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ACCURACY ANALYSIS OF THE NUMERICALLY CALCULATED DISCHARGE CHARACTERISTICS OF FLOW-MEASURING WEIRS AND FLUMES

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A comparative analysis of the empirical and numerically found discharge characteristics of the most common flow-measuring structures was performed. The results of testing the developed technical solutions by comparison with known experimental and theoretical dependencies are presented.

Keywords: computational fluid dynamics; flow-measuring structure; discharge characteristic; thin-plate weir; broad-crested weir; crest with a sloping front wall; critical-depth flume; Crump weir.

The flow of water through a flow-measuring structure has a complex three-dimensional character. The discharge characteristics of such structures depend both on the individual size and shape and on the flow structure in the approach canal. The discharge characteristics of almost all flow-measuring structures are obtained on the basis of experimental studies. The range of application and reliability of the existing empirical discharge dependences are limited by the accuracy and amount of experimental studies, on the basis of which these dependences are constructed. For example, the values of water discharge through a thin-plate weir, calculated using known formulas, can differ by up to 5%. The presence of the approach velocity coefficient C_v in modern empirical formulas, which is a function of the ratio of the total and geometric heads, does not ensure that the influence of the approach flow velocity profile is taken into account, which is essential for structures with a low headrace backwater.

The known theoretical methods for calculating the flow, that are based on the solution of the St. Venant's system of hyperbolic differential equations, are inapplicable for flow-measuring structures due to the presence of supercritical flow mode with the Froude code $Fr > 1$. The known numerical solutions based on the Navier – Stokes equations without taking into account the flow history have insufficient accuracy of determining the flow discharge for water metering. For example, in published studies, the error in calculating the flow discharge in the Parshall flume is 4 – 16% [5, 6].

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In this research, a comparative analysis of the empirical and numerically found discharge characteristics of the most common flow-measuring structures is performed. The results of testing the developed technical solutions by comparison with known experimental and theoretical dependencies are presented.

MATERIALS AND METHODS

The numerical calculation of the discharge characteristics of flow-measuring structures was carried out using the software and computing complex DisCo4 [1 – 3]. The operating principle of the complex is based on the numerical solution of the motion equations (Navier – Stokes) and preservation in a three-dimensional formulation by the finite volume method. The operations scope is standard: generation of a three-dimensional grid of the computational domain, setting the boundary and initial conditions, hydraulic calculation and post-processing of computational data. The computational grid includes a flow-measuring structure and sections of the inlet and outlet canals. The flow velocity formed as a result of the calculation is a function of the assigned pressure (depth in the headrace). The water discharge is calculated by integrating the velocity field in the supply canal.

The peculiarities of the data processing methodology include dividing the inlet boundary of the computational domain into separate patches, on which individual boundary conditions are set, and performing a hydraulic calculation in several automatically repeating cycles with the adjustment of

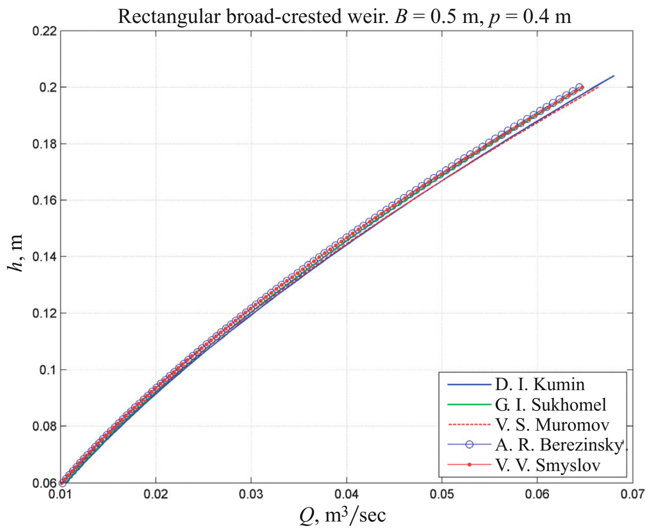

Fig. 1. Discharge characteristics of a broad-crested weir calculated according to Eqs. (2) – (6).

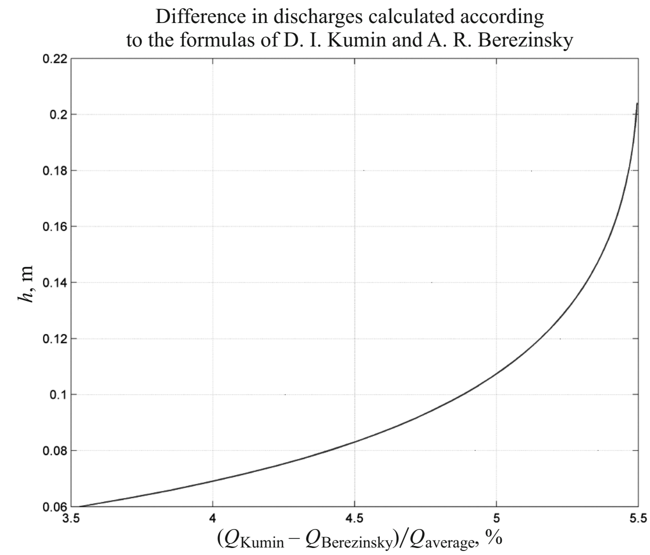

Fig. 2. Relative difference in discharges calculated by the formulas of D. I. Kumin and A. R. Berezinsky.

the boundary conditions according to the results of the previous calculation.

RESULTS

1. Broad-crested weir

The formula for the dependence $Q = f(H)$ has the form

$$Q = mb\sqrt{2g}H^{1.5}, \tag{1}$$

where m is the discharge coefficient; b is the width (in the longitudinal direction); H is the head.

Despite the fact that experimental and theoretical studies of a broad-crested weir (BCW) have been performed for over 100 years, due to a large number of influencing factors, there is still no generally accepted theory of flow through a weir. The greatest contribution to the study of this weir type was made by D. I. Kumin, G. I. Sukhomel, V. S. Muromov, A. R. Berezinsky, and V. V. Smyslov, who developed the following empirical formulas for the discharge coefficient of an clear overflow broad-crested weir with vertical front and rear walls [4]:

D. I. Kumin

$$m = m_s + \frac{0.385 - m_s}{1 + 2P/H},\ m = 0.32; \tag{2}$$

G. I. Sukhomel, I. L. Rozovsky, and V. V. Smyslov

$$m = 0.32 + 0.014H/P; \tag{3}$$

V. S. Muromov

$$m = 0.306\sqrt{\frac{P/H + 1}{P/H + 0.5}}; \tag{4}$$

A. R. Berezinsky

$$m = 0.32 + 0.01\frac{3 - P/H}{0.46 + 0.75P/H}; \tag{5}$$

V. V. Smyslov

$$m = 0.3 + \frac{0.08}{1 + P/H}. \tag{6}$$

To analyze the accuracy of determining the discharge, a numerical calculation of the discharge dependence of the BCW with the geometry and the heads range, selected on the base of the general limitations on the values of the limit head and the crest height for the given formulas, was carried out. Figure 1 shows the discharge characteristics of the BCW, calculated by Eqs. (2) – (6). Figure 2 shows the relative difference in discharges calculated according to the formulas of the authors who used the largest amount of experimental data to construct their formulas (D. I. Kumin and A. R. Berezinsky).

Taking into account the significant difference in discharges (3.5 – 5.5%) calculated by the empirical formulas, the axis of the corridor between the discharges calculated by Eqs. (2) – (6) was taken as a reference for comparison with the numerical calculations results (Fig. 3).

The relative difference in the discharge values found by numerical and empirical methods is $\delta_{mean} = 1.8\%$. The discharge characteristic, found by the developed method,

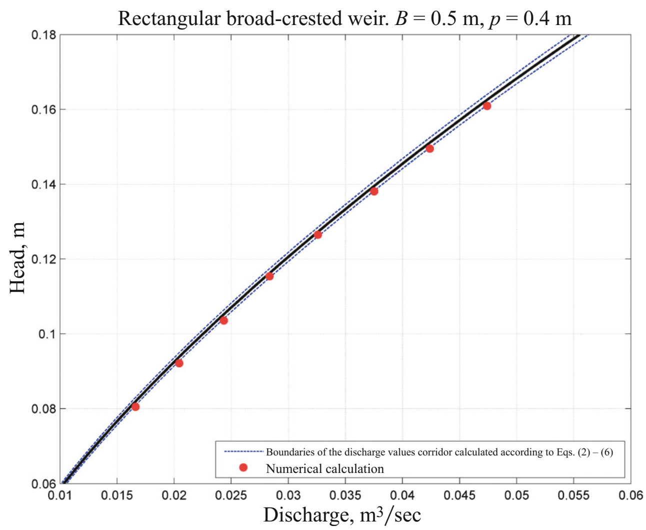

Fig. 3. Calculated values of the discharge through the broad-crested weir in the corridor of empirical discharge dependences calculated according to Eqs. (2) – (6).

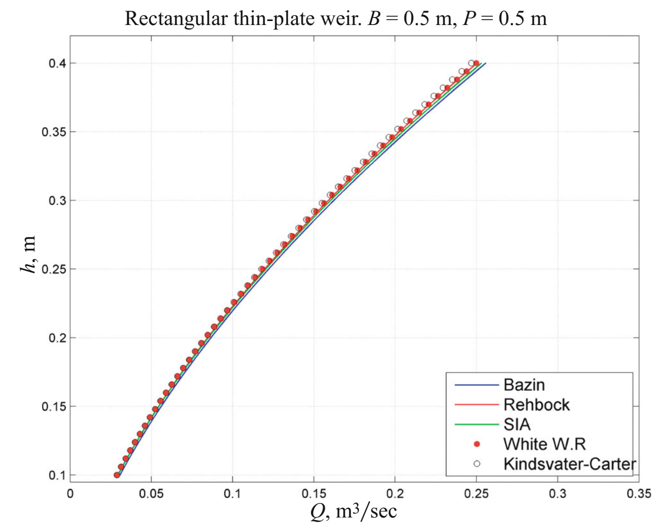

Fig. 4. Empirical discharge characteristics of a thin-plate weir calculated according to Eqs. (8) – (12).

most closely coincides with the empirical dependence of D. I. Kumin (δ_{mean} = 0.25%).

2. Thin-plate weir

Despite a number of significant disadvantages, due to the simplicity of implementation, thin-plate weirs (TPW) are widely used in land reclamation systems. In most cases, rectangular weirs are used, since, in comparison with triangular weirs, they have a wider range of discharges and a lower headrace backwater. An additional advantage is that they are well studied. The amount of theoretical and experimental studies devoted to this weir exceeds the number of studies of other TPW types.

The TPW in a rectangular canal without lateral flow compression was chosen as a reference for assessing the reliability of the calculation by the developed method, since, as preliminary calculations showed, the known formulas for the discharge dependence with lateral compression give significantly different results. Empirical equations for the discharge coefficient m in the formula for the discharge Q

$$Q = mb\sqrt{2gh}^{1.5} \tag{7}$$

have the following form [5].

H. Bazin

$$m = \left(0.405 + \frac{0.0027}{h}\right)\left[1 + 0.55\left(\frac{h}{h+P}\right)^2\right]. \tag{8}$$

Limitations: $P \geq 2H$; $b = 5h$; $h_{\min} = 0.06$ m; $h_{\max} = 0.65$ m.

T. Rehbock

$$m = 0.5689\left(1 + 0.1389\frac{h}{P}\right), \tag{9}$$

where $h = h_m - 0.0012$; h_m is the measured value.

Limitations: $h_{min} = 0.03$ m; $h_{max} = 0.75$ m; $P_{\min} = 0.1$ m; $(h/P)_{\max} = 1$; $b_{\min} = 0.3$ m.

SIA (Swiss Society of Engineers and Architects)

$$m = 0.580\left(1 + \frac{0.001}{h + 0.0016}\right)\left[1 + 0.5\left(\frac{h}{h+P}\right)^2\right]. \tag{10}$$

Limitations: $h_{min} = 0.025$ m; $h_{max} = 0.8$ m; $P_{\min} = 0.3$ m; $(h/P)_{\max} = 1$; $b_{\min} = 0.3$ m;

W. R. White (Hydraulic Research Station, England)

$$m = 0.562\left(1 + 0.153\frac{h}{P}\right). \tag{11}$$

Limitations: $h_{min} = 0.013$ m; $h_{max} = 0.34$ m; $P_{\min} = 0.15$ m; $(h/P)_{\max} = 2.2$; $b_{\min} = 0.2$ m.

C. Kindsvater, R. Carter, ISO 1438

$$m = \frac{2}{3}C_e,\ h = h_m + K_h;\ b = B + K_b, \tag{12}$$

where h_m is the measured value; B is the canal width; $K_b = -0.0009$ m (for weir without lateral compression); $K_h = 0.001$ m at $h < 0.1$ m, otherwise $K_h = 0$; $C_e = 0.602 + 0.075h/P$.

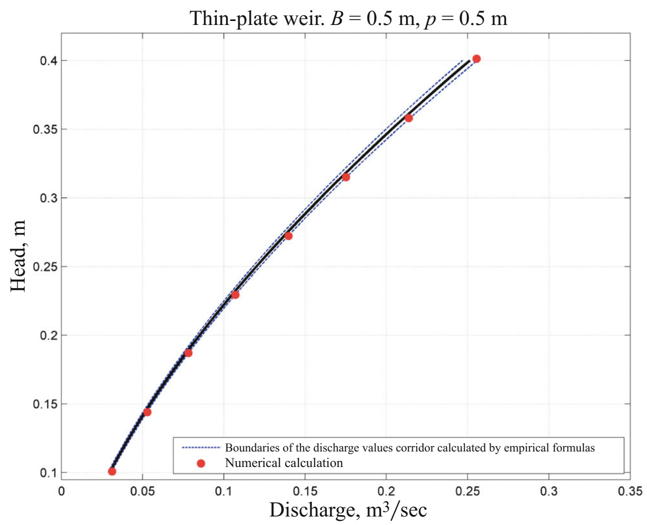

Fig. 5. Calculated values of the discharge through a thin-plate weir in the corridor of empirical discharge dependences calculated according to Eqs. (8) – (12).

Limitations: $h_{min} = 0.03$ m; $P_{min} = 0.1$ m; $b > 0.15$ m; $(h/P)_{max} = 2.5$.

The geometry parameters of the model and the head range are selected on the base of the general limitations of the given formulas. Figure 4 shows the discharge characteristics of the specified weir, calculated according to the empirical formulas above. Figure 5 shows a joint graph of the discharge values calculated by numerical and empirical methods.

The average relative difference in the values of the discharge through the monitored thin-plate weir, found by numerical and empirical methods, is $\delta_{mean} = 2.0\%$.

3. Crest with a sloping front wall

The weir in the form of a crest with a sloping front wall can have a trapezoidal or rectangular cross-section (Fig. 6) [6].

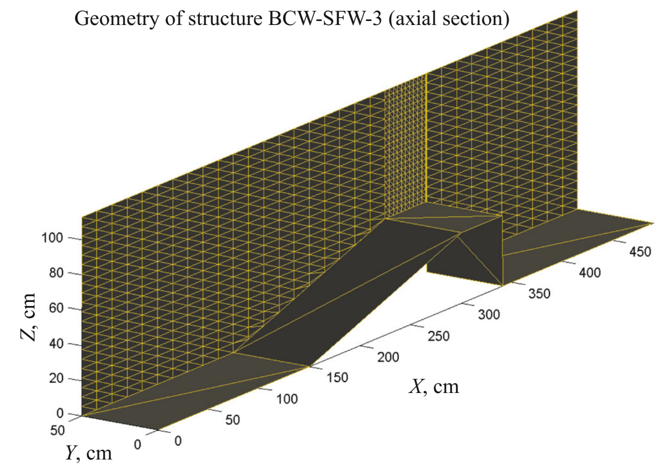

Fig. 7. Geometry of the crest with a sloping front wall in a rectangular canal (axial section).

The empirical dependence $Q = f(h)$ has the form

$$Q = C_V C_D C_F b\sqrt{g} h^{1.5}, \tag{13}$$

where the discharge coefficient $C_D = (0.523 + 0.0707h/P)$ for the crest with a rectangular cross-section; $C_D = (0.523 + 0.056h/P)$ for the crest with a trapezoidal cross-section; the form coefficient $C_F = 1 + h\tan\alpha/b$ for the crest with a trapezoidal cross-section; $C_F = 1$ for the crest with a rectangular cross-section.

The values of the coefficient of accounting the approach velocity C_V are calculated according to the dependence $C_V = (H/h)^{1.5} = f(C_D bh/(A\omega))$ for a trapezoidal form (H is the total head; $A\omega$ is the cross-sectional wet area) or $C_V = f(C_D h/(h + P))$ for a rectangular form according to MI 2406-97 [6]. The regulatory document provides two values of the canal slope coefficient ($m = 0$ and $m = 1$). Limitations: $b \geq 0.3$ m; $b/P \geq 2$; $P \geq 0.15$ m; $h/P \leq 1.3$; $\alpha = 45°$ or $0°$; $L_1 = (3 - 4)h_{max}$; $L \approx 0.8P$.

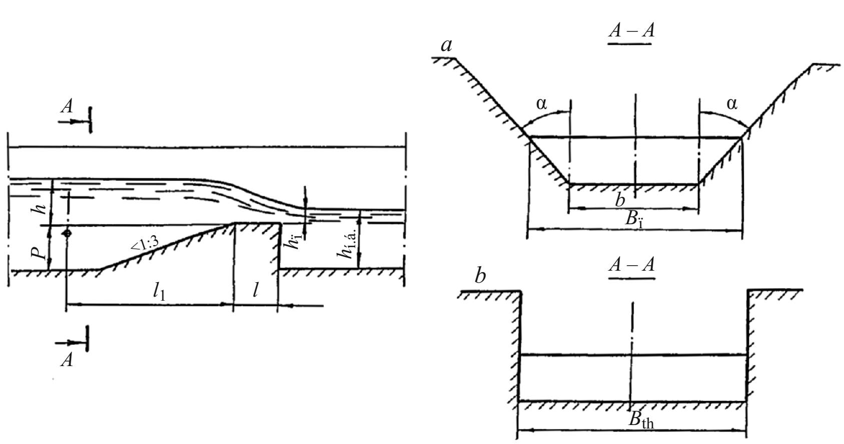

Fig. 6. Geometry parameters of the crest with a sloping front wall (MI 2406-97).

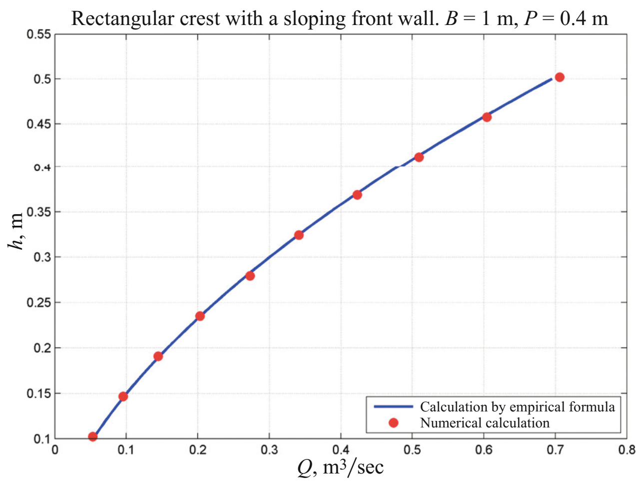

Fig. 8. Dependence $Q = f(h)$, determined the empirical formula of Eq. (13) and by the numerical method.

The accuracy of the numerical calculation of the discharge dependence was estimated by comparison with the empirical characteristic for a structure with the following parameters: width $b = 1$ m; slope coefficient $m = 0$; crest height $P = 0.4$ m; head range 0.1 – 0.5 m. Figure 7 shows the geometry of the structure, and Fig. 8 shows the dependence $Q = f(h)$, determined by the empirical formula of Eq. (13) and by the numerical method.

The average difference in discharge values in the head range 0.1 – 0.5 m is 1.2%. In calculating a structure that is half the size ($B = 0.5$ m, $P = 0.2$ m) in the head range of 0.1 – 0.2 m, the average difference in the calculated discharges was 1.6%.

4. Crump weir

The empirical discharge characteristic has the form [6]

$$Q = \frac{2\sqrt{2}}{3\sqrt{3}} C_D C_V b \sqrt{g} h^{1.5}, \tag{14}$$

where C_D is the discharge coefficient; C_V is the velocity coefficient.

Limitations: $P_{\min} = 0.1$ m; $b_{\min} = 0.3$ m; $h/P \le 3$; $b/h \ge 2$; $h_{min} \ge 0.06$ m. Methodical calculation error according to the given formula is 2% ($dQ/Q = 10C_V - 9\%$) [7]. Figure 9 shows the geometry of the structure, and Fig. 10 shows the discharge characteristic in comparison with the empirical formula.

5. Critical-depth flume

The accuracy of the numerical calculation of the critical-depth flume was verified by comparison with the measurements results in the laboratory flume of FGBNU VNIIGiM. The geometry of the studied lateral compression

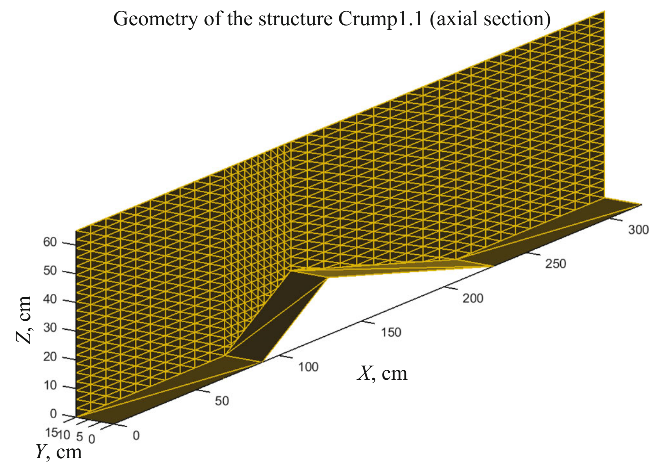

Fig. 9. Geometry of the Crump weir. Structure No. 1 ($B = 0.3$; $P = 0.2$; $dH = 0.06 – 0.15$). Axial section XOZ.

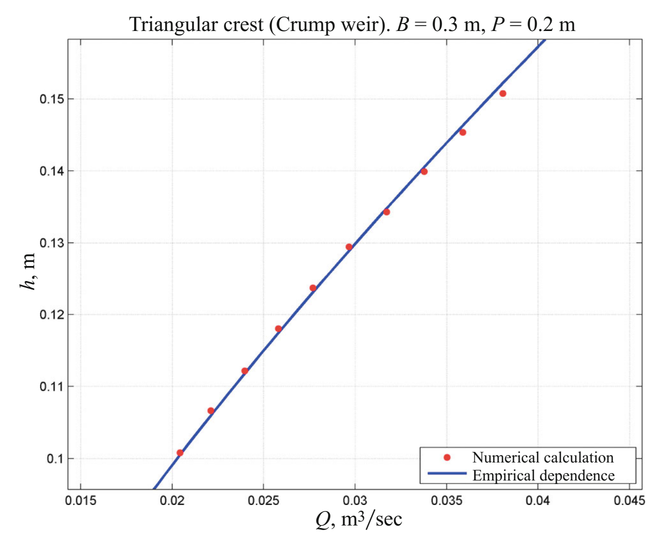

Fig. 10. Empirical and calculated discharge characteristics of the Crump weir (structure No. 1).

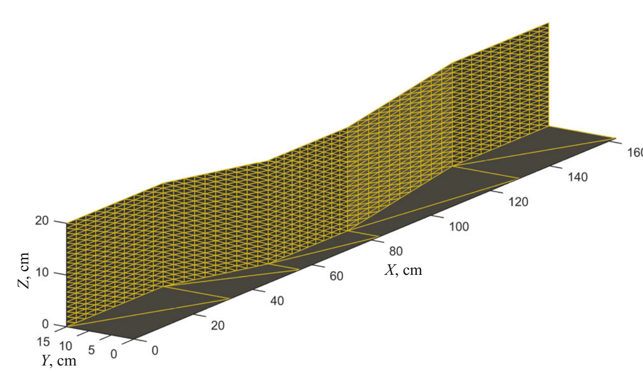

Fig. 11. Geometry of the critical-depth flume.

flume is shown in Fig. 11, the discharge curve is shown in Fig. 12.

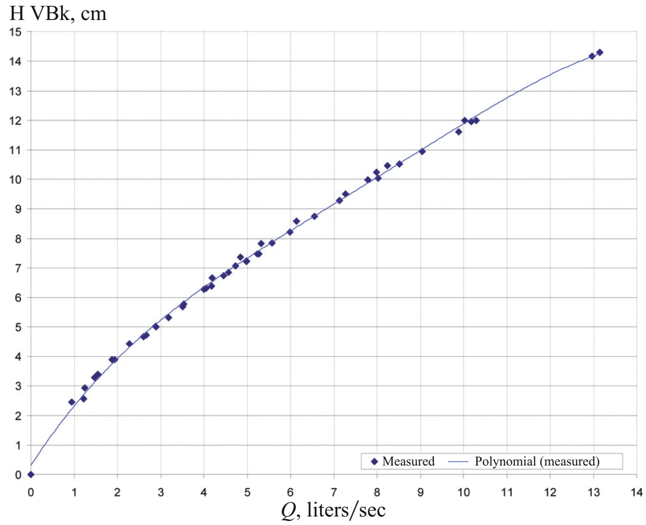

Fig. 12. Discharge dependence of the studied critical-depth flume.

The difference between the calculated discharge dependence and the function of Eq. (15), which approximates the experimental data, does not exceed 2%:

$$Q = -1.389h^3 + 0.6368h^2 +$$

$$+ 0.02944h - 6.313e^{-005} \text{ m}^3/\text{sec}. \tag{15}$$

CONCLUSIONS

The developed software and computing complex can be used to improve the accuracy and expand the measurement range of existing flow-measuring structures, to develop new types and for in-depth research of the operation of water metering devices. According to the results of testing at low sampling values of the operating area, the accuracy of calculating the discharge characteristic is not worse than 2 – 4%. The dependence of the calculation accuracy on discretization in the transverse direction was not found, which confirms the conclusion of the boundary layer theory about the small thickness of the near-wall boundary layer (and, hence, the weak influence of the roughness of solid walls) at a positive gradient of the longitudinal velocity. The accuracy of the numerical calculation of flow-measuring weirs and flumes is sufficient for commercial water metering in open reclamation canals.

REFERENCES

1. A. M. Kusher, "Application of numerical modeling in the analysis of hydrometric structures in open water channels," *Vestn. MGSU*, **13**[8(119)], 1008 – 1015 (2018).
2. A. M. Kusher, "Flow-measuring flumes for canals of water-use systems," in: *Materials of the International Scientific Forum* "*Problems of Water and Land Resources Management 2015*," Moscow, pp. 105 – 115 (2016).
3. A. M. Kusher, "Flow-measuring flumes for irrigation canals," *Prirodoobustroistvo*, No. 5, 78 – 85 (2016).
4. A. R. Berezinsky, *Throughput Capacity of a Broad-Crested Weir* [in Russian], Gosudarstvennoe Izdatel'stvo Stroitel'noi Literatury, Moscow – Leningrad (1950).
5. E. G. Filippov, *Hydraulics of Flow-Measuring Structures for Open Flows* [in Russian], Gidrometeoizdat, Leningrad (1990).
6. *MI 2406–97. Fluid Discharge in Open Canals of Water Supply and Sewerage Systems. Measurement technique Using Standard Weirs and Flumes* [in Russian], Gosstandart RF, Moscow (1997).
7. M. G. Bos, *Discharge Measurement Structures. 3rd Edition*, ILRI, Wageningen, 1980.