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NUMERICAL MODELING OF A CASCADE HYDRODYNAMIC BREAKDOWN AT THE VERKHNEURAL'SK AND MAGNITOGORSK DAMS

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The results of numerical modeling are presented for a complex cascade hydrodynamic breakdown (not based on an actual threat) caused by a dam break in the body of the Verkhneural'sk embankment dam. The studies were carried out by the methods of numerical hydrodynamic modeling using the Russian software package STREAM 2D CUDA.

Keywords: numerical modeling; cascade hydrodynamic breakdown; flood; dam break; embankment dam.

The object and the research problem. The object of study was a section of the upper course of the Ural River, including the Verkhneural'sk and Magnitogorsk reservoirs, as well as the retaining hydraulic structures on them. The objective of this work is the numerical modeling of a hydrodynamic breakdown on the embankment dam of the Verkhneural'sk reservoir and the spread of the breakthrough wave downstream, resulting in a cascade breakdown on the Magnitogorsk hydraulic unit.

A special feature of the stated task consisted of the fact that, first, the Verkhneural'sk and Magnitogorsk reservoirs are used jointly and are arranged closely to each other, and therefore it will not be possible to eliminate a cascade breakdown. Secondly, there are very many objects of infrastructure in the downstream of the Verkhneural'sk dam, namely highway and railway bridges and embankments leading to them that can be destroyed by a flood. In addition, the Magnitogorsk reservoir has a rather large buffer zone that has been walled off from the basic part by dams and has a connection with it through canal locks and a regulator.

The Verkhneural'sk hydraulic unit contains (Fig. 1): an embankment dam, a flood spillway, and bottom drainage, combined with a hydraulic power plant (HPP) and an HPP structure. In accordance with the declaration of safety [1], the Verkhneural'sk hydraulic unit is treated as class I.

Key parameters of the alluvial sand-and-gravel dam of the Verkhneural'sk hydraulic unit: dam length along the crest, 1480 m; dam width along the crest, 10 m; maximum width of the dam at the foot, 240 m; maximum height of the dam, 27 m; and maximum pressure, 22 m.

The Magnitogorsk hydraulic unit includes: a stone-and-embankment dam, a flood spillway, bottom drainage, a gravity dam, and capital class III.

Key parameters of the stone-earth embankment dam of the Magnitogorsk hydraulic unit: dam length along the crest, 728.0 m; dam width along the crest, 6.0 m (expanded to 14.0 m at the spillway); maximum width of the dam at the foot, 85.0 m; and maximum height of the dam, 17.5 m.

Methodology of the study. Numerical hydrodynamic models of the advance of breakthrough waves in pools of hydraulic units began to develop intensively from 1960s and 1970s after publication of a well-known analytical solution by J. A Stoker [2] on dam failure and in connection with the development of computer equipment.

From among Russian scientists, Academician of the AN SSSR O. F. Vasil'ev [3, 4]; M. T. Gladyshev [3], B. L. Istorik [5], V. A. Prokofiev [6], S. Ya. Shkolnikov [7], B. V. Ostapenko [8, 9], and others, introduced a major contribution to the development of mathematical and numerical models of breakthrough waves.

As far back as late 1950s, Academician of the AN SSSR S. K. Godunov developed an effective and explicit in time numerical framework for the solution of the equations of fluid dynamics [10, 11]. In 1985, this was adapted by V. V. Belikov and A. Yu. Semionov for solution of two-dimensional equations of shallow water on hybrid networks, taking into account the precise solution for the breakdown of an arbitrary hydrodynamic discontinuity (the so-called Riemann problem) on a horizontal bottom [12, 13]. This algorithm is also explained in sufficient detail in [14]. Abroad, practically at the same time simplified alternatives of Godunov's plan for the equations of shallow water began to be de-

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Fig. 1. Photo of the structures of the Verkhneural'sk hydraulic unit.

veloped. Hence Roe's approximate method [15] was adapted by Glaister for the equations of the theory of shallow water [16], and is widely used for modeling various types of flows. The Hartena – Osher plan [17] is also used.

However, the presence of an irregular bottom in the equations of shallow water did not permit extending automatically the gas-dynamic algorithms to this class of problems. And the use of the approximation approaches to the solution of the problem on the breakdown of an arbitrary discontinuity drew some researchers aside from the determination of a precise and unique solution of a Riemann problem for the equations of shallow water on a discontinuous bottom. The precise method for solution of the problem of the breakdown of a discontinuity was developed for the case of an irregular (discontinuous) bottom in [18, 19], which makes it possible to enhance the effectiveness and precision of modeling and also ensure the existence and uniqueness of a solution of the Riemann problem for any initial data.

Godunov's plan for the equations of shallow water, taking into account the solution of the problem of the breakdown of an arbitrary discontinuity on an irregular bottom turned out to be the most effective for modeling the breakthrough waves. This plan is situated as the foundation of the Russian software package STREAM 2D CUDA [20], which has been used for modeling in this work.

The most important component of the algorithm for calculating a breakthrough wave is also the possibility of calculating the scouring of the embankment dam and embankment structures located in the downstream of the hydraulic unit (for example, automobile and railway causeways, and also other dams). There are a number of methodologies for calculating the formation and development of a dam break in a embankment dam, among which one of the most reliable and useful is the methodology of A. M. Prudovskii [21]. However, the latter does not permit calculating scenarios with a spillover through the dam crest or a traffic-bearing surface of major length.

The calculation of a dam break in the body of the embankment dam of the Verkhneural'sk reservoir was executed using the physical and mathematical model of the development of dam breaks in embankment dams that is included in STREAM 2D CUDA. The model is based on the hydromorphological method of numerical modeling of the development of a dam break in dams composed of homogeneous and inhomogeneous soil. The description, justification, verification, and examples of the application are featured in [22].

The physical and mathematical model of the development of a dam break describes the convective transposition of soil particles by the flow, taking account of scouring and sedimentations of deposits, and is augmented by diffusion terms that account for the change of marks of the bottom in time, based on the known effect of transverse flattening out of an underwater slope. For above-water slopes (dam break side edges), a diffusion model that begins to operate when the angle of the dry slope exceeds the limiting angle of stability is selected. This approach makes it possible to model the development of a dam break in embankment dams without the use of additional hypotheses on the form of the dam break, the ratio of the intensity of the removal of soil from the bottom and side slopes of the dam break, and so forth [22].

Numerical model of an object. In order to account for the formation and spread of a breakthrough wave during a hydrodynamic breakdown on the Verkhneural'sk embankment dam, there was developed a two-dimensional (lateral) uniform model of the Verkhneural'sk reservoir, a section of the Ural River valley, the Magnitogorsk reservoir, the city of Magnitogorsk within the boundaries of expected waterflooding, and a section of the downstream of the Magnitogorsk hydraulic unit. The extent of the numerical model along the Ural River channel was about 70 km.

A detailed three-dimensional digital contour model (DCM) was constructed in the form of a uniform editable triangulation surface (TIN). The contour of the bottom land is formulated based on a georaster of resolution 30 × 30 m, obtained from the resource <https://www.eorc.jaxa.jp>. Bathymetry of the Magnitogorsk reservoir is based on the measurements of 2022. The bathymetry of the Verkhneural'sk reservoir was made "provisional," proceeding from the principle of matching the total volume of water in the model with actual (by RUWR [23]) for an NWL level equal to 601 million m^3. Such an approach is fully permissible, since in mod-

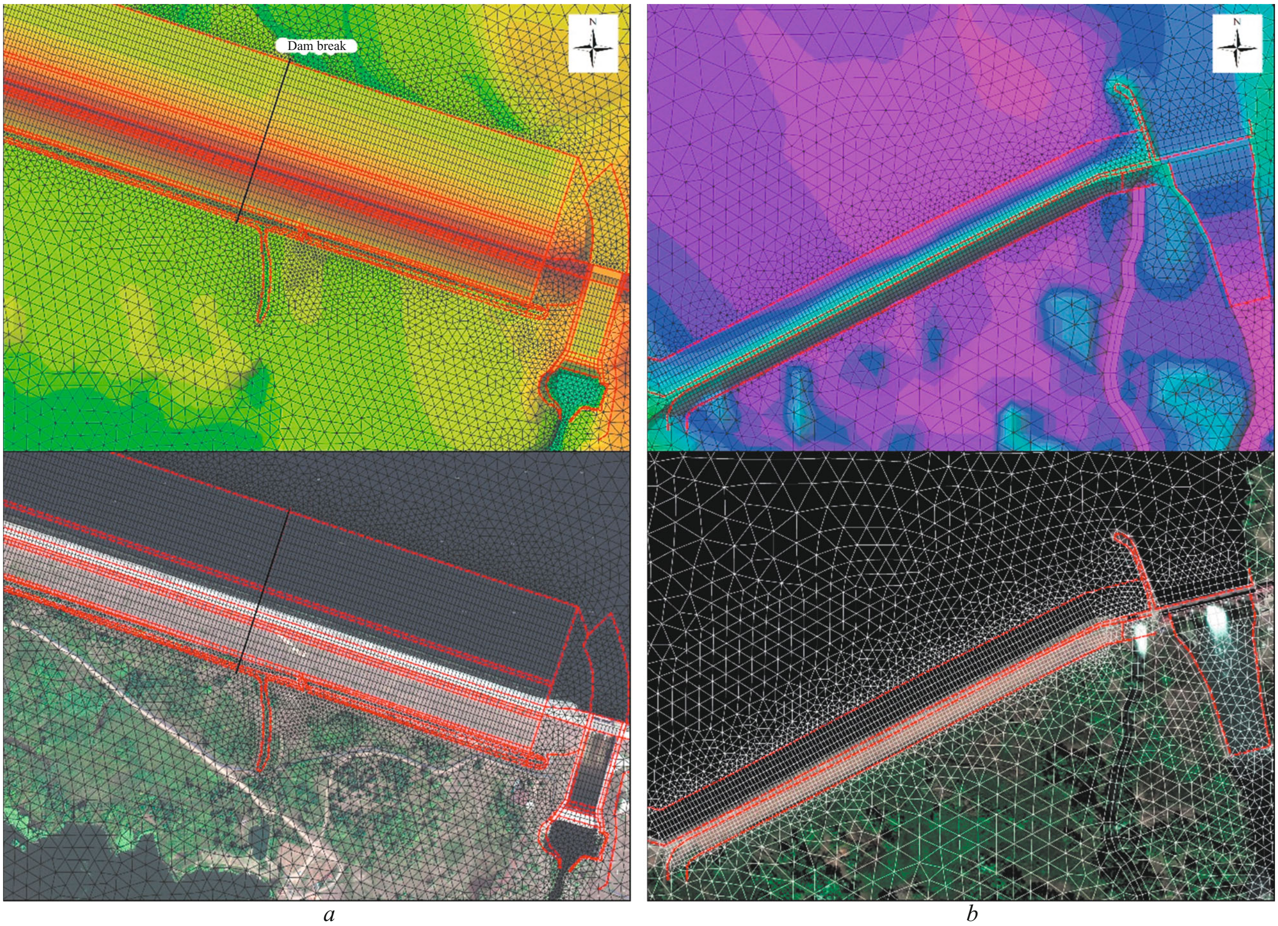

Fig. 2. Degree of detailing of the contour and the computational grid and fragments: *a*, Verkhneural’sk hydraulic unit; *b*, Magnitogorsk hydraulic unit.

eling the development of a dam break, the defining factors in the headwater are the volume of water in the reservoir and its configuration in the plan, and the contour of the bed plays a subsidiary role. However, for more complete correspondence with natural conditions, it is desirable to verify not only the total volume at the NWL, but also the intermediate values of the volumes of water for the specified levels. After several sequential iterations, a satisfactory correspondence of the volume-level dependences for the model and the data of RUWR was achieved.

The channel of the Ural River in the section between the Verkhneural’sk and Magnitogorsk reservoirs, as well as downstream of the Magnitogorsk dam, was formed of a trapezoidal profile with a mean depth of 2 m. The shoulders of the channel were determined by entering the topography of the georaster of the bottom land. Necessary adjustment of the contour by elevation coordination of the points of the channel and the bottom land was done.

Later, from the proprietary original procedure, a computational grid of irregular structure, adapted from plan outlines of the contour and structural components of the GTS, was constructed from triangular and quadrilateral cells,.

The computational grid and DCM are created with a rather high level of detailing (Fig. 2):

— the contour of the Verkhneural’sk embankment dam is built according to sketches of the safety declaration: practically all design features are included, and the cells of the computational grid are 5 × 5 and 5 × 10 m;

— 4 bridges and road embankments on the Ural River section from Verkhneural’sk to the Magnitogorsk reservoir, + 4 bridges on the Magnitogorsk reservoir, and the heights of the embankment are specified taking account of non-transfusion at 1% flow rate, and the grid cells are 5 × 10 and 5 × 15 m (1 – 2 cells on the crest and the embankment slopes);

— dams of the Magnitogorsk reservoir (in total about 15), grid cells of width from 5 to 10 m and length 15 – 20 m are predominantly rectangular, and on some slopes triangular;

— the Magnitogorsk dam was formulated more provisionally as a simple trapezoid, and the placements of the upper and lower slopes are taken from the mean values from the declaration (the sketches were lost), and the size of a grid cell is 5 × 10 m;

— along the Ural River channel, a rectangular grid was constructed, of a quadrangular network from 5 to 3 cells across the channel with side lengths from 5 to 15 m, and from 15 to 30 m along the channel;

— in the region of the Verkhneural'sk and Magnitogorsk reservoirs, as well as floodplains, a triangular grid of irregular structure was constructed, with a compression towards the channel and structures and with an increase in the lengths of sides of the cells in remote sections (varied from 15 to 120 m).

The total number of cells of the computational grid was more than 240,000, with side lengths from 5 m to 150 m.

In the numerical model, 20 constraints were set: 6 for representation of the various input and output conditions, and 14 interior, to monitor the parameters during the calculation.

The values of coefficients of roughness (*n*) were assigned from experience with calculations of other plants. For the channel of the Ural River, *n* was taken as 0.025; for concrete parts of spillway structures, taking into account the duration of the service life, 0.022; for the earthen components of dams, bulkheads, embankments, banks of roads, and islands with moderate vegetation, 0.03; overgrown, 0.04; and bottom land sections, 0.04 – 0.045.

Scenarios of calculations. Implementation of the scenarios presented in the Declaration of Safety of the GTS [1] on a numerical model was performed as follows:

Scenario 1 (the most probable one) occurs on the background of a low-water period, with water level in the Verkhneural'sk reservoir equal to the NWL [normal water level], and the flow rate of inflow to the reservoir is 15 m^3/sec.

Scenario 2 (the weightiest) occurs against the background of a flood with provision of 0.01% (the verification case for structures of class I of the degree of durability) with water level in the Verkhneural'sk reservoir equal to the HWL [high water level], and the flow rate of inflow to the reservoir is set according to conventional hydrography.

All road embankments specified in the model, bulkheads, and the Magnitogorsk embankment dam were set as being scoured. Since the Verkhneural'sk and Magnitogorsk embankment dams are constituted of soils that are various in fractional composition, it was decided in the model to specify soil with three fractions of various mean diameter (Table 1) and percentage relation of the particles in the overall volume (Table 2).

TABLE 1. Fractional Composition of the Soil of the Model

Fraction number	Diameters of particles, mm	
	d_{50}	d_{90}
1	300	500
2	2	10
3	0.25	0.7

Initial conditions for scenario 1 were steady-state streams with a flow rate of 15 m^3/sec, and the thresholds for discharges of the water of the Verkhneural'sk and Magnitogorsk hydraulic units were selected at the passage of the given flow rate and observance of the requirement to maintain the NWL in the headwaters of reservoirs, and there is no backflow in the buffer part of the Magnitogorsk reservoir. At the crest of the Verkhneural'sk embankment dam, the initial dam break was recessed to a height 0.5 m lower than the NWL. The water flow at the initial dam break results in dam washout, and the maximum width of the opening of the dam break is determined in the process of calculation. It is assumed that 3 h after the occurrence of the failure on the Verkhneural'sk dam, the spillway dam at the Magnitogorsk hydraulic unit will be opened and discharge of the Magnitogorsk reservoir will begin.

Initial conditions for scenario 2 were steady-state streams with a flow rate of probability 1% (866 m^3/sec). The conditions on the spillway dams of the Verkhneural'sk and Magnitogorsk hydraulic units are analogous to scenario 1. In the course of 24 h, the flow rate of the current into the Verkhneural'sk reservoir is increased up to 0.01% (2280 m^3/sec). The Verkhneural'sk reservoir is gradually filled, and the level of the headwater reaches the HWL. The initial dam break on the crest of the Verkhneural'sk embankment dam was recessed 0.5 m below the mark of the HWL. The water flow at the initial dam break results in a dam scour, and the maximum width of the dam break is determined by calculation.

Results of calculations. In the course of calculating a dam break in the body of an embankment dam of the Verkhneural'sk hydraulic unit, three stages of its development are derived: 1) an incut, 2) intensive sideways spread, 3) stabilization on width.

In the execution of scenario 1 (most probable) of a hydrodynamic breakdown on the Verkhneural'sk dam, an incut of the dam break proceeds with insignificant side extension for 2.2 h. Intensive side extension lasts for the following 5 – 6 h, and after 8 h from the occurrence of the failure, the dam break is stabilized on width, and an insignificant reorganizations of the base of dam and scours of the upper slope are noted. As a result, the dam break will be the maxi-

TABLE 2. Percentage Ratio of the Soil Particles of the Model

No.	Name of structure	Index number of fraction	Ratio of fractions, %		
			1	2	3
1	Verkhneural'sk dam	2; 3	0	70	30
2	Magnitogorsk dam	1; 2; 3	90	5	5
3	Automobile and railway banks	2; 3	0	60	40
4	Dams of the Magnitogorsk reservoirs	2; 3	0	50	50

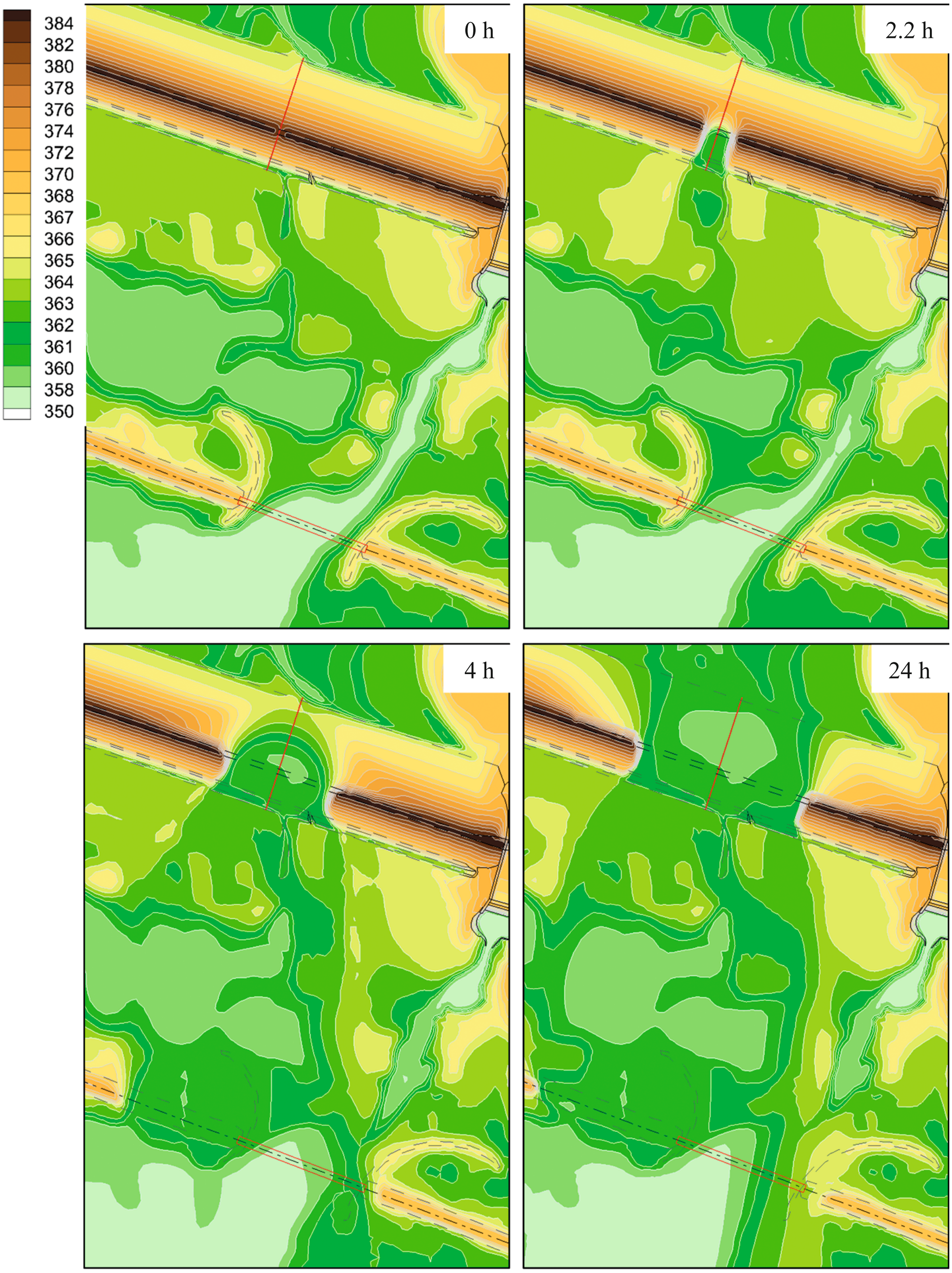

Fig. 3. Process of the failure of the Verkhneural'sk dam and bridge No. 1 (scenario 1).

mum width of 360 m (Fig. 3). The maximum flow rate at the spillway on the Verkhneural'sk dam will be 27,674 m^3/sec, at the site of the Magnitogorsk hydraulic unit 24,860 m^3/sec, and at the exit boundary of the model 19,300 m^3/sec (the flow rate decreases downstream because of the accumulation of part of the water on the bottom land).

In the execution of scenario 2 (weightiest) of a hydrodynamic breakdown on the Verkhneural'sk dam, for 32 h there will be filling of the Verkhneural'sk reservoir. When the water level equal to the HWL is reached near the dam at the crest of the embankment dam, spillover by a thin jet through the site of the initial dam break is generated, this process takes about 2 – 2.5 h, then begins the stage of an incut of the dam break with insignificant extension and in 37 h from the beginning of the calculation (or 5 h from the beginning of the spillover) the lower part of the dam break reaches the base of

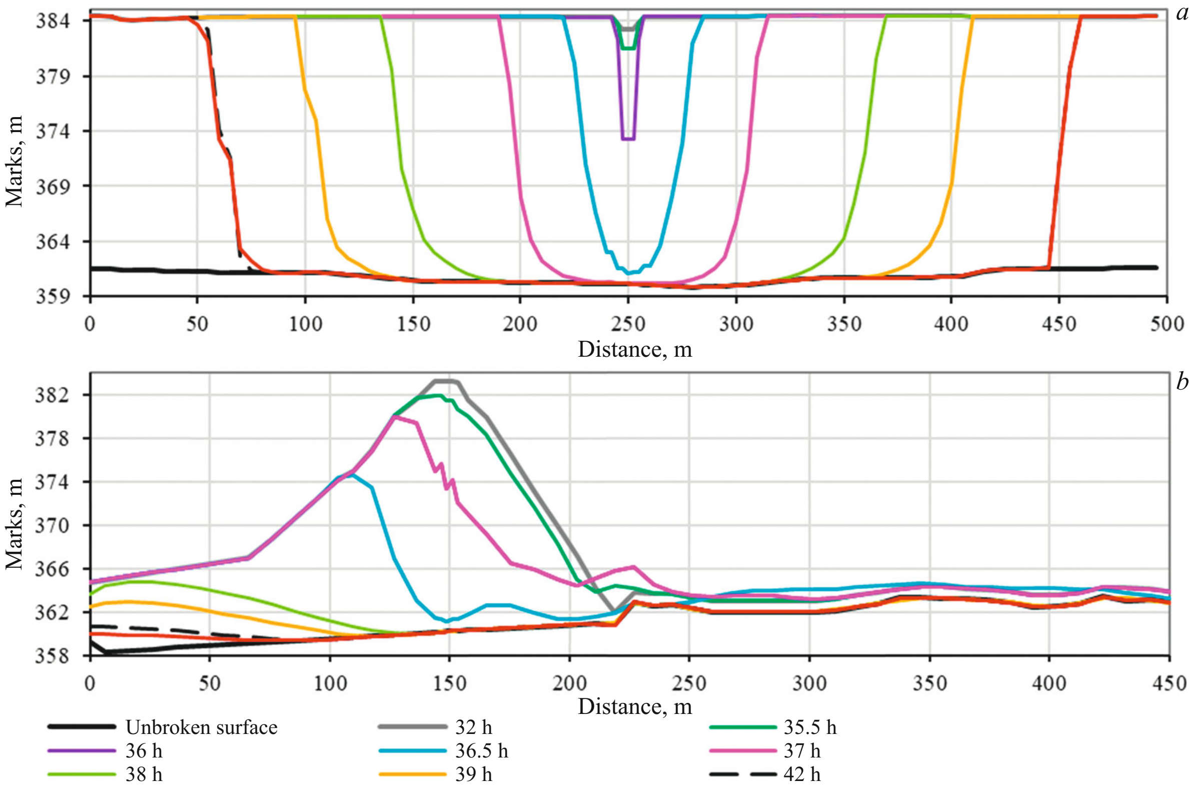

Fig. 4. Profiles of the bottom on a dam break of the Verkhneural'sk dam for scenario 2: *a*, longitudinal cross-section; *b*, transverse cross-section at site 1.

the dam. The dam break transitions to the stage of intensive side extension which lasts about 8 h more. The maximum width of the opening, equal to 415 m (Fig. 4), will be reached in 42 h from the beginning of the calculation (or 10 h from the beginning of the spillover), after which there will be free outflow from the upper to the lower pool lasting 28 h. The maximum flow rate of the outflow at the Verkhneural'sk dam will be 33,217 m^3/sec, at the site of the Magnitogorsk hydraulic unit 27,940 m^3/sec, and at the exit boundary of the model 21,600 m^3/sec.

For both scenarios, the road embankments and dams of the Magnitogorsk reservoir will be partially destroyed in the zones of the action of fast velocities (greater than 5 m/sec) due to water spillover through their crests. Erosion was modeled with continuous calculation according to the original procedure included in the STREAM 2D CUDA model. Flow velocities at the sites of bridge crossings over the Ural River are quite high, and their maximum values will be 5 – 8 m/sec and locally up to 10 m/sec (Fig. 5). With such velocities, all bridges will be destroyed.

At the Magnitogorsk hydraulic unit, the height of water will reach the crest of the stone-earth embankment dam crest within 9 and 10 h from the beginning of the breakdown on the Verkhneural'sk dam according to scenarios 1 and 2, respectively. Spillover over the crest will begin, with erosion of the lower slope at current velocities equal to 10 — 11 m/sec (Fig. 6), and partial (by height) damage of the stone-earth dam will occur on the entire overflowing front. This process will happen rather quickly, and after 0.3 h the stone-earth dam will be washed away to a height of 11 – 11.4 m, with the greatest height of the dam being 17.5 m (Fig. 7).

CONCLUSIONS

1. Numerical modeling of the calculated section was built with a sufficiently high level of contour detailing, taking account of practically all design features of the Verkhneural'sk embankment dam, automobile and railway banks of 8 bridge passages, dams of the Magnitogorsk reservoir, and the Magnitogorsk stone-embankment dam.

2. On the unified numerical model including the upper and lower pools of the cascade of the Verkhneural'sk and Magnitogorsk hydraulic units on the Ural River, it was possible to fulfill, with continuous calculation, modeling of the development of the primary dam break at the Verkhneural'sk embankment dam and subsequent scourings of the road and railway banks of bridge passages across the Ural River, and the partial failure of the Magnitogorsk stone-earth dam due to spillover across its crest.

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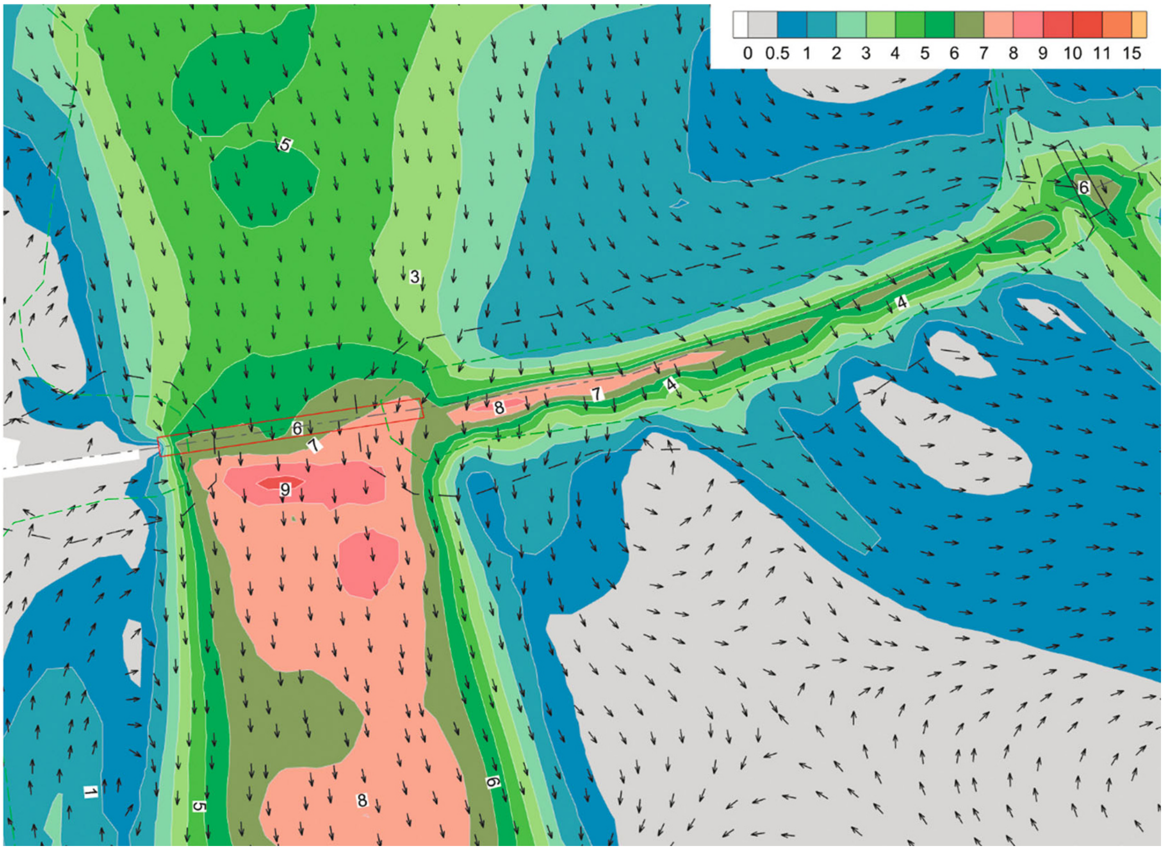

Fig. 5. Field of maximum current velocities at the site of bridge No. 8 under scenario 2.

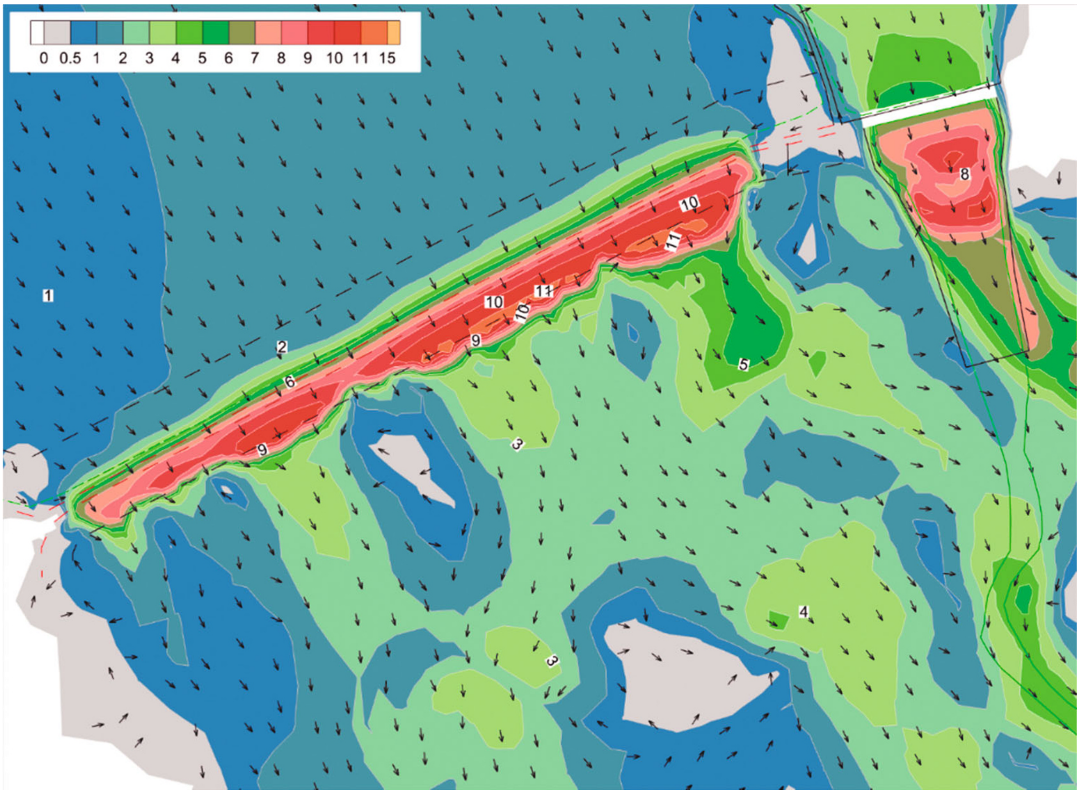

Fig. 6. Field of current velocities at the site of the Magnitogorsk hydraulic unit for scenario 1 at the initial moment of a spillover.

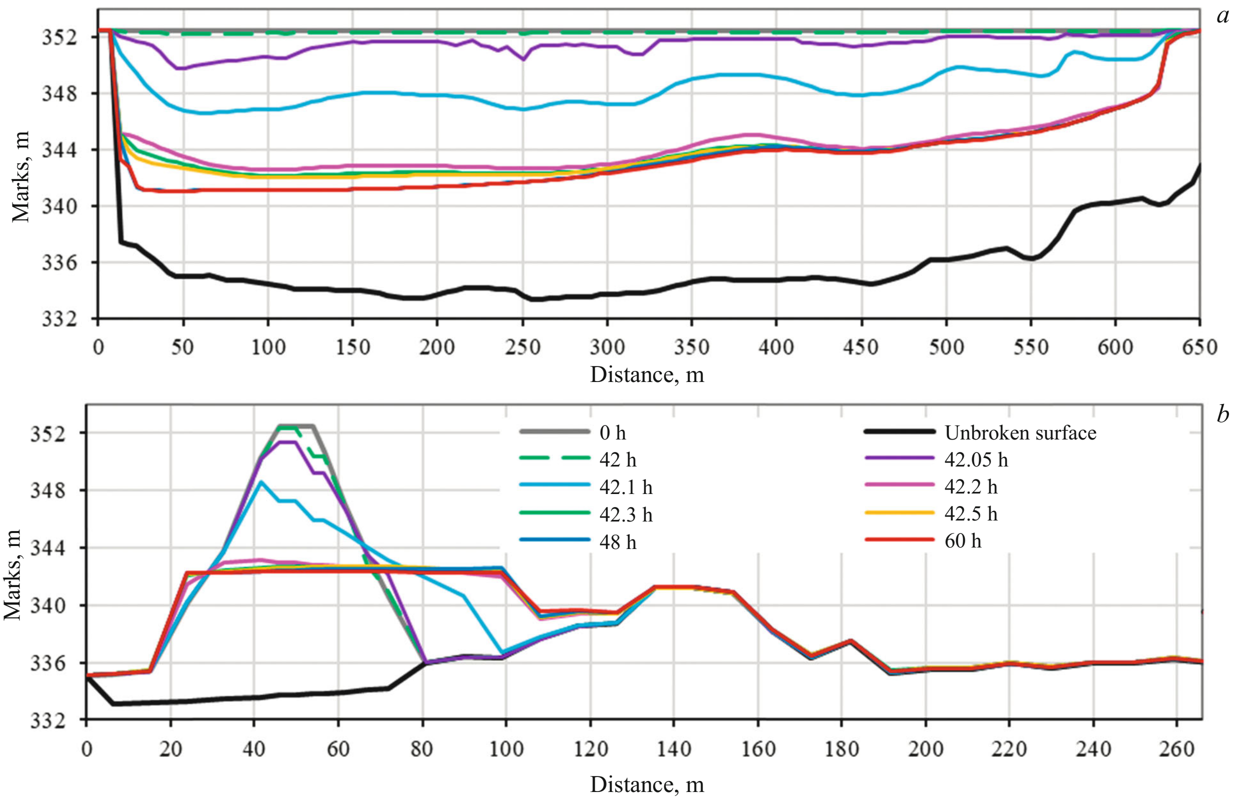

Fig. 7. Bottom profiles at the dam break of the Magnitogorsk dam for scenario 2: *a*, longitudinal section; *b*, cross-section at site 1.

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