ability changed, most children became adept at quickly modeling quantities, rarely slowing to pay attention to changing numeral tiles, decreasing the degree of access to simultaneous linking. Other children altered their approach as their ability changed, attending to simultaneous linking only when nearing their answer or being faced with a challenging task. Similarly, while interacting with the Place Value Cards app, six of the 31 children who initially used the addition frame to organize their answers stopped doing so, and 12 children began answering before the voice finished reading the number.

During interactions with Zoom Number Line, focused constraint prevented children from accidentally zooming in or out to extremes when attempting to reach a given interval and curtailed left to right movement to keep children within a specific range on the number line. As children's abilities changed via honing input gestures and improving mathematical understandings to accurately navigate the number line, most decreased the degree of accessing this focused constraint. However, a few children purposefully changed approaches to use focused constraint to aid navigation. Thus, not only did affordance–ability relationships change over time, these changes often linked to variations by approach and degree.

3.2.4 Outcomes of Affordance Access Varied

Outcomes of affordance access also varied, akin to findings by Tucker (2015, in press). During interactions with Base-10 Blocks, some children changed their access to simultaneous linking, while other children continued with the same approach. Children who were less fluent in modeling quantities may have tracked the numeral tiles linked to their actions, altering their understandings and at times increasing their efficiency, which could lead to changes in approach to accessing the affordance. Children who were already somewhat fluent in modeling quantities may have become confused by the mismatched representations, including four children whose efficiency decreased. However, other children began to ignore some of the representations and maintained or increased their efficiency. Whether or not affordance access changed, children could increase or decrease in efficiency while demonstrating changes in understanding.

For the majority of children, accessing efficient precision requiring exact placement and pauses while interacting with Frog Number Line encouraged accurate skip counting, but even similar access led to different outcomes. Most children continued using the same strategies. However, several children used the same approach yet showed new skip counting understandings, such as the child who exclaimed that he found a "secret trick" in which one skipped two numbers to count by 3 and one skipped three numbers to count by 4. Other children switched from dual counting (i.e., counting on their fingers or counting the numbers 1, 2, 3) to counting the number skipped. These children had ability such that accessing efficient precision facilitated the outcome of improved skip counting understandings. Thus, using the same approach to accessing an affordance could lead to different outcomes, depending on a child's corresponding ability.

However, five children who demonstrated strong skip counting understandings before interacting with the app often attempted to bypass the same efficient precision affordance by making correct skip counting jumps without pausing. This forced these children to decrease their speed and pause to indicate an answer, which for three children required repeatedly reselecting intended responses the app did not recognize. These three children showed similar levels of accuracy on many tasks as children who did not attempt to bypass the efficient precision affordance but had not previously shown a strong understanding of skip counting. Thus, different approaches to affordance access could lead to similar apparent outcomes. Therefore, children's ability influenced outcomes related to accessing the affordance.

3.2.5 Context Influenced Affordance Access

Context also influenced affordance access. When interacting with researcher-designed Base-10 Blocks tasks, some children's approaches to accessing simultaneous linking depended on whether they perceived the task as easy or difficult. Furthermore, 15 children modeled the same quantity on at least one pair of pre-assessment and post-assessment tasks where multiple solutions were possible, and only one child did not consistently apply a chosen modeling strategy (e.g., hundreds, then tens, then ones). During app-generated Place Value Cards tasks, all 33 children consistently modeled regular numbers (e.g., 721) by placing the hundreds, then the tens, and finally the ones. Of the 27 children who encountered irregular numbers (e.g., 711, 719), 22 children used their regular strategy, three children appealed for interviewer assistance, and two children placed the hundreds, then the ones, and finally the tens. Both of these suggest limited access to creative variation that varied based on children's abilities. During researcher-designed 100s Board tasks, 20 children mentioned patterns when counting by nine, but only two children mentioned patterns when counting by six. Therefore, context, such as characteristics of the tasks, influenced children's affordance access.

4 Discussion

The results of this study build upon prior research and have implications for theory, empirical research, and practice. While interacting with the touchscreen virtual manipulative apps in this study, children accessed all five categories of affordances of virtual manipulatives proposed by Moyer-Packenham and Westenskow (2013). Supporting findings from previous studies, affordance access in this study varied by approach and degree depending on a child's ability Tucker (2015, in press) and outcomes of affordance access varied (e.g., Moyer-Packenham and Suh 2012; Tucker 2015, in press). Consistent with implications from previous studies, affordance–ability relationships changed over time (e.g., Bartoschek et al. 2013; Paek 2012). Results also indicated that context influenced affordance access. However, this study involved a unique combination of user population, sample size, mathematical content focus, and app choice, suggesting that these themes may be consistent across many types of user-tool interactions.

4.1 Theoretical Implications

Theoretical implications of the results concern the complexity of affordance–ability relationships related to the five main themes: (a) affordance access could vary by approach, (b) affordance access could vary degree, (c) affordance–ability relationships changed as ability changed, (d) outcomes of affordance access varied, and (e) context influenced affordance access. Together, these findings demonstrate that an affordance alone does not dictate children's actions; rather, these interactions involve complex affordance–ability relationships. Therefore, these findings support prior theorists who asserted that affordance–ability relationships link the environment and the agent (e.g., Burlamaqui and Dong 2014; Chemero 2003; Greeno 1994). Importantly, these results extend beyond previous theoretical assertions from Gibson (1986) and Greeno (1994). Variations by approach and degree suggest that affordances are not simply graded properties, whereas the influence of previous or ongoing outcomes on the affordance–ability relationship imply that subjective values may influence outcomes that feed back into the continuous affordance–ability relationship, rather than affordances being completely separate from their objective outcomes.

The findings also imply that children's perception of potential affordance access may change during interactions as part of the changing affordance–ability relationships, such as when children increased efficiency by directly using the sum box in Place Value Cards and when children used navigation restrictions to aid task completion in Zoom Number Line. This builds on previous research indicating that perception of affordances is influenced by characteristics of both the tool and the user (e.g., Burlamaqui and Dong 2015). However, the results of this study suggest that perception may develop as part of the continuous affordance–ability relationships that connect the attributes of tools and users. Therefore, these findings support emergent characterizations of affordance–ability relationships Tucker (2015, in press), implying that they warrant further investigation and integration into affordance theories. In particular, this research could focus on how affordance–ability relationships change over time through interactions, as few authors have focused specifically on this area. Considering these characterizations and how they manifest over time may be productive for advancing theory, as it acknowledges the continuous nature of affordance–ability relationships.

4.2 Implications for Empirical Research

The results also have implications for empirical research, including methodology and future directions. The qualitative methods led to complementary yet at times divergent findings that shed light on the related quantitative data (Moyer-Packenham et al. 2015). This illustrated the value of using multiple methods to examine data to provide in-depth examinations of phenomena (Greene 2007). It is also important to consider how research design influences results. In this study, researchers designed some tasks (e.g., Base-10 Blocks modeling quantities) but set parameters for app-generated tasks (e.g., Zoom Number Line levels). In some cases, researchers also controlled which features of the apps children could access (e.g., Base-10 Blocks using blocks, not cupcakes) or provided structures that were not otherwise present (e.g., 100s Board skip counting sequences). When researchers design tasks, they gain control over the mathematical content and sequencing, which may allow for comparisons among children on the same task. However, researchers should be aware that when they set the context by structuring tasks, they influence the affordances children can access. For example, the 100s Board app heavily relied on children's internal motivation, providing no evaluative feedback, with the researcher-generated tasks adding structure and affording motivation external to the sandbox (i.e. free play) format (Zanchi et al. 2013) the app presented.

The results should also influence future empirical research. Four of the five categories of affordances of virtual manipulatives were prevalent in children's interactions with the apps in this study, with most apps affording multiple examples within each category. However, creative variation was uncommon in these interactions, which may be due in part to the design of the apps and tasks (i.e., specific goals, rather than free play). These findings, along with other research applying these categories to inform analysis (e.g., Moyer-Packenham et al. 2016) and to structure coding (e.g., Tucker 2015) support the application of Moyer-Packenham and Westenskow's (2013) affordance categories as a frame for studying how children interact with virtual manipulatives, including those that are part of touchscreen tablet apps. This is important because research on touchscreen tablet virtual manipulatives was not yet widely available during the meta-analysis that identified the affordance categories (Moyer-Packenham and Westenskow 2013), while the follow-up meta-analysis indicated that touchscreen devices were becoming a prevalent mode of interacting with virtual manipulatives (Moyer-Packenham and Westenskow 2016). Avenues for future research include identifying apps and tasks where creative variation may be more likely to occur, or what paths of access to particular affordances lead to specific learning outcomes in various content areas. Additionally, in-depth qualitative studies, such as micro-longitudinal examinations of individual cases, may provide additional empirical evidence to support continued theory development concerning characterizations of affordance–ability relationships and how they change over time.

4.3 Implications for Practice

The results of this study have practical implications for those who design and implement virtual manipulatives. Developers may implicitly consider examples of affordance categories and potential affordance–ability relationships when creating or choosing virtual manipulatives, but detailed examinations of specific affordance–ability relationships could be undertaken. For example, the delayed simultaneous linking in Base-10 Blocks may have been beneficial for some children who worked slowly, but children who attempted to move quickly often became confused or attempted to ignore this affordance and may have preferred a different form of scaffolding. This aligns with research suggesting that users do not always access affordances as designers intended (Burlamaqui and Dong 2015). Developers could consider making affordances readily modifiable, in this case by using the menu to toggle instant changes instead of relatively lengthy animations. Teachers could then consider which version of the affordance is appropriate for a given child, or if external scaffolding is required. Examining affordance–ability relationships and their outcomes may also help teachers analyze children's interactions with virtual manipulatives and other educational technology, potentially revealing children's mathematical understandings (e.g., Tucker 2015, in press) and whether interventions are required to help children develop alternative strategies (e.g., Baccaglini-Frank and Maracci 2015). Thus, while developers should base design in sound research, the teacher's role as the knowledgeable implementer influencing classroom context is critically important.

5 Conclusion

These results indicate that affordance access is as complex and multidimensional as the children themselves due to nuances in the corresponding ability, based on relevant attributes. Rather than an amount of ability simply facilitating a degree of affordance access, it may be that variations in ability and contextual factors inform approaches to and degrees of affordance access as part of a continuous affordance–ability relationship that can lead to an array of outcomes depending on a child's corresponding ability. Thus, theoretical and empirical research should continue to explore the intricacies of these affordance–ability relationships. These results also have implications for practice, as those who design and implement educational technology tools should be aware of affordance categories and affordance–ability relationships when developing, choosing, and evaluating these tools and children's interactions with the tools. Importantly, these results indicate to researchers, designers, and implementers of educational technology that attributes of users, tools, and contexts will vary, prompting different mathematical learning experiences by users based on affordance–ability relationships.

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Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

See Table 3.

Table 3 Mathematics apps selected for the pre and post assessments and learning activities

App name in article	Full app name	App section used	App developer
100s Board	100s Board	*Full app*	Matthew Thomas
Frog Number Line	Teaching Number Lines	Skip Counting Test	Aleesha Kondys
Counting Beads	Montessori Bead Skip Counting	*Full app*	MontessoriTech
Base-10 Blocks	Montessori Numbers—Math Activities For Kids	Quantity (100–999)	L'Escapadou
Zoom Number Line	Motion Math: Zoom	Levels 2–6	Motion Math
Place Value Cards	Montessori Place Value	3-Digit Numbers Without Zeros	MontessoriTech

Full app name and app developer appear as listed on iTunes App Store when the study was conducted

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