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An optimal decision making model for supporting *week hospital* management

Domenico Conforti · Francesca Guerriero · Rosita Guido · Marco Matucci Cerinic · Maria Letizia Conforti

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Abstract Week Hospital is an innovative inpatient health care organization and management, by which hospital stay services are planned in advance and delivered on week-time basis to elective patients. In this context, a strategic decision is the optimal clinical management of patients, and, in particular, devising efficient and effective admission and scheduling procedures, by tackling different requirements such as beds' availability, diagnostic resources, and treatment capabilities. The main aim is to maximize the patient flow, by ensuring the delivery of all clinical services during the week. In this paper, the optimal management of Week Hospital patients is considered. We have developed and validated an innovative integer programming model, based on clinical resources allocation and beds utilization. In particular, the model aims at scheduling Week Hospital patients' admission/discharge, possibly reducing the length of stay on the basis of an available timetable of clinical services. The performance of the model has been evaluated, in terms of efficiency and robustness, by considering real data coming from a Week Hospital Rheumatology Division. The experimental results have been satisfactory and demonstrate the effectiveness of the proposed approach.

Keywords Health care services · Week hospital · Patient scheduling · Decision making problem · Mathematical programming

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D. Conforti · F. Guerriero · R. Guido (✉)
Laboratory of Decisions Engineering for Health Care Delivery, Department of Electronics, Computer Science and Systems, University of Calabria, Ponte Pietro Bucci 41C, 87036 Rende Cosenza, Italy
e-mail: rguido@deis.unical.it

D. Conforti
e-mail: mimmo.conforti@unical.it

F. Guerriero
e-mail: guerriero@deis.unical.it

M. Matucci Cerinic · M. L. Conforti
Rheumatology Division, Department of Internal Medicine, University of Florence, Viale Pieraccini 18, 50139 Firenze, Italy

M. Matucci Cerinic
e-mail: marco.matuccicerinic@unifi.it

1 Introduction

Scientific and technological innovations, aging population, and limited economic resources are factors that, among the others, have generally affected the health care scenarios of recent years since their remarkable impact to services planning, organization, and management. The same factors raised also need for a substantial improvement of hospital services organization and management. In fact, some important issues have been related to a greater integration with primary care and continuous assistance services, internal reorganization matching criteria of efficiency and effectiveness, stronger involvement of health professionals in change processes.

Under this respect, the basic orientation of health care policies at national and regional level moves to define and test new organizational models that may effectively impact on care workflows and, on the other side, optimise the use of resources. For instance, several clinical domains (e.g., gastroenterology, diabetes, endocrinology, rheumatology) periodically require that a patient carries out diagnostic tests and therapeutic treatments, usually performed during a hospitalization time with night stay. Following these issues, a novel organizational model of inpatient hospital care services planned and delivered on a weekly time horizon, seems to fully respond to these requirements, mainly allowing a reduction of inpatient length of stay with a better benefit for patient health. We refer to this model as Week Hospital (WH for short) system.

A WH system is a planned hospital stay whose maximum duration is five weekdays. It basically aims at performing inpatient diagnostic tests and clinical care in a more effective and efficient way with respect to ordinary hospitalization, by exploiting the available time horizon of one week. Each elective patient, waiting to be admitted to WH, is characterized by a clinical pathway, defined by physician and detailing the set of diagnostic tests and therapeutic treatments. Moreover, effective WH patient management implies also the assignment of a suitable clinical priority.

It is our aim, by this paper, to introduce and propose WH as a new standard, which extends the taxonomy of hospitalization: day hospital, *week hospital*, ordinary/long stay hospital.

The main issues of WH organization concern the admission policies and the patient scheduling. The admission planning problem aims at defining the set of admitting patients, whereas the patients scheduling problem deals with the management of diagnostic tests and clinical care. In the following, we briefly refer to diagnostic tests, therapeutic treatments, and related resources as “clinical services”.

In order to allow an efficient use of clinical services and increase, if possible, the admission rate to WH, we have developed an optimization model for supporting the area manager in planning and scheduling patients and clinical services into a WH division. Furthermore, our contribution lies also on defining a novel rule able to manage several dynamic aspects characterizing this context.

Even though, to the best of our knowledge, this specific problem has not been yet addressed in the scientific literature, it shares some common features with the outpatients appointment scheduling problem. An extensive review on this topic is due to Cayirli and Veral [1]. Interesting contribution has been given by the paper of Gupta and Denton [2], which describes the most common types of health care delivery systems with focusing on the factors that make rather challenging appointment scheduling problems. Several authors [3–7] recognized that demand and capacity have to be balanced otherwise patient dissatisfaction is high and wasting of resources is unavoidable; in addition, also timely access and patient no-show need to be considered [3, 8, 9]. In most of the cases, queueing models have been proposed for appointment-driven health care systems [10].

It is worth observing that these existing approaches cannot be applied to a WH system, since the main requirements are rather different. In fact, the WH management aims at satisfying time constraints (i.e., patient gets admission care if and only if all prescribed clinical services are booked during the week) and specific resource constraints (given by several clinical services and bed availability), which are not considered in outpatient appointment scheduling.

On the other hand, it is worth observing that bed availability has been taken into account to address patients admission planning problem in a general hospital [6]. Usually, the main decision is to select waiting patients by effective and efficient utilization of nursing staff and resource and bed availability; however, time constraints are generally not considered. Recently, a hospital appointment scheduling model, characterized by time and resource constraints, has been proposed by Chern et al. [11], by assuming that every resource is always available. This assumption is rather unrealistic since hospital resources are limited and makes the problem rather difficult to solve. In fact, the proposed model has been solved with a heuristic approach. The appointments are scheduled for checkups without overnight stay (thus bed availability is not needed); consequently, a patient does not have admission/discharge date since all procedures are performed in a “day hospital” outpatient manner. This is substantially the main difference with the WH management problem addressed in this paper.

The paper is structured as follows. In Section 2, we give a general description of the relevant characteristics of WH, with an analysis of the main management problems. In Section 3, we present the optimization model, which is tested in Section 4 within a pilot setting in the Rheumatology Division of “Careggi” University Hospital (Florence, Italy). A discussion on the results and the conclusion section complete the paper.

2 Week hospital: definition and problem statement

Generally, hospital admission classification is related to length of stay (LOS). As a matter of fact, we typically refer to “day hospital”, “ordinary hospital”, and “long stay” according to the relevant inpatient care.

We now remark that several clinical domains, typically characterized by chronic diseases, very often require that patients periodically carry out specialized diagnostic tests and personalized therapeutic treatments. To this end, the physician prescribes inpatient care based on hospital stay with a given minimum LOS, typically independent from the required diagnostic tests, but aimed at obtaining, for example, a detailed monitoring of overall patient’s conditions, an effective tuning of drug dosage, and an assessment of therapy side effects.

In order to meet these requirements, WH is a new way of planned hospital stay allowing an effective and efficient inpatient clinical management along a time horizon of five consecutive weekdays. Since maximum hospital stay is a week, all decisions about appointment time have to be made before the admission phase, in order to efficiently schedule clinical services.

It is worth while to observe that one of the aims of WH system is to avoid frequent hospital re-admissions (following a discharge) since they have high cost implications for a general health care system. As we already pointed out, from a clinical point of view, WH is typically motivated for the management of chronic diseases. In this case, periodically planned WH stays are also aimed at avoiding or anticipating possibly worsening events, which very often imply frequent acute hospital admissions. During a WH stay, prescribed diagnostic tests, specific therapeutic treatments and clinical assessment of the patient conditions can be efficiently carried out. Hence, an optimal management can beneficially impact on the overall economic and social costs associated to inpatient care.

Generally, the organizational layout of WH is based on a dedicated theater composed of a specific ward with a dedicated nurse staff, a given number of available beds and commonly sharing the overall clinical services available in the hospital. WH ward typically works from Monday morning to Friday midday and WH patients usually stay more than 24 h.

Several issues in developing a WH affect the internal organizational dimension of each involved division, also raising some logistic problems. The main aspects are the following:

- a detailed planning and standardization of admission/discharge procedures;
- the respect of the operative times, also through an efficient management of the internal connections;
- the organization of the activities by a logic of division clinical services aiming at improving welfare levels;
- the enlargement of the hospital pre-admission;
- the prevision of the possibility to be not discharged by Friday morning and then the singling out of beds that are available at a continuous cycle;
- the guarantee of the availability during weekends of emergency services and assistance for the discharged patients;
- the definition of suitable care protocols, shared with the primary care, for patients needing continuous assistance.

A typical WH process consists of the following main phases

1. *pre-admission*: usually a pre-admission phase includes a review of patient’s health care history. During a baseline visit, the health care team, by following specific guidelines, prescribes a complete list of diagnostic tests and treatments to be performed assigning also a clinical priority. A set of clinical services is then allotted to evaluate patient’s condition, performing also appropriate care. In case of not immediate hospitalization, the patient is inserted into a waiting list;
2. *patient hospitalization*, during which all prescribed clinical services are delivered. The admission date is planned in such a way the prescribed clinical services can be delivered by (at most) five consecutive weekdays. Worsening of conditions or acute events may require that the planned hospital stay be rescheduled. In this case, the patient is immediately transfered to a dedicated care unit;
3. *follows up*: a first check follows patient discharge, which can be followed by next checks if necessary;
4. *continuous assistance*: appropriate integration of primary, secondary, and tertiary care for selected groups of patients after discharge.

The date and time of patient’s discharge is defined in advance so that the activity of responsible personnel (i.e., planning admission/discharge and scheduling of the several clinical services) is organized accordingly. It is quite evident that this complex problem requires a centralized planning and scheduling function.

The number of weekdays between the admission and the discharge dates of a patient defines the related LOS. Obviously, shorter hospital stays, by making a more efficient use of hospital resources, are preferred in terms of overall system efficiency, since more patients can get care treatment during the same period, improving hospital service, and allowing cost reduction [12]. In this context, an important issue is to ensure, by a more efficient planning of clinical services during the hospitalization period, that patients recover more quickly, leaving hospital as soon as possible. Under this respect, we remark that the overall quality of care from the patient's point of view is guaranteed by the required minimum LOS prescribed by the physician during the baseline visit.

WH management typically involves the following three hierarchical decision levels:

Strategic level is devoted both to clinical service capacity sizing and allocation of the clinical resources;

Tactical level concerns the assignment of the required clinical resources to a specific clinical specialty, typically according to a block time strategy (clinical services timetable);

Operational level is devoted to WH patients planning and scheduling. The main goal is to decide when a waiting patient is hospitalized, that is the definition of the admission/discharge date.

The planning problem considered in this paper is to generate an admission profile for a specified clinical specialty when target patient throughput and hospital resource availability are known. Moreover, a planned admission requires an appropriate coordination of involved staff members and resources. In detail, two different issues affect patients scheduling: (1) a timetable related to the availability of clinical personnel and equipments, in order to carry out the prescribed clinical services, and (2) the number of available beds. Both issues could potentially represent a bottleneck for patients flow within a WH organization. The first is solved at the tactical decision level: a five weekdays timetable of clinical services is developed, typically on the basis of retrospective data and estimated demand. The resulting timetable is obtained at the end of a suitable negotiation phase between clinical division and hospital health care manager.

In this paper, we focus our attention on a simplified case study setting, since we take into account a WH organization based on only one specialty. The specific objective of this study is to design an effective and efficient WH admission/discharge scheduling system. Furthermore, we will show that both tactical and operational decisions affect the efficiency level of WH with some important consequences on patients waiting list and clinical management. In particular, the proposed optimization model allows to schedule patients by efficiently booking all the resources for performing the several clinical services during his/her LOS. Available beds and clinical service capacity are taken into account for defining the "optimal" schedule.

As final remark, we observe that WH management resembles, in terms of structure, the operating rooms management [13] even though it is more complex. In general, it requires a multiple assignment of several clinical resources shared among several divisions.

3 The optimization model

In order to effectively address the described problem, we have developed an integer linear programming model allowing suitable management of the workload of inpatients flow. The decisions are related to which waiting patient is hospitalized and when, on the basis of several patient's clinical paths and hospital resource availability, including beds. Moreover, only one admission date is possible for every hospitalized patient during the planning week. A bed is assigned to a patient only if he/she can undergo all the prescribed clinical services in the planning week (i.e., the assignment of a bed to a patient and then his/her hospitalization implies the booking of all the prescribed services); if this is not the case, patient is not hospitalized.

A crucial aspect that affects a schedule is the planning horizon definition. Herein, we consider a week as planning horizon so that the waiting list is weekly updated. This may allow the respect of equity criteria in the health care system. Furthermore, future analysis of various situations are useful for providing recommendations (such as a reallocation of some resources).

To simplify the mathematical formulation, we introduce the so-called "blocks". We assume that each weekday is divided in two blocks, i.e., morning and afternoon. Since the services of a WH end on Friday midday, there are nine ordered blocks (from Monday morning to Friday morning) in a planning week. A schematic representation of a planning week is reported in Fig. 1. For example, the morning block could start at 8.00 a.m. and end at 2 p.m., whereas the afternoon block starts at 2 p.m. and includes overnight stay. Moreover, every block is split in a fixed number of time slots (TSs) during which specific clinical

	Monday	Thuesday	Wednesday	Thursday	Friday
morning	1	3	5	7	9
afternoon	2	4	6	8	

Fig. 1 Representation of a planning week with 9 blocks

services can be delivered (for the sake of simplicity, it is assumed that the time slots have the same duration). However, this assumption does not affect the validity of the approach in a different situation (e.g., a different number of blocks and different duration of TSs). In each "cell" of such a timetable the clinical service availability and the related capacity per time slot are settled. In particular, we observe that, in most of the cases, the resources for the WH clinical services are shared among several hospital divisions/wards. This implies that the clinical services timetable, which has been defined at the tactical level for a given WH division, denotes the clinical service availability in each time slot of a related block. Moreover, even though for most of the clinical domains, a specific order for services is not prescribed, a timetable implicitly could define a sequence for performing clinical services. This occurs in presence of limited resources. The admission-discharge planning problem is thus represented as a block-scheduling resource-constrained problem. In this way, the number of admitted/discharged patients during each weekday is known per each block.

Before going into the details of the mathematical model formulation, we introduce the used notation and suitable assumptions. More specifically, we have:

- blocks set $\mathcal{B} = \{b : b = 1, \ldots, 9\}$;
- time slots set $\mathcal{K} = \{k : k = 1, \ldots, l\}$;
- clinical services set $\mathcal{S} = \{i : i = 1, \ldots, m\}$;
- patients' ordered waiting list $\mathcal{P} = \{p : p = 1, \ldots, n\}$;
- number of available beds D.

As already mentioned, we basically assume that both the timetable and the capacity of clinical services are defined and known in advance, as well as the number of beds into the hospital division. We thus know the following data:

- for each time slot $k \in \mathcal{K}$ of a block $b \in \mathcal{B}$, the capacity $\mu^i_{bk} \geq 0$ of the clinical service $i \in \mathcal{S}$ is given: for the sake of simplicity, μ^i_{bk} is defined as the number of patients that can undergo the clinical service i in the slot k of the block b;
- for each waiting patient $p \in \mathcal{P}$: the date of the baseline visit D^p_0, the assigned clinical priority pr_p, the minimum LOS nb_p (in terms of number of blocks) required by the physician, and the set of prescribed clinical services expressed as
  $$s^i_p = \begin{cases} 1 & \text{if the service } i \text{ is prescribed to patient } p \\ 0 & \text{otherwise.} \end{cases}$$
  We denote as $ns_p = \sum_{i \in \mathcal{S}} s^i_p$ the number of prescribed clinical services to patient p.

Decision variables

$$x^i_{pbk} = \begin{cases} 1 & \text{patient } p \text{ undergoes the clinical service } i \text{ during slot } k \text{ of block } b \\ 0 & \text{otherwise} \end{cases}$$

$$y_{pb} = \begin{cases} 1 & \text{if patient } p \text{ occupies a bed during block } b \\ 0 & \text{otherwise} \end{cases}$$

$$adm_{pb} = \begin{cases} 1 & \text{if patient } p \text{ is admitted during block } b \\ 0 & \text{otherwise} \end{cases}$$

rs_{pb} = number of the remaining clinical services to which patient p has to be undergone during block b.

rb_{pb} = number of the remaining blocks with respect to the required minimum LOS (i.e., nb_p) for patient p

Constraints For each patient p, every prescribed clinical service has to be booked and performed only once during the planning week:

$$\sum_{b \in \mathcal{B}} \sum_{k \in \mathcal{K}} x^i_{pbk} \leq s^i_p, \quad \forall i \in \mathcal{S}, \forall p \in \mathcal{P}. \tag{1}$$

Obviously, a patient p can undergo at most only one clinical service during slot k of block b:

$$\sum_{i \in \mathcal{S}} x^i_{pbk} \leq 1, \quad \forall p \in \mathcal{P}, \forall b \in \mathcal{B}, \forall k \in \mathcal{K}. \tag{2}$$

The number of patients undergoing the clinical service $i \in \mathcal{S}$ during slot k of block b is upper bounded by the related capacity μ^i_{bk}, that is:

$$\sum_{p \in \mathcal{P}} x^i_{pbk} \leq \mu^i_{bk}, \quad \forall i \in \mathcal{S}, \forall b \in \mathcal{B}, \forall k \in \mathcal{K}. \tag{3}$$

Furthermore, only one admission is possible for each hospitalized patient. The first booked clinical service defines the patient admission block:

$$\sum_{b \in \mathcal{B}} adm_{pb} \leq 1 \quad \forall p \in \mathcal{P}, \tag{4}$$

and no clinical service can be performed before the admission:

$$\sum_{j=1}^{b} adm_{pj} \leq \sum_{j=1}^{b} \sum_{k \in \mathcal{K}} x^i_{pjk} \quad \forall p \in \mathcal{P}, \forall b \in \mathcal{B}. \tag{5}$$

As already stated, each clinical service prescribed to a patient is booked only during his/her hospitalization time. Consequently, by constraints 6–8, we impose that each patient occupies a bed, even though some services cannot be performed during a defined block:

$$x^i_{pbk} \le y_{pb} \qquad \forall i \in \mathcal{S}, \forall p \in \mathcal{P}, \forall b \in \mathcal{B}, \forall k \in \mathcal{K} \tag{6}$$

$$rs_{p1} = ns_p adm_{p1} \qquad \forall p \in \mathcal{P} \tag{7}$$

$$rs_{pb} = ns_p \sum_{j=1}^{b} adm_{pj} - \sum_{i \in \mathcal{S}} \sum_{j=1}^{b-1} \sum_{k \in \mathcal{K}} x^i_{pjk} \qquad \forall p \in \mathcal{P}, b = 2, \ldots, |\mathcal{B}|. \tag{8}$$

Moreover, by the following constraint, a bed can be occupied not before the relevant patient's admission block:

$$\sum_{j=1}^{b} y_{pj} \le b \sum_{j=1}^{b} adm_{pj}, \quad \forall p \in \mathcal{P}, \forall b \in \mathcal{B}. \tag{9}$$

We now remind that LOS of each hospitalized patient has to be at least equal to the number of blocks required by the physician, and obviously a bed is occupied during the related period:

$$rb_{p1} = nb_p adm_{p1} \qquad \forall p \in \mathcal{P} \tag{10}$$

$$rb_{pb} \le nb_p \sum_{j=1}^{b} adm_{pj} - \sum_{j=1}^{b-1} y_{pj} \qquad \forall p, b = 2, \ldots, |\mathcal{B}| \tag{11}$$

$$nb_p y_{pb} \ge rb_{pb} \qquad \forall p \in \mathcal{P}, \forall b \in \mathcal{B} \tag{12}$$

$$\sum_{j=b}^{b+nb_{p-1}} y_{pj} \ge nb_p adm_{pb} \qquad \forall p \in \mathcal{P}, b = 1, \ldots, |\mathcal{B}| - nb_p + 1 \tag{13}$$

$$y_{pb} = 0 \qquad \forall p \in \mathcal{P}, b > |\mathcal{B}| - nb_p + 1 \tag{14}$$

We remark that a patient p has admission care only and only if all clinical services are booked. If a patient is admitted during the planning week, the corresponding binary decision variable $adm_p = 1$ else $adm_p = 0$, that is:

$$\sum_{i \in \mathcal{S}} \sum_{b \in \mathcal{B}} \sum_{k \in \mathcal{K}} x^i_{pbk} = ns_p \sum_{b \in \mathcal{B}} adm_{pb} \quad \forall p \in \mathcal{P}. \tag{15}$$

Finally, the number of beds D is the upper bound for the number of hospitalized patients in each block:

$$\sum_{p \in \mathcal{P}} y_{pb} \le D, \quad \forall b \in \mathcal{B}. \tag{16}$$

Objective function The objective function aims at maximizing the number of admitted patients during the planning week and at scheduling the patients with higher value of weight w:

$$\max \sum_{p \in \mathcal{P}} \sum_{b \in \mathcal{B}} w_p adm_{pb}. \tag{17}$$

The weight w is a value assigned to each waiting patient and, for the specific clinical specialty considered in this paper, represents a score computed by taking into account priority to clinical services, elapsed waiting time, and maximum allowed waiting time. As we will see in the following section, this score is weekly updated for those patients that remain on the waiting list for over one week.

The complete mathematical formulation is reported in the following (where it is not explicitly specified, $i \in \mathcal{S}$, $p \in \mathcal{P}$, $b \in \mathcal{B}$, $k \in \mathcal{K}$):

$$\max \sum_{p \in \mathcal{P}} \sum_{b \in \mathcal{B}} w_p adm_{pb} \tag{18}$$

s.to

$$\sum_{p \in \mathcal{P}} x^i_{pbk} \le \mu^i_{bk} \qquad \forall i, \forall b, \forall k$$

$$\sum_{b \in \mathcal{B}} \sum_{k \in \mathcal{K}} x^i_{pbk} \le s^i_p \qquad \forall i, \forall p$$

$$\sum_{i \in \mathcal{S}} x^i_{pbk} \le 1 \qquad \forall p, \forall b, \forall k$$

$$\sum_{b \in \mathcal{B}} adm_{pb} \le 1 \qquad \forall p$$

$$\sum_{j=1}^{b} adm_{pj} \le \sum_{i \in \mathcal{S}} \sum_{j=1}^{b} \sum_{k \in \mathcal{K}} x^i_{pjk} \qquad \forall p, \forall b$$

$$x^i_{pbk} \le y_{pb} \qquad \forall i, \forall p, \forall b, \forall k$$

$$rs_{p1} = ns_p adm_{p1} \qquad \forall p$$

$$rs_{pb} = ns_p \sum_{j=1}^{b} adm_{pj} - \sum_{i \in \mathcal{S}} \sum_{j=1}^{b-1} \sum_{k \in \mathcal{K}} x^i_{pjk} \qquad \forall p, b = 2, \ldots, |\mathcal{B}|$$

$$\sum_{j=1}^{b} y_{pj} \le b \sum_{j=1}^{b} adm_{pj} \qquad \forall p, \forall b$$

$$rb_{p1} = nb_p adm_{p1} \qquad \forall p$$

$$rb_{pb} \le nb_p \sum_{j=1}^{b} adm_{pj} - \sum_{j=1}^{b-1} y_{pj} \qquad \forall p, b = 2, \ldots, |\mathcal{B}|$$

$$nb_p y_{pb} \ge rb_{pb} \qquad \forall p, \forall b$$

$$\sum_{j=b}^{b+nb_{p-1}} y_{pj} \ge nb_p adm_{pb} \qquad \forall p, b = 1, \ldots, |\mathcal{B}| - nb_p + 1$$

$$y_{pb} = 0 \qquad \forall p, b > |\mathcal{B}| - nb_p + 1$$

$$\sum_{i \in \mathcal{S}} \sum_{b \in \mathcal{B}} \sum_{k \in \mathcal{K}} x^i_{pbk} = ns_p \sum_{b \in \mathcal{B}} adm_{pb} \quad \forall p$$

$$\sum_{p \in \mathcal{P}} y_{pb} \leq D \qquad \forall b$$

$$x^i_{pbk} \in \{0, 1\} \qquad \forall i, \forall p, \forall b, \forall k$$

$$y_{pb}, adm_{pb} \in \{0, 1\} \qquad \forall p, \forall b$$

We observe that the proposed formulation represents an "application driven" discrete (i.e., integer linear) optimization model, specifically developed for the robust and efficient solution of the considered problem. The computational complexity of this class of optimization problems is generally exponentially affected by the size of the instance to be solved, and, in our case, this is mainly related to the cardinalities of the sets $\mathcal{B}$, $\mathcal{K}$, $\mathcal{P}$ and $\mathcal{S}$. Under this respect, we remark that we are mainly interested to obtain a quite reliable solution process, in order to have an effective optimal solution, and we do not need to determine the solution of the model in "real time". As we will see in the next section, the typical size of the above mentioned sets is of the order of tens. Hence, the choice to use theoretically well sound general purpose methods based on exact solution approaches is well-founded.

4 Pilot study within a rheumatology division

The Rheumatology division of "Careggi" University Hospital (Florence, Italy) is one of the most important national clinical site within its own field. During 2009, the division performed 5,400 outpatient visits and 11,786 clinical treatments on day-hospital basis. Sclerosis systemic (i.e., a systemic disorder that affects the connective tissue of the skin, internal organs, and the walls of blood vessels), rheumatoid arthritis, psoriatic arthritis, and spondylitis rheumatology are the main treated pathologies. Patients can be admitted to the WH for many reasons, such as advanced diagnostic tests, drug dosage tuning, stabilization and monitoring of current health conditions.

Rather recently, the division started to experiment WH services, devoted to both diagnostic tests and therapeutic treatments (i.e., clinical services); these are prescribed by physician by considering patient's health history and current pathological conditions. Data collected during the pilot study concerns patients affected by different rheumatic diseases; each patient has a proper set of prescribed clinical services to be performed. Currently, WH organization is based on a timetable of available clinical resources, exploiting the availabilities of only four dedicated beds. Patient flow management is carried out manually, with a considerable effort of area manager, and so far this has implied that trivially only four patients per week have been admitted to the WH.

Generally speaking, it is well known that variability inherently characterizes different health care processes for several reasons. In order to suitable assess the proposed model, we have tackled both clinical variability and flow variability, through simple but rather realistic test cases. A clinical variability is defined by various patient's pathological conditions; this means that the clinical pathway (represented here as a set of prescribed clinical services and minimum required LOS) is typically different for every waiting patient. A flow variability is given by patients arriving during a given period. Moreover, the rheumatologist has typically a close working relationship with other medical doctors (e.g., orthopedists, ophthalmologists, physical and occupational therapists, child psychologists, and nephrologists); this particular aspect increases the difficulty in efficiently tackling the considered problem. As we will show, we have tackled also this variability in the third test case by changing the capacity, defined per time slot, of some clinical services.

To capture some dynamic aspects (which are typical in this context, as already stated) and to manage the waiting list $\mathcal{P}$, we have designed the following rule that assigns a score to each waiting patient:

$$w_p = pr_p\left(D_1 - D_0^p\right)\left(\bar{W} - MaxWait_p\right), \forall p \in \mathcal{P}, \tag{19}$$

where D_1 is the date of the planning-scheduling, $D_1 - D_0^p$ denotes the number of elapsed waiting days for patient p until D_1, pr_p is the clinical priority and $MaxWait_p$ denote the maximum waiting time (in days) allowed for patient p, typically fixed on the basis of the clinical protocols of the relevant specialty. $\bar{W}$ is an appropriate upper bound on the value of $MaxWait_p$ ($\bar{W} \geq \max_{p \in \mathcal{P}}\{MaxWait_p\} + 1$).

The choice of the multiplicative form of Eq. 19 is motivated by its capability to sharper represent (with respect, for example, to an additive form) the differences among the patients in terms of the related values of pr_p, $D_1 - D_0^p$, and $MaxWait_p$.

To differentiate the several rheumatic pathologies that affect patients, three priority classes have been defined by taking into account illness severity. In Table 1, we report these classes with the corresponding priority value and the maximum waiting time for a given patient. It is worth noting, that the maximum waiting time for a patient of class A is smaller than the

Table 1 Priority classes and maximum waiting time (in days) of patients into a Rheumatology division

Class	Priority	MaxWait (days)
A	3	10
B	2	30
C	1	60

waiting time of a patient of class B and class C, due to the higher priority.

In our proposed approach, patient's weight is computed by considering clinical priority, elapsed waiting time (defined as the elapsed time between the date of baseline visit and the date of planning-scheduling), and the maximum number of waiting days. Moreover, patient's waiting time depends on several factors, as we will see in the following. Specifically, we have managed patient's variability and flow variability by considering scenarios concerning the following aspects:

1. different diseases/pathologies with different severity;
2. different minimum required LOS of hospitalized patients;
3. variable patients arrival rate along the planning period;
4. variable capacity of some clinical services.

We suppose all care providers are equal in their ability to provide quality health care.

4.1 Experimental setting and computational results

In this section, we assess the efficiency of the proposed optimisation model 18. To this end, computational experiments have been carried out on realistic instances, by taking as use case the Rheumatology division of "Careggi" University Hospital. The typical set of clinical services, mainly based on diagnostic tests, is reported in Table 2.

The overall health care organization of the hospital is such that the health care manager defines a specific timetable for every division. The relevant clinical services timetable of the Rheumatology division, fixed at the tactical level, is reported in Table 3, where the clinical services are numbered according to the notation of Table 2. Note that, for each slot of every block, the cell entries of Table 3 show the available clinical services, where the reported enumeration does not indicate a prescribed order for service. In fact, as we have already stated, in the case of rheumatic diseases clinical domain, a prescribed order for services is not strictly requested. On the other hand, since we assume that the clinical services timetable is already provided by the hospital health care management, an order for services is implicitly defined by the same timetable.

As mentioned in the previous section, for the sake of simplicity, every time slot (TS) of the timetable has the same duration. However, this does not impair the applicability of the optimisation model to other settings because an important requirement for patients scheduling problem is the defined clinical resource capacity per TS. Indeed, commonly the hospital manager has previously computed a resource capacity per TS, either by taking into account if there are more than one resource for a specific clinical service per TS or if it is possible to execute more than one in the same time. In this way, the capacity of a given resource per TS defines the maximum number of patients that can carry out the relevant clinical service during the defined TS; thus, the time to be spent for executing a specific clinical service is not specified here.

In the following, we have considered three realistic and different test cases (coming out from data of "Careggi" Rheumatology division) representing the decision making process during a planning horizon of two consecutive weeks. A schematic representation of the patient flow and related decision making process, along two consecutive weeks, is reported in Fig. 2.

The number of available beds is 4 in each block. If not specified, the capacity of each clinical service, available into a defined TS, is assumed equal to 2 (i.e., at most two patients can carried out the related clinical service during this TS) and the minimum LOS required by physician equal to 2 blocks for each patient (i.e., $nb_p = 2, \forall p \in \mathcal{P}$). The main aim of WH, for this specific division, is to monitor the course of diseases through suitable clinical tests and assess signs and symptoms for a prompt titration of drug treatments. We remark that the minimum LOS imposed by physician is mainly due to specific medical treatments/procedures that a patient needs to undergo. As we will see, this value typically affects the optimal schedule.

Table 2 Clinical services of rheumatology division

1) Computed tomography scan	10) Scintigraphy
2) X-rays	11) Eco heart
3) Nuclear magnetic resonance	12) Holter heart
4) Biopsia	13) Esophageal manometry
5) Ecography	14) Gastroscopy
6) Eco-doppler	15) Eyes exams
7) Hematochemical analysis	16) Mammography
8) Human leucocyte antigens	17) Angiography
9) Pulmonary function test	18) Colonoscopy

Table 3 Clinical services timetable for the Rheumatology division (m = morning, a = afternoon). Note that the enumeration does not imply a defined order for service

	Slot								
	1	2	3	4	5	6	7	8	9
Monday m	3,5,6,7	3,5,7,8	2,7,8,14	4,5,7,8,10	3,5,6,8,11,18	11,13,14,18	1,14	6,16	17
Monday a	7,8	8	13,16	17		8		7	
Tuesday m	6,13			4	4,6,11	11		7	
Tuesday a	5	5	12	5,12				16	
Wednesday m			2		11	8,11	1,8	8	17
Wednesday a		7	7,12,16	12					
Thursday m		7		4	9			9,15	
Thursday a			12	12					
Friday m			18	18	9		1,3		

4.2 Test case 1

Within this test case (which consists of two consecutive planning weeks), we assume a waiting list of 20 patients at the beginning of the first week. The related information (i.e., prescribed clinical services, priority value, and computed score) are reported in Table 4. We remark that the score of patient p is computed by considering the elapsed time (measured in days) between current date D_1 of planning and scheduling process and D_0^p (date of the baseline visit).

Even though the small size of this instance, it is quite evident the strong difficulty in manually determining an efficient patients scheduling with only "paper and pencil", basically due to the complex combinatorial nature of the WH management problem. The process is time-consuming due to the lack of an efficient software support system and the schedule is, in some cases, far to being optimal. On the contrary, both patients admission/discharge planning problem, and thus the scheduling of clinical services problem, is solved to the optimality, in a few seconds by implementing and solving the proposed optimisation model 18 with the standard MIP solver of ILOG CPLEX 10 (www.ilog.com), on a PC MS Windows XP Professional SP3, Intel Core duo Processor T2400, 1.83 GHz CPU with 1GB RAM. The model is able to select the waiting patients with high score by matching efficiently both the requirements of availability and capacity of the clinical services.

The obtained solution defines the set of patients to hospitalize during the considered week, the related admission and discharge blocks, and the related appointments for performing the required clinical services. In this specific week, eight waiting patients in $\mathcal{P}$ (highlighted in Table 4 with grey color) are selected. The optimal schedule is reported in Fig. 3: for each hospitalized patient both admission and discharge blocks, and thus also his/her LOS, are defined. For

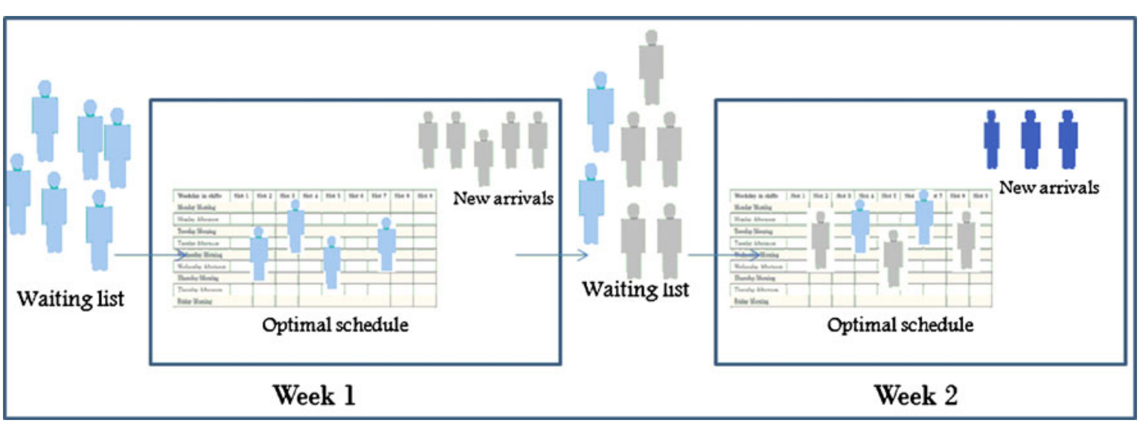

Fig. 2 Patient flow and decision making process for two consecutive weeks. Given a waiting list, some patients are scheduled in the first week. The unscheduled patients with new arrivals form a new waiting list, used to plan the second week

Table 4 Waiting list of the first week of test case 1: waiting time, priority, score, and set of prescribed clinical services

Patient	Priority	Score	Prescribed Clinical Services
1	1	12	1,2,3,5,8,10
2	1	12	3,7,8,10,18
3	1	12	8,11,12
4	2	26.4	1,2,3,5,6,7,8,10,16
5	2	26.4	1,2,3,5,7,8,14,18
6	2	21.12	3,6,7,8,9,16,17
7	3	33.6	7,8
8	1	9.6	7,8
9	2	21.12	8,9
10	1	9.3	3,5,7,8,14,18
11	2	15.84	8,13
12	3	12.6	7,8,13
13	3	11.55	7,8,13
14	2	6.6	5,6,7,8,11,13,14,16,17
15	1	2.7	1,3,4,8,11,12,15,16,18
16	1	2.4	1,2,7,8,10
17	2	4.62	1,2,3,8,16,17
18	3	6.3	1,2,3,8,10
19	3	5.25	1,2,3,7,8
20	3	4.2	1,2,3,5,7,8,12

the sake of brevity, the booked appointments (for this and the other instances) are not reported. We observe that some patients (e.g., P_4, P_5, P_7, P_9) have LOS of only two blocks: these patients undergo all prescribed clinical tests during the minimum required LOS (which is 2 blocks in this specific instance). On the other hand, the LOS of other patients (e.g., P_{11} that has to undergo only 2 clinical services) is longer than 2 blocks. These particular cases can be explained by remarking that not only the specific waiting list and the prescribed clinical services affect the optimal schedule, but also the availability and capacity of the required resources to perform such services. Furthermore, we observe that patients P_1, P_2, P_3 have the same score and P_3 is scheduled. As it has been verified, the schedule still remains optimal if P_2 is hospitalized instead of patient P_3. On the other hand, if P_1 is scheduled, a worse solution (measured in terms of number of hospitalized patients, resources utilization, and total score) is obtained. Finally, we notice that the number of available beds is the upper bound on the number of the scheduled patients. Of course, all these considered issues could not be drawn when a manually scheduling approach is used.

The next step of a good WH management concerns the updating phase of data used to plan the subsequent week. The dynamic aspects of the problem are captured by using the rule 19 and by updating the waiting list with the new arrivals: the set of not currently scheduled patients (e.g., P_1, P_2, and so on) have an updated score due to a further wait (D_1 is now different), whereas the score of new arrivals is defined by their priority class. The value of parameter $\bar{W}$, used to update the score of patients is set to 360. Obviously, a new arrived patient could be scheduled while some patients could remain still waiting. This specific case is provided by the second week, which is characterized by 11 new arrivals. These patients have been visited during the previous week by

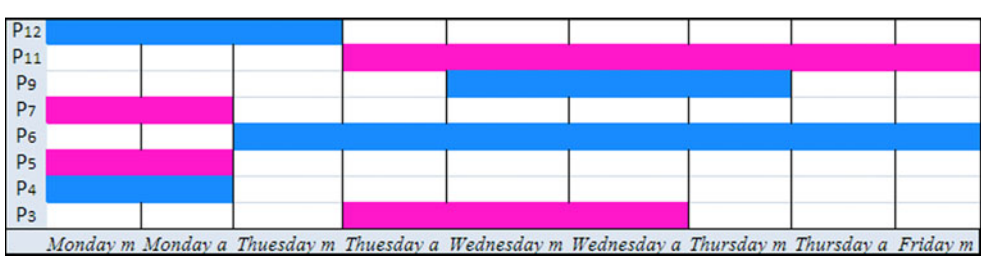

Fig. 3 Test case 1: scheduled patients and LOS of the first planned week (m = morning, a = afternoon)

Table 5 Waiting list of the second week of test case 1: waiting time, priority, score, and prescribed clinical services. The scheduled patients are in grey color

Patient	$D_1 - D_0$	Priority	Score	Prescribed Clinical Services
1	46	1	13.8	2,3,7,8,10,18
2	46	1	13.8	7,8,11,12
3	38	1	11.4	7,8
4	37	1	11.1	3,5,7,8,14,18
5	17	3	17.85	7,8,13
6	16	2	10.56	5,6,7,8,11,13,14,16,17
7	15	1	4.5	1,2,7,8,10,12,15,16,18
8	14	1	4.2	1,2,3,7, 8,16, 17
9	13	2	8.58	5,6,7,8,11,13,14,16,17
10	12	3	12.6	1,3,4,8,11,12,15,16,18
11	11	3	11.55	1,2,7,8,10
12	10	3	10.5	1,2,3,8,16,17
13	10	1	3	1,2,3,5,7,8,12
14	8	1	2.4	1,2,3,5,8,10
15	8	2	5.28	3,7,8,10,18
16	8	1	2.64	8,11,12
17	8	2	5.6	1,2,3,5,6,7,8,10,16,17
18	7	3	7.35	3,6,7,8,9,16,17
19	4	3	4.2	7,8
20	4	1	1.2	7,8
21	4	1	1.2	8,9
22	4	1	1.2	3,5,7,8,14,18
23	2	3	2.1	1,2,3,5,7

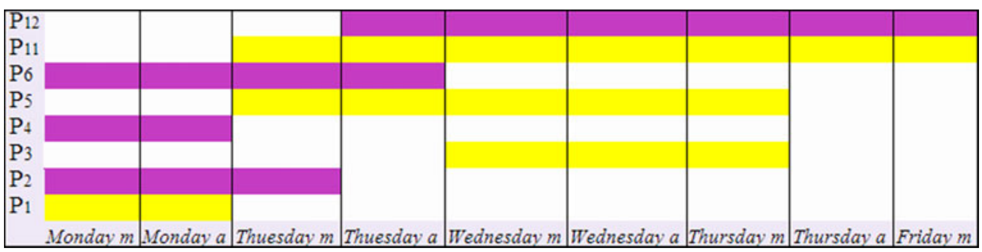

Fig. 4 Test case 1: scheduled patients and LOS of the second planned week (m = morning, a = afternoon)

Table 6 Waiting list of the first week of test case 2: waiting time, priority, score, minimum LOS ($LOS_{\min}$) required by physician, and set of the prescribed clinical services. The scheduled patients are in grey color

Patient	Priority	Score	LOS_{min}	Prescribed Clinical Services
1	1	12	1	1,2,3,5,8,10
2	1	12	2	3,7,8,10,18
3	1	12	2	8,11,12
4	2	26.4	3	1,2,3,5,6,7,8,10,16
5	2	26.4	3	1,2,3,5,7,8,14,18
6	2	21.12	3	3,6,7,8,9,16,17
7	3	33.6	1	7,8
8	1	9.6	1	7,8
9	2	21.12	1	8,9
10	1	9.3	2	3,5,7,8,14,18
11	2	15.84	2	8,13
12	3	12.6	1	7,8,13
13	3	11.55	1	7,8,13
14	2	6.6	3	5,6,7,8,11,13,14,16,17
15	1	2.7	3	1,3,4,8,11,12,15,16,18
16	1	2.4	2	1,2,7,8,10
17	2	4.62	3	1,2,3,8,16,17
18	3	6.3	1	1,2,3,8,10
19	3	5.25	1	1,2,3,7,8
20	3	4.2	2	1,2,3,5,7,8,12

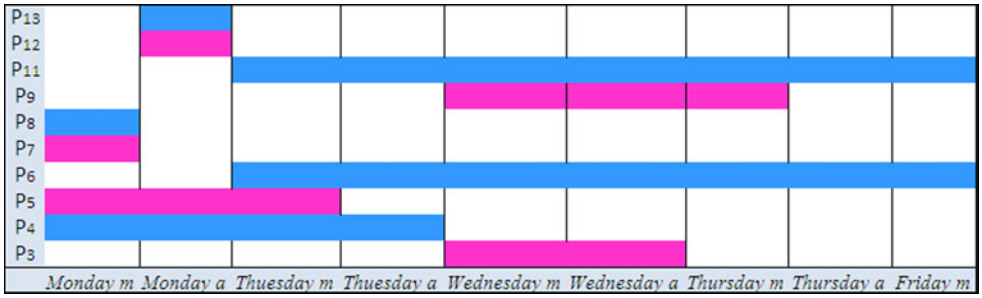

Fig. 5 Test case 2: scheduled patients and LOS of the first planned week (m = morning, a = afternoon)

Table 7 Waiting list of the second week of test case 2: waiting time, priority, score, minimum LOS ($LOS_{\min}$) required by physician, and prescribed clinical services. The scheduled patients are in grey color

Patient	$D_1 - D_0$	Priority	Score	LOS_{min}	Prescribed Clinical Services
1	46	1	13.8	1	2,3,7,8,10,18
2	46	1	13.8	2	7,8,11,12
3	37	1	11.1	2	3,5,7,8,14,18
4	16	2	10.56	3	5,6,7,8,11,13,14,16,17
5	15	1	4.5	3	1,2,7,8,10,12,15,16,18
6	14	1	4.2	2	1,2,3,7, 8,16, 17
7	13	2	8.58	3	5,6,7,8,11,13,14,16,17
8	12	3	12.6	1	1,3,4,8,11,12,15,16,18
9	11	3	11.55	1	1,2,7,8,10
10	10	3	10.5	2	1,2,3,8,16,17
11	10	1	3	2	1,2,3,5,7,8,12
12	8	1	2.4	2	1,2,3,5,8,10
13	8	2	5.28	1	3,7,8,10,18
14	8	1	2.64	4	8,11,12
15	8	2	5.6	3	1,2,3,5,6,7,8,10,16,17
16	7	3	7.35	3	3,6,7,8,9,16,17
17	4	3	4.2	2	7,8
18	4	1	1.2	1	7,8
19	4	1	1.2	2	8,9
20	4	1	1.2	2	3,5,7,8,14,18
21	2	3	2.1	1	1,2,3,5,7

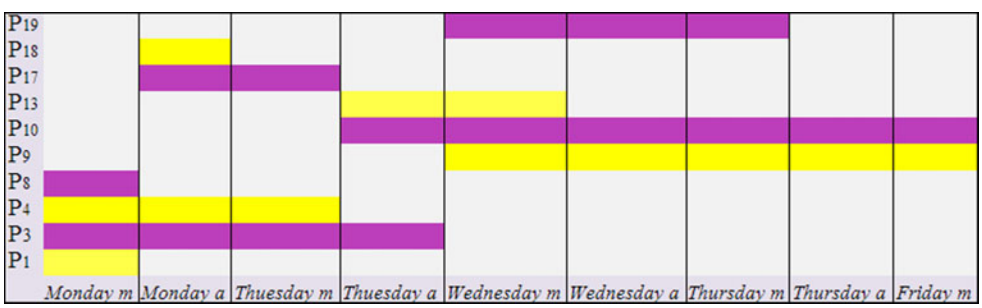

Fig. 6 Test case 2: scheduled patients and LOS of the second planned week (m = morning, a = afternoon)

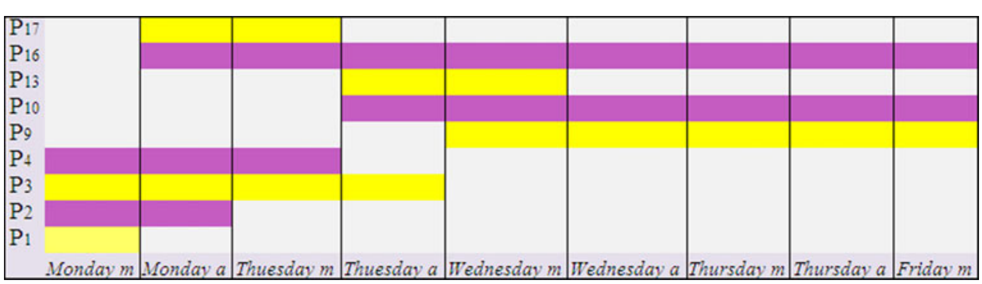

Fig. 7 Test case 3: scheduled patients and LOS (m = morning, a = afternoon)

the rheumatology staff. The corresponding waiting list is thus constructed by updating the score of patients not scheduled during the first week, according to the described rule 19 and by inserting the 11 new patients. Hence, the new waiting list has 12 patients (from P_1 to P_{12}) with the updated score, and 11 new patients (from P_{13} to P_{23}). The related data are reported in Table 5.

By solving the optimisation model 18, the new set of patients to hospitalize is determined (patients are highlighted in grey color in Table 5). The optimal schedule is reported in Fig. 4. We observe that also in this case there are patients with LOS of only two blocks (e.g., P_1, P_4); some of the scheduled patients have a high score since the elapsed waiting time is higher than that of patients arrived during the previous week. Moreover, it may happen that patients belonging to class A are not scheduled despite the highest priority class (e.g., P_{10} and P_{18}): this is due both to the score of the other patients and the availability of clinical resources (i.e., timetable structure).

4.3 Test case 2

This second instance, consisting of two consecutive planning weeks, has been defined with the aim of showing as a small variation in the data may result in a significant change in the optimal schedule. In particular, the same data of the first week of test case 1 are used, with the only exception of the minimum required LOS, which depends now on patient, as reported in Table 6.

By solving the optimisation model 18, a set of hospitalized patients different from that determined for the test case 1 is obtained (see Fig. 5).

The second week is characterized by the same 11 new arrivals of test case 1. Now, the waiting list consists of 10 patients (from P_1 to P_{10}), with updated score, and 11 new patients (from P_{11} to P_{21}). The corresponding data are reported in Table 7. The optimal schedule determined in this case is given in Fig. 6, from which it is evident that the number of scheduled patients (i.e., 10) is higher than that obtained in the test case 1, and this is mainly due to a different value of the minimum required LOS. It is worth observing that, also in this simple case, a manually drawn schedule requires a strong human effort and, in any case, it is difficult to define an optimal schedule.

4.4 Test case 3

The third test case has been defined with the aim of investigating the influence of the clinical resources availability on the patient scheduling. This instance has been defined by considering the second week of the test case 2 and modifying the capacity of clinical services 3, 7, and 8, which has been increased to 3.

The corresponding optimal schedule is reported in Fig. 7. We exploit this schedule to highlight a very important aspect, that is just the number of scheduled patients does not allow to have a clear indication of how the system is performing.

By comparing the optimal schedule of the test case 3 with that obtained on the second week of the test case 2 (see Fig. 6), we observe that a lower number of patients is scheduled (i.e., 9 scheduled patients instead of 10), but the optimal value of the objective function is greater. This improvement, in terms of objective function is due to the increased capacity of some clinical services allowing to schedule patients with higher weights: the patients P_8, P_{18}, and P_{19} (that are scheduled in the second week of the test case 2 and not in the test case 3) have an overall weight 13.5, whereas the patients P_2 and P_{16} (scheduled in test case 3 and not in test case 2) have an overall weight 21.15. Since the optimal decision criterion of the optimization model 18 is the maximization of the total weighted sum of all scheduled patients, the increased availability of some clinical resources (test case 3) allows to further increase the optimal value of the objective function.

Hence, the efficiency of a schedule cannot be measured by only counting the scheduled patients; as it has showed in this simple test case, both the total score of scheduled patients and the clinical resources' utilization rate need to be taken into account.

5 Discussion and conclusions

In this paper, a novel WH management has been considered by exploiting quantitative optimisation approaches. In particular, the proposed optimal decision making model may represent an effective tool for supporting the hospital health care manager aiming at improving the quality of health care delivered to patients.

By comparing the real schedules of the Rheumatology division of "Careggi" University Hospital and the optimal schedules obtained by solving the optimization model 18 on the depicted test cases, there is a clear improvement in terms of number of scheduled patients. Due to the high complexity of the problem, actually the manual schedule is able to hospitalize only four patients per week. Furthermore, we have shown that it is possible to provide a more efficient use of the clinical services with respect to a manually defined schedule. When the values of some parameters change (e.g., the minimum required LOS of patients and capacity of clinical services) also the optimal schedule could change but the use of clinical service is always optimised as possible.

We remark that the number of available beds is the crucial determinant affecting patients scheduling. Indeed, the beds availability represents potentially a bottleneck for patients flow and is a very tight constraint in case of manual scheduling, even if resource capacity increases.

Thus the proposed test cases give an idea about the complexity of the WH management and motivate the need of effective and efficient tools for optimally supporting the decision making process.

On the other hand, WH organization can be beneficial also in terms of remarkable reduction of economic costs. From a recent study made in Italy considering cost analysis in sclerosis systemic [14], it is possible to estimate a reduction of direct and indirect costs of about four times with respect to the ordinary inpatient hospitalization.

On the basis of the reported experimental validation phase and its results, the following general conclusions can be drawn.

- We remark that the optimal schedule obtained by the optimisation model can be interpreted from different point of view:
  - from a patient perspective, since he/she knows in advance admission and discharge date (if no complication occurs) and the clinical services appointments. Any repeated admission is avoided with a consequence reduction of stress and social costs;
  - from a health care operator since he/she knows in advance the clinical procedures per TS to perform;
  - from a manager perspective, since costs and utilization of all resources are known in advance. He/she analyzes if current utilization is in line with best practice and is able to identify critical resources. A timetable is modified when changes in volume of clinical service demands or other requirements occur.
- The application of the optimisation model may support the management of health care personnel. Since every clinical service requires a specialized medical staff, activities of personnel/staff can be planned in advance (a medical doctor/staff constraint is solved by manager when he/she defines the timetable).
- The proposed model can be also applied to verify if a patient could be admitted in WH. In fact, during the baseline visit, it is possible to confirm whether a patient may undergo all the clinical services during a week on the basis of the given timetable. If this is not possible, patient will not be added into the WH waiting list.
- As far as the structure of the objective function of our model is concerned, we remark that the maximization of the sum of the weight of scheduled patients allows a LOS reduction for some patients. On the other hand, it could be a conflicting goal in some cases with LOS minimization; this specific problem, can be handled by considering a bi-objective integer programming model whose objective function is a linear combination (with appropriate weights) of the two mentioned goals. It is our intention to further investigate this issue in a forthcoming paper.
- Finally, a waiting time reduction could be possible by allocating resources in a different manner or changing their capacity. Since a timetable of resources is a factor that strongly affects the optimal schedule, our future work will be an integration of tactical decision level.

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