Int J Comput Math Learning (2009) 14:203–215
DOI 10.1007/s10758-009-9153-y

# Mathematics Education & Digital Technologies: Facing the Challenge of Networking European Research Teams

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Published online: 22 December 2009


**Abstract** This paper introduces the IJCML Special Issue dedicated to digital technologies and mathematics education and, in particular, to the work performed by the European Research Team TELMA (Technology Enhanced Learning in Mathematics). TELMA was one of the initiatives of the Kaleidoscope Network of Excellence established by the European Community (IST-507838—2003–2007) to promote the joint elaboration of concepts and methods for exploring the future of learning with digital technologies. TELMA addressed the problem of fragmentation of theoretical frameworks in the research field of mathematics education with digital technologies and developed a methodology based on the in field cross-experimentation of educational digital environments for maths. Six European research teams engaged in cross-experimentation in classrooms as a means to begin to develop a common language and to analyse the intertwined influence played, both explicitly and implicitly, by different contextual characteristics and theoretical frames assumed as reference by the diverse teams participating in the initiative.

**Keywords** Technology enhanced learning · Networks of excellence · Collaboration · Cross-experimentations · Learning environments

## 1 Introduction

Research in mathematics education with digital technologies began around 40 years ago with the prominent work of Papert and has developed into a field of study with its own identity, language and set of theoretical frameworks and constructs (Cornu and Ralston 1992; Crowe and Zand 2000; Hoyles and Lagrange 2010). The emerging knowledge has drawn from both the fields of mathematics education and digital technologies in education. In both of these fields, however, there has recently been a growing recognition that, despite

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the abundance of theoretical frameworks and constructs which have been developed, the overall picture remained fragmented and this has a negative impact on the growth of knowledge and the clarity of communication beyond the respective research communities.

In the field of mathematics education, research has not followed a unified paradigm but appears as a field following a multiplicity of different trends that develop more or less independently and that are inextricably bound to different contexts from which they have emerged. The origin of the expertise of a research team, for example, has an important bearing on the epistemological assumptions, the methods, and the scope of validity of the results obtained. The socio-systemic context has a great influence on the mathematical teaching and learning activities a research study is about, especially in Europe, where a dense pluralism of different educational systems and cultures is present.

The consequence of this is that a variety of poorly connected conceptual and methodological tools are produced, thus making it difficult for researchers to orient themselves in the field and to develop an overall sense of which are the main results and the open problems, even if it is evident that some common trends exist. In spite of the multiplicity of international conferences and workshops, exchanges among different groups often remain superficial since each group refers to their own theoretical frameworks and methodologies. This situation gives an overall impression of division, and, what is more, does not engender an impression that results obtained are convincing and valuable (Arzarello et al. 2008; Prediger et al. 2008). For this reason, in the recent years, the research community of mathematics education devoted increasing effort to comparing and connecting between different theoretical frames and work methodologies. This is illustrated, for instance, in two recent issues of the Journal Zentralblatt für Didaktik der Mathematik (ZDM 2005, Vol. 37(6) and ZDM 2006, Vol. 38(1)), and by the establishment of a working group especially devoted to these issues at the last two conferences of the European Association for Research in Mathematics Education (ERME).

Researchers involved in mathematics education with digital technologies are particularly sensitive to the problems raised by the current fragmentation of the field (as evidenced, for instance, by the meta-study of Lagrange et al. 2001) since they strongly perceived the need of designing and implementing tools and methodologies that can have a wide scope of application and that are not restricted to a particular community or context.

The general problem of integration of different approaches, tools, and theoretical frameworks has been identified also at the level of the European Commission as a crucial concern for the field of Information Society Technologies (IST). Inevitably, this field, which involves studies in computer science, cognitive science, social sciences, learning, and brain physiology as well as systems and software development, is widely open to the problem of theories and knowledge fragmentation. In the European scene, there has been an explicit attempt to bring together people from diverse areas especially through productive networking initiatives such as the so-called 'Networks of Excellence' (NoE). Such networks are funded in order to promote integration of a meaningful variety of the most important European research teams and institutions involved in the IST field. In particular, the NoE Kaleidoscope was established by the European Community (IST-507838—2003–2007) to promote the joint elaboration of concepts and methods for exploring the future of learning with digital technologies. It brought together 80 different European teams involving more than 1,100 researchers and PhD students distributed over 23 Countries (more details can be found at the following URL: www.noe-kaleidoscope.org).

Within Kaleidoscope, a number of different joint research activities, covering a wide range of topics, have been carried out. Among these, the TELMA initiative was specifically focused on Technology Enhanced Learning in MAthematics. Its main aim was that of networking a selected number of teams[1] having a consolidated tradition in mathematics education with digital technologies with the aim of building a shared view of key research topics in the area, proposing joint research activities, and developing common research methodologies. In this paper, we firstly set the scene with respect to the different approaches that can be assumed when considering the influences of digital media in mathematics education. We, then, describe the main characteristics of the work methodology developed by the TELMA group to foster integration and collaboration among research teams, ending with a reflection on its positive results and on future perspectives.

This work introduces the subsequent papers of this special issue that will each examine a specific aspect of the work performed within TELMA.

## 2 Setting the Scene: Mathematics, Digital Technologies and Education

When considering the research field of digital technologies and mathematics education, it is possible to identify various orientations for research that have been assumed and that can broadly be synthesized as follows:

- Analysis of the changes undergone in mathematics curricula as a consequence of the widespread diffusion of new technologies.
- Teaching of mathematical topics in computer science courses and also outside the education system, e.g., in the workplace.
- Design and use of ICT tools as mediators of teaching and learning processes in mathematics.
- Study of mathematics educational process in technology-rich learning environments.

These orientations are only apparently disjointed since, even if with different approaches and at different educational levels, they answer to a similar need, that of re-addressing teaching and learning approaches in mathematics. This need has emerged from both the problems and difficulties usually encountered in teaching and learning mathematics and from the epistemological and methodological challenges appearing at a dramatically rapid pace along with the widespread diffusion of new technologies.

The first two research orientations are more concerned with curricula and content choices. They involve, on the one hand, the epistemological discussion of new mathematical knowledge and skills that are necessary in the so called “knowledge society” and, on the other, the changing nature of knowledge itself, what Papert calls the “what” as opposed to the “how” of learning (Papert 2006). New kinds of mathematics become learnable with digital technologies and new meanings of traditional mathematical concepts and relations are constructed by students using malleable and dynamic representations of digital media. One can think, for example, of the increasing necessity of using information technology to process quantitative or symbolic data and of the consequences that this can have on mathematics curricula, with the introduction of topics like statistics and probability, and also, of basic elements of computer science. As observed by Noss et al. (2007), a

[1] TELMA teams (whose acronyms are indicated in brackets) belong to the following Institutions: Consiglio Nazionale delle Ricerche, Istituto Tecnologie Didattiche, Italy (CNR-ITD); Università di Siena, Dipartimento di Scienze Matematiche ed Informatiche, Italy (UNISI); University of Paris 7 Denis Diderot, France (DIDIREM); Grenoble University and CNRS, Leibniz Laboratory, Metah, France (LIG); University of London, Institute of Education, United Kingdom (UNILON); National Kapodistrian University of Athens, Educational Technology Laboratory, Greece (ETL-NKUA).

new “techno-mathematical literacy” is needed to cope with the dramatic increase in the deployment of information technologies within workplace practices.

The debate on the relationship between mathematics and computer science dated back to the early 1980s, when, following the advent of the microcomputer, many research studies explored the opportunities brought about by new technologies to the teaching and learning of mathematics. Initially, a significant part of these studies was concerned with the natural links that exists between mathematics and computer science (Goldstine 1972), and this, at the school level, led to consideration of the affinity of some educational objectives and abilities of such disciplines: decoding of problems, modelling of situations, use of correct and not ambiguous language, passage from natural languages to artificial ones. An important survey in this regard was the one performed for the first ICMI Study by Cornu and Ralston (1992), which suggested that it was possible to introduce computer science elements in school mathematics without relevant additions in content and radical substitutions. How and to what extent this has changed teachers’ conceptions of mathematics itself and classroom practice was also a subject of concern discussed among researchers (see, for example, Bottino and Furinghetti 1996).

Of the four orientations outlined above, the first two focused on curriculum content rather than the study of learning processes and the affordances of digital media themselves. However, the evolution of hardware and software and the widespread diffusion of digital technologies in society, together with the evolution of theoretical frameworks of reference, provoked a progressive shift of focus from content to digital media design methods and principles as well as teaching and learning processes. Inevitably, the analysis of the mediation that technologies can offer to the teaching and learning of mathematics became a predominant focus in the research community. The main bulk of research so far has thus focused on the third and fourth research orientations envisaged at the beginning of this section: the design and/or use of ICT-based tools for supporting innovative teaching and learning processes in mathematics and the study of these processes including the support that has to be given to teachers. Such perspectives have been considered at different educational levels and have given rise to various research studies concerning the design, use, and analysis of mathematics educational environments in which ICT tools were integrated. In the following we thus analyse the latter two orientations with some more detail, since these are the ones that TELMA teams assumed in their work.

## 3 Mathematics Learning Environments Integrating Technologies

Many digital tools produced commercially for mathematics education are essentially electronic versions of textbooks or drill and practice, sometimes disguised as games. However, researchers have mostly addressed the different opportunities provided by ICT-based tools for added value in educational practice. They have mainly studied the potential for changes in the teaching and learning of mathematics starting from the consideration that, while mathematics is traditionally perceived as abstract and formal, ICT can facilitate access to mathematical concepts by means of the exploration and the manipulation of concrete representations. ICT, considered in such context, can thus provide added value to the engagement of students with experimental mathematics in conjunction with the understanding of and practice with formal mathematics. One can think, for example, of experimentation with the use of well-known tools such as spreadsheets, computer algebra systems, dynamic geometry systems, or of the use of software specifically implemented for mathematics educational purposes (e.g., microworlds targeted to the exploration of specific topics). Such studies underwent a deep transformation in the course of time, due to the parallel evolution of pedagogical and cognitive science theories, and to the technological progress which constantly opens up new opportunities (Bottino 2004).

An early focus was on the use of digital media as a means to enhance the potential for developing constructivist learning environments (Papert 1980), i.e., those which would be rich in opportunities for students to generate mathematical meanings (see, for example, Laborde and Strasser 1990). This was widely taken as a recipe for quick radical changes in schools, which was challenged by grounding comparative studies doubting the potential of quick changes in schools and in students' mathematical abilities (De Corte 1996; EC 2004). In the 1990s, epistemological analyses of the mathematics embedded in a range of software tools emerging from the rapid developments in technology, as well as domain-specific studies of these tools in use, gave a picture of a more complex world (Balacheff and Sutherland 1994; Laborde et al. 2006). It has been pointed out that providing environments where students could explore mathematics concepts and methodologies was not enough to have an actual impact on school education, and that it has been necessary as well to revise aims, strategies, methods, and activities.

Despite the positive results produced in a number of experimental settings and the budget invested by many governments for equipping schools, actual use of ICT tools in real school environments is still having a limited impact (see, for example, Sutherland 2004; Ruthven 2007). The use of technologies has simply not scaled up and the changes promised by the case study experiences have not really been far noticed beyond the empirical evidence given by the studies themselves. This is true in particular for mathematics (Artigue 2000; Lagrange et al. 2001).

The changes in classroom practices involved in the use of technology seem to pose a real challenge to administrators, curriculum designers, teachers and students. There is thus a great need for a deeper understanding of how the potential suggested by research studies in the 1980s and the 1990s can be grounded both in classroom practices with respect to systemic schooling, in other institutional environments such as the workplace, and in informal situations for children and adults. The quest for such understanding oriented researchers towards the actual and potential influence of the teacher.

In the course of time a significant stream of research studies has addressed the teachers' potential to contribute to and to guide the emergence of innovative learning environments based on the use of digital media in the classroom. An important strand of this research has focused on the ways of supporting teachers in teaching with technological tools, and on their evolving conceptions and concerns (Bottino and Furinghetti 1999; Mumtaz 2000). One main area of study has been that of the methods and the outcomes of teachers' initial and in-service professional development, especially with respect to the kinds of perturbations and challenges posed to teachers as they engage in the use of digital media to design and implement activities in their classrooms. This kind of upset posed to traditional teaching is seen more and more as a tool for reflection and professional development for teachers rather than an implicit obstacle to implementing technologies in the classroom (Laborde 2001; Kynigos 2007a, b).

As a matter of fact, careful attention has progressively been paid not only to digital tools but also to the teaching and learning situations and contexts where those tools are used. This is a sign of an increasing interest in the whole teaching and learning situation and not only in the relationship between the student and the technological environment. Another area of interest has thus been to identify actual uses of technology in mathematics classrooms and the ways in which contextual issues influence the kinds of teaching and the ways in which digital media are used (Ruthven 2007; Funglestadt et al., in press).

As said before, important consideration has been given to the definition of meaningful practices through which technology can be used effectively. This, at the theoretical level, has been supported by a progressive move from cognitive theories that emphasize individual thinkers and their isolated minds to theories that emphasize the social nature of cognition and meaning (Resnick 1987). The focus shifts from the individual as the centre of learning to the idea of learning as a social process based on negotiation of meaning, something that has been a wider trend in mathematics education and indeed within educational research as a whole.

This evolution has affected the design of ICT-based tools for mathematics education. From a mathematics education research perspective, such tools are often seen as malleable digital environments lending themselves to exploration and are characterized by an embedded mathematical knowledge domain, acting as a coherent world, inside which a student can explore alternatives, test hypotheses, and discover facts that are true for this world. The basic characteristic of these systems is, thus, that they are designed as rule-governed environments, which are accessible for manipulation and exploration, by the learner (Kynigos 2007a, b). Moreover, since these systems are considered not as single entities but as part of the whole learning environment, attention is paid to the ways in which they support the re-elaboration of personal experience and its sharing with other students and with the teacher. That is, research interest has been focused on how these systems mediate the different relationships that are established between participants during a teaching and learning activity in order to set up flexible and context-sensitive learning environments (Bottino 2000).

Up till now we addressed what we perceive to be core issues in the evolution of research orientations regarding digital media design and teaching and learning practices. These issues span from how media is designed to support empirical mathematical activity for students to the problems met in understanding this kind of educational practice and infusing such practice at a large scale in educational systems. Perceiving mathematical learning as an essentially social activity and focusing on how to empower teachers in changing teaching methods and coping with their changing role in the classroom have been two important trends. Looking deeper into how mathematical objects and relationships are embedded in dynamic digital media, how traditional and new representations are linked and used by students and how mathematical meanings can be generated by means of activities with these tools has been another trend.

In brief, we can thus say that in the design of a technology-based system for mathematics education, the following issues have progressively assumed a crucial importance:

- The computational objects and interactivity that the system makes available.
- The different representations of mathematical concepts it offers.
- The ways in which student's actions are validated.
- The modalities in which the setting up of a social context is able to assist students' performance and the evolution of competencies and knowledge are supported.
- The ways in which teaching methods can facilitate such learning and the methods by which teacher can be supported to adopt such methods.

The aim is to offer tools and functionalities for problem exploration, for the construction of mathematical meanings, for representing solution strategies, for communicating them, and for supporting learning evaluation.

From these considerations, it follows that pedagogical design and technological implementation are strictly linked and are to be considered together. As a matter of fact, the influences that advanced technologies can have in mathematics education are equally to be considered within the wider field of research in Technology Enhanced Learning (TEL). These two worlds—mathematics education and TEL—are often fragmented from each other. There are projects, for example, which address the development of software for mathematical learning clearly from a computer science point of view (focusing on what the advances in technology made possible) without visible grounding on the knowledge emerging from the mathematics education community. Conversely, there is an abundance of little pieces of mathematical software that constitute the embodiment of theoretical constructs from the mathematics education community which are clearly in need of help from a computer science and technological perspective. There is, thus, a growing recognized need to find ways and methods to cope with the complexity of mathematical education with digital media and with the variety of perspectives, contextual characteristics and theoretical frameworks coexisting in the field.

Within Kaleidoscope, TELMA teams have specifically addressed the problem of integration in a mathematics education perspective. The aim of TELMA is to build a research strategy in the field and a long lasting collaboration among its teams. The methodology adopted to meet this aim is presented in the following section.

## 4 TELMA: How to Develop a Collaborative Research Methodology in Technology Enhanced Mathematics Education?

At the beginning, the TELMA teams faced the challenge of networking research teams that were very different. Each team brought with it particular focuses and theoretical frameworks, adopted and developed over a quite long period of time. Most of them had also implemented and studied learning environments integrating digital technologies for use in mathematics learning, designed, developed and tested in accordance with their own theoretical perspectives. The TELMA group refers to these as Interactive Learning Environments (ILEs).

The starting phase of TELMA was thus very demanding, requiring six teams with different backgrounds, work methodologies, and ILEs, to begin to share knowledge, with the long-term aim to develop a common language and common topics of research.

TELMA researchers quickly saw that a pre-requisite to any kind of productive collaboration between them was a clarification of their respective cultures, theoretical frames and perspectives. Moreover, since it was clear from the beginning that, to approach integration, it was necessary to find some common perspectives from which to look at the different approaches adopted by each team, to find similarities and to clarify differences, it was decided to focus on school practices and to concentrate the analysis on three interrelated topics considered important for mutual knowledge and comparison: the theoretical frameworks within which the different research teams approached research in mathematics education with technology, the role assigned to representations provided by technological tools, and the characteristics of the different contexts within which each team designed and developed tools and theories, addressed schools and carried out innovative projects in real classrooms.

The first analysis of these topics was carried out considering the most representative publications of each team but left the researchers partially unsatisfied for several reasons. In the research papers provided by each team, theoretical references were explicitly mentioned but it was very difficult to infer from what was written, the exact role these had played from an operational point of view in the design and management of the research projects, and thus, in the results obtained. The same was true for the impact of contextual characteristics, and it made it difficult to figure out up to what point the experience and knowledge gained in one team could be useful for the others, and on what basis collaboration could be undertaken.

It was evident that depending on descriptions of previous work at an abstract level was not enough to create a meaningful networking mechanism. What was needed was some kind of joint experience that would engage the teams in collaborative design and implementation of concrete research studies so that the inevitable need for negotiation of operational details would allow a deeper communication and understanding in order to build a joint language about similarities and differences in each team. A new methodological approach was thus jointly conceived and implemented by TELMA teams: the cross-experimentation methodology (Bottino et al. 2009; Artigue et al. 2007).

## 4.1 The Cross-Experimentation Methodology

The key idea around which this methodology was built was the design and the implementation by each TELMA team of a short-term experiment, in a real classroom setting, making use of an ICT-based tool developed by another team. Such experiments were constructed in order to provide a systematic way of gaining insight into theoretical and methodological similarities and differences in the work of the various TELMA teams. This is a new approach to collaboration that seeks to facilitate common understanding across teams with diverse practices and cultures and to elaborate integrated views that transcend individual team cultures. There are two principal characteristics of the cross-experimentation methodology elaborated by TELMA that distinguish it from other forms of collaborative research:

- The design and implementation by each research team of a teaching experiment making use of an ICT-based tool developed by another team.
- The joint construction of a common set of questions to be answered by each team (both before and after the teaching experiment) in order to frame the process of cross-team communication.

Moreover, in the development of cross-experiments an important role was given to TELMA young researchers and doctoral students (Cerulli et al. 2007a). This choice was congruent with the general philosophy of Kaleidoscope and was suggested also by the wish to have “fresh” eyes looking at teams’ approaches, theoretical frameworks, and consolidated practice, in order to better make explicit those factors that often remain implicit in the choices made by more experienced researchers.

Each team was asked to select an ICT-tool among those developed by the other teams, as shown in Table 1. This decision was expected to make more visible the influence of theoretical frames and to favour exchanges between the team that had developed an interactive learning environment for mathematics education and the team(s) that had to implement a teaching experiment using it.

To facilitate the comparison between the different experimental settings, it was also agreed to address common mathematical knowledge domains (Arithmetic and Introduction to Algebra), to carry out the experiments with students between the 5th and 8th grade, and to perform them for about the same amount of time (1 month).

Experimental guidelines were collectively built for monitoring the whole process: from the design and the a priori analysis of the experiments to their implementation, the collection of data, and the a posteriori analysis. Guidelines contained all the research questions to be addressed and the plans for experimentation developed by each team (Cerulli

**Table 1** The ICT-based tools employed by TELMA teams in the cross-experiments

| Interactive learning environment (ILE) | Team having developed it | Teams having experimented it |
|---|---|---|
| Aplusix<br>http://aplusix.imag.fr | LIG (France) | CNR-ITD (Italy)<br>UNISI (Italy) |
| E-Slate<br>http://etl.ppp.uoa.gr | ETL-NKUA (Greece) | UNILON (UK) |
| ARI-LAB 2<br>http://www.itd.ge.cnr.it/arilab_english/index.html | CNR-ITD (Italy) | LIG (France)<br>DIDIREM (France)<br>ETL-NKUA (Greece) |

et al. 2007b). These plans included information on the experimental settings, on the modalities of employment of the tool, and on the methods used to collect and analyse data. The research questions included in the guidelines were both questions to be addressed before the experiments and questions to be addressed after them.

At the end of the experiments, reflective interviews with the researchers based on stimulated recall were conducted in order to make clear the exact role theoretical frames and contextual characteristics had played in the different phases of experimental work, explicitly, or in a more naturalized and implicit way.

## 5 Reflections and Links

Cross-experiments were developed with the aim of acquiring a better understanding of what happens when an ICT-based learning environment is implemented using a tool that has been designed under theoretical frameworks and in a context different from that of the experimenting team. Moreover, since most tools were experimented with by two different teams, it was also possible to compare their designs, implementations and analysis. All these comparisons were expected to contribute to the visibility of the role played by theoretical frames and contexts, and help understand their respective influence.

To complement the work carried out in the cross-experiments, TELMA teams also felt the need to get a wider insight to the landscape of interactive learning environments for arithmetic and algebra. Consequently, they decided to conduct an analysis of a selected number of systems that are currently available. Even if various methods have been proposed in the course of time to study computer-based learning tools, they are often concerned only with the artefact, and many issues of interest for TELMA, such as the underpinning theoretical frames, rationale for the design choices, contextual issues, difficulties encountered in the development process, remain hidden. For this reason the group found it interesting to collaboratively develop its own methodology in order to shed light also on such implicit issues.

These two complementary approaches, cross-experiments of tools "internal" to the group and analysis of a selection of "external" tools, according to jointly developed methodologies, allowed us to go further in the analysis of the complexities involved in designing and implementing learning environments integrating technology. Within TELMA, the following issues have been, in particular, considered and will be discussed in the others chapters of this Special Issue:

- the role played by different theoretical frameworks and how theoretical reflection can be addressed in tight connection with practice;
- the role played by representations provided by technological tools in the construction of meaning for mathematical concepts;
- the way in which each team perceived the contextual issues as having a bearing on how the tools were approached and used;
- the way in which the analysis of technological tools for mathematics learning can be collaboratively approached.

In the first paper, ‘Connecting and integrating theoretical frames: the TELMA contribution’, M. Artigue, M. Cerulli, M. Haspekian and M. Maracci analyse the approach followed in TELMA to face the problems posed by the accumulation of a multiplicity of conceptual and methodological tools poorly connected. Rather than looking for a unified theory, the paper addresses theory from an operational point of view. The notions of “didactical functionality” of an ICT-based tool and of “key concerns”, that is, issues functionally important, are, thus, presented and discussed making reference to two case studies taken from the cross-experimentation project and to some of the results obtained.

In the next paper, ‘Representation in computational environments: epistemological and social distance’, C. Morgan, M. A. Mariotti and L. Maffei, address the problem of fragmentation in the perceptions and terminologies used by the TELMA teams for the notion of external mathematical representations. The paper illustrates how a set of specific questions were designed within the framework of cross-experimentation to tease out coherences and differences in the ways in which representations are perceived and put to use by the different teams. The paper uses the notion of epistemological distance between different representations and the ways in which this may affect meaning making. In particular, it focuses on the dynamic nature of the different representations used and on the kind of feedback provided. The notion of social, curricular, and pedagogical distance of computational representations is also introduced with respect to those used in educational practices. These ideas are analytically discussed through the description of three case studies taken from the cross-experiments.

In the following paper, ‘The role of context in research involving the design and use of digital media for the learning of mathematics: boundary objects as vehicles for integration’, C. Kynigos and G. Psycharis discuss the way in which fragmentation can be addressed while addressing context-related issues. The paper looks at context as a complex notion and gives a brief overview of how it has been hitherto addressed in the research community. It adopts a pragmatic view of context focusing on some of its components such as the organisational pragmatics of design and development of digital media, the researchers’ epistemological and pedagogical assumptions and approaches, the school contexts and the wider socio-systemic milieu within which empirical interventionist design research is carried out. The paper uses the notion of boundary objects to analyse how the production of and response to key questions addressing context may operate as a medium for the discrimination of contextual differences between teams and on how these may affect the researchers’ perspectives. Case studies from the cross-experimentations are used to operationally show how the questions were used as a boundary object. The paper proposes the intentional design of a variety of such objects as tools for integrated approaches to research involving diverse communities of practice.

In the final paper titled ‘An analysis of interactive learning environments through an integrative perspective’, J. Traglova, D. Bouhineau and J. F. Nicaud provide an overview of interactive learning environments dedicated to arithmetic and algebra with the aim to identify and compare didactical functionalities of these systems in terms of available (or not available) features and functionalities. The paper refers to a methodological tool elaborated within TELMA to frame analysis by inspection of a number of aspects of the considered systems. It helps to better positioning the ILEs developed by TELMA teams in the wide landscape of digital tools devoted to the teaching and learning of arithmetic and algebra.

Results and considerations from all the papers presented in this Special Issue revealed the scope of this style of collaborative research as a means to forge links between theoretical frameworks by developing a language amongst the researchers necessary to communicate at such a detailed and functional level. We realized that even at this level of small experiments with existing tools such a language began to develop within the pragmatics of cross-experimentation. Although we found the TELMA studies illuminative in their own right, we felt that engaging in a more holistic collaborative experience would help us elaborate our integrating language of mutual understanding and connectivity. The TELMA study thus constituted the conceptual and methodological basis for a subsequent larger study titled 'the REMATH project—Representing Mathematics with Digital Media, 2005–2009' (Artigue 2008; Artigue et al. 2009). This study was co-funded by the European Commission (IST-4-26751) and had two main goals: the development of ICT-based tools for mathematics education at secondary school level and the design and experimentation, in different school contexts, of learning activities involving the use of such tools (http://remath.cti.gr). The aim here was to gain shared experience at the scale of a project transcending the whole iterative cycle of ILE development: design, implementation, and classroom practice. It was felt that building tools to be explicit about the process of co-production of ILEs and research would greatly help with the production of a language that could operationally help make constructive connections between fragmented theoretical frameworks from within both the fields of mathematics education and ICT in education.

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