Hyperfine Interactions (2019) 240: 74
https://doi.org/10.1007/s10751-019-1620-9

Effect of quadrupole deformation & temperature on bubble structure in $N = 14$ nuclei

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Published online: 27 June 2019

Abstract

The anti-bubble effect of the quadrupole deformation in the light nuclei is investigated by applying the relativistic mean-field (RMF) plus state dependent BCS approach. We perform a systematic study of $N = 14$ isotonic chain to understand the influence of deformation on the occupancy and depletion fraction (D.F. = $(\rho_{max} - \rho_c)/\rho_{max}$, where ρ_{max} and ρ_c are maximum and central densities, respectively). The quenching effect of deformation is found very predominant in light nuclei. In view of the fact that apart from deformation, temperature is also expected to hinder or rather completely wash out the bubble effect, we investigate the interesting role of deformation and temperature together in the quenching of proton bubble in the well deformed ^{24}Ne and ^{32}Ar.

Keywords Relativistic mean-field plus BCS approach · Bubble nuclei · Depletion fraction · Quadrupole deformation

This article is part of the Topical Collection on *Proceedings of the International Conference on Hyperfine Interactions and their Applications (HYPERFINE 2019), Goa, India, 10-15 February 2019*
Edited by S. N. Mishra, P. L. Paulose and R. Palit

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1 Introduction

Recently, the bubble structure [1–7] has become an interesting topic of the nuclear physics research which is identified by the depletion in the central nucleonic density related to the shell effects associated with the occupation of s-orbit in the light nuclei. In heavy nuclei, it is mainly due to the Coulomb repulsion and strong forces where the shell effects may only have a subtle role [8–13]. In recent works, the pairing correlations, deformation and the temperature have been indicated to hinder the bubble effect which needs some detail investigation. After the very, first microscopic model of the bubble structure [14] and recently the first experimental evidence of the bubble effect in ^{34}Si [15], many theoretical works [1–7] using various models like the relativistic Hartree-Fock-Bogoliubov (RHFB) theory [16], ab-initio self consistent Green's function many-body method [17], nuclear density functional theory [8], Skyrme Hartree-Fock mean-field [18], and the relativistic mean-field models [19–21] by us, have provided reasonable amount of information on the bubble effect. The equilibrium deformation has been indicated [17, 21] to quench the bubble effect whereas the temperature is expected to eliminate the bubble structure completely [22]. Hence the purpose of this work is to reveal the role of deformation and temperature on the bubble effect in the light nuclei. A systematic study of the depletion in deformed bubble candidates at temperature (T) ≥ 0 which has not been done so far except for a few recent works [18, 22], is presented.

The s-orbit having zero orbital angular momentum ($\ell = 0$) give rise to large central density when occupied. Its unoccupancy may result in the central density depletion which is believed to be the predominant cause of formation of bubble. Sometimes, the unoccupied s-orbit and nearby single-particle shells can favor collective correlations and thus lower or even wash out the central density depletion. Hence the maximum bubble effect comes by s-orbitals, which is surrounded by the orbitals with $\ell \neq 0$ near the Fermi level, and is well separated in energy from its nearby single-particle states so that the dynamical correlations are weak. Some recent works [2, 7, 16, 23, 24] have shown that the dynamical quadrupole shape effects hinder the bubble formation. The nuclear tensor-force and the pairing correlations also have been conjectured to have important implications in the shell evolution and the bubble structure [2, 3, 24, 25]. The pairing correlations hinder the proton bubble structure in 46Ar [3, 25] whereas the tensor force seems to favour it [2, 3, 24, 25]. Relativistic Mean-Field theory with TMA parameter [26–28] has shown significant depletion due to inversion of 2 s and 1d states without including the tensor force [1–3, 24] as shown in our recent work [20]. Calculations with the ab-initio many-body method for the experimentally identified bubble nucleus ^{34}Si [17] showed that the dynamical correlations reduce the depletion fraction from 0.34 to 0.15 without washing out the bubble structure entirely. The influence of binding energy and the deformation on the bubble structure has also been discussed in one of our recent works [21]. Encouraged with the above mentioned attempts on the quenching of bubble, the study on the quenching of neutron bubble due to deformation is still lacking, which is the objective of this work.

2 Relativistic mean-field theory

RMF calculations have been carried out using the model Lagrangian density with nonlinear terms both for the σ and ω mesons as described in detail in Refs. [27, 28].

$$
\begin{aligned}
\mathcal{L} &= \overline{\psi}\left[\iota\gamma^{\mu}\partial_{\mu}-M\right] \\
&+\frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma-\frac{1}{2}m_{\sigma}^{2}\sigma^{2}-\frac{1}{3}g_{2}\sigma^{3}-\frac{1}{4}g_{3}\sigma^{4}-g_{\sigma}\overline{\psi}\sigma\psi \\
&-\frac{1}{4}H_{\mu\nu}H^{\mu\nu}+\frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu}+\frac{1}{4}c_{3}\left(\omega_{\mu}\omega^{\mu}\right)^{2}-g_{\omega}\overline{\psi}\gamma^{\mu}\psi\omega_{\mu} \\
&-\frac{1}{4}G_{\mu\nu}^{a}G^{a\mu\nu}+\frac{1}{2}m_{\rho}^{2}\rho_{\mu}^{a}\rho^{a\mu}-g_{\rho}\overline{\psi}\gamma_{\mu}\tau^{a}\psi\rho^{\mu a} \\
&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-e\overline{\psi}\gamma_{\mu}\frac{(1-\tau_{3})}{2}A^{\mu}\psi
\end{aligned}
\tag{1}
$$

where the field tensors H, G and F for the vector fields are defined by

$$
\begin{aligned}
H_{\mu\nu} &= \partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}, \\
G_{\mu\nu}^{a} &= \partial_{\mu}\rho_{\nu}^{a}-\partial_{\nu}\rho_{\mu}^{a}-2g_{\rho}\epsilon^{abc}\rho_{\mu}^{b}\rho_{\nu}^{c} \\
F_{\mu\nu} &= \partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}
\end{aligned}
$$

and other symbols have their usual meaning. The corresponding Dirac equations for nucleons and Klein-Gordon equations for mesons obtained with the mean-field approximation are solved by the expansion method on the widely used axially deformed Harmonic-Oscillator basis [29, 30]. The quadrupole constrained calculations have been performed for all the nuclei considered here in order to obtain their potential energy surfaces (PESs) and determine the corresponding ground-state deformations [29, 31]. In the calculations, we use pairing interaction as delta force, i.e., $V = -V_0\delta(r)$ with the strength $V_0 = 350$ MeV fm^3 which has been used in Refs. [28, 32] for the successful description of drip-line nuclei. Apart from its simplicity, the applicability and justification of using such a δ-function form of interaction has been discussed in Ref. [33], whereby it has been shown in the context of HFB calculations that the use of a delta force in a finite space simulates the effect of finite range interaction in a phenomenological manner (see also [34] for more details). In the present work, the single-particle states subject to the pairing interaction are confined to the region satisfying

$$\epsilon_i-\lambda \ \leq E_{cut} \tag{2}$$

where ϵ_i is the single-particle energy, λ the Fermi energy, and E_{cut} = 8.0 MeV. The center-of mass correction is approximated by

$$E_{cm} = -\frac{3}{4}41A^{-3/4} \tag{3}$$

which is often used in the relativistic mean field theory among the many recipes for the center-of-mass correction [35]. For further details of these formulations we refer the reader to Refs. [27, 29, 30].

3 Results and discussions

After the validation of proton bubble ($Z = 14$) in ^{34}Si [15], it is an obvious choice to check the possibility of a neutron bubble with neutron number $N = 14$. In oneof our work [20], ^{34}Ca has already been pointed out as a strong neutron bubble candidate similar to the proton bubble candidate ^{34}Si. To check the effect of quadrupole deformation on neutron bubble structure, we

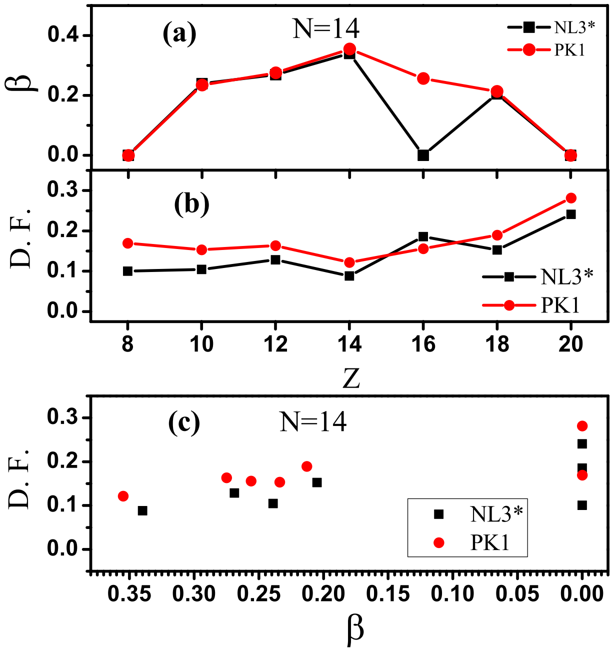

Fig. 1 (Colour online) (**a**) Quadrupole deformation (β) vs. Z (**b**) Depletion fraction (D.F.) vs. Z (**c**) Depletion fraction (D.F.) vs. β

perform our quadrupole constrained calculations for the full isotonic chain with neutron number $N = 14$ and plot quadrupole deformation (β) vs. proton number Z in Fig. 1a, neutron depletion fraction (D.F.) vs. Z in Fig. 1b and neutron depletion fraction vs. β in Fig. 1c. From Fig. 1a, it is observed that except ^{22}O and ^{34}Ca, all other nuclei in N = 14 isotonic chains are found deformed with both NL3* [36] and PK1 [37] parameters. However, the value of quadrupole deformation (β) for ^{30}S is found model dependent. The value of β is found maximum for the case of ^{28}Si in this isotonic chain. The influence of deformation on the depletion fraction (D.F.) is seen in Fig. 1a and b where β and D.F. show inverse dependance. In Fig. 1c also, we find that D.F. has wide range for spherical nuclei and it is found maximum for the spherical nucleus ^{34}Ca. By enlarge, for deformed nuclei, D.F. is found lesser than few spherical nuclei and minimum for well deformed nucleus^{28}Si. The above discussion indicates the anti-bubble influence of deformation.

Another important parameter which can quench the bubble is the temperature which has been shown recently [18, 22]. To understand the quenching of bubble effect under the influence of deformation and temperature, we have picked up well deformed potential bubble nuclei ^{24}Ne and ^{32}Ar from the $N = 14$ isotonic chain that have also been reported in [38] recently. These candidates are found possible deformed candidates of proton bubble. Therefore, we evaluate and plot in Fig. 2 (i) the deformation (ii) occupancy of 2 s orbit and (iii) depletion fraction of ^{24}Ne and ^{32}Ar as a function of temperature T varying from 0 to 4 MeV. It is observed that at $T = 0$, 24Ne is deformed and the occupancy of 2 s orbit is zero and the value of D.F. is highest which indicates the bubble effect even though the nucleus is well deformed. With increasing T, deformation reduces to almost zero at $T > 1$ MeV where the occupation probability starts increasing. As T increases, D.F. decreases and reaches a minimum value at a

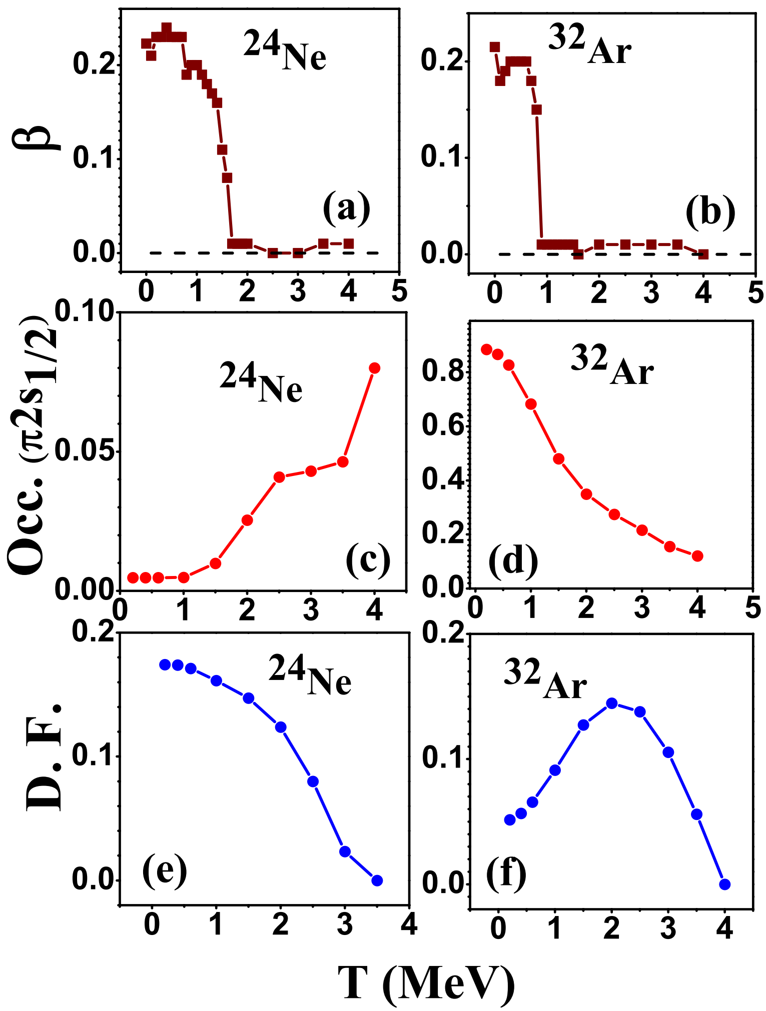

Fig. 2 (Colour online) (**a**) and (**b**) deformation β, (**c**) and (**d**) proton occupation probability, and (**e**) and (**f**) proton D.F. for ^{24}Ne and ^{32}Ar vs. temperature (T)

critical T around 3–4 MeV showing the vanishing of central depletion due to temperature. Although the quenching of bubble effect should decrease with the decreasing deformation due to increasing temperature, but the influence of deformation appears to be effective in ground state only and as T increases, the deformation starts vanishing and the quenching of bubble effect mainly caused by the temperature only. The Variation of occupancy and the D.F. of proton rich ^{32}Ar appears to be entirely different from that of ^{24}Ne. It may be noted from Fig. 2d and f, that at $T = 0$, the proton occupation probability is high around 0.8 and the D.F. shows a very low value. As T increases, occupancy in 2 s orbit decreases due to the particle occupying higher levels and hence the D.F. increases. High D.F. at $T \approx 2$ MeV indicates bubble structure which appears to be a unique feature. It is important to mention here that the deformation is able to only reduce the central depletion whereas the temperature T completely washes out the bubble effect as also suggested by [17]. The possible reason for this behaviour of ^{32}Ar may be drawn from the shell structure effects and further structural transitions taking place due to deformation and temperature. For$T>$ 2 MeV, D.F. starts decreasing which indicates the usual quenching of bubble effect as seen in ^{24}Ne deformed nucleus in Fig. 2e.

4 Conclusions

Quenching of neutron bubble structure has been explored in the light mass region using the relativistic mean-field plus BCS approach using NL3* and PK1 parameters. A systematic study of quenching of bubble structure due to deformation has been performed for $N=14$ isotonic chain. Deformation appears to play very significant role in the central depletion in this region. ^{34}Ca is found to be the best candidate of neutron bubble structure because of its zero deformation. Deformation is found maximum for ^{28}Si and consequently the depletion is minimum in ^{28}Si among all the nuclei studied in $N=14$ isotonic chain. Temperature effect along with deformation on proton bubble structure is also studied for the case of ^{24}Ne and ^{32}Ar from $N=14$ isotones. Deformation only quenches the bubble structure whereas the temperature completely wash them out.

Acknowledgements G. Saxena and M. Aggarwal acknowledge the support by SERB for YSS/2015/000952 and WOS-A schemes respectively.

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