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An efficient scenario penalization matheuristic for a stochastic scheduling problem

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Abstract

We propose a new scenario penalization matheuristic for a stochastic scheduling problem based on both mathematical programming models and local search methods. The application considered is an NP-hard problem expressed as a risk minimization model involving quantiles related to value at risk which is formulated as a non-linear binary optimization problem with linear constraints. The proposed matheuritic involves a parameterization of the objective function that is progressively modified to generate feasible solutions which are improved by local search procedure. This matheuristic is related to the ghost image process approach by Glover (Comput Oper Res 21(8):801–822, 1994) which is a highly general framework for heuristic search optimization. This approach won the first prize in the senior category of the EURO/ROADEF 2020 challenge. Experimental results are presented which demonstrate the effectiveness of our approach on large instances provided by the French electricity transmission network RTE.

Keywords Mixed integer programming · Local search · Stochastic scheduling · Mean-risk · Quantiles

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1 Introduction

The practice of decision making under uncertainty frequently resorts to two criteria mean-risk models (cf. Markowitz and Todd 2000) where the mean represents the expected outcome, and the risk corresponds to a scalar measure of the variability of outcomes, in which the mean is maximized and the risk is minimized. The standard mean-variance model, called the Markowitz model, uses the variance as the risk measure in the mean-risk analysis. The mean-variance model under a set of linear constraints can be formulated as a quadratic programming problem (Markowitz 1952). Sharpe (1971) proposed a linear programming (LP) approximation to the mean-variance model for the general portfolio analysis problem. Several authors (cf. Ogryczak and Ruszczynski 2002; Ogryczak and Ruszczyński 2002) have pointed out that in general the mean-variance model is not consistent with stochastic dominance rules (Whitmore and Findlay 1978). To overcome this flaw of the mean-variance model, we consider a mean-risk model involving q-quantiles in risk measure. Note that the measure of value at risk (VAR) is defined as the maximum loss at a level q, consequently, VAR is a widely used quantile risk measure (cf. Jorion 1997, and references therein).

We consider the stochastic scheduling problem (SSP) proposed by Réseau de Transport d'Electricité, usually known as RTE. RTE is the electricity transmission system operator of France, responsible for the operation, maintenance and development of the French high-voltage transmission system, which with approximately 100,000 kms, is the largest of Europe. Maintenance policies have a major economic impact in many areas of the industry such as the electricity sector (Froger et al. 2016), the manufacturing industry and civil engineering. The main goal is to build an optimal maintenance schedule to ensure the delivery and supply of electricity. This problem corresponds to the competition of ROADEF/EURO challenge 2020 (ROADEF 2020).

The stochastic scheduling problem consists in determining the start time of maintenance activities (called interventions) in a high-voltage transmission network over a given time horizon. Each intervention needs a certain number of time units to be achieved without interruption that depends on its starting time. All interventions must be planned and finished before the end of the time horizon. A feasible plan must be consistent with all activity related restrictions such as resource constraints (e.g. each intervention consumes resources and the total amount of resources used at each time step is bounded from below and above) and exclusions between interventions (e.g. some interventions cannot take place at the same time). Given a set of scenarios, where the risk value of each intervention is known at each time step and for each scenario, the goal is to minimize a convex combination of the expectation and the quantile of the risk.

The stochastic scheduling problem can be formulated as a non-linear Mixed Integer Programming (MIP) model with a non-linear (generally non convex) objective function and linear constraints. Note that SSP optimization problem is similar to mean-VAR portfolio problem. Benati and Rizzi (2007) showed that mean-VAR portfolio problem is NP-hard and proposed MIP formulation using Cplex to solve medium size instances. It is well known that in general the exact methods require exponential growth of computational effort for large scale instances. These practical limitations have occasioned a considerable research effort focusing on approximation approaches such as heuristics, metaheuristics and matheuristics (cf. Hashimoto et al. 2011; Gavranović and Buljubašić 2016; Buljubašić et al. 2018; Hanafi and Todosijević 2017). Gouvine (2021) proposed a hybrid approach combining a branch and cut algorithm (Padberg and Rinaldi 1991) with a constraint generation method based on Benders decomposition (Rahmaniani et al. 2017). Cattaruzza et al. (2022) presented a finite convergent adaptive scenario clustering algorithm that guarantees an optimal solution but is only useful for problems of small size. They also developed an overlapping alternating direction method (Glowinski and Marroco 1975; Gabay and Mercier 1976) that serves as a primal heuristic for quickly computing feasible solutions of good quality for problems in the size range examined. Zholobova et al. (2021) developed a hybrid approach combining the Covariance Matrix Adaptation Evolution Strategy (Hansen et al. 2003) and Variable Neighborhood Search (Hansen et al. 2017) metaheuristics.

In this paper, we introduce a new matheuristic for the SSP optimization problem, called Scenario Penalization Matheuristic (SPM). The proposed matheuritic involves a parameterization of the objective function that is progressively modified to generate feasible solutions which are improved by a local search procedure and which is applicable to applications of practical sizes.

The SPM is based on the ghost image process (GIP) approach by Glover (1994) which is a highly general framework for heuristic search optimization. Note that GIP is a generalization of many relaxation methods (e.g. Lagrangian, surrogate and composite relaxations) and self-organizing neural networks of Kohonen (1988) as applied to optimization problems. GIP has been applied to some interesting optimization problems. For example, Woodruff (1995) developed a GIP application to the problem of computing the minimum covariance determinant estimators. With regard to the parameterized objective function, the GIP approach is related to the concept of slope scaling applied within the context of fixed-charge networks (see e.g. Kim and Pardalos 1999; Gendron et al. 2003; Crainic et al. 2004; Glover 2005; Gendron et al. 2018). This approach SPM won the first prize in the senior category of the EURO/ROADEF 2020 challenge. Experimental results are presented which demonstrate the effectiveness of our approach on large instances provided by RTE.

The remainder of this paper is organized as follows. A description of the stochastic scheduling problem is provided in Sect. 2 and the scenario penalization matheuristic development for the stochastic scheduling problem is presented in Sect. 3. Computational results obtained from the available set of instances, provided by RTE, are provided in Sect. 4. Finally, Sect. 5 summarizes our conclusions.

2 Stochastic scheduling problem

In this section, first we will describe the input parameters of the stochastic scheduling problem (SSP). We refer the reader to the challenge subject in ROADEF (2020) for a more thorough description of the maintenance of electricity transmission lines and the implications of the problem from business and environmental perspectives. Additional details of the global RTE strategy and risk management can be found at Crognier et al. (2021). Next we propose a mathematical programming formulation of this problem.

2.1 Input parameters of the problem

We consider a finite set of maintenance activities $\mathcal{I}$ (called *interventions* in the challenge application) to be scheduled over a discrete time horizon $\mathcal{T}$. Interventions are not equal in terms of duration or resource requirements. Because of days off (weekends...), the duration of a given intervention is not fixed in time and depends on when it starts. Therefore, $\delta_{i,t} \in \mathbb{N}$ denotes the actual duration of intervention $i \in \mathcal{I}$ if it starts at time $t \in \mathcal{T}$.

To carry out the different interventions, a workforce is necessary, split into teams (or resources) of various sizes and specific skills. Each team has different specific skills and can potentially be required for any task. The available resources are always limited and vary over the time horizon. Let $\mathcal{J}$ denote the set of resources. The amount of resource $j \in \mathcal{J}$ used at time $t \in \mathcal{T}$ by intervention $i \in \mathcal{I}$ starting at time t' is given by $a_{i,t'}^{j,t}$. So for every resource $j \in \mathcal{J}$ and every time step $t \in \mathcal{T}$, the amount of resource consumed must be between the lower bound l_t^j and the upper bound u_t^j.

When an intervention is being performed, the power lines involved must be disconnected, causing the electricity network to be weakened at this time. This implies a certain risk for `RTE`, which is highly linked to the grid operation: if another nearby site were to break down (due to extreme weather for example), the network may not be able to handle the electricity demand correctly. Even if such events have an extremely low probability of occurring, they must be taken into account in the schedule. In order to financially quantify these risks, `RTE` previously conducted simulations for various scenarios with different time steps. Let $\mathcal{S}_t$ be the set of scenarios at time $t \in \mathcal{T}$ and $\mathcal{S} = \cup_{t\in\mathcal{T}}\mathcal{S}_t$ be the set of all scenarios. The risk value depends on the intervention concerned and on the time period, as it is often much less risky to perform interventions in summer (when there is less demand on the electricity network) rather than in winter. So the risk value (expressed in Euros in the challenge application) is denoted by $risk_{i,t'}^{s,t} \in \mathbb{R}$ for time period $t \in \mathcal{T}$, scenario $s \in \mathcal{S}$ and intervention $i \in \mathcal{I}$ when i starts at time step $t' \in \mathcal{T}$. Further, some maintenance activities cannot be performed at the same time. This exclusion between interventions is given by a set $\mathcal{E}$ of triplets (i, i', t) such that interventions i and i' cannot be both in process at time t.

Intervention preemption is not allowed, and each intervention must be terminated at time $|\mathcal{T}|$. To more formally derive the optimization models and their associated solutions, we require some additional notation. Let $\mathcal{T}(i) = \{t \in \mathcal{T} : t+\delta_{i,t} \leq |\mathcal{T}|+1\}$ denote the set of feasible starting times of intervention $i \in I$, and let $\mathcal{T}^+(i,t) = \{t' \in \mathcal{T} : t \leq t' < t + \delta_{i,t}\}$ denote the set of times for which the intervention i is in process if it starts at time t. We further let $\mathcal{T}^-(i,t) = \{t' \in \mathcal{T} : t' \leq t < t' + \delta_{i,t'}\} = \{t' \in \mathcal{T} : t \in \mathcal{T}^+(i,t')\}$ denote the set of starting times of intervention i for which the intervention is in process at time t, and let $\mathcal{I}(t) \subseteq \mathcal{I}$ denote the set of interventions in process at time $t \in \mathcal{T}$. For any real value $v \in \mathbb{R}$, let $\lceil v \rceil = \min\{v' \in \mathbb{N} : v' \geq v\}$.

2.2 Mathematical programming formulation

A schedule (or solution) consists of a list of starting times of interventions. Hence a solution will be represented by an integer vector $\sigma \in \mathbb{N}^{|\mathcal{I}|}$ where each component σ_i

represents the starting time of intervention $i \in \mathcal{I}$. An alternative is to define a solution by a binary matrix $x \in \{0, 1\}^{|\mathcal{I}|\times|\mathcal{T}|}$ such that

$$x_{i,t} = \begin{cases} 1 & \text{if intervention } i \in \mathcal{I} \text{ starts at time step } t \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$

for $i \in \mathcal{I}$ and $t \in \mathcal{T}$. The compact integer representation σ of a feasible schedule will be used in heuristic procedures and the binary representation x will be exploited in the mathematical programming formulation.

2.2.1 Non linear objective function

The evaluation score of a feasible schedule depends only on the distribution of risks. Two criteria are taken into account: the average and the excess of the risk values. The excess is defined from the *quantile* value of the risk distribution.

More formally, given a feasible schedule represented by the integer vector σ or the binary matrix x, the mean risk $Mean(x) = Mean(\sigma)$ and the expected excess risk $Excess(x) = Excess(\sigma)$ are evaluated as follows.
Mean cost The cumulative planning risk at $t \in \mathcal{T}$ for a scenario $s \in \mathcal{S}_t$, denoted by $risk^{s,t}(x)$ or $risk^{s,t}(\sigma)$, is the sum of risks in scenario s over the in-process interventions at t:

$$risk^{s,t}(x) = \sum_{i\in\mathcal{I}(t)} \sum_{t'\in\mathcal{T}^-(i,t)} risk_{i,t'}^{s,t} \times x_{i,t}$$

Then the mean cost overall planning risk is

$$Mean(x) = \frac{1}{|\mathcal{T}|} \sum_{t\in\mathcal{T}} \frac{1}{|\mathcal{S}_t|} \sum_{s\in\mathcal{S}_t} risk^{s,t}(x)$$

Excess cost The planning quality also takes into account the cost variability. Computing the mean risk over all scenarios induces a loss of information, and critical scenarios inducing extremely high costs may not be adequately captured by the mean. To prevent this kind of outcome from happening, a metric exists to quantify the variability of the scenarios. The expected excess indicator relies on the τ quantile values where $\tau \in]0, 1]$. For every time period t, we define the quantile value Q_τ^t as follows:

$$Q_\tau^t(x) = \min\{q \in \mathbb{R} : \exists E \subseteq \{1, \dots, |\mathcal{S}_t|\} : |E| \geq \lceil \tau \times |\mathcal{S}_t| \rceil; \forall s \in E, risk^{s,t}(x) \leq q\}$$

Note that if we sort elements of the set $\mathcal{S}_t$ in increasing order, then $Q_\tau^t(x)$ will be equal to the element at position $\lceil \tau \times |\mathcal{S}_t| \rceil$ in this sorted set. The expected excess of a planning is:

$$Excess(x) = \frac{1}{|\mathcal{T}|} \sum_{t\in\mathcal{T}} max \left(0, Q_\tau^t(x) - \frac{1}{|\mathcal{S}_t|} \sum_{s\in\mathcal{S}_t} risk^{s,t}(x)\right)$$

Table 1 Example of non-convexity: 4 scenarios $\mathcal{S} = \{s_1, s_2, s_3, s_4\}$, $\alpha = 0.5$ and $\tau = 0.7$

scenarios	$\mathtt{r}_1$	$\mathtt{r}_2$	$\mathtt{r}_3 = 0.5 \times (\mathtt{r}_1 + \mathtt{r}_2)$
s_1	1.000	1.000	1.000
s_2	3.000	1.000	2.000
s_3	4.000	9.000	6.500
s_4	9.000	4.000	6.500
$Mean(r)$	4.250	3.750	4.000
$Excess(r)$	0.000	0.250	2.500
$Obj(r)$	2.125	2.000	3.250

Two risk vectors: $\mathtt{r}_1$, $\mathtt{r}_2$ and a linear combinaison of them: $\mathtt{r}_3 = 0.5 \times (\mathtt{r}_1 + \mathtt{r}_2)$. Bold values correspond to the Q_τ values. $Obj(\mathtt{r}_3) \neq 0.5 \times (Obj(\mathtt{r}_1) + Obj(\mathtt{r}_2))$

The two metrics $Mean(x)$ and $Excess(x)$ described above cannot necessarily be compared directly, as they depend on risk aversion (or risk policies). That is why a scaling factor $\alpha \in [0, 1]$ is needed. Then the final score of a planning is:

$$Objective(x) = \alpha \times Mean(x) + (1 - \alpha) \times Excess(x).$$

In general, the quantile function $Q_\tau^t(x)$ is non-convex as shown by Gouvine (2021). Hence, the objective function $Objective(x)$ is non-convex. Moreover, the objective function $Obj(r)$ of the vector of decision variables risk r associated with a solution x (i.e. $r = (risk^{s,t}(x))_{t \in \mathcal{T}, s \in \mathcal{S}_t})$ is a non-convex function as can be observed from Table 1.

The goal is to find a feasible schedule which minimizes the objective function $Objective(x)$. A schedule x is feasible if satisfies all the linear constraints presented below.

2.2.2 Linear constraints

Schedule Interventions have to start at the beginning of a period. Moreover, as interventions require shutting down some lines of the electricity network, once an intervention starts, it cannot be interrupted. More precisely, if intervention $i \in \mathcal{I}$ starts at time $t \in \mathcal{T}$, then it must end at $t + \delta_{i,t}$. All interventions must be executed and completed no later than the end of the horizon. If intervention $i \in \mathcal{I}$ starts at time $t \in \mathcal{T}$, then $t + \delta_{i,t} \leq |\mathcal{T}| + 1$. Hence, the following multiple choice constraints must be satisfied:

$$\sum_{t \in \mathcal{T}} x_{i,t} = 1, \quad \forall i \in \mathcal{I} \tag{1}$$

From the definition of this schedule problem, we have the reduction

$$x_{i,t} = 0, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} - \mathcal{T}(i)$$

which can be incorporated in the preprocessing step.

Resource limitation The resources needed cannot exceed the resource capacity but have to be at least equal to the minimum workload, and hence the resource constraints are:

$$l_t^j \leq \sum_{i \in \mathcal{I}} \sum_{t' \in \mathcal{T}} a_{i,t'}^{j,t} \times x_{i,t'} \leq u_t^j, \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{2}$$

Exclusion between interventions The exclusion constraints can formally be written as:

$$i \in \mathcal{I}(t) \implies i' \notin \mathcal{I}(t) \quad \forall (i, i', t) \in \mathcal{E}$$

This implication can formulated using the binary variables $x_{i,t}$ as follows:

$$\sum_{t' \in \mathcal{T}^-(i,t)} x_{i,t'} + \sum_{t' \in \mathcal{T}^-(i',t)} x_{i',t'} \leq 1, \forall (i, i', t) \in \mathcal{E} \tag{3}$$

2.3 `MIP_full` formulation

Finally, the stochastic scheduling problem `SSP` can be expressed as a non-linear binary `MIP` model, denoted `MIP_full`, given by

$$(\texttt{MIP\_full}) \begin{cases} \min \ obj = \frac{\alpha}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \frac{1}{|\mathcal{S}_t|} \sum_{s \in \mathcal{S}_t} risk^{s,t}(x) \\ \qquad + \frac{(1-\alpha)}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \max(0, Q_\tau^t(x) - \frac{1}{|\mathcal{S}_t|} \sum_{s \in \mathcal{S}_t} risk^{s,t}(x)) \\ \text{s.t.: } (1), (2) \text{ and } (3), \\ \quad x \in \{0, 1\}^{|\mathcal{I}| \times |\mathcal{T}|} \end{cases}$$

The quantile value $Q_\tau^t(x)$ for each scenario s in time step t can be expressed by introducing a binary variable $y_{s,t}$ which indicates whether $Q_\tau^t(x)$ is greater than or equal to the total risk for scenario s at time step t, i.e.

$$y_{s,t} = \begin{cases} 1 & \text{if } Q_\tau^t(x) \geq risk^{s,t}(x) \\ 0 & \text{otherwise} \end{cases}, \quad \forall t \in \mathcal{T}, s \in \mathcal{S}_t \tag{4}$$

with additional linear constraints requiring that the number of $y_{s,t}$ binary variables that take the value 1 must be greater than or equal to $\lceil \tau \times |\mathcal{S}_t| \rceil$, i.e.

$$\sum_{s \in \mathcal{S}_t} y_{s,t} \geq \lceil \tau \times |\mathcal{S}_t| \rceil, \quad \forall t \in \mathcal{T} \tag{5}$$

The implicit definition (4) of auxilary binary variables y for quantile can be formulated as quadratic constraints

$$Q_\tau^t(x) \geq y_{s,t} \times risk^{s,t}(x), \quad \forall t \in \mathcal{T}, s \in \mathcal{S}$$

Linear `MIP` formulations are proposed by Gouvine (2021) and Cattaruzza et al. (2022). To strengthen the linear relaxation of the quantile model, they generate a set of valid inequalities derived from the expression of the quantile function and linear programming duality theory. The existence of binary variables x and y with $max(0, \cdot)$ function and the non-convexity of quantile function $Q^t_\tau(x)$ makes the `MIP_full` model very difficult to solve optimally with state-of-the-art `MIP` solvers like Cplex and Gurobi except for small size instances.

3 Scenario penalization matheuristic

Our new scenario penalization matheuristic (`SPM`) for solving the `SSP` optimization problem is based on four main steps. Each solution obtained throughout **Step 1** and **Step 2** of the `SPM` (Algorithm 1) is evaluated as a candidate for the best solution x^* currently found.

Algorithm 1 `SPM()`

Step 0: Create an initial approximation `MIP_mean`(c^0) of `MIP_full` model by ignoring the excess cost and set $c = c^0$ and $Objective(x^*) = \infty$.
while termination criterion is not met **do**
  Step 1: Solve `MIP_mean`(c) yielding a solution x.
  Step 2: Starting from x, apply a Local Search to obtain a better solution x'.
  Step 3: Update the coefficient matrix c to take into account the excess from x and update the best solution $x^* = \mathtt{argmin}\{Objective(y) : y \in \{x, x', x^*\}\}$.
end while
return x^*

In the following, we give details of these steps as adapted to the present context.

3.1 Step 0: initial `MIP_mean`(c^0) approximation generating initial solution x^0

As noted above, the `MIP_full` model is too hard to solve for real world instances. Hence, the initial approximation we consider corresponds to the mean model ignoring the excess. Formally, let $c^0_{i,t}$ denote the initial resulting mean risk if intervention $i \in \mathcal{I}$ starts at time step $t \in \mathcal{T}$, defined as follows

$$c^0_{i,t} = \frac{1}{|\mathcal{T}|} \sum_{t' \in \mathcal{T}^+(i,t)} \frac{1}{|S_{t'}|} \sum_{s \in S_{t'}} risk^{s,t'}_{i,t} \quad (6)$$

Then the initial linear approximation of the `MIP_full` model without excess cost is given below.

$$(\mathtt{MIP\_mean}(c^0)) \begin{cases} \min \; Mean(x) = \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} c^0_{i,t} \times x_{i,t} \\ \text{s.t.: } (1), (2) \text{ and } (3) \\ \quad x_{i,t} \in \{0, 1\} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \end{cases}$$

Required computational time to solve MIP_mean(c^0) varies across the data sets: it is usually quite fast but can take up to a few minutes for largest instances. Consequently, solving MIP_mean(c^0) optimally or approximatively with an MIP solver will generate an initial feasible solution x^0 for the MIP_full model (i.e. x^0 corresponds to the first solution generated by SPM() Algorithm.

3.2 Step 3: scenario penalization heuristic updating the current MIP_mean(c) approximation

Our goal in this step is to modify the current objective coefficient matrix c of the MIP_mean(c) model, stating with $c = c^0$ to make the resulting model closer to the original MIP_full model. To achieve this, let x be an optimal or best solution of the current MIP_mean(c) model and define the excess risk cost at time step $t \in \mathcal{T}$ by:

$$Excess^t(x) = max(0, Q^t_\tau(x) - \frac{1}{|\mathcal{S}_t|} \sum_{s \in \mathcal{S}_t} risk^{s,t}(x)) \tag{7}$$

Hence, $Excess(x) = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} Excess^t(x)$. We first provide observations that describe the behavior of the function $Excess^t(x)$. For any step time $t \in \mathcal{T}$, we define the set of scenarios $S^+_t(x) = \{s \in \mathcal{S}_t : risk^{s,t}(x) > Q^t_\tau(x)\}$.

It is obvious that this modification increases the mean value $\frac{1}{|\mathcal{S}_t|} \sum_{s \in \mathcal{S}_t} risk^{s,t}(x)$. Moreover, this modification involves only binary variables $y_{s,t} = 0$ since $S^+_t(x) = \{s \in \mathcal{S}_t : y_{s,t} = 0\}$, and the constraints (5) imposed on the quantile remain satisfied. Consequently, the quantile value $Q^t_\tau(x)$ is unchanged and the value $Excess^t(x)$ will decrease after this change. Hence for any time step $t \in \mathcal{T}$, increasing the risk value $risk^{s,t}(x)$ for any scenario $s \in S^+_t(x)$ decreases the value $Excess^t(x)$.

Let $Risk^t_{mean}(x)$ (resp. $Risk^t_{max}(x)$) denote the mean (resp. max) risk at time step $t \in \mathcal{T}$:

$$Risk^t_{mean}(x) = \frac{1}{|\mathcal{S}_t|} \sum_{s \in \mathcal{S}_t} risk^{s,t}(x) \tag{8}$$

$$Risk^t_{max}(x) = \max\{risk^{s,t}(x) : s \in \mathcal{S}_t\} \tag{9}$$

Then we can state

Proposition 1 *For any $t \in \mathcal{T}$ such that $|\mathcal{S}_t| = \lceil \tau \times |\mathcal{S}_t| \rceil$, we have*

$$Excess^t(x) = Risk^t_{max}(x) - Risk^t_{mean}(x). \tag{10}$$

Proof If $|\mathcal{S}_t| = \lceil \tau \times |\mathcal{S}_t| \rceil$ then from the τ quantile constraint (5) we deduce that the indicator binary variables $y_{s,t}$ for all $s \in \mathcal{S}_t$ are set to 1. Consequently, from (4) we have $Q^t_\tau(x) \geq risk^{s,t}(x)$ for all $s \in \mathcal{S}_t$ and the operator $\max(0, .)$ can be dropped from the expression to compute $Excess^t(x)$, i.e. now we have

$$Excess^t(x) = Q^t_\tau(x) - \frac{1}{|\mathcal{S}_t|}\sum_{s\in\mathcal{S}_t} risk^{s,t}(x)$$

Moreover, from the definition of the τ quantile $Q^t_\tau(x)$ in Sect. 2.2.1, in this case we have

$$Q^t_\tau(x) = \min\{q \in \mathbb{R} : \forall s \in \mathcal{S}_t, risk^{s,t}(x) \le q\}$$

Or equivalently

$$Q^t_\tau(x) = Risk^t_{max}(x) = \max\{risk^{s,t}(x) : s \in \mathcal{S}_t\}$$

This completes the validation of Eq. (10). □

Next, observe that the objective coefficients $c_{i,t}$, can be written:

$$c_{i,t} = \frac{1}{|\mathcal{T}|}\sum_{t'\in\mathcal{T}^+(i,t)}\sum_{s\in\mathcal{S}_{t'}} risk^{s,t'}_{i,t} \times \frac{1}{|\mathcal{S}_{t'}|} \quad (11)$$

We replace $\frac{1}{|\mathcal{S}_{t'}|}$ values in formula (11) by non-constant values $\mu_{s,t'}$, to yield the following formula for calculating $c_{i,t}$ values:

$$c_{i,t} = \frac{1}{|\mathcal{T}|}\sum_{t'\in\mathcal{T}^+(i,t)}\sum_{s\in\mathcal{S}_{t'}} risk^{s,t'}_{i,t} \times \mu_{s,t'} \quad (12)$$

The $\mu_{s,t}$ values, for $s \in \mathcal{S}$ and $t \in \mathcal{T}$, are set to $\frac{1}{|\mathcal{S}_t|}$ initially and penalizing scenario s at time step t will be done by increasing $\mu_{s,t}$ value. Note that the sum of these values over all scenarios in the time step equals 1 initially ($\sum_{s\in\mathcal{S}_t}\frac{1}{|\mathcal{S}_t|} = 1$) and we will keep the sum fixed at 1 when doing the modifications. This condition is imposed because we want the sum of values/coefficients to be the same in each time step in order to not prioritize some time steps over others—only the importance of scenarios inside the time step will change.

Formally, we calculate $\mu_{s,t}$ values in the following way. Let $Penalty_{s,t}$ be a current penalty for scenario s at time step t and let $Freq_{s,t}$ denote how many times scenario s at time step t is penalized. Penalty and frequency values are initialized to 0 and they will be updated in each iteration i.e. after each call of `MIP_mean`(c) as follows. For each time step $t \in \mathcal{T}$ with excess i.e. for which $Q^t_\tau(x) > Risk^t_{mean}(x)$ we update the penalties and frequencies for $|\mathcal{S}_t| - \lceil \tau \times |\mathcal{S}_t| \rceil$ scenarios with the highest $risk^{s,t}(x)$ values (those are scenarios with risk values at least $Q^t_\tau(x)$):

$$Penalty_{s,t} = Penalty_{s,t} + \frac{(Q^t_\tau(x) - Risk^t_{mean}(x)) \times |\mathcal{S}_t|}{|\mathcal{S}_t| - \lceil \tau \times |\mathcal{S}_t| \rceil} \quad (13)$$

$$Freq_{s,t} = Freq_{s,t} + 1 \quad (14)$$

Then the $\mu_{s,t'}$ values are calculated by setting:

$$\mu_{s,t} = \frac{\beta_{s,t}}{\sum_{s' \in \mathcal{S}_t} \beta_{s',t}}, \quad \forall t \in \mathcal{T}, s \in \mathcal{S}_t \tag{15}$$

where

$$\beta_{s,t} = \begin{cases} 1 & \text{if } Freq_{s,t} = 0 \\ 1 + Penalty_{s,t}/Freq_{s,t} & \text{otherwise} \end{cases}, \quad \forall t \in \mathcal{T}, s \in \mathcal{S}_t \tag{16}$$

We increase the penalties (in formula (13)) such that no excess appear in the solution obtained by `MIP_Mean`(c) (i.e. $Q^t_\tau(x) = Risk^t_{mean}(x)$). Note that if $|S^+_t(x)| = 0$, no penalties are given to the scenario coefficients of this time step.

3.3 Step 2: local search (LS)

Step 2 constitutes an improvement method that transforms a feasible solution into one or more best solutions. The efficacy of Local Search (LS) (also called neighborhood search) for solving a wide variety of optimization problems (cf. Kirkpatrick et al. 1983; Glover and Laguna 1997; Hoos and Stützle 2004; Hansen et al. 2017) depends strongly on the characteristics of one or more neighborhood structures involved and their combination. We are interested in two well-known moves, namely Shift and Swap moves. The shift neighborhood is the set of solutions that can be obtained by changing the start time of one intervention, while the swap neighborhood is the set of solutions that can be obtained by interchanging the start times of two interventions. Formally, given a feasible schedule σ, for any intervention $i \in \mathcal{I}$ and any period $t \in \mathcal{T}$, the shift neighbor solution $\sigma' = \texttt{Shift}(\sigma, i, t)$ is defined by

$$\sigma'_i = t \quad \text{and} \quad \sigma'_{i'} = \sigma_{i'} \quad \text{for} \quad i' \in \mathcal{I} - \{i\}$$

and for two interventions $i_1, i_2 \in \mathcal{I}$ the swap neighbor solution $\sigma' = \texttt{Swap}(\sigma, i_1, i_2)$ is defined by

$$\sigma'_{i_1} = \sigma_{i_2}, \sigma'_{i_2} = \sigma_{i_1} \quad \text{and} \quad \sigma'_{i'} = \sigma_{i'} \quad \text{for} \quad i' \in \mathcal{I} - \{i_1, i_2\}$$

Consequently, the `Shift` and `Swap` neighborhood sets are defined by

$$\texttt{Shift}(\sigma) = \{\sigma' = \texttt{Shift}(\sigma, i, t) : \sigma' \text{ is feasible}, i \in \mathcal{I}, t \in \mathcal{T}\}$$
$$\texttt{Swap}(\sigma) = \{\sigma' = \texttt{Swap}(\sigma, i_1, i_2) : \sigma' \text{ is feasible}, i_1, i_2 \in \mathcal{I}\}$$

where the statement that σ' is feasible means that the solution σ' satisfies the linear constraints (1), (2) and (3).

There are several ways to combine two or more neighborhoods and computational results show that certain combinations are superior to others (cf. Di Gaspero and Schaerf 2006; Lü et al. 2011; Mjirda et al. 2017; Hansen et al. 2017). The three basic combinations of two neighborhood structures are strong neighborhood union, selective neighborhood union and token-ring search. For strong neighborhood union, the `LS` algorithm picks each move (according to the algorithm's selection criteria) from all the `Shift` and `Swap` moves. For selective neighborhood union, the `LS` algorithm selects one of the two neighborhoods to be used at each iteration, choosing the neighborhood `Shift` with a predefined probability p and choosing `Swap` with probability $1 - p$. Note that an `LS` algorithm using only `Shift` or `Swap` is a special case of an algorithm using selective neighborhood union where p is set to be 1 and 0 respectively. In token-ring search, the neighborhoods `Shift` and `Swap` are alternated, applying the currently selected neighborhood without interruption, starting from the local optimum of the previous neighborhood, until no improvement is possible.

For the `SSP` challenge problem, the token-ring search implemented in the `LS`(σ, TL, TL_1, TL_2) procedure is described in Algorithm 2. Specifically, the `LS` procedure starts from a given feasible solution σ, and uses one neighborhood until a best solution is determined, subject to time limits imposed on the search (TL_1 (resp. TL_2) for `Shift` (resp. `Swap`) neighborhood exploration. Then the method switches to the other neighborhood, starting from this best solution, and continues the search in the same fashion. The search comes back to the first neighborhood at the end of the second neighborhood exploration, repeating this process until time limit TL is reached. Neighborhoods `Shift` and `Swap` are explored by randomly selecting a candidate feasible move which is performed if it improves the cost. However, performing only the moves that improve the objective function can quickly lead to a local optimum. Escaping a local optimum is therefore achieved by occasionally allowing non-improving moves. This is done in a simple way: from time to time (for example once in 1000 iterations) a 'bad' move is accepted.

Algorithm 2 `LS`(σ, TL, TL_1, TL_2)

```
Input: feasible solution σ, running time limits TL, TL1 and TL2
while time limit TL not exceeded do
    while time limit TL1 not exceeded do              ▷ explore Shift neighborhood
        select random σ′ ∈ Shift(σ)
        if σ′ is accepted then
            σ = σ′
        end if
    end while
    σ = best solution found in SHIFT
    while time limit TL2 not exceeded do              ▷ explore Swap neighborhood
        select random σ′ ∈ Swap(σ)
        if σ′ is accepted then
            σ = σ′
        end if
    end while
    σ = best solution found in SWAP
end while
return σ
```

3.4 Full SPM algorithm

In this section, we specify the full description of the SPM algorithm. The preliminary initialization (Step 0) constructs the coefficient matrix $c = c^0$ of the initial MIP_Mean(c) approximation ignoring the excess cost and set the penalties and frequencies to 0 (i.e. $Penalty_{s,t} = Freq_{s,t} = 0, \forall t \in \mathcal{T}, s \in \mathcal{S}_t$). A each iteration, the current MIP_Mean(c) approximation is solved to generate a current feasible solution x (i.e. Step 1). The LS heuristic procedure tries to generate an improved solution x' (i.e. Step 2). Step 3 updates the coefficient matrix c of the current MIP_Mean(c) approximation. The process is repeated until the time limit allocated is exceeded. The full algorithm pseudo-code is given in Algorithm 3.

Algorithm 3 SPM(TL)

```
for i ∈ I do                                                   ▷ Step 0: Initialization
    for t ∈ T do
        c_{i,t} = c^0_{i,t}
    end for
end for
for t ∈ T do
    for s ∈ S_t do Penalty_{s,t} = Freq_{s,t} = 0
    end for
end for
while time limit TL not exceeded do
    x = MIP_Mean(c)                  ▷ Step 1: Solve the current approximation MIP_Mean(c)
    x = LS(σ(x))                                               ▷ Step 2: Local Search
    for t ∈ T do                               ▷ Step 3: Update penalties and frequencies
        for s ∈ S_t do
            if risk^{s,t}(x) > Q^t_τ(x) then
                Penalty_{s,t} = Penalty_{s,t} + ((Q^t_τ(x) − Risk^t_{mean}(x)) × |S_t|) / (|S_t| − ⌈τ × |S_t|⌉)
                Freq_{s,t} = Freq_{s,t} + 1
            end if
            β_{s,t} = 1
            if Freq_{s,t} > 0 then
                β_{s,t} = 1 + Penalty_{s,t} / Freq_{s,t}
            end if
        end for
        for s ∈ S_t do
            μ_{s,t} = β_{s,t} / Σ_{s'∈S_t} β_{s',t}
        end for
    end for
    for i ∈ I do                        ▷ Update the coefficient matrix of MIP_mean(c)
        for t ∈ T do
            c_{i,t} = (1/|T|) Σ_{t'∈T+(i,t)} Σ_{s∈S_{t'}} risk^{s,t'}_{i,t} × μ_{t',s}
        end for
    end for
end while
return best solution found
```

To clarify the procedure, we consider the small example provided in the next section.

Table 2 $risk^{s,t}_{i,t'}$ for 7 scenarios

	Intervention i_1			Intervention i_2			Intervention i_3			Intervention i_4			
scenario	$\binom{s,1}{1,1}$	$\binom{s,2}{1,2}$	$\binom{s,3}{1,3}$	$\binom{s,1}{2,1}$	$\binom{s,2}{2,2}$	$\binom{s,3}{2,3}$	$\binom{s,1}{3,1}$	$\binom{s,2}{3,2}$	$\binom{s,3}{3,3}$	$\binom{s,1}{4,1}$	$\binom{s,2}{4,1}$	$\binom{s,2}{4,2}$	$\binom{s,3}{4,2}$
s_1	10	9	9	11	12	7	10	9	8	9	11	9	11
s_2	12	12	13	10	11	10	7	7	8	10	12	10	12
s_3	8	9	10	11	10	5	13	14	12	12	20	16	19
s_4	5	6	6	7	8	4	8	9	7	7	8	8	9
s_5	12	11	10	7	8	8	10	10	10	8	10	8	10
s_6	7	7	6	9	8	8	9	10	10	9	9	9	9
s_7	10	11	10	9	9	10	8	8	8	12	13	12	13

Table 3 Objective coefficients ($c_{i,t}$) evolution in 8 iterations of `SPM` procedure without local search: column Full obj. corresponds to $\alpha \times Mean + (1-\alpha) \times Excess$

Iter.	Intervention i_1			Intervention i_2			Intervention i_3			Intervention i_4		Mean	Full obj.
1	3.048	3.095	3.048	3.048	3.143	2.476	3.095	3.190	3.00	7.143	7.381	7.833	8.333
2	3.396	3.125	3.528	3.115	3.146	2.796	2.760	3.250	3.00	7.604	7.361	8.021	9.000
3	3.302	3.229	3.428	3.246	3.156	2.644	3.016	3.458	3.17	8.291	8.702	8.590	8.500
4	3.333	3.491	3.389	3.250	3.351	2.524	2.981	3.134	3.26	8.028	8.558	8.433	8.333
5	3.280	3.321	3.238	3.043	3.267	2.475	3.135	3.125	3.17	7.492	8.017	8.165	8.238
6	3.273	3.321	3.323	3.050	3.267	2.593	3.140	3.125	3.10	7.521	7.989	8.245	8.333
7	3.358	3.323	3.366	3.079	3.268	2.633	3.021	3.108	3.11	7.546	7.832	8.261	8.500
8	3.364	3.447	3.361	3.077	3.364	2.628	3.018	3.072	3.12	7.690	7.920	8.349	8.167

3.5 A small example

We illustrate Algorithm 3 for an instance with four interventions $\mathcal{I} = \{i_1, i_2, i_3, i_4\}$, three time steps $\mathcal{T} = \{1, 2, 3\}$, seven scenarios in each time step $\mathcal{S} = \{s_1, \ldots, s_7\}$, $\tau = 0.7$ and $\alpha = 0.5$. The duration of the first three interventions is 1 (i.e. $\delta_{i_1,t} = \delta_{i_2,t} = \delta_{i_3,t} = 1$) and the duration of the last intervention is 2 ($\delta_{i_4,t} = 2$). All risk values are given in Table 2.

For example, starting at time step 1, the $risk^{s_4,2}_{i_4,1}$ value of intervention i_4 for scenario s_4 at time step 2 is equal to 8 (bold cell). The initial objective function coefficients $c_{i,t}$ are computed using formula (6): we obtain the following value for intervention $i = 1$ starting day $t = 1$: $c_{1,1} = \frac{1}{3} \times \frac{1}{7} \times (10 + 12 + 8 + 5 + 12 + 7 + 10) = \frac{64}{21} = 3.048$. The evolution of these coefficients during 8 iterations of `SPM` procedure is given in Table 3.

Solving `MIP_Mean`(c) with these $c_{i,t}$ values produces the solution with optimal mean risk. Since we do not have any resource or exclusion constraints, the optimal solution can be obtained by simply selecting the time step with the smallest $c_{i,t}$ value for each intervention i. Selected time steps are bold at iteration 1 given the solution $x_{1,1} = x_{2,3} = x_{3,3} = x_{4,1} = 1$, and in italic at iteration 2 given the solution $x_{1,2} = x_{2,3} = x_{3,1} = x_{4,2} = 1$. We have calculated new scenario coefficients, update penalties and frequencies only for the two scenarios with greatest risk ($|\mathcal{S}_t| - \lceil \tau \times |\mathcal{S}_t| \rceil = 7 - 5 = 2$)—achieved in Algorithm 3 by adding 1 to the previous frequency values and adding $\Delta_{penalty} = \frac{(Q^t_\tau(x) - Risk^t_{mean}(x)) \times |\mathcal{S}_t|}{|\mathcal{S}_t| - \lceil \tau \times |\mathcal{S}_t| \rceil}$ to the previous penalty values, giving us the values shown in line 2 of Table 3. In the last line of Table 3 bolditalic values correspond to the solution $x_{1,3} = x_{2,3} = x_{3,1} = x_{4,1} = 1$ which is optimal.

Table 4 Characteristics of the 30 instances with $|\mathcal{J}|= 9$ and $\alpha = 0.5$

Inst	Data set C					Data set X				
	$\|\mathcal{I}\|$	$\|\mathcal{T}\|$	$\|\mathcal{E}\|$	$\|\mathcal{S}_t\|$	τ	$\|\mathcal{I}\|$	$\|\mathcal{T}\|$	$\|\mathcal{E}\|$	$\|\mathcal{S}_t\|$	τ
1	120	53	54	169–207	0.95	120	53	48	169–207	0.80
2	120	53	43	169–207	0.80	706	53	1234	56–69	0.85
3	706	53	1223	56–69	0.85	280	53	162	169–207	0.80
4	706	53	1194	56–69	0.90	426	25	490	175–203	0.80
5	706	53	1377	56–69	0.95	467	220	604	84–103	0.85
6	280	53	183	169–207	0.80	528	300	703	45–55	0.95
7	120	42	38	113–138	0.95	209	300	80	56–69	0.90
8	426	25	340	175–207	0.80	209	300	57	56–69	0.90
9	110	53	38	169–207	0.90	548	30	820	141–173	0.80
10	522	102	705	56–69	0.95	460	35	527	146–173	0.95
11	89	102	35	171–207	0.90	521	131	725	56–69	0.95
12	298	191	195	84–103	0.80	522	131	723	56–69	0.95
13	505	230	533	56–69	0.95	336	212	248	84–103	0.90
14	465	220	620	84–103	0.85	613	180	951	56–69	0.95
15	528	300	624	45–55	0.95	613	180	917	56–69	0.95

4 Experimental results in the context of ROADEF-EURO challenge

Our `SPM` matheuristic obtains the best results in the final phase of the ROADEF/EURO 2020 challenge. In this phase, 13 teams (i.e. algorithms), denoted by $\mathcal{A} = \{J3, J24, J43, J49, J73, S14, S19, S28, S34, S56, S58, S66, S68\}$, were qualified as finalists over 74 registered junior and senior teams. The `SPM` matheuristic corresponds to the algorithm proposed by team $S34$ composed by the two first authors of this paper.

Platform The computer used to evaluate the programs of the teams is a Linux OS machine with 2 CPU, 16 GB of RAM, and the allowed list of MILP solvers are CPLEX, Gurobi and LocalSolver. Our winning `SPM` algorithm was implemented in `C++` language and used Gurobi as an MIP solver to solve the `MIP_Mean`(c) approximations of `MIP_full`.

Dataset To evaluate the 13 finalist algorithms, a set of 30 industrial problem instances was provided by `RTE`, divided into two data sets (Data set C and Data set X) with 15 instances for each one. Those data sets are available at the website of the challenge (ROADEF 2020). The characteristics of these 30 instances are given in Table 4.

Time limit Two stopping criteria of each algorithm on each instance are used to differentiate the 13 algorithms by imposing two time limits of 15 and 90 min.

Objective evaluation Given a time limit TL and a dataset $K = C$ or X. For each instance $k \in K$ and each algorithm $a \in \mathcal{A}$, the evaluation function of the returned solution x by algorithm a is given by $obj_{k,a}^{TL} = Objective(x)$ if x is feasible, and otherwise $obj_{k,a}^{TL} = \infty$. The final score attribuated to an algorithm $a \in \mathcal{A}$ on an instance $k \in K$ uses the convex weighting

$$obj_{k,a} = 0.8 \times obj_{k,a}^{15} + 0.2 \times obj_{k,a}^{90}.$$

Finally, given a dataset K, for each algorithm $a \in \mathcal{A}$, the number of times where algorithm a fails to find a feasible solution or crashes with time limit TL is given by

$$Crash_{K,a}^{TL} = |\{k \in K : a \text{ is infeasible or crashes with time limit } TL\}|.$$

Ranking method Let $Better_{k,a}$ be the number of algorithms with a result strictly better than the result of algorithm a on instance k, i.e.

$$Better_{k,a} = |\{a' \in \mathcal{A} : a' \neq a, obj_{k,a'} < obj_{k,a}\}|.$$

For each dataset K, we compute the sum of $Better_{k,a}$ over $k \in K$, i.e.

$$Better_{K,a} = \sum_{k \in K} Better_{k,a}$$

and the number of instances with $Better_{k,a} = 0$ over $k \in K$, i.e.

$$Better_{K,a}^{0} = |\{k \in K : Better_{k,a} = 0\}|.$$

The score of an algorithm a for the instance k is defined by

$$Score_{k,a} = \begin{cases} max(0; Better^{*} - Better_{k,a}) & \text{if x is feasible} \\ 0 & \text{if x is unfeasible or crash} \end{cases}$$

where $Better^{*}$ is the maximal score that an algorithm can earn from one instance. During both the qualification and final phases, $Better^{*}$ is equal to 10. The global score of an algorithm a, on a given dataset K denoted as $score(a)$ is defined by

$$Score_{K,a} = \sum_{k \in K} Score_{k,a}.$$

The score of an algorithm a for the dataset K is given by

$$Score_{K,a}^{*} = |\{k \in K : Score_{k,a} = Better^{*}\}|.$$

Given a time limit TL and a dataset K, for each algorithm $a \in \mathcal{A}$, the number of times where the algorithm a is the best in terms of the relative gap value, is defined as follows

$$Best_{K,a}^{TL} = |\{k \in K : \Delta_{k,a}^{TL} = 0\}|$$

where

$$\Delta^{TL}_{k,a} = \left\lfloor 1000 \times \frac{obj_{k,a} - obj^*_k}{obj_{k,a}} \right\rfloor$$

and where obj^*_k is the best value over the algorithms $a \in \mathcal{A}$, i.e. $obj^*_k = \min\{obj_{k,a} : a \in \mathcal{A}\}$.

Table 5 presents the values of the 6 parameters ($Score_{K,a}$, $Score^*_{K,a}$, $Better_{K,a}$, $Best^{TL}_{K,a}$, $Crash^{TL}_{K,a}$) used to evaluate an algorithm $a \in \mathcal{A}$ over a dataset $K \in \{C, X, C + X\}$. The values provided in this table are obtained from disggregated information described in the 4 matrices ($Score_{k,a}$, $Better_{k,a}$, $\Delta^{15}_{k,a}$, $\Delta^{90}_{k,a}$) presented in the Appendix.

The final ranking of the ROADEF-EURO challenge is based on the $Score^*_{C+X,a}$ values for the submitted algorithms $a \in \mathcal{A}$. The winning algorithm a^* is the one that receives the highest score, i.e.

$$Score^*_{C+X,a^*} = max\{Score^*_{C+X,a} : a \in \mathcal{A}\}$$

.
Our SPM algorithm (i.e. $a^* = S34$) strictly dominated all the other algorithms $a \in \mathcal{A} - \{a^*\}$ on the 18 comparison criteria ($Score_{K,a}$, $Score^*_{K,a}$, $Better_{K,a}$, $Best^{TL}_{K,a}$) for $K \in \{C, X, C+X\}$ and $TL \in \{15, 90\}$, (which justifies the word "efficient" in the title of this paper!). However, there is no dominance among the remaining 4 best algorithms, i.e. $\mathcal{A}^* = \{a \in \mathcal{A} : Score^*_{C+X,a} \geq 150\}$ with or without the 6 criteria $Crash^{TL}_{K,a}$ for $K \in \{C, X, C+X\}$ and $TL \in \{15, 90\}$. For example, the second algorithm $a^2 = S66$ is dominated by the third algorithm $a^3 = S56$ on the two criteria $Best^{15}_{K,a}$) and $Crash^{15}_{K,a}$. The fourth algorithm $a^4 = S19$ dominates the second algorithm a^2 and the third algorithm a^3 on the 4 criteria ($Score^*_{C,a}$, $Better^*_{C,a}$, $Crash^{90}_{X,a}$, $Crash^{15}_{C+X,a}$).

Basic lower and upper bounds: Since for any feasible solution x the value $Excess(x)$ is non negative and $\alpha \in [0, 1]$ we have $Objective(x) \geq Mean(x)$. Consequently, a lower bound on the optimal value of `MIP_full` can be obtained by solving the `MIP_mean`(c^0), i.e.

$$LB = \alpha \times v(\texttt{MIP\_mean}(c^0)) \leq v(\texttt{MIP\_full})$$

where $v(P)$ is the optimal value of a given optimization problem P.

From Proposition 2, for any $t \in \mathcal{T}$ such that $|\mathcal{S}_t| = \lceil \tau \times |\mathcal{S}_t| \rceil$, we have $Excess^t(x) = Risk^t_{max}(x) - Risk^t_{mean}(x)$ which is valid for $\tau = 1$, the objective function can be expressed as follows:

$$Objective(x) = \frac{2 \times \alpha - 1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} Risk^t_{mean}(x) + \frac{(1 - \alpha)}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} Risk^t_{max}(x)$$

Table 5 Final results of the ROADEF/EURO 2020 challenge on datasets C, X and $C+X$

Algorithm a	J3	J24	J43	J49	J73	S14	S19	S28	S34	S56	S58	S66	S68
$Score_{C,a}$	21	16	51	55	60	13	103	7	142	122	48	124	66
$Score^*_{C,a}$	0	0	0	0	0	0	3	0	10	1	0	2	1
$Better_{C,a}$	135	139	105	98	90	139	47	154	8	28	109	26	88
$Better^0_{C,a}$	0	0	0	0	0	0	3	0	10	1	0	2	1
$Best^{15}_{C,a}$	1	1	1	1	1	0	4	0	14	10	7	9	4
$Best^{90}_{C,a}$	3	1	1	2	2	0	5	0	14	9	4	9	3
$Crash^{15}_{C,a}$	0	0	2	1	0	0	0	3	0	0	0	0	1
$Crash^{90}_{C,a}$	0	0	0	1	0	0	0	3	0	0	5	0	2
$Score_{X,a}$	27	33	26	22	79	27	84	6	149	103	90	112	73
$Score^*_{X,a}$	0	0	0	0	0	0	0	0	14	0	0	1	0
$Better_{X,a}$	126	121	126	129	71	123	66	155	1	45	62	38	75
$Better^0_{X,a}$	0	0	0	0	0	0	0	0	14	0	0	1	0
$Best^{15}_{X,a}$	0	0	0	0	0	0	0	0	15	5	4	6	2
$Best^{90}_{X,a}$	0	0	0	0	1	0	0	0	14	6	6	6	3
$Crash^{15}_{X,a}$	1	0	7	9	0	5	0	11	0	2	2	3	2
$Crash^{90}_{X,a}$	0	0	1	6	0	1	1	10	0	2	0	0	2
$Score_{C+X,a}$	48	49	77	77	139	40	187	13	291	225	138	236	139
$Score^*_{C+X,a}$	0	0	0	0	0	0	3	0	24	1	0	3	1
$Better_{C+X,a}$	261	260	231	227	161	262	113	309	9	73	171	64	163
$Better^0_{C+X,a}$	0	0	0	0	0	0	3	0	24	1	0	3	1
$Best^{15}_{C+X,a}$	1	1	1	1	1	0	4	0	29	15	11	15	6
$Best^{90}_{C+X,a}$	3	1	1	2	3	0	5	0	28	15	10	15	6
$Crash^{15}_{C+X,a}$	1	0	9	10	0	5	0	14	0	2	2	3	3
$Crash^{90}_{C+X,a}$	0	0	1	7	0	1	1	13	0	2	5	0	4

In this case (i.e. $\tau = 1$), the non-linear optimization `MIP_full` can be stated as the linear mixed integer programming problem:

$$(\texttt{MIP\_Q1}) \begin{cases} \min obj = \frac{2\times\alpha-1}{|\mathcal{T}|} \sum_{t\in\mathcal{T}} Risk^t_{mean}(x) + \frac{(1-\alpha)}{|\mathcal{T}|} \sum_{t\in\mathcal{T}} z_t \\ \text{s.t.: } (1), (2) \text{ and } (3), \\ \quad z_t \geq Risk^{s,t}(x), \forall t \in \mathcal{T}, s \in S_t, \\ \quad x \in \{0,1\}^{|\mathcal{I}|\times|\mathcal{T}|}, z \in \mathbb{R}^{|\mathcal{T}|} \end{cases}$$

where z_t represents $Risk^t_{max}(x)$ for $t \in \mathcal{T}$. Since $Risk^t_{max}(x) \leq Risk^t_{mean}(x)$, a basic upper bound

$$UB = v(\texttt{MIP\_Q1}) \geq v(\texttt{MIP\_Full}).$$

Table 6 gives the lower bound (*LB*) and the upper bound (*UB*) for *C* and *X* datasets. MIP solver has been used to compute the bounds: we have set parameters gap to 0.0005 and time limit to 3600 s. All solutions produced by `SPM` algorithm have a better objective than *UB*. But non exact approaches can produce worse solutions than *UB*.

Behavior of `SPM` *algorithm* Results have been obtained by running the *SPM* algorithm with 10 seeds. Last column gives the best values obtained by the challenge competitors. Table 6 below summarizes the results obtained by our approach.

5 Conclusion

Our new scenario penalization matheuristic (`SPM`), for the mean-risk model involving quantiles related to value at risk measure (`VAR`) is an instance of the highly general framework of ghost image processing (`GIP`) proposed by Glover (1994), and is based on both mixed integer programming (`MIP`) models and local search (`LS`) methods. In this paper, we considered the application of the mean-risk model arising in the stochastic scheduling problem (`SSP`) formulated as a MIP model (`MIP_full`) with binary decision variables where the objective function is a non-linear (generally non-convex) function and the constraints are linear. The SPM matheuritic involves a parameterization of the objective function that is progressively modified to generate feasible solutions which are improved by local search. The initial parameterization provides an approximation `MIP_mean`(c) that corresponds to the `MIP_full` model by ignoring the excess cost (the non linear part of the original objective function). At each iteration, from an optimal or best solution of the current approximation `MIP_mean`(c), the parameter cost matrix c is modified in order to improve the next approximation by penalizing the scenario risks. The current feasible solution is improved by a local search procedure based on the combination of shift and swap neighborhoods. This approach won the first prize in the senior category of the EURO/ROADEF 2020 challenge. Experimental results are presented which demonstrate the effectiveness of our approach on large instances provided by the French electricity transmission network `RTE`.

Table 6 Behavior of SPM algorithm

Instance	*LB*	15 min			90 min			*UB*
		Average	Best	Challenge	Average	Best	Challenge	
C01	2004.38	8515.90	8515.90	8515.90	8515.90	8515.90	8515.90	8677.32
C02	2197.87	3541.27	3540.24	3541.65	3539.67	3538.16	3539.80	4168.92
C03	11,202.71	33,514.49	33,513.15	33,511.70	33,513.48	33,512.63	33,512.26	33,795.93
C04	11,076.18	37,592.72	37,589.07	37,585.73	37,590.55	37,587.86	37,586.31	37,867.05
C05	1035.68	3168.29	3167.07	3166.89	3167.23	3166.67	3166.18	3214.66
C06	4584.21	8401.94	8399.82	8396.00	8400.59	8398.13	8394.48	9921.36
C07	1742.22	6085.07	6083.04	6083.27	6083.04	6083.04	6083.04	6193.27
C08	6397.48	11,163.64	11,158.58	11,162.84	11,157.97	11,152.15	11,155.64	12,778.86
C09	1927.08	5600.41	5596.45	5586.98	5598.75	5596.82	5585.65	6560.18
C10	12,923.01	43,343.11	43,342.48	43,342.49	43,342.15	43,341.83	43,341.84	43,599.30
C11	2306.20	5749.95	5749.95	5749.95	5749.95	5749.95	5749.95	5890.23
C12	6916.38	12,726.23	12,720.54	12,721.13	12,725.77	12,719.95	12,718.79	14,062.89
C13	12,730.23	42,490.05	42,488.24	42,487.99	42,486.11	42,484.17	42,484.56	42,722.61
C14	10,696.36	26,484.12	26,476.34	26,467.22	26,475.58	26,462.26	26,457.11	31,291.59
C15	13,913.86	39,759.93	39,759.43	39,758.03	39,758.25	39,757.03	39,757.54	39,936.22
X01	2059.00	4014.69	4013.07	4014.37	4014.67	4014.37	4011.38	4934.86
X02	12,084.05	32,233.07	32,232.61	32,231.44	32,232.19	32,230.50	32,228.64	32,516.23
X03	4392.66	8106.03	8101.47	8104.54	8101.43	8099.46	8102.59	9567.29
X04	5954.28	11,306.58	11,297.59	11,315.95	11,299.80	11,294.90	11,303.40	13,471.33
X05	10,089.50	22,862.03	22,856.17	22,858.11	22,852.29	22,844.25	22,837.42	25,672.49
X06	12,513.95	47,034.63	47,034.33	47,032.96	47,033.09	47,029.01	47,032.16	47,224.30
X07	5567.17	13,221.59	13,221.50	13,221.62	13,221.58	13,221.50	13,221.36	13,376.11
X08	5340.95	13,731.07	13,726.46	13,717.37	13,714.12	13,709.56	13,707.29	16,006.74
X09	8747.08	20,203.38	20,191.66	20,195.41	20,192.59	20,180.70	20,180.45	25,261.84
X10	6733.26	17,285.42	17,276.33	17,289.32	17,269.76	17,245.75	17,267.82	19,960.94
X11	12,345.36	39,120.02	39,119.46	39,121.52	39,116.58	39,115.05	39,115.27	39,330.49
X12	11,589.49	47,582.00	47,546.70	47,502.81	47,531.48	47,500.68	47,441.37	57,727.16
X13	8789.77	15,788.38	15,785.95	15,784.25	15,786.07	15,782.77	15,784.25	17,458.48
X14	15,683.41	79,424.23	79,422.33	79,417.03	79,419.54	79,413.79	79,416.87	79,896.10
X15	14,802.74	45,540.78	45,526.50	45,491.81	45,459.32	45,428.37	45,422.29	52,577.95

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Annexes

See Tables 7, 8, 9 and 10.

Table 7 Score matrix: $Score_{k,a}, k \in C + X, a \in \mathcal{A}$

k/a	J3	J24	J43	J49	J73	S14	S19	S28	S34	S56	S58	S66	S68
C01	4	0	6	3	7	2	5	1	10	9	0	8	0
C02	2	0	7	4	1	0	8	0	10	9	5	6	3
C03	0	1	6	0	4	3	5	2	10	9	0	8	7
C04	0	3	5	8	2	0	10	0	9	6	4	7	1
C05	2	1	3	4	6	0	7	0	9	10	0	8	5
C06	0	0	5	4	2	0	9	1	10	7	6	8	3
C07	3	1	4	2	5	0	6	0	7	9	0	8	10
C08	0	0	4	5	2	1	8	0	10	9	6	7	3
C09	2	1	6	0	3	0	10	0	8	7	4	9	5
C10	0	1	3	4	6	2	5	0	10	8	0	9	7
C11	7	2	0	4	3	0	10	1	10	6	5	10	0
C12	0	3	0	7	2	0	6	1	10	8	5	9	4
C13	0	0	0	3	5	2	4	1	10	9	6	8	7
C14	0	1	2	3	6	0	5	0	10	8	7	9	4
C15	1	2	0	4	6	3	5	0	9	8	0	10	7
X01	1	0	7	2	3	0	6	0	10	8	5	9	4
X02	3	2	1	0	6	1	4	0	10	7	9	8	5
X03	2	1	0	0	4	3	6	0	10	7	8	9	5
X04	7	6	0	0	8	4	5	0	10	0	9	4	0
X05	2	3	4	0	6	1	5	0	10	9	8	0	7
X06	1	2	0	4	7	3	5	0	10	8	0	9	6
X07	0	0	0	3	5	2	4	1	10	9	7	8	6
X08	0	0	0	3	5	1	7	2	10	8	6	9	4
X09	3	2	1	0	6	4	5	0	10	0	8	9	7
X10	1	3	4	0	5	2	7	0	10	9	6	8	0
X11	1	2	3	0	5	0	4	0	10	6	8	9	7
X12	3	5	4	1	2	1	7	0	10	8	1	9	6
X13	0	2	0	4	5	1	7	3	10	8	0	9	6
X14	0	1	0	3	6	2	4	0	9	7	8	10	5
X15	3	4	2	2	6	2	8	0	10	9	7	2	5

Table 8 Better matrix: $Better_{k,a}, k \in C + X, a \in \mathcal{A}$

k/a	J3	J24	J43	J49	J73	S14	S19	S28	S34	S56	S58	S66	S68
C01	6	10	4	7	3	8	5	9	0	1	11	2	12
C02	8	12	3	6	9	10	2	11	0	1	5	4	7
C03	10	9	4	11	6	7	5	8	0	1	11	2	3
C04	10	7	5	2	8	11	0	12	1	4	6	3	9
C05	8	9	7	6	4	11	3	10	1	0	12	2	5
C06	12	11	5	6	8	10	1	9	0	3	4	2	7
C07	7	9	6	8	5	10	4	11	3	1	12	2	0
C08	11	12	6	5	8	9	2	10	0	1	4	3	7
C09	8	9	4	12	7	10	0	11	2	3	6	1	5
C10	10	9	7	6	4	8	5	12	0	2	11	1	3
C11	3	8	11	6	7	10	0	9	0	4	5	0	12
C12	11	7	12	3	8	10	4	9	0	2	5	1	6
C13	11	10	12	7	5	8	6	9	0	1	4	2	3
C14	11	9	8	7	4	10	5	12	0	2	3	1	6
C15	9	8	11	6	4	7	5	12	1	2	10	0	3
X01	9	12	3	8	7	10	4	11	0	2	5	1	6
X02	7	8	9	11	4	9	6	11	0	3	1	2	5
X03	8	9	10	11	6	7	4	11	0	3	2	1	5
X04	3	4	8	8	2	6	5	8	0	8	1	6	8
X05	8	7	6	11	4	9	5	11	0	1	2	10	3
X06	9	8	11	6	3	7	5	11	0	2	10	1	4
X07	11	12	10	7	5	8	6	9	0	1	3	2	4
X08	12	10	11	7	5	9	3	8	0	2	4	1	6
X09	7	8	9	10	4	6	5	10	0	10	2	1	3
X10	9	7	6	10	5	8	3	10	0	1	4	2	10
X11	9	8	7	10	5	10	6	12	0	4	2	1	3
X12	7	5	6	9	8	9	3	12	0	2	9	1	4
X13	10	8	11	6	5	9	3	7	0	2	12	1	4
X14	10	9	11	7	4	8	6	12	1	3	2	0	5
X15	7	6	8	8	4	8	2	12	0	1	3	8	5

Table 9 Gap matrix: $\Delta^{15}_{k,a}$, $k \in C + X$, $a \in \mathcal{A}$

k/a	J3	J24	J43	J49	J73	S14	S19	S28	S34	S56	S58	S66	S68
C01	1	28	1	3	0	13	1	16	0	0	0	0	∞
C02	11	23	2	2	11	12	1	20	0	0	2	2	9
C03	50	40	1	6	1	21	1	28	0	0	0	0	0
C04	2	0	0	0	1	2	0	2	0	0	0	0	1
C05	7	56	5	5	1	103	1	88	0	0	1	0	2
C06	22	19	6	8	14	19	1	18	0	5	5	3	12
C07	2	35	1	5	1	50	0	59	0	0	0	0	0
C08	28	32	8	8	20	23	3	25	0	1	3	4	19
C09	16	19	7	∞	14	32	0	35	3	4	7	2	7
C10	56	42	21	13	1	24	3	∞	0	0	0	0	1
C11	0	10	16	1	2	11	0	9	0	0	0	0	0
C12	11	7	17	2	6	10	2	10	0	2	4	1	6
C13	42	28	∞	12	1	19	3	21	0	0	0	0	0
C14	37	24	21	9	4	26	5	∞	0	1	4	1	6
C15	50	23	∞	9	1	15	2	∞	0	0	95	0	1
X01	23	37	1	12	10	26	2	26	0	1	4	1	7
X02	69	106	∞	∞	1	∞	5	∞	0	0	0	0	3
X03	24	33	∞	∞	17	20	8	∞	0	5	4	4	12
X04	35	57	∞	∞	19	∞	6	∞	0	∞	4	∞	∞
X05	25	25	23	∞	5	29	7	∞	0	1	3	∞	4
X06	30	16	∞	12	1	13	1	∞	0	0	51	0	1
X07	20	21	14	10	1	12	2	12	0	0	0	0	0
X08	37	25	30	13	8	22	3	21	0	2	8	1	9
X09	65	70	∞	∞	7	40	11	∞	0	∞	5	0	6
X10	∞	49	12	∞	7	34	5	∞	0	1	5	1	∞
X11	37	28	23	∞	1	∞	3	∞	0	0	0	0	0
X12	63	36	39	∞	98	∞	6	∞	0	3	∞	1	16
X13	17	13	22	9	4	13	3	12	0	1	∞	1	4
X14	40	26	∞	19	1	20	3	∞	0	0	0	0	3
X15	62	30	∞	∞	10	∞	4	∞	0	1	8	∞	9

Table 10 Gap matrix: $\Delta^{90}_{k,a}, k \in C + X, a \in \mathcal{A}$

k/a	J3	J24	J43	J49	J73	S14	S19	S28	S34	S56	S58	S66	S68
C01	1	22	1	0	0	11	1	13	0	0	∞	0	∞
C02	10	17	2	3	12	10	1	15	0	1	2	1	9
C03	20	34	1	∞	1	19	1	18	0	0	∞	0	0
C04	0	0	0	0	1	1	0	1	0	0	0	0	1
C05	5	42	7	5	0	98	0	71	0	0	∞	0	2
C06	15	16	6	7	13	17	1	16	0	1	5	2	12
C07	1	24	1	5	1	28	0	43	0	0	∞	0	0
C08	20	27	8	9	18	23	3	23	0	1	4	3	19
C09	14	19	4	4	11	22	0	32	2	3	7	2	7
C10	0	32	17	10	1	22	2	∞	0	0	∞	0	1
C11	0	6	17	1	2	9	0	7	0	0	0	0	∞
C12	9	2	10	1	6	9	2	9	0	1	3	1	6
C13	25	23	15	9	1	13	2	14	0	0	0	0	0
C14	27	16	17	7	5	23	3	∞	0	1	4	1	5
C15	16	16	68	7	1	9	1	∞	0	0	0	0	1
X01	21	34	3	10	9	23	1	27	0	0	5	0	8
X02	31	94	1	∞	1	24	4	∞	0	0	0	0	0
X03	20	31	6	∞	17	17	5	∞	0	1	3	1	12
X04	24	58	∞	∞	19	25	∞	∞	0	∞	5	2	∞
X05	22	19	18	∞	6	23	5	∞	0	1	3	1	5
X06	18	11	40	6	1	8	1	7	0	0	0	0	1
X07	10	13	11	6	0	11	1	10	0	0	0	0	0
X08	35	23	25	8	7	21	2	19	0	2	5	1	9
X09	51	63	7	∞	8	39	12	∞	0	∞	5	1	7
X10	43	51	7	∞	7	∞	6	∞	0	1	8	1	∞
X11	1	25	22	8	1	17	1	∞	0	0	0	0	0
X12	54	33	33	16	46	37	6	∞	1	3	0	2	17
X13	15	11	21	5	4	12	2	12	0	1	3	1	4
X14	23	23	32	10	1	15	3	∞	0	0	0	0	2
X15	46	27	22	15	7	33	4	∞	0	1	4	1	11

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