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Investigating the Role of Context in Experimental Research Involving the Use of Digital Media for the Learning of Mathematics: Boundary Objects as Vehicles for Integration

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Abstract The paper describes a study of the contexts of six teams, expert in research and development of digital media for learning mathematics, who cross-experimented in classrooms with the use of each other's artefacts. Contextual issues regarding the designed tasks and technologies, the socio-systemic milieu and the ways in which the researchers worked with the teachers were in focus. We analysed the ways in which a set of mutually constructed and negotiated questions aiming to illuminate otherwise tacit contextual issues operated as boundary objects amongst the teams. We discuss the need to develop special tools such as these boundary objects in order to elicit issues of context and the ways they may affect the production of theory.

Keywords Boundary objects · Context · Digital media · Cross-experimentation

1 Introduction

The development and use of digital tools as well as the research on technology enhanced learning of mathematics have limited impact on the educational practices developed in schools. It has been pointed out that this situation is due partly to the fragmentation of the existing theoretical frameworks in the field of mathematics education (Artigue 2008). Theoretical constructs on learning mathematics have inevitably been affected by the contexts within which the respective research was carried out. We suggest that the fragmentation amongst these and the lack of visible coherent differences in the use of terms, connections, complementarities and incommensurabilities amongst them is largely due to both that they emerged from different contexts and that the languages developed to describe these contexts has hitherto been rather weak. Research on the use of digital

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media—often characterised by a design to question or even to make changes to actual contexts in school—exacerbates the bluntness in our ways of incorporating the role of context and the language we use for that purpose.

In this paper we discuss the notion of context in mathematics education research highlighting why and how the use of digital media enhances the importance of investigating ways to address contextual issues seriously in order to avoid fragmentation in the emerging collective theoretical knowledge. This work is part of the collaborative work developed within the European TELMA project aiming to promote integration amongst the approaches followed by the six participating research teams when engaged in experimental research (Bottino and Kynigos, this issue). The aim was to enhance the cohesion of the emerging knowledge concerning the design and use of digital technologies—termed as Interactive Learning Environments (ILEs) within the project—in the teaching and learning of mathematics. In this respect our aim was not to address the influence of context in the productions of research teams theoretically, but rather to focus on the ways in which contextual issues and concerns may influence researchers' practices when they design, implement and analyse teaching experiments based on the use of ILEs.

As described in Bottino and Kynigos (this issue) the six European research teams engaged in cross-experimentation research where each team carried out one teaching experiment with an ILE developed by another TELMA team. We were engaged in cross-team communication to co-design concrete research experiments as a step towards integrating the developed research approaches. The process required being explicit about research choices, theories, contexts and use of representations when collaborative design and implementation is taking place. We wanted to focus on the reality in which the teams conduct research and to capture the explicitness emerging from the need to communicate over a variety of issues concerning digital tool and experiment design and the analysis of tool use. We took into account the contexts within which the different teams designed digital tools, developed theories, addressed schools and carried out research in real classroom settings. Thus, we adopted a pragmatic view of context involving diverse features such as the organisational pragmatics of the design and use of digital media, the ways in which didactical interventions are materialised in schools and the nature of communication between teachers and researchers, the modalities of employment of specific ILEs in real classroom situations and the wider milieu within which experimental work takes place. In this paper we describe the development of a methodological tool for context analysis, and the ways in which we used it to investigate the role of contextual issues in the design, implementation and analysis of the cross-experiments developed by the TELMA teams.

Initially, we attempted to map out the operational details which would allow a deeper understanding of the diverse practices and cultures underlying the research approaches followed by each team which were in principle very different. We then transformed them into questions which were categorised in clusters of relevant contextual concerns aiming to address aspects of the design and/or use of digital artefacts and their possible effects on practice. The methodological tool for context analysis consisted of these questions which played the role of research tools for integration. They were mutually constructed and negotiated to achieve a higher level of integration through joint research experience by addressing aspects of design, implementation and analysis at an operational level of practical research needs. Our aim was to directly address the problem of how a group of researchers, working in diverse teams, can strive to understand and make use of each other's work. As we subsequently discuss, we perceived the methodological tool for context analysis as an example of an artefact designed as a 'boundary object' (in the sense of Star and Griesemer 1989) between such teams in the attempt to make a research study understood in depth from outside the context in which it took place. The changes made to this tool through negotiation amongst the six teams as well as the ways in which it helped the teams to communicate and understand each other are thus discussed. Although this task was very concrete and with well defined time and resource constraints we hoped that it might contribute to the development of research tools for addressing the complexities involved in integrating different research approaches in technology enhanced learning of mathematics. It is important to mention that we perceive this process of refining the tool as continuous; these changes continued after the end of this particular cross-experimentation in the framework of a subsequent project involving the same teams (project ReMath[1]) where more comprehensive experimentations were carried out including the design and development of new ILEs.

After briefly discussing the main interpretations of context in educational research in the last 30 years we refer to how this notion has been taken into account in the mathematics education community. The first section of the paper ends up with justifying why the design and use of technological tools seems to provide a fertile terrain for studying the role of contextual issues in experimental research on the teaching and learning of mathematics. Then, we refer to the notion of boundary objects and how this was taken into consideration in the process of developing the methodological tool for context analysis. Next, there is a brief description of the TELMA approach, consisting of the main group's objectives in relation to context and the cross-experimentation in terms of the ILEs used and the experiments set up by the teams. We then give an account of the development of the methodological tool for context analysis in experimental research. In the next section of the paper, we focus on some exemplary cases within the cross-experimentation and compare the different ways by which the six diverse teams coped with contextual issues described by the aforementioned methodological tool. Finally, we discuss the results highlighting the potential and constraints of our approach to study the impact of contextual issues on research practice. It is important to note that in developing and using the methodological tool for context analysis we did not intend to simplify the notion of context nor disregard its inherent complexity. Rather, our analysis indicates no more than the possibilities for gaining insight into the complex contextual issues involved in integrating research approaches taking place in different experimental settings.

2 The Notion of Context

Although the notion of context is at the core of modern educational research, there is no generally accepted definition of it in the literature. Definitions of context are not necessarily made explicit in the respective papers, and even when this is the case the reader is not provided with enough information about the ways in which it has influenced the respective research work. Researchers, for instance, often use the terms 'context of school' or 'context of the classroom' in written or spoken form of language in a rather unproblematic way. The problematic nature of these terms, however, becomes apparent in cases when researchers attempt to transcend specific educational environments in specific research approaches in a variety of didactic cultures of different countries.

Context has been described as "perhaps the most prevalent term used to index the circumstances of behaviour" (Cole 1996, p. 132). Cole (1996) brought into play two

[1] 'ReMath': Representing Mathematics with Digital Media FP6, IST-4, STREP 026751 (2005–2009), http://remath.cti.gr.

metaphors for defining it. The first one considers context as embedded and refers to that which surrounds and the second one considers context as interactive/dynamic and refers to that which weaves together. As far as teaching in the classroom is concerned, the first metaphor of context situates specific teachers and their classes in a specific type of school (e.g., scientific, vocational), using particular resources (e.g., sets of artefacts) and following a curricula of a national educational system. While indicating a concrete external situation, context in this view can be considered as inextricably linked with human activity taking place within particular surrounding layers (Cole 1996). The participants in the classroom, however, develop their activities through their engagement in performing particular tasks and subsequently in meaning-generation processes which cannot be considered as predetermined or static. Human actions and interactions in this case constitute aspects of context emerging in a social setting. According thus to the second metaphor of an interactive/dynamic context, any attempt to focus on particular aspects of the teaching/learning process demands a more dialectic and dynamic conception of context as interrelated with the interaction between the individual and the social within particular surroundings. In this view context can be considered as weaving together rather than surrounding, i.e. spreading between people and their activities within particular institutional settings.

Historically, there has been a significant shift in the ways by which the mathematics education research community took into account the notion of context in the respective studies. In the past, the importance of contextual concerns was not emphasized by research since the research paradigm within the education field was mainly experimental aiming to describe learning phenomena through generalized models. In this perspective context-dependency was something to be ignored or even 'minimised' as much as possible. It seemed as though there was only implicit recognition of the complexity of human learning and the processes for supporting this learning. In the cases where it was recognized that there might be factors influencing some educational process outside the ones under scrutiny, the researcher's stance was to try to 'neutralize' them rather than to include them in the analysis of the data as for example in conventional control and experimental group situations (Pea and Kurland 1984). In the wake of this initial trend, most of the research was of a 'diagnostic' character, aiming to identify learning difficulties and student misconceptions by means of research 'instruments' (e.g., questionnaires, tests).

From as far back as the end of the 1970s (for a critique of experimental design see Papert 1987), a questioning of this approach led to alternative research paradigms including factors influencing the learning process, such as the situated cognition movement (Lave 1988). Since then, there has been a vast move towards the recognition of the complexity of human thinking and learning, perceiving humans as social beings using cultural artifacts such as oral language, written expression and technology, for communication (Bartolini Bussi 1996). Every explanation of context, then, is essentially related with some kind of a physical and cultural environment and learning is considered as embedded in social situations calling for tool-mediated activity and active involvement in meaning generation.

A basic tenet of the tool-mediated activity approach to the mathematics education field is that a contextual "view" of learning has been adopted by several scholars for a relatively long time now (Bishop 1988; Bauersfeld 1988). Several studies have focused on the influence of context on students' mathematical behaviour incorporating broader analyses of social and cultural factors in any meaning-making process (e.g., the language of the problem at hand, the resources available, the discourse developed within particular social settings). An early focus was on the situated nature of human cognition which is bound by specific settings with their own discourse and meaning-making mechanisms (a notable example is the research on ‘street mathematics’ reported by Nunes et al. 1993). Similar insights confirming the centrality of context in student’s mathematical activity have been at the core of new movements such as ethnomathematics (Keitel et al. 1989) and the political dimensions underlying mathematics education (Skovsmose 1994). In the course of time a significant number of research studies highlighted the need to specify the level of context to which they attend aiming to examine the role of particular contextual aspects in the formation of mathematical knowledge by the students. Wedege (1999) summarised that two main meanings of context have been brought into play in the respective studies: the ‘situation context’ (e.g., workplace, classroom social context, computer-based learning environments, etc.) (see for instance Educational Studies in Mathematics ‘Special Issue’: cf. Vol. 31, Nos. 1–2, 1996) and the ‘task context’ (e.g., everyday life situations used for bridging the gap between the students’ informal knowledge and formal mathematics, see for instance studies involved in the Realistic Mathematics Education movement, Gravemeijer 1994; Gravemeijer and Doorman 1999). Clarke and Helme (1998) pointed out that task context and situation context act complementarily and they are constructed individually by an agent every time she/he gets involved in a setting. In that sense context apart from being considered as a synonym for a concrete situation it can be characterised as a conceptual framework for mathematical reasoning as well.

A common characteristic of any consideration of context as it emerges from the above as well as other subsequent studies is its dynamic nature. Van Oers (1998) drawing on Vygotsky, Leont’ev and their followers proposed an activity approach that goes beyond “overly simplistic notions of context as ‘situation’ or ‘some configuration of circumstances’” (p. 482). His approach, rather, places context making (‘contextualising’) as a central element for understanding context in terms of dynamical features of the activities to be carried out by the participants in a setting. These are related with determining particular goals, examining prior experiences, identifying available means and making sense of actions to be performed. Thus, context constituted by the actions of the participants in a community of learners cannot be articulated by a pre-given structure but emerges as a dynamic process of discursive structuring (i.e. construction and negotiation of meanings during the activity) which affect fundamentally the respective research approaches in terms of design and analysis.

The above view is similar to that of Paul Cobb and his group (Cobb and Yackel 1996) who coined the term ‘emergent perspectives’ to signify how analyzing the data through one lens left the group unhappy about the extent to which they could explain teaching and learning phenomena in mathematics classrooms. This resulted in the group going back to the same data and looking at it from different perspectives, in this case, bringing into account the classroom social norms influencing teacher and student behaviors. Kynigos and Theodossopoulou (2001) used this approach by studying the same data concerning small group work in mathematics with computers in the classroom through an ‘emic’ a dialogical and a social perspective.

We intend to highlight two points that stem from the above review of the literature on context and they are related to the present study. First, the shift of the focus of investigation in the field of mathematics education community to study context not as a backdrop of the teachers’/students’ activities but rather as an integral part of the teaching/learning process that influences the research objectives, the followed approach as well as the analysis of the results. Second, the lack of direct references to wider aspects of context that underlie the design, implementation and analysis of a teaching experiment in real educational settings. Although context seems to be considered by the researchers as a means to describe and understand the learning environment, it has not been explicitly related to their research practice before, during and after the implementation of a teaching experiment. Nevertheless, important aspects of context which remain implicit in most published papers might influence the ways in which the educational processes are studied, the ways in which educational environments are generated and sustained as well as the methods, collaboration, interaction and organizational structure behind researchers' productions (e.g., research projects, ways of developing educational materials). Additionally, there is a lack of research attempts aiming to address the diversity of approaches towards context in different didactic cultures and settings.

The present study involves different research teams which were engaged in co-designing concrete research experiments based on the use of digital tools as a step towards integrating the developed research approaches. We were interested in gaining further understanding on how diverse contextual issues related to the wider milieu within which experimental work with digital tools takes place (e.g., how didactical interventions are materialised in different school settings, aspects of communication between teachers and researchers) may influence researchers' practices when they design, implement and analyse teaching experiments based on the use of ILEs.

2.1 The Use of Technology as a Research Field for Contextual Issues in Mathematics Education

Our choice to focus on context and to use research on teaching and learning mathematics with technologies as a window for studying it results from several reasons related to (a) the progressive sensitivity of mathematics education community towards context and (b) the exacerbation of the importance of addressing contextual issues in research with digital media in real educational settings.

First, the mathematics education community appears more and more sensitive to the role of context as a critical issue underlying existing difficulties in terms of appropriation of research rationales and results, in terms of exchanges and communication between different communities (e.g., researchers, educators, policy makers), in terms of exploitation of knowledge, in terms of relationships between research and practice. This sensitivity is attested by the increasing number of projects and publications addressing these issues. The second reason is linked to the fact that research involving the use of digital technologies seems an interesting terrain for investigating such issues. Aspects of context are challenged, actors are perturbed, new kinds of mathematical teaching and learning and new kinds of mathematical meanings are generated. We will briefly mention some key characteristics of using digital media in classrooms which enhance the importance of addressing context.

2.1.1 The Innovative Character of Digital Technologies and the Interventionist Design of New Settings for Teaching and Learning

As pointed out by Artigue (2008) digital technologies have been considered as a tool for educational change, a tool for enabling the implementation of innovative didactic strategies more in line with the principles underlying educational research for opening new windows in teaching and learning. From a research point of view, the use of digital tools in the teaching and learning process signified a need to explore the nature of computationally-mediated settings with a dual purpose to gain insight into meaning-making processes and simultaneously to design new computational environments that can act as a support for students to develop new meanings (Hoyles and Noss 2003). Thus, the respective research objectives in the field were centered around what was learned, in which ways and in which context. This has taken researchers along paths which, for example, involve re-addressing the relationship between quantitative and qualitative research techniques and developing new research tools, coining new terms to describe emerging phenomena as well as to theorize on the contingency of specific technologies and tools in knowledge construction.

2.1.2 The Resistance of Schooling to the Educational Legitimacy of Computer Technologies

Artigue (2000) discusses legitimacy as a central issue underlying the opposition between the poor educational exploitation of digital technologies in contrast to their high social and scientific appreciation. She mentions that the high social status of computer technology has been contributed to relatively high expectations for rapid improvement of teaching and learning in schools. Thus, computer technologies have been 'asked' to guarantee that norms and practices would be quickly challenged into progress through their use although "values and norms of mathematics learning and teaching are still defined with respect to the mathematical needs and practices associated with paper and pencil environments." (ibid., p. 8). Since early expectations of that kind have been frustrated over years, the tension between schooling and the use of computers in mathematics tends to generate "an inevitable vicious cycle of disillusion." (Hoyles et al. 2004, p. 311). It is as if learning environments based on the use of digital media were originally designed as elastic bands which would 'pull' educational practices into a higher rate of development. Not only is this not happening but instead the distance between designed and actual practices is increasing as our knowledge and technological means and infrastructure improve.

2.1.3 The Underestimation of the Complicated Ways in Which Mathematical Knowledge is Transformed by the Use of Computers

This is a key issue concerning the complex ways in which mathematical knowledge is transformed by the computers' presence in the classroom. At a first level, the use of computers in the teaching of mathematics requires an analysis of tool characteristics and functionalities as well as the identification of the links between the technological representation of mathematical objects and its relation to the traditional means of representation. Indeed, there is growing evidence that discrepancies of the computer-based representations with the standard mathematical knowledge complicate further the issue of legitimacy of the mathematical meanings generated by the students' interaction with computational tools (Artigue 2000). At the second level, there is a need to recognize the balance between the technical and the conceptual aspects underlying the students' mathematical work in computer-based environments. Although the enhanced wealth, specificity and dynamic-consequential nature of representations and functionalities afforded by digital media provides a catalyst for richer experience with mathematics there is a difficulty to estimate the complexity of instrumental genesis which, in turn, necessitates a conception of the changed student body of mathematical knowledge (Artigue 2002).

2.1.4 The Complex Strand of Contextual Issues Characterizing the Research Related to the Design and Use of ICT Tools

In research involving the development and use of technological tools there are specific contextual issues other than the ones usually treated in education research which have to do with the educational process in general. A large number (if not the majority) of technology-based research in education involves design of some educational process, intervention in the everyday life and habits within learning situations and the infusion of some kind of innovation (i.e. change) with respect to practices, tools, learning domain, social milieu etc. (see Nardi 1996 for an activity theory perspective to illuminate context through human–computer interaction in real settings). Moreover, since most of the research teams are involved in design and development of technological tools, the contextual complexity should not be overlooked. The one involving the types and methods of design and development work and how these influence the characteristics of the tools and the nature of intervention in the school, as well as the research questions themselves. Apart from the educational process, aspects of context, other types of aspect—such as socio-systemic and technology development—are seldom explicitly described in educational research publications. However, as argued by diSessa et al. (2004), these issues do matter and greatly influence the types of technology emerging from the teams and the kinds of use they are put to in the educational context.

Research in the use of digital technologies in teaching and learning situations has thus taken on the issue of design very seriously to the extent where the recently articulated 'design research' method (Cobb et al. 2003) is predominantly used in this field. The paradigm of this research is that an innovation is designed by the researcher who then takes part in its implementation in an educational organisation. This requires the aspects of how the organisation responds to the innovation to be made explicit. Implementation of the innovation creates some kind of 'perturbation' to use Laborde's term (2001) articulated in the context of teacher education. The difficulty is thus to identify which aspects of the teaching and learning activity are influenced by the perturbation and in which way. The researcher also has the daunting task to dissociate from her/his personal experience of engagement in the design and implementation in order to get a clearer view of the teaching and learning processes and gain some disaffection with the issue of the extent to which the implementation was 'successful'. This issue poses an inherent difficulty in the process of making contextual issues explicit and new tools and methods on the one hand, but on the other the process itself is such that the issue cannot be avoided.

Taking into account the existing fragmentation related to the contexts within which experimental research is situated, in this paper we adopt a pragmatic view of context focusing on diverse features such as the organisational pragmatics of design and development of digital media, the researchers' epistemological and pedagogical assumptions and approaches, the school contexts and the wider socio-systemic milieu within which empirical interventionist design research is carried out.

3 Boundary Objects as Vehicles for Integration

In the previous sections we highlighted the relatively recent and gradual recognition of the critical role of context in learning mathematics both at a local task and classroom level and at an institutional and wider cultural level. We also mentioned that research approaches on uses of digital media often create perturbation in schools which brings to the foreground aspects of context, but at the same time makes it much more complicated to understand what we are seeing in terms of the potential of using digital tools for the learning of mathematics. It is thus crucial to find tools, techniques and methods to facilitate the emergence of an explicit language to discuss the critical aspects of context and to make connections amongst different contexts.

The cross-experimentation methodology adopted by the TEMA group was designed as a setting for such a language to emerge. Within the framework of cross-experimentation a deeper communication between diverse teams became an operational need. We felt that it was necessary, however, to develop tools to facilitate the emergence of such a language. In that sense the research tools first mentioned in the introduction were designed in order to bring about interactions between the groups which would enhance mutual understanding of each other's perspectives, activities, constructs and digital media. These tools have been playing the role of artefacts mutually designed constructed and argued over to achieve a higher level of integration. A useful way to think of those tools is to associate them with the notion of 'boundary objects'.

The notion of 'boundary objects' was originally coined by Star and Griesemer (1989) to denote artifacts over which different communities engage in debate, argumentation or negotiation, originally perceiving these artifacts in distinct ways. Different communities, such as for instance, developers of digital media and educational designers, are often dismayed as they realize that when put together to construct a piece of software for mathematical learning, they are faced with intense miscommunication. The different epistemologies, histories of thinking, terminology used and know-how have resulted in years of frustration within the European setting when respective research teams have received funding to jointly construct digital media to learn mathematics (Kynigos 2002). However, the fact that these communities have inevitably been engaged in argumentation and communication in the process of the joint construction of an artifact places that artifact in the role of a boundary object. Communication and mutual understanding is generated in conjunction with and as a corollary to the building of the artifact. Communal design can generate the need to be explicit, to reflect and to express meanings through argumentation (Kynigos 2007). Issues of artifact functionalities, such as the creation and handling of mathematical objects, properties and relations, the representations used, the ways in which they are expected to be used by students, the pertinent aspects around this kind of use and this kind of mathematical activity all come to the foreground. Issues of roles between communities (who decides what), timing of activities, degree of positivism in design also arise and need to be dealt with. What makes an artifact a boundary object in this case is that the communities collaboratively work towards a common goal to jointly construct and make changes to the artifact in question. They may have an ideal artifact designed in detail from the start and argue over how to create it, or they may realize that in the process of creating it hitherto implicit issues of design emerge giving the artifact the status of an *improvable* object (Scardamalia and Bereiter 2003). It is that status which can generate explicitness amongst the communities. This artifact is both the centre of the activity, and also functions as a communicational tool to shape a common language within the community.

Cobb et al. (2003) extend this notion by proposing that the term "boundary objects" can refer to specific objects within different communities, which are relatively transparent means of conveying meaning among the members of the community who created them. They can also be the centre around which community members organize their activities and can additionally operate as tools for communication among the members of the same community, and the members of other communities.

In TELMA we were interested in the issue of how to design artifacts so that they can best facilitate emergent explicitness amongst research communities. Hoyles et al. (2004), mention that digital artifacts can be designed to play the role of boundary objects in reference to the ways they are used to generate communication over mathematical ideas between learners and teachers. These ideas would otherwise be bound to the situations in which they emerged. In this paper we discuss the research tools we designed to play that role in the cross-experimentation activities as boundary objects intentionally used for the integration process. These were in the form of a set of questions pertaining to contextual issues which were meant to sharpen as a result of the process of answers given by the different teams and criticism provided by them with respect to how well they were understood.

4 The TELMA Approach

4.1 The TELMA Group's Objectives

As mentioned by Bottino and Kynigos (this issue), the initial part of the collaborative work of the TELMA teams was based on the theoretical analysis of the team's prior work. This proved useful for improving mutual understanding and allowed identifying the TELMA teams' common concerns, e.g., contextual, social and cultural dimensions of learning, instrumental aspects of the use of ILEs, etc. (we use the term 'concerns' similarly to the sense of Artigue et al., this issue). However, it also revealed a diversity of ways in which these concerns were dealt with. This was mainly due to the variety of theoretical frameworks used by the teams (ibid.), the different contexts concerning the complex system of both immediate and broad goals, social and cultural values, individual and institutional relationships, tools and finally due to those situational, social and cultural elements within which individuals act and which influence the individuals' activity itself (Kynigos 2005). Furthermore, from a methodological point of view, it also made it evident that the diversity of the ways each team dealt with contextual concerns in the design and/or use of digital artefacts remained largely implicit in most published papers, and the data one would like to access in order to understand this role better were rarely provided. Our understanding of the possible effects on practice thus remained superficial.

The cross-experimentation methodology was thus developed in order to overcome the limitations of trying to discriminate contextual issues by analysing academic publications. It was felt that engaging with the cross-experiments would allow us to gain more intimate insight into each team's respective research and design practices since it would be based on some common research experience. In the domain of context the primary aims of the cross-experimentation were to investigate:

- the nature of the contexts within which the research teams function in design, development and learning research and
- the ways that the use of technology influences the production of tools, the study of learning and the learning process itself.

In this process, the collaborative design of the cross-experimentation, the elaborations by the teams of their methodologies, and the comparison of analyses by different teams of the same ILE was expected to elucidate the ways in which each team perceived the contextual issues as having a bearing on how the tool was approached and used. For instance, the choices made by each team to "tune" the use of the same tool according to their own pedagogical perspectives and research frameworks; coherences and differences in the ways in which particular representations were perceived and put to use in the different teams; the explicitation of the role played by implicit contextual assumptions that affect the design and implementation of an experiment involving a tool.

Although this methodology was not intended to provide a tool for comparing the teams' approaches with regard to the above issues in general, but rather a methodological tool for studying their role when enacted in actual practice, it may contribute to the development of tools for integrating different research approaches. In the following paragraph we provide some brief information on the cross-experimentation in terms of the ILEs used and the experiments set up by the teams since we refer to both of these in subsequent parts of the paper.

4.2 ILEs Used and Cross-Experiments

The cross-experimentation involved a rich diversity of ILEs, educational contexts and research approaches. In order to allow as much comparability as possible between the different experimental settings, it was also agreed to address common mathematical knowledge domains (arithmetic and introduction to algebra), with students between years 7 and 12 of schooling in experiments lasting approximately 1 month. Table 1 summarises the ILEs employed by TELMA teams in the cross-experimentation.

4.2.1 Aplusix

Aplusix (http://aplusix.imag.fr) allows students to build and transform numerical and algebraic expressions and statements (e.g., those that include the symbols "=", "<", ">"), based on solving computational tasks such as *expand* and *simplify, factorize, solve equation* or *inequality*. It also provides feedback on the correctness of calculations and transformations and verifies the end of the exercises.

– In case numerical or algebraic expressions are inserted, the tool provides feedback that basically constantly informs the user about the equivalence or not between the original expression and the expression that is being produced by the user. Particularly, the tool can function in two different ways depending on the kind of the given tasks: 'with control' mode and 'without control' mode. In the former case, the tool provides three kinds of immediate feedback on the equivalence of two successive expressions: in cases of two equivalent consecutive expressions the lines connecting them are black ('correct'); in cases of non-equivalent expressions the connecting lines are red and crossed ('incorrect'); and in case one of the expressions is not well-formed from the mathematical point of view the connecting lines are blue and crossed ('unknown') (see Fig. 1).

In the latter mode, the equivalence of two consecutive expressions is not indicated by any kind of feedback (in this case two consecutive expressions are connected with a single black line, see Fig. 2) but the tool provides the opportunity for users to observe a-posteriori

Table 1 The tools employed by TELMA teams in the cross-experimentation

ILE	Developers' team	Experimenting team
Aplusix	LIG (France)	CNR-ITD (Italy)
		UNISI (Italy)
E-Slate	ETL-NKUA (Greece)	UNILON (UK)
Ari-Lab 2	CNR-ITD (Italy)	LIG (France)
		DIDIREM (France)
		ETL-NKUA (Greece)

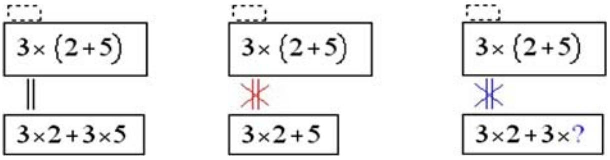

Fig. 1 The three kinds of feedback on the equivalence of two successive arithmetic expressions in the "with control" mode of Aplusix. The double lines appear *black* (on the left), *red* (in the centre) and *blue* (on the right)

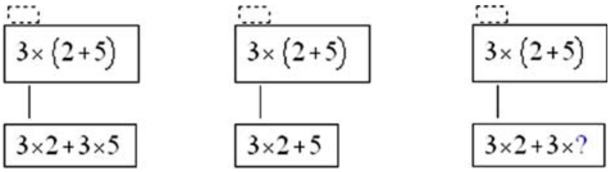

Fig. 2 The equivalence of two consecutive arithmetic expressions in the "without control" mode of Aplusix

step by step and, in case, modify their own work using the above described control feedbacks coupled with specially designed functionalities (i.e., the 'self-observation' and the 'self-correction' environments). In both modes of functioning the tool indicates when an expression is not mathematically well-formed (e.g., a question mark appears when an argument of an expression is missed) (see Figs. 1 and 2 on the right).

– In case of statements are inserted (i.e. $2/3 > 0$), the control on the 'correctness', and the related feedback, is based on the truth values associated to the statements. Particularly, if the produced statement has the same truth values of the original statement, the step is indicated as correct, and vice versa is indicated as incorrect if the truth values are not the same. As the truth values can be only 'true' and 'false', any 'true' statement can be followed by any other 'true' statement which may not even be related to the original one; in the same way any 'false' statement can be followed by any other 'false' statement which may not even be related to the original one.

A central design feature of Aplusix is that the validation of students' solutions rest on the tool both in the 'with control' and 'without control' mode.

In the cross-experimentation, the UNISI team took a Vygotskian perspective and designed their experiment with the aim to study how the control offered by Aplusix and the respective feedback can enhance students' self control and consciousness about their own difficulties and errors when manipulating numerical expressions and ordering of rational numbers (integers, fractions and decimals). The study involved two classes of Grade 9: one experimental class and one control class. The research started from detecting all the difficulties encountered by the whole class, and then, the focus was on how each pupil—with her/his own specific difficulties—may take an advantage from the interaction with the tool. In order to enhance students' motivation to take responsibility of detecting and overcoming their errors, the students worked in pairs without the teachers' support in solving the given tasks.

In designing their experiment, the ITD-CNR researchers took an approach based on Activity Theory (Cole and Engeström 1993) and used Aplusix as a means for supporting students in revisiting and consolidating aspects concerning fractions already taught during the previous school year (e.g., equivalence, operations with fractions, and ordering of fractions). The experiment was carried out with two Grade 7 classes (11–12 years old students) in two different schools and one experimenting teacher in each school. The lessons took place in the computer laboratories of the two schools and the students worked in groups of two.

4.2.2 Ari-Lab2

AriLab2 (http://www.itd.cnr.it/arilab/_english/micro_ari.html) is a stand-alone version of an open system which is composed of several interconnected microworlds based on the idea of integrated multiple representations and functionalities designed to support activities in arithmetic problem solving and in the introduction to algebra. AriLab2 involves also a communication environment that enables exchange of messages and solutions among users. In TELMA, three microworlds were used: The 'Fractions Microworld' (FM), the 'Abacus Microworld' and the 'Euro Microworld'.

– The FM is a piece of software which provides the user with the opportunity to construct fractions which are represented graphically and symbolically as points on the number line and further explore their properties concerning topics such as comparison/ordering of fractions, operations with fractions and equal fractions. Within the microworld the provided feedback combines arithmetic as well as geometric aspects of the notion of fraction represented by using two half number lines: one horizontal (called number line) and one slanted (called multiplication or partition number line) (Fig. 3). The construction of a fraction in the FM is based on two different types of representations: the first one is associated with Thales Theorem (i.e., the projection principle), while the second one with the notion of fraction as a quotient of a division in which the divider (numerator) and divisor (denominator) are selected by clicking on the number line and the partition number line respectively. The geometrical representation is visible only when the user selects the numbers for constructing a fraction from the number line and the partition line respectively. When the construction is completed and the fraction appears—graphically and symbolically—on the number line, the parallel lines constituting the geometrical representation of the partition automatically disappear. The symbolic notation of each constructed fraction is automatically given near its representing point (on the horizontal line) in a 'post-it' form (see Fig. 3).

The number-measure interpretation of fractions is at the core of the available representational infrastructure, since for each particular fraction the representation is provided as a number, i.e. the rational number that the fraction represents as well as a measure, i.e. the distance from 0 represented by a point on the number-line. By trying out different combinations of numerators and denominators, the user may construct various types of fractions and consequently explore their position on the number line as well as their properties. The tool also provides labels keeping track of the various ways by which a number could be built (e.g., 7/3, 2 + 1/3, 14/6, …).

– The Abacus Microworld is for representing numbers in the positional decimal system. It offers the opportunity to build the abacus, to decide how many rods will compose it, to produce cue balls on the rods, to execute sum and subtraction operations involving beads.
– The Euro Microworld is for producing and manipulating coins and notes of the European monetary system, for representing situations referred to problems related to the money use (money counting, sales, exchange, etc.). It offers opportunities to

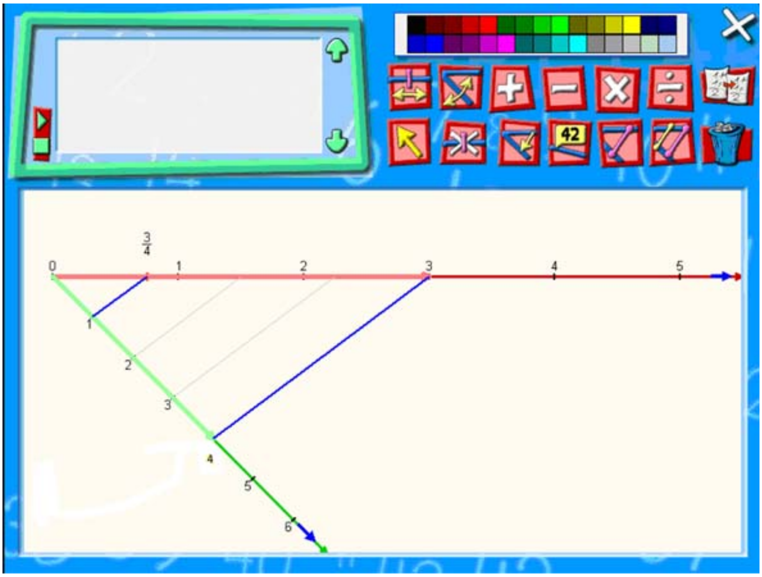

Fig. 3 3/4 represented on the FM number line

reproduce coins and notes and to exchange them with coins and notes whose values are equivalent.

In the cross-experimentation, the general aim of the LIG team was to evaluate the effects of the use of the FM on student's competencies concerning the notion of fraction at the end of the elementary school (i.e. various expressions of equal fractions, comparison and ordering of fractions, operations with fractions). The LIG experiment was carried out in one elementary school class comprising Grades 4 and 5. Grade 4 pupils have just been introduced to the notion of fraction (sharing a unit in equal parts), while Grade 5 pupils have been taught the meaning of fractions in the previous school year. The LIG researchers refer to Theory of Didactic Situations (TDS) (Brousseau 1997) as their main theoretical framework.

The general aim of the ETL experiment was to support students in constructing mathematical meanings for the notion of fraction as number-measure when engaged in activities involving comparison and ordering of fractions as well as operations with fractions. Task design was based on integrating the visualisation of fractions as numbers indicated by specific points and labels on the number line with distances of places in the space (e.g. between one's house and a playground). The ETL experiment was carried out as a case study with two groups of 6th grade students (last grade of the primary level in Greece) who have already been taught about fractions in the traditional classroom. ETL researchers refer to Constructionism (Harel and Papert 1991) as their main theoretical framework.

DIDIREM researchers decided to carry out their experimentation as a case study with two groups of 6th grade students, working with the Abacus Microworld and the Euro Microworld. The team used these microworlds to approach the solving of arithmetic problems which involved subtractions beyond the size of numbers the pupils were familiar with, and the development of two kinds of subtraction techniques: one based on the decimal system and another one based on the decomposition of the first term of the subtraction in order to obtain the second and its complement as the result of the subtraction. The main theoretical frames that underpin the research approach of the DIDIREM team are TDS and Anthropological Theory (Chevallard 1992).

4.2.3 E-Slate

E-slate (http://etl.ppp.uoa.gr) is a construction kit for educational software. It provides a variety of generic pieces of software (called components) which can be linked and manipulated by users through the use of prefabricated connections or through a programming language to develop microworlds. In the TELMA cross-experimentation, UNILON team made use of the 'Fraction-sliders Microworld', built within E-Slate, provided by the developers. This microworld includes 'sliders' in the form of number lines whose behaviour is governed by a set of Logo procedures. The numbers entered in a Logo procedure determine the relationships between the values displayed on the sliders. Dragging the pointer on the 'control' slider, in particular, causes proportionate changes in position of pointers on one or more dependent sliders. While this microworld could in principle be used to explore a wide range of types of functional relationship, for the purposes of the cross-experimentation UNILON team decided to focus only on multiplicative relationships between the 'control slider' value and the values of the dependent sliders, using these specifically to address the area of fractions. The research aim was articulated to investigate representations of fraction and how they are used in lessons with the Fraction-sliders Microworld. The team used classroom-based research, studying two 'mixed ability' classes—year 7 (11–12 years) and year 8 (12–13 years) respectively—in different schools. The UNILON team's research approach was shaped by a socio-cultural and social semiotic theoretical framework (Bernstein 1996). That is, any learning incident must be understood in the context of wider social structures within which it is embedded and which the participants bring with them.

In the next part of the paper we present the development of the methodological tool for context analysis in experimental settings aiming (a) to capture how contextual issues are addressed by the TELMA teams in tight connection with their research practice, and (b) to highlight how the contextual issues addressed by the teams can provide a basis for moving towards integrating the followed research approaches. These two complementary aims, diversity of the contextual issues enacted in experimental research and the use of these as a means to network research approaches developed by different teams, allows making some step further in the analysis of the complexities involved in developing an integrated approach to the research on technology enhanced learning of mathematics.

5 Method and Research Tool

Within TELMA work, the methodological tool for context analysis in experimental research was developed in two successive chronological phases (see also Kynigos et al. 2006). The first was during the analysis of the most representative publications of each team in relation to the contexts in which the use of technology was embedded (Phase A). The second was during the design and realization of the cross-experimentation when the issue of integration of the teams' research approaches appeared at a more practical level (Phase B).

In phase A, we aimed to provide a synthetic report on the teams' previous research work based on common sensitivities to specific contextual characteristics and the evidence of differences in the ways these were dealt with. The analysis of the teams' published papers brought to the foreground a number of contextual concerns that we used to deal with the complexity of the respective educational contexts. These were structured around the following clusters:

- an *educational environment cluster,* which addresses how the educational processes are studied;
- a *socio-systemic cluster*, which includes concerns about the relations and collaborations between the organizations and between the actors involved in the educational process;
- a *teacher communication and support cluster,* which addresses the teacher's role and ways of communication between teachers-researchers;
- a *technology design and development cluster*, whose importance clearly emerges within a project in which research teams are involved in design and use of technological tools.

To each cluster, we associated a set of questions corresponding to specific concerns that seemed relevant and shared by the teams and seemed to express the main sensitivities evidenced by the analysis at this phase.

Considering, then, the design of the cross-experimentation and the analysis of the collected data (Phase B), our aim was to determine in depth if and how the selected contextual concerns were enacted in each team's research practice. For instance, if they were addressed or not, the importance given to them if addressed and the effect of these on the researchers' choices. Thus, the tool was further enriched with more specified questions explicitly referring to the process of realization of a learning environment (i.e. design, implementation and analysis/reflection). These questions were mutually constructed and negotiated through cross-team communication in different times of the TELMA activities but mainly during the collective construction of common guidelines (Cerulli et al. 2007) for monitoring the whole process of cross-experimentation (see Bottino and Kynigos, this issue). For instance, the research questions for contextual issues included in the guidelines (i.e. addressed by the teams before and after the cross-experiments) had to be clarified and/or modified according to the participant researchers' exchange of opinions and ideas during the cross-team communication. Our joint effort in designing the tool assumed that as a boundary object based on our common research experience it would help to identify interrelations between contextual concerns and practices developed by the teams in different experimental settings.

5.1 The Research Tool

The methodological tool for context analysis in experimental research is the following:

5.1.1 (a) Questions with Respect to Educational Environments

The concerns related to educational environments appear more directly linked to the theoretical frameworks which the research teams referred to (e.g., for an investigation of the relations between particular theoretical frameworks and task design in computer-based environments, see Ainley et al. 2006). Indeed any theoretical framework, even though not explicitly addressing them, may provide elements to deal with concerns like the social dimension of learning, or the role of teacher mediation, and so on. In a sense, this cluster represents the point on which explicit elements of the theoretical frameworks and implicit elements of context might hinge in the following five sub-clusters:

5.1.1.1 Social Aspects of Learning

What are the ways by which the social aspects are addressed (i.e. interaction, participation in communities, groups) in the classroom?

5.1.1.2 Nature of Activities and Tasks

What is the nature and the type of the activities and tasks (i.e., structured, game-like, scenarios, projects) given to students?

What characteristics of the activities support the generation of meanings?

5.1.1.3 Process of Mathematical Reasoning

How is the mathematical reasoning integrated in educational environments?

How do you capture/analyze the role of the tools in students' problem solving processes or solutions?

5.1.1.4 Teacher Mediation

How is the teacher's role related to different aspects of educational environments?

5.1.1.5 Use of Language

What is the role of language in educational environments?

5.1.2 (b) Questions with Respect to the Intervention into a Socio-Systemic Milieu

In most of the work with ICT tools a didactical intervention is designed and implemented within particular educational sites usually through design experiments (Cobb et al. 2003). The interventionist character of this process raises not only the need to understand the contextual issues involved in order to carry out the experiment but also the need to identify the reactions of the actors involved as they are challenged to question many aspects of the context in which they act. One example is the perturbation in the classroom involving practical issues, such as everyday schedules and technology use management, but also much deeper issues at the socio-systemic contexts of the actors involved (Jaworski 2004). These are, for instance, the organizational pragmatics of the University or the lab, the relationship between the researchers' organization and the educational sites, how this intervention is materialized between organizations and in what type of organizational context this collaboration takes place, e.g. teacher-student roles, social orchestration in the classroom, epistemologies and beliefs about mathematics and the educational process. Thus, the articulation of the questions finally decided by the TELMA teams for this cluster were:

What type of research is followed (e.g., classroom based, case studies) and how is it related to the kind of research focus?

How is the lab situation/structure taken into account in the research design?

How are the socio-systemic factors addressed: administration, teachers in daily action, roles and relationship with researchers, daily program (time, curriculum, method)?

What are the organizational pragmatics of the University or the lab, the relationship between the researchers' organization and the educational sites (existence or absence of institutional mechanism, e.g., part of an institutionalized pairing of University—school etc.)?

When the researchers approach a site, what is their perceived role by the administration of that site and the actors to be involved? How much personal contact do the researchers have with the actors? Does it have any effect on the research?

What kind of 'perturbation' does the implementation of the research imply (e.g., not only practical issues but also much deeper issues like teacher-student roles, social orchestration in the classroom, epistemologies and beliefs about mathematics and the educational process)?

5.1.3 (c) Questions with Respect to Teacher Communication and Support

The presence of the computer in the classroom inevitably represents a perturbation element in the context of the classroom. The teacher has to elaborate a new relationship to mathematical knowledge, together with the whole set of relations which link this knowledge to the use of technology in general and of specific software in particular. At the same time the teacher has to adapt his/her role of mediator taking into account the new elements offered by the software. All of these issues involve not only teachers' time and energy but also some kind of perception of the teaching profession as a developing one and of engagement in professional development activity as a normal part of the teachers' job. Furthermore, the ways in which the intervening researchers are perceived and their respective role, i.e. their official 'capacity', their actual contribution to the teachers' work, highly influences the ways in which the technology will be used. Indeed, the teacher communication and support aspect of context appears inadequately treated in the corresponding research agendas. Although teachers are often taken into account explicitly in the research projects, there is very scant information about contextual issues involving their relationship with the research teams. The key questions that were selected for this cluster of contextual concerns are:

What is the context of communication with the teachers (e.g., institutionalized channel or ad-hoc project)?

Are there specific courses for teachers? In that case which is the frequency, duration? How do the teachers use the course?

What is the influence of the ways in which the intervening researchers are perceived (their official 'capacity', as well as their actual contribution to the teachers' work) on the use of technology?

Are there indications of teachers elaborating a new relationship to mathematical knowledge, to the use of technology in general and of the specific software in particular?

5.1.4 (d) Questions with Respect to Technology Design and Development

This cluster of the methodological tool concerns mainly the investigation of relations and the kind of cooperation among the tool developers and the users of computational tools (e.g., educators, researchers in mathematics education), each having her/his own field of experience, frame of reference, epistemology and methods, as well as the phenomenon of the emergence of hybrid expertise and actors. This is an issue that appears greatly underrated in mathematics education research. The questions we selected for this cluster are:

What is the scheme of collaboration with companies or other development institutions (on–off collaboration, discrete sequence of projects ad-hoc, long-term sustainable collaboration)?

In case of in-house development—how is it paid for and sustained vis-à-vis persons and know-how?

In the next section of the paper we provide some results of the first use of the above methodological tool in analysing the data of the cross-experimentation. Our aim was to elucidate the role played both explicitly and implicitly by specific elements of context enacted in actual research practice.

6 Investigating Contextual Concerns Enacted in Actual Research Practice

The task to address issues within the clusters of the contextual concerns as the teams engaged in practice was one of the main features that characterized the cross-experimentation process either as a way of comparing research approaches, or as a means to investigate details of how to employ ILEs in mathematics education. The need to communicate to the other teams how each of these issues determined the design, conduct and analysis of classroom experiments, forced each team to address them explicitly, and to leave as few unexplained choices as possible. Moreover, the fact that each team had to address common questions formulated jointly by the group provided further discussion on how the questions were understood by each, the questions operating as boundary objects within the cross-team communication. In this section, we outline some of the most striking results of a comparative analysis of the local experiments by TELMA teams in terms of the impact of contextual concerns on the design, implementation and analysis of the experiments. The analysis was based on the groups' elaborations—design and analysis—on their experiments as reported in the respective TELMA documents (i.e. deliverables and experiments guidelines, see Bottino and Kynigos, this issue).

6.1 Context of Educational Environments

6.1.1 Nature of Activities and Tasks

Tasks constitute a fundamental element of the teaching and learning process since they embody the researchers' expectations of the learning environment to be created in the classroom (i.e. how the implementation of these might work out in practice). Thus, the design of activities involving the use of computational tools can be seen as terrain in which multiple aspects of the educational environment come into play, such as the choice of the tools that would be used to foster mathematics learning, the roles of the participants and the learning objectives (diSessa et al. 1995). An important factor underlying the design of activities and tasks by a research group is the adopted theoretical frames (Ainley et al. 2006) which are also connected to the conception of an experiment with respect to the posed educational goals. In this section we aim to investigate if and how the types of tasks—and consequently the conception of the experiments by the teams—were influenced by their theoretical frames. For that reason we compare the tasks designed by the teams which chose to experiment with the same ILE, based on compatible theoretical frames (or not) and working within the same didactic culture (or not).

In the teams' experiments the tasks have been basically described in the form of answers to the research questions contained in the guidelines document, and in the form of plans to be contained in the same document. The teams' activities ranged from well structured defined tasks aiming to identify students' reasoning on specific curriculum-based concepts (UNISI, DIDIREM, LIG) to loosely defined exploratory activities aiming to elicit the generation of meanings in a more experimental way (CNR-ITD, NKUA-ETL). However, the reasons for the respective choices have been differently supported by each team.

6.1.1.1 Activities and Tasks When Using the Same ILE Under Compatible Theoretical Frames UNISI and CNR-ITD experiments were designed under two compatible theoretical frameworks—respectively Vygotskian Theory (as for the construction of higher psychological functions) and Activity Theory—and centered their experiments on the use of the same ILE, namely Aplusix. From the ILE analysis, the teams identified different educational aims for their experiments, which resulted in two teaching experiments, both consistent with the respective theoretical frames, but deeply contrasting as far as the use and set up of the tool in general and on the kind of tasks given to students in particular.

The tasks designed by the UNISI team—concerning arithmetic calculations and ordering of integers and rational numbers—can be characterised as 'closed' since most of them corresponded to just a single correct answer. For instance:

– Write the smallest and the greatest numbers out of the following. Write the smallest first and separate the number with the symbol <. $5/3; 2; 5/4; 4/3$
– Compute the following expression in at least two different ways (use the tree-structure): $(-2 + 3/8) \times 3/8 - 3/4$

This choice of the researchers, however, seemed to be indirectly motivated by the adopted theoretical frame and the educational goals of the experiment. Rather, it was directly related to features of the tool for editing tasks and the provided feedback by which the tool checks (or not) the equivalence between an expression or a statement entered by the user and an expression or a statement produced by the user in the next step ('with control' mode or 'without control' mode).

Two different kinds of tasks were identified by the team:

- Tasks in the 'with control'-mode environment endowed with the control feedback described previously. In addition, in this case the tool was set up in such a way that users could not ignore these feedbacks. Particularly, students were allowed to write down a third expression only if the two previous ones (in the same tree branch) were equivalent (Fig. 1).
- Tasks in the 'without control'-mode environment without the control feedback (Fig. 2). In this case students were required to observe and, in case, correct their own work a-posteriori through the use of the self-correction environment which was set up to function as described in the previous item (i.e. 'with control' mode). For instance, in Fig. 4 there is an example of computing an expression which was initially given in the 'without control'-mode. Figure 4 shows a student's work as displayed in the self-correction environment in which the non-equivalence of the first two expressions is red marked.

This kind of tasks seems interrelated with the researchers' choice to address a couple of possible educational goals for their experimentation: (a) to reinforce students' operational and ordering skills in arithmetic, and (b) to enable students to develop abilities for controlling and reflecting on their own work, to develop strategies for detecting and overcoming errors, and for anticipating possible difficulties. Taking a Vygotskian perspective, which emphasises the development of general abilities concerning consciousness and control of one's activity, the UNISI researchers considered the above aims consistent with

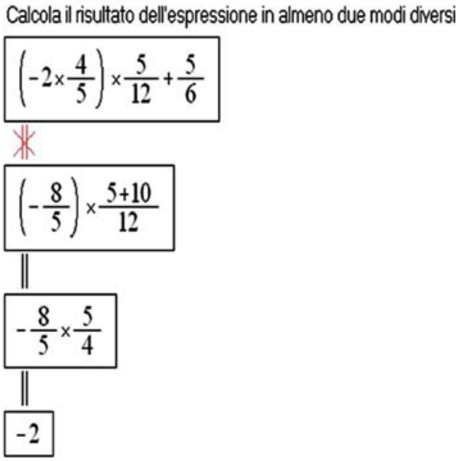

Fig. 4 An example of a student's work as displayed in the self-correction environment of Aplusix. The connecting lines of the first two non-equivalent expressions appear red and crossed ('incorrect')

tasks designed for students-tools interaction without the teachers' mediation and validation of solutions.

Using the same ILE, however, the CNR-ITD team designed its experiment using a set of activities of open-ended nature. Though Aplusix is designed to support specific pre-defined tasks of standard computational procedures (e.g., *expand and simplify, solve, factorize*) the research aim of the team was to develop and test new didactical functionalities of Aplusix in order to use it as a means for supporting students in revisiting and consolidating aspects concerning fractions already taught during the previous school year (e.g., equivalence, operations with fractions, and ordering of fractions). The CNR-ITD researchers did not aim at the production of correct solutions but rather at the development of solving strategies by students which they assumed to be a means for abstracting the properties of fractions. Aplusix may favour students following trial and error strategies in order to validate the solution of a problem (i.e. which fraction is bigger than another one) without the intervention of the teacher. The problem that arises is how students might go beyond specific results and be able to justify their choices and search for more efficient strategies. At this point the CNR-ITD researchers concentrated on the possibilities offered by Aplusix to express and explore equivalence statements by means of open-ended activities, and they individuated some more key features (see Fig. 5):

- The software allows leaving 'unfilled' some boxes in constructing expressions with placeholders (represented by a square with a question mark inside), thus it allows the design of tasks where students will be asked to fill the placeholders.

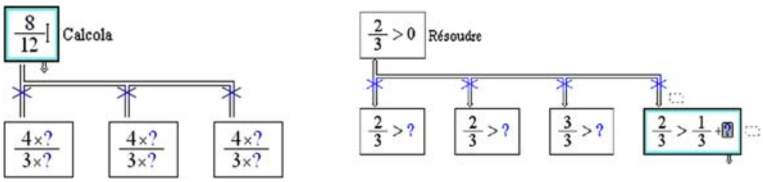

Fig. 5 Examples of trees with empty placeholders. Students were asked to fill them in a way to obtain expressions (*left*) or statements (*right*) equivalent to the given expression (fraction 8/12) or statement (2/3 > 0)

- The software allows the construction of trees of expressions (which could be ‘unfilled’ too) showing the equivalence relationships among them. Moreover, when the root of a tree is a statement, instead of an expression, if the leafs of the tree are statements too (these too can be left ‘unfilled’), then the “equivalence” feedback can be interpreted in terms of equivalence of truth values of the statements. Since the tool does not provide any indication on the truth values of a single statement, this implies that in order to assess if a statement is true one can firstly introduce a statement that is surely true, and then introduce the considered statement and see if these two statements hold the same truth values. This, for CNR-ITD team, has strong implications on the nature of tasks to be given to students. For instance, “if one wants pupils to produce a fraction that is smaller then 2/3, in order to exploit the feedback provided by Aplusix, one has to previously introduce a true statement and propose the statement with the unknown fraction as a follow up” (Cerulli et al. 2007, p. 9) (see Fig. 5 on the right).

Under this perspective the analysis of the functionalities of the tool and the diversity of its utilisation lead the CNR-ITD researchers to enlarge the class of pre-defined tasks embedded in the design of Aplusix to tasks including comparison and exploration of ordering statements between fractions. The team designed thus (a) open ended activities, such as ‘fill the boxes’, that exploit the validation/equivalence feedback of Aplusix; (b) verbalization activities, such as ‘write your strategy’, ‘try out your strategy’ or other tasks provided ad hoc by the teacher to question/validate pairs’ strategies; (c) class discussions highlighting and discussing the emergent strategies.

To provoke the emergence or the evolution of the students’ strategies, the teacher needed to intervene and ask them to formulate the strategies, either using comments in Aplusix or just paper and pencil. However, the tool not being able to validate the described strategies, the teacher was expected to take a more active role in the realisation of the experiment by taking the validation in charge and organizing classroom sessions aiming at discussing and validating or not students’ strategies (Bottino and Cerulli 2007). The emphasis on the constitutive role of the teacher in the implementation of the activity is an indicative element underlying task design that brings in the foreground the different ways by which the same theoretical framework of the two teams was enacted in different ways by each team in actual research practice. CNR-ITD team referred mainly to Activity Theory which considers the community (involving teachers and/or researchers) in mutual relationship with the subject (e.g., the learner) and the object (e.g., the task solution) in every activity (Cole and Engeström 1993). Thus, the team gave high priority to the social construction of knowledge under the guidance of the teacher assuming that all the details that were left ‘undefined’ would have been addressed by the teachers during the classroom activities (Bottino and Cerulli 2007). The UNISI team—referring to the Vygotskian theoretical ideas of control and consciousness—set up an experiment where the rupture of automatisms and the contextual emergence of obstacles in computational processes (which students may experience when making errors within Aplusix) were exploited as means for achieving specific educational goals without making explicit which methodological tools were used from the Vygotskian framework for putting this idea into practice.

The CNR-ITD and UNISI teams chose to experiment with the same ILE by adopting compatible theoretical frames and acting within the same (Italian) didactic culture. So far, we have compared the two different perspectives in which these teams coped with task design. The comparison indicates that the types of tasks that they designed were part of different teaching experiments, consistent with the respective theoretical frames, but deeply contrasting as far as other aspects of the research design such as the validation of students' responses and the role of the teacher. We suggest that the above comparison contributes to the clarification of the links between contextual issues of educational environment and the designed tasks by focusing on the role of theoretical frames. In this case the influence of theoretical frames to the design of tasks appears not to be 'given' or predefined. Since the CNR-ITD and UNISI teams worked within the same didactic culture the comparison indicates that the design decisions might have been determined by the teams' usual habits and experience or didactical preference and not under the control of theory.

6.1.1.2 Activities and Tasks When Using the Same ILE Under Distinct Theoretical Frames In the following example the differences in task design were more directly linked to the team's theoretical assumptions. The LIG and NKUA-ETL teams used in their experiments the FM of AriLab2, a software which combines graphical and symbolic notation of fractions represented as points on the number line. LIG and NKUA-ETL teams act in the French and Greek didactic cultures respectively and follow distinct theoretical frameworks, namely TDS and Constructionism.

The general aim of the LIG team was to evaluate the effects of the use of the FM on the learning of different aspects of the notion of fraction at the end of the elementary school (i.e. various expressions of equal fractions, comparison and ordering of fractions, operations with fractions).

When identifying the didactical functionalities of the FM, the LIG researchers paid attention to the fact that the tool provides labels keeping trace of the various ways by which a number could be built (e.g. 7/3, 2 + 1/3, 14/6, …). This kind of feedback seemed particularly interesting for the LIG team because apart from the novel way to construct a fraction it allowed the introduction of the idea that different expressions can represent the same number. Considering the anticipation phase of the answers before validating them as an essential part of the learning process, the LIG researchers based the implementation of the experimental activities on the complementary use of the paper and pencil environment with the FM. More specifically, students were initially asked to anticipate their answers in the paper and pencil environment based on their existing knowledge and these were validated afterwards in the FM. For example, in the second activity (Table 2) students were asked to answer question (a) in paper and pencil environment and then to verify their answers using the FM (question b). Finally, students were asked to reflect on their mistakes (if any).

In line with the theoretical frame of TDS the LIG team designed an a-didactical situation focusing primarily on the nature of interactions between the students and the 'milieu' (Brousseau 1997). According to the LIG researchers there is a search for adding constrains to the 'milieu' combining the need to adapt to it with pupils' motivation to question

Table 2 Part of the LIG activities

II. Different expressions representing the same number
(a) Among the following expressions, encircle with a blue pen those that represent a same number
(i) 3/4 9/12 15/16 6/10 45/60
(ii) 2 + 1/3 3/5 7/3 21/9 1 + 5/3
(b) Verify your answer with the computer. Correct your errors, if any, in red
(c) Explain how you decide whether two expressions represent a same number or not

themselves about the reasons for their errors (Cerulli et al. 2007). In such a setting learning is expected to result from the student's adaptation to an antagonist 'milieu'.

The NKUA-ETL team using the same ILE took into account the novel character of the available representations of fractions and supported the idea of focusing particularly on students' interaction with these representations to construct meanings for fraction as number-measure (Psycharis et al. 2009). In line with its constructionist theoretical approach, the team positioned considerations of meaningfulness and motivation high on the agenda for the design of tasks that were likely to facilitate primary students' encounter with the idea of 'measuring distances' by providing them with opportunities to *use* comparison and ordering of fractions as well as operations with fractions in the number line context. In this perspective understanding is expected to emerge through students' active engagement in using the respective mathematical concepts before learning about its procedures and relationships (the *power principle*, Papert 1996).

The team's approach on task design was thus centered on the utilization of the different representations and the feedback that they can provide so as to provoke multiple decisions within open-ended exploratory tasks concerning the comparison and ordering of fractions as well as operations with fractions. The main idea underlying the design of tasks was that instead of seeing the numbers on the number line as static measures to view them as representing distances and engage students in measuring the distances between specific authentic places (e.g., their homes, their schools, a playground, a supermarket etc.). This choice led to a set of activities based on integrating aspects of the representation of fractions as points-numbers on the number line with students moving ('walking') between them to reach specific places (see Table 3).

The different kinds of tasks that the teams designed (i.e. 'closed' as opposed to 'exploratory') in their own experiments brings to the fore the relation between the proposed tasks and the different theoretical frameworks adopted by each team as well as the respective didactic cultures. Taking a comparable view of the above examples we can see that theoretical frameworks determine only partially the identification of didactical functionalities and the design of the experiments. Teams using the same ILE and the same theoretical frameworks designed very different experiments, and as stressed in the reports, there exists a real gap between most macro-level theories and the decisions to be taken in the design phase and especially in task design. In two cases (UNISI and CNR-ITD) two compatible theoretical frames led to substantially different tasks signalling the implicit ways in which some theoretical origins are 'translated' in actual research practice. In the case of the NKUA-ETL and LIG teams the influence of the theoretical frames was much more obviously reflected in the tasks: the LIG team based on the TDS approach—representative of the French didactic culture—designed an a-didactical situation consisting of closed tasks tightly related with specific objectives of the curriculum. The NKUA-ETL

Table 3 Part of the NKUA-ETL activities

Activity 2: Constantina's school is 1 km away from her house. On her way to school she sees a kiosk at 6/7 km, a super market at 2/5 km and a playground at 3/4 km. Which is the order she sees them on her way to school?
Activity 3: Lazarus is Constantina's best friend; his house is between the playground and the super market. Can you find some fractions indicating the position of his house?
Activity 4: Efie and Constantina are friends. They meet each other at the playground. Efie says to Constantina: "You are very lucky. Your house is closer to the playground than mine." Discuss about the position of Efie's house

team chose to design open tasks—not strictly defined by the curriculum—so as to explore if and how fraction as number-measure could be approached by the students in meaningful ways.

These results highlight also the operational character of the questions selected to play the role of tools for integrating research approaches developed in different experimental settings. The need of the teams to be explicit coupled with the comparative analysis of the teams' elaborations allowed us to investigate if and how theoretical frames were related to the types of the tasks designed by the teams acting in different didactic cultures. Without this kind of activity these influences may not have been visible by the members of the teams since they were used to consider the design of specific types of tasks as a 'natural' consequence of their research approaches. The cross-team communication that preceded the construction as well as the refinement of the proposed questions challenged the researchers to describe more precisely their choices in designing tasks and elaborate on the compatibility between their theoretical frames and their didactic culture taking also into account the implementation of their tasks in the classroom.

For instance, during cross-team communication NKUA-ETL researchers characterised their tasks—directly linked to their constructionist theoretical frame—as 'not compatible' to the highly centralised Greek didactic culture and curriculum. These researchers clarified that in order to implement their designed activities in the classroom they cooperated with specific schools and teachers 'open to innovations'. In a similar elaboration LIG researchers confirmed that although they found "the 'distance' between the fraction implemented in the FM of Ari-Lab2 and the fraction taught and learned in the French primary school is rather big" (Cerulli et al. 2007, p. 43) the activities they designed were transformed so as to fit the teaching sequence suggested by the French curriculum. Thus, another contextual issue that is brought here concerns the factors that determine the compatibility between a specific team's research approach that involves intervention in the classroom setting and the national curriculum.

6.2 Context of Intervention into a Socio-Systemic Milieu

Mathematics education research involving the use of ILE in schools is inevitably faced with issues concerning the socio-systemic context of the given intervention in the school setting. Such issues affect the ways in which researchers can approach a school (or other institution within which the intervention is implemented) with respect to their agenda of introducing new practices, epistemological frameworks as well as the relation of this intervention to curricular constraints and normal school practice. The questions involved in the methodological tool for investigating issues related to this aspect of context required the TELMA researchers to jointly reflect and challenge each other in an attempt to gain understanding while comparing how this was done within each team.

As far as the types of intervention are concerned, most of the teams' approaches included some kind of participatory research in school sites where tools were studied in educational contexts. Inevitably, there were differences in the ways in which each team approached the respective school communities taking into account that the duration of the experiments was commonly agreed to be nearly 1 month. The UNISI team, for instance, was used to conduct research in the framework of long-term teaching experiments in close collaboration with specific schools and teachers. This kind of research was located by the team in the 'research for innovation' paradigm, in which action in the classroom is both a means and a result of the evolution of research analysis (Bartolini Bussi 1996). Though they considered the short term nature of the cross-experimentation as restricted, the UNISI researchers followed their usual research practice taking a collaborative perspective for intervention in the school culture. This involved a close collaboration with a group of teachers whose interaction with researchers had been constant in all phases of the project as well as a joint analysis of the material produced by students. The activities were implemented in the classroom as part of the regular curriculum.

The DIDIREM and NKUA-ETL teams considered that 1 month for the duration of the implementation was very tight for a class to get used to the microworlds of AriLab2 and thus modified their usual type of interventionist design research from classroom experiment to case studies. This was not usual in the practice of NKUA-ETL. Though acting in a typical centralized educational system with minimal opportunities for the use of technology in mathematics teaching, the ETL-NKUA team had a long tradition in collaborating with schools through personal contacts with school heads and teachers who would work with research students and reciprocally participate in teacher training courses at the university. This type of collaboration aimed at supporting the implementation of design experiments involving innovative uses of computational tools in real classroom settings as a means to generate and enhance meaning-making situations. The analysis shows that the change of the research approach from classroom to case study in the cross-experimentation influenced the conception of the experiment by the NKUA-ETL researchers in terms of implementation and analysis. By answering a set of questions concerning a general account of their experiment in the cross-experimentation they clearly expressed their concerns about the adopted case study methodological approach:

"In retrospect, we think that the choice of conducting case studies deprived our experiment of the more fruitful social interactions that could have taken place in the context of classroom related to the interpretation of the phenomena observed on the screen not only within the teams but also among the teams." (Cerulli et al. 2007, p. 75).

In the case of the DIDIREM team, however, the kind of intervention appeared interrelated to methodological issues of high priority based on the team's theoretical assumptions. This team—referring mainly to TDS and Anthropological Theory—decided to conduct case studies after examining the nature of the 'milieu' related to the AriLab2 microworlds (i.e. Euro Microworld and Abacus Microworld) which had been chosen for the respective experiment. The analysis of the characteristics of the two microworlds revealed that there was a 'distance' between the mathematical objects and representations provided by the tool and those familiar to the students and used in French schools at the primary level. For instance, the correspondence between the numbers involved in the required subtractions with banknotes and coins (in the Euro Microworld) or the need to translate subtracting schemes involving exchanges (in the Abacus Microworld) to the ordinary decimal system. Thus, the team considered the time available very limited for a classroom experiment and decided to conduct case studies with five students chosen by their teacher ("supposed not to have too much difficulties", Cerulli et al. 2007, p. 61). In the words of the DIDIREM researchers:

"We always think what we have planned is adequate in terms of distance. In our experiment the time for the instrumental genesis is a 'limiting factor' for achieving an adequate use of the tools by the pupils and to permit them to go beyond their actual capabilities of computing with the whole numbers." (ibid., p. 53).

However, when implementing the designed activities the DIDIREM researchers realised that even in the case study setting the effects of the instrumental issues appeared much more influential for the evolution of the students' activity than expected by the team when planning the experiment. The team underestimated also the fragility of the students' existing numerical knowledge and the cognitive demands required for adapting it to new artefacts involving new objects, techniques and semiotic systems. This caused unexpected perturbation in the sessions indicating that what was expected by the students was certainly too much for a short term experiment. It is interesting, however, that in an a posteriori report of their experiment the DIDIREM researchers acknowledged a need for taking into account the collaborative aspects of students' activity as well as the communication part of the environment which allows students' exchange their works.

Alongside this global view to the teams' intervention into a socio-systemic milieu reveals a need for a finer elaboration of some aspects of institutional constraints emerging when adapting the way in which an ILE is used to a different context. For instance, the researchers acknowledged the relation between the proposed tasks and the curriculum as implemented in the traditional mathematics classroom. Some teams encountered difficulties using specific ILEs in their school context when considering inconsistencies between the ways that specific mathematical notions were represented through technological tools and the ways (or the chronological order) these notions were introduced in the curriculum. As an example we consider the FM of AriLab2. As mentioned earlier, in this microworld the construction of a fraction is realized as a quotient between two numbers selected from the number line and the partition line respectively. After the selection of numbers, concurrently with the arithmetic representation of the constructed fraction, a geometrical representation underlying the construction technique based on the projection method involved in Thales Theorem is also provided instantly (see Fig. 3).

Within the cross-experimentation methodology of TELMA, teams coped differently with the fact that the geometrical representation of the FM based on Thales Theorem is not commonly familiar to students at the primary level. For the developing Italian team (CNR-ITD), the socio-constructivist perspective permits the use of such representations as 'black boxes' (i.e., not explaining their meaning to the students) giving responsibility to the teacher to manage the didactic situation and students' activities. Other teams such as the DIDIREM team which works within the French didactic culture, could not accept such an approach by raising issues related to its theoretical orientation as well as to curricular constraints which had to be respected. Finally, along with the LIG team, the NKUA-ETL team chose the black box approach, but only due to the time limits of the cross-experimentation. Consistent with their constructionist theoretical framework the NKUA-ETL researchers considered this specific representation of fractions as offering a novel way to introduce primary students to the geometrical aspects of fractions before—and independent of—the learning of it in the traditional classroom in a future study and under specially designed tasks.

Thus, the use of technological tools designed by one research team and used by another research team in a new socio-systemic milieu highlights differences in the ways in which different cultural/didactic contexts and theoretical frameworks handle epistemological aspects underlying the provided computational representations. One helpful point here concerns the notion of 'distance' between the computational objects and the means of manipulating them provided by a tool and those used in paper-and-pencil based work within a particular target domain. Morgan, Mariotti and Maffei (this issue) offer an interesting elaboration of this notion and develop it in a way that can be of use in the design and employment of technological tools in mathematics education and in research into their use. More particularly, they use it to consider first epistemological issues related to representation of mathematical objects and then issues arising from contextual differences at institutional levels and in relation to teacher-student relationships and interactions.

The above synthetic results—based on the teams' answers to the questions concerning their intervention in the school settings—appear productive in the sense that they reveal a complex set of contextual issues that arise by the teams' common research activity and influence at some level their decisions in research practice. These issues involve, for instance, the duration of the experiment, the time available for instrumental genesis to take place and the educational legitimacy of the black box approach in the didactical exploitation of particular representations available in specific ILEs. They also involve the relation between the proposed tasks and the curriculum. Examples are the epistemological nature of the available representations or more practical issues such as the time in which specific notions are introduced to students according to the curriculum.

The questions operating as boundary objects, can be seen as providing a means to think and talk about aspects of context without the necessity of any one team adopting the perspective of the other. From this point of view, different contextual issues have been brought into association through communication and analysis around a boundary object. This happened in two levels: first, research teams were asked to make explicit particular contextual aspects concerning their intervention in the school settings by analyzing/interpreting their experiments; second, these analyses were (re)-interpreted by other researchers within the same community. For instance, one question concerning the socio-systemic factors involved in school-based research was: 'What type of research is followed (e.g., classroom based, case studies) and how is it related to the kind of research focus?' The answers of some teams provided further information on their practical need to address a multiplicity of other issues which emerged when they approached specific school sites such as curriculum issues, teaching methods, available time, mathematical content and instrumental issues related to the use of particular ILEs in the classroom. This kind of sharing of ideas and analyses stimulated further the communication between the teams and challenged them to enrich their analyses by reflecting on similar issues from their own perspective. At the same time the methodological tool for context analysis was further refined by adding new—and in a sense more acute—questions aiming to capture the described diversity such as: 'How are the socio-systemic factors addressed: administration, teachers in daily action, roles and relationship with researchers, daily program (time, curriculum, method)?' In this way new contextual issues seemed to have emerged. For instance, the issue of 'distance' between traditional and computational representations as a parameter influencing the research practice emerged as a result of the teams' references to the criteria on which they had based their decision to use or not a specific ILEs in their experiments.

6.3 Teacher Communication and Support

The TELMA teams recognized the importance of communicating with and supporting the teachers taking part in the experiments. There was diversity however in the teams' perceptions of the status and the objectives of their relations with those teachers. There was also diversity in the mathematical and pedagogical knowledge each team assumed the teachers possessed and in the extent to and the way in which part of the teams' role contributed to the teachers' professional development.

Teachers' engagement in research was one of the first concerns addressed by all teams in the course of joint construction of the questions pertaining to context. However, detailed issues underlying the communication between teachers and researchers (i.e. respective roles, timing of activities, degree of readiness to incorporate new tools in the lessons, ethos of a particular school), were not fully elaborated from the beginning of the cross-experimentation. Rather, these issues arose during the research. They emerged for instance as a result of the process of reciprocal demanding of clarity in the answers given by the different teams to specific questions. Such processes resulted to the collective elaboration of the respective questions which challenged the teams to further reflect on particular aspects of the teachers' engagement in cross-experimentation which might have not been taken into account in the design phase.

In this paragraph we provide a concrete example based on the comparison of two teams' approaches in dealing with teachers to illustrate how particular contextual issues influenced their research both at the implementation and the analysis level. It is important to note that while the first team (IOE) addressed practical aspects of the cooperation with the teachers *explicitly* in the research design, the second team (ITD-CNR) considered *implicitly* that similar aspects would be addressed by the participating teachers and researchers during the implementation of the activities in the classroom. However, the ITD-CNR team reflected on this issue a-posteriori through the above mentioned cross-team communication. Apart from indicating areas of shade and light in the two groups' approaches, the comparison contributes to making sense of 'unexpected' events occurring during the implementation of the experiments. This allows us think of consistencies and inconsistencies and to seek their sources.

Taking a social semiotic theoretical approach, the IOE team used the E-Slate Fraction-sliders Microworld to investigate the introduction of a novel representation of fractions (i.e. as a dynamic functional relationship between values on two number lines) to provide possibilities for conceptual development in the area of fractions as students explored the tool. The team cooperated with two teachers-researchers both of whom were at the time students on the MA course in Mathematics Education at the Institute of Education (University of London). The participant teachers were engaged in research having a dual role as practitioner researchers as well as teachers in the respective lessons in two different schools.

The design of the experiment was conceived in collaboration with the teachers through communication and sharing of ideas around the available representations of fractions in the microworld, the educational goals and the modes of use as well as the effective implementation of the activities in the classroom. However, there was a deliberate lack of design of pedagogy so as for the teachers-researchers to feel free to adapt the microworld to their needs and plan the details of their lessons by themselves. We note that the theoretical ideas available to the teachers through their engagement in the design of the experiment and their recent participation in an MA module on the role of digital technologies in mathematics learning involved social semiotics and constructionism. The research aim was conceived at two levels: at the first level the teachers-researchers investigated the use made by students of the tool and at the second level the IOE researcher studied the influences of each of the teachers' institutional and cultural contexts to the types of meanings constructed by both students and teachers while using the same tool. In fact, the identification of the different modes of employment of the tool by the two teachers and how they integrated it into their lessons was a main issue in the experiment. In classroom activity two rather different teaching styles have emerged concerning the teachers' choices in critical parameters of the implementation such as the student-computer configuration, the introduction to the software and the presentation of tasks in the classroom as well as the control maintained over student participation in whole class interaction (Morgan 2007).

Teacher 1, for instance, despite described his planned student activity as 'exploration'—implying in a way the theoretical ideas behind the IOE team research approach—took a more teacher-centred approach to his lesson choosing to present many examples so as to facilitate student's generalization. In contrast, Teacher 2 seemed to focus more on the active construction of meanings by the students as resulted through their interaction with the provided tools. Apart from discussing the resulting differences in the specific forms of pedagogy in the two classroom settings as related to the teachers' implicit or explicit theoretical frameworks, the analysis also indicated these differences as consistent with the institutional differences, i.e. at the organisational and the cultural specificities of the two classrooms. Notably, the researchers initially identified the school of the Teacher 1 as having a strongly controlled 'traditional' ethos while the school of the Teacher 2 as being relatively liberal or 'progressive' (Morgan 2007).

Following a socio-constructivist and Vygotskian theoretical perspective, the ITD-CNR team gave high priority to the role of the teachers in the social construction of knowledge and set up a classroom experiment involving group activities, such as class discussions, where concepts would be shared among the groups of students. However, the team followed a research approach that did not involve the two participant teachers in the design of the experiment. During the implementation of the activities the team identified differences in the ways in which the teachers perceived the details of their cooperation with the research team which influenced the roles they had adopted in the classroom. At that time the researchers were confronted with difficulties to define in practice the teachers' role, since their assumed theoretical assumptions provided little reference to practical details. The reciprocal roles of the teachers and the researchers were not made explicit in the task design nor negotiated in some way when researchers approached the school. So, during data collection the researchers realized that one of the teachers was quite passive during the sessions, leaving them the responsibility to manage all the activity. This was an unexpected occurrence by the team that posed some problems to the researchers who were not prepared to have the responsibility of 'doing the lesson' and working 'alone' with the students. In contrast, the behaviour of the other teacher was different: she performed 'spontaneously' the institutionalisation of the concepts dealt with during the computer sessions. Since the researchers did not clearly explain to the teachers what they expected from them during the experiment, the team attributed this difference to the previous experience of this teacher in developing some experimental projects in her classes following the same approach (students' group-work, interaction between students, whole class discussions following the work at the computers, etc.).

Summing up, it can be said that the CNR-ITD team left undefined some important aspects of the activity which need to be directly addressed during the classroom work by the participating teachers and researchers. In practice, however, it appeared that both teachers and researchers seemed to consider implicitly that their reciprocal roles were 'given' (i.e. according to their own interpretations of the activity) without an explicit negotiation around them.

The above comparison of the approaches of the two teams that followed the analysis of their experiments highlights important aspects of the relation between contextual issues concerning teacher communication and support and research practice. In particular: (a) the types of roles and teaching styles that teachers elaborated or not during their lessons and (b) their relationships with the researchers during the design and implementation of the activities in the classroom. In the ITD-CNR case it turned out that critical methodological decisions concerning the implementation phase of the experiment may need a finer specification by the researchers and call for the teachers' engagement in the design of the activities or a more direct communication on the teachers' role in the classroom. In the IOE case the two experimenting teachers were engaged to co-design the implementation of the activity in the classroom through collaborative work with the researchers. The important differences in the pedagogic stance that the teachers developed in the classroom brought to the fore institutional and cultural aspects characterising the specific school communities within which they acted. We suggest that the above findings indicate that studying the teachers' role in classroom-based research while taking into account and analyse both their relations with the researchers and their practices yields insight into the nature of institutional and cultural influences associated with the use of computational tools for the teaching and learning of mathematics.

At the same time the questions related to teacher communication and support aspect of context challenged the teams to highlight explicitly the role of the respective issues in actual research practice. During the implementation phase of the cross-experimentation these questions were used to highlight the respective areas of light and shade in each teams' approach. However, the sharing of answers in cross-team communication that followed the analysis of the experiments (e.g., during the preparation of deliverables) challenged some teams—like the CNR-ITD in this case—that did not pay much attention to these aspects during the design of their experiment to address them in retrospect. In this way the methodological tool for context analysis—viewed as boundary object—helped the teams to start to understand each other's perspectives indicating a developing focus on contextual aspects underlying their research approaches.

7 Summary and Perspectives

This study perhaps raises more questions than answers about the ways in which contextual issues and concerns may influence researchers' practices and productions when they design and analyse teaching experiments based on the use of ILEs.

As far as contextual issues are concerned, the TELMA teams directly approached the practical problem of how different groups of researchers, working in different contexts, are able to understand and make use of each other's work. The main results of the cross-experimentation generated a somewhat pragmatic definition of context and the development of an analytical tool for identifying and describing contextual issues. The development of the methodological tool for context analysis has been concerned with:

- the identification of contextual issues which had not been focused explicitly in the mathematics education community and
- the comprehensive inclusion of these issues into the analysis as an integral part of the research paradigm.

The main focus was centered on the issue of the diversity of the various teams' contexts in which tasks were designed and technologies used. For that reason, the notion of context was understood in a very wide sense as evidenced by the descriptions produced in the preceding paragraphs in which contexts may be conceived as to have disclosed the complexity of the notion of context itself.

The use of this methodological tool in the analysis of the cross-experimentation in the role of a boundary object shows that all the contextual concerns mentioned were addressed, but with evident differences in emphasis and extent. The use of the tool as a boundary object made it legitimate, appropriate and necessary for the teams to express their concerns, theoretical approaches and contextual specificities while making explicit characteristics of their research which would not otherwise be visible to the other teams. We suggest that mechanisms enhancing sensitivity and awareness of contextual issues in design research such as that of boundary objects may prove to be necessary for the produced knowledge to be better understood more widely. What is important is that through the use of boundary objects, exemplifying contextual issues gained a social legitimacy in the sense that it was operational to the process of the joint research. The analysis shows how such a process may reveal tacit aspects of context which nevertheless have an important bearing on productions and knowledge emerging from a research experiment. How for instance the two teams working with Aplusix developed such different tasks even though they had compatible theoretical frames and experimented in the same culture. How the CNR-ITD and NKUA-ETL teams' differing perceptions of time restriction resulted in circumventing engagement with the respective schools as organizations. How the IOE and CNR-ITD teams' approach to the teachers taking part in the study inadvertently influenced the classroom milieu and resulting analyses. It also became apparent to us that the ways in which these issues affected the whole process of research and inevitably its outcomes were very complex. Further research on the ways context influences the whole cycle of digital tool design and production, design research and theory building, we feel will contribute to understanding theoretical constructs and frames themselves and also the kinds of impact they may have on pedagogy across cultures.

As a result of this study we felt that some reworking is necessary in order to make the notion of context more technical and operational. Especially in the case of design research, interventions in a specific context require dynamic perceptions of context, where situated cognition theory, street mathematics, ethno-mathematics and even political dimensions of mathematics education have an important bearing. In the case of digital media however, there are particular issues to bear in mind. The dynamics of the production of these media is one, the dynamics of researchers intervening to infuse innovations in the educational system and directly addressing teachers operating within that system is another. One possibility is to distinguish only between two main dimensions of context: *the context of situation* (for instance, for a student at school the immediate situation context—the classroom, the activity, etc—within he/she engages in the education experience) where an individual is immediately embedded and a more general *context of culture* which includes the former (for instance, for the student, but also for the teacher, this means school institution, but also the world outside the school) (Morgan 2006). Let us point out that, even when introducing only these two dimensions, one can nevertheless think of context in at least two different but complementary ways:

- as centred on subjects—individuals and communities as well—and as an enlarging sphere that progressively includes and organizes in a consistent system more and more elements;
- as centred on activities, based on the articulation, distinction and relation between different activities, for instance between the design and the possible uses of a tool (use intended in a very general sense, for instance in the school practice, or in the experimental situation).

The TELMA teams felt that this approach would warrant further cross-experimentation in situations where ample time could be spent on design and implementation in schools and would also involve the development in digital media. This kind of work can be found in a subsequent venture, mentioned also in the introduction of this paper, a European project titled 'ReMath'.

References

Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal, 32*(1), 23–38.

Artigue, M. (2000). Instrumentation issues and the integration of computer technologies into secondary mathematics teaching. *Proceedings of the Annual Meeting of the GDM*. Potsdam, 2000. http://webdoc.sub.gwdg.de/ebook/e/gdm/2000).

Artigue, M. (2002). Learning Mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*(3), 245–274.

Artigue, M. (2008). Digital technologies: a window on theoretical issues in mathematics education. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the 5th Conference of the European Society for Research in Mathematics Education* (pp. 68–82). Larnaca, Cyprus.

Artigue, M., Cerulli, M., Haspekian, M., & Maracci, M. (this issue). Connecting and integrating theoretical frames: the TELMA contribution. *International Journal of Computers for Mathematical Learning*. Special Issue on TELMA work.

Bartolini Bussi, M. (1996). Mathematical discussion and perspective drawing in primary school. *Educational Studies in Mathematics, 31*(1–2), 11–41.

Bauersfeld, H. (1988). Interaction, construction and knowledge: Alternative perspectives for mathematics education. In D. A. Grouws, T. J. Cooney, & D. Jones (Eds.), *Effective mathematics teaching* (pp. 27–46). Hillsdale, New Jersey: National Council of Teachers of Mathematics, Lawrence Erlbaum Associates.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research, critique*. London: Taylor & Francis.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht: Kluwer Academic Publishers.

Bottino, R. M., & Cerulli M. (2007). Design and analysis of technology based field experiments in mathematics: A methodology for networking research teams. *IFIP TC3-WG3.3 Working Group on Research into Educational Applications of ICT*. Budapest.

Bottino, R. M., & Kynigos, C. (this issue). Mathematics education and digital technologies: Facing the challenge of networking European research teams. *International Journal of Computers for Mathematical Learning*. Special Issue on TELMA work.

Brousseau, G. (1997). *The theory of didactical situations in mathematics*. Dordrecht: Kluwer Academic Publishers.

Cerulli, M., Pedemonte, B., & Robotti, E. (Eds.) (2007). *TELMA Cross Experiment Guidelines*. TELMA Internal Report, CNR-ITD, Genoa. http://www.itd.cnr.it/telma/documents.php.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique: perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques, 12*(1), 77–111.

Clarke, D. J., & Helme, S. (1998). Context as construction. In O. Bjorkqvist (Ed.), *Mathematics teaching from a constructivist point of view* (pp. 129–147). Vasa, Finland: Abo Akademi University: Faculty of Education.

Cobb, P., Confrey, J., diSessa, A., Lehrer, P., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent and sociocultural perspectives in the context of developmental research. *Journal of Educational Psychology, 31*, 175–190.

Cole, M. (1996). *Cultural psychology: A once and future discipline*. Cambridge, MA: Harvard University Press.

Cole, M., & Engeström, Y. (1993). A cultural–historical approach to distributed cognition. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 1–47). Cambridge: Cambridge University Press.

diSessa, A., Azevedo, F. S., & Parnafes, O. (2004). Issues in component computing: A synthetic review. *Interactive Learning Environments, 12*, 109–159.

diSessa, A., Hoyles, C., & Noss, R. (1995). *Computers and exploratory learning*. Berlin: Springer.

Gravemeijer, K. P. E. (1994). *Developing realistic mathematics education*. Utrecht: CD-ß. Press, Freudenthal Institute.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39*, 111–129.

Harel, I., & Papert, S. (1991). *Constructionism: Research reports and essays*. Norwood, New Jersey: Ablex Publishing Corporation.

Hoyles, C., & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education*. Dordrecht: Kluwer.

Hoyles, C., Noss, R., & Kent, P. (2004). On the integration of digital technologies into mathematics classrooms. *International Journal of Computers for Mathematical Learning, 9*, 309–326.

Jaworski, B. (2004). Grappling with complexity: co-learning in inquiry communities in mathematics teaching development. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17–36). Bergen Norway: Bergen University College.

Keitel, C., Damerow, P., Bishop, A. J., & Gerdes, P. (1989). *Mathematics, education, and society*. Paris: UNESCO.
Kynigos C. (2002). Generating cultures for mathematical microworld development in a multi-organisational context. *Journal of Educational Computing Research*, Baywood Publishing Co. Inc. (1 and 2), 183–209.
Kynigos, C. (Ed.) (2005). *Learning contexts.* TELMA Deliverable 20.3.2, Kaleidoscope Network of Excellence. http://telma.noe-kaleidoscope.org.
Kynigos, C. (2007). Half-baked Logo microworlds as boundary objects in integrated design. *Informatics in Education, 6*(2), 1–24. Institute of Mathematics and Informatics, Vilnius.
Kynigos, C., Artigue, M., Bottino, R., M., Cerulli, M., Mariotti, M.-A., Morgan, C., Psycharis, G. (2006). *Methodological tools for context analysis*. TELMA Deliverable 20.3.3, Kaleidoscope Network of Excellence. http://telma.noe-kaleidoscope.org.
Kynigos, C., & Theodosopoulou, V. (2001). Synthesizing personal, interactionist and social norms perspectives to analyse student communication in a computer-based mathematical activity in the classroom. *Journal of Classroom Interaction, 36*(2), 63–73.
Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-Geometry. *International Journal for Computers in Mathematical Learning, 6*, 283–317.
Lave, J. (1988). *Cognition in practice*. Cambridge: Cambridge University Press.
Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics, 61*(1–2), 219–245.
Morgan, C. (2007). Variations on a theme: introducing new representations of fraction into two classrooms. In D. Küchemann (Ed.), *Proceedings of the British Society for Research into Learning Mathematics, 27*, 1. London South Bank University.
Nardi, B. (1996). *Context and consciousness: Activity theory and human–computer interaction*. Cambridge: MIT Press.
Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics*. Cambridge: Cambridge University Press.
Papert, S. (1987). Computer criticism versus technocentric thinking. *Educational Researcher, 16*(1), 22–30.
Papert, S. (1996). An exploration in the space of mathematics educations. *International Journal of Computers for Mathematical Learning, 1*(1), 95–123.
Pea, R. D., & Kurland, M. (1984). On the cognitive effects of learning computer programming. *New Ideas in Psychology, 2*(2), 137–168.
Psycharis, G., Latsi, M., & Kynigos, C. (2009). Meanings for fraction as number-measure by exploring the number line. *International Journal for Technology in Mathematics Education, 16*(3), 91–108.
Scardamalia, M., & Bereiter, C. (2003). Knowledge building environments: Extending the limits of the possible in education and knowledge work. In A. DiStefano, K. E. Rudestam, & R. Silverman (Eds.), *Encyclopedia of distributed learning* (pp. 269–272). Thousand Oaks, CA: Sage Publications.
Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer.
Star, S. L., & Griesemer, J. (1989). Institutional Ecology, 'translations', and boundary objects: Amateurs and professionals in Berkeley's Museum of vertebrate zoology, 1907–1939. *Social Studies of Science, 19*, 387–420.
Van Oers, B. (1998). From context to contextualizing. *Learning and Instruction, 8*(6), 473–488.
Wedege, T. (1999). To know or not to know—mathematics, that is a question of context. *Educational Studies in Mathematics, 30*, 205–227.