



A Novel Robust Flexible Minimum Cost Consensus Model with Consensus Granule

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Abstract

In the consensus reaching process of group decision making (GDM), consensus measures do not require the consensus opinions of all decision makers. Meanwhile, unit adjustment cost is one of the important and often uncertain factors that affect consensus in GDM. Due to the uncertainty of unit adjustment costs, the moderator may not be able to provide each decision maker with an accurate unit adjustment cost. To overcome these problems, a novel class of group consensus decision models is proposed in this paper. First, fuzzy consensus measures are defined to make the consensus flexible using the specificity and coverage of the consensus granule. Secondly, to describe the uncertainty of the cost of unit adjustment, three uncertainty scenarios are created by the robust optimization approach is introduced. In the end, the feasibility and applicability of the method are verified by taking the classical GDM problem as an example, and sensitivity and comparative analyses are also performed.

Keywords Group decision making · Minimum cost consensus · Robust optimization · Flexible consensus · Granule

1 Introduction

GDM is a collective decision-making process aimed at identifying the optimal decision solution by aggregating individual opinions of decision-makers (DMs). (Herrera-Viedma et al. 2002; Wang et al. 2020; Zhang et al. 2019a). In the early days, GDM was mainly applied to the design of voting mechanisms and voting paradoxes. But with more and more scholars studying GDM, it is now widely used in the fields of failure mode and effects analysis (FMEA), supplier selection,

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emergency management, and environmental governance. GDM mainly consists of a CRP and a selection process. In the CRP, the DMs' opinions often diverge greatly due to subjective factors such as knowledge, information, and preferences of DMs. Therefore, DMs need to continuously revise their individual opinions to reach consensus. In the feedback adjustment process, a certain cost is generally spent by the moderator to negotiate with the DM. In the CRP, we aim to consider uncertain unit adjustment costs and a new flexible consensus measure under the condition of consensus.

In GDM, there are two main models. One is the group consensus decision model based on the identification rule and direction consensus rule (Aguaron et al. 2016; Altuzarra et al. 2010; Cabrerizo et al. 2010; Dong et al. 2010; Perez et al. 2014; Wu and Xu 2016). The other is the group consensus decision model with optimization-based rule (Ben-Arieh et al. 2009; Gong et al. 2015a, 2015b; Parreiras et al. 2010; Xu et al. 2015; Yu et al. 2021; Zhang et al. 2020, 2012, 2019b). With the increasing research on GDM, the application of consensus models has become more widespread. In the context of multi-criteria GDM, Li et al. (2023a) introduced stochastic multi-attribute acceptability analysis to investigate the consensus-building process for multi-criteria social network GDM problems with manipulative behavior. Li and Zhang (2023) proposed a consensus-building model based on minimum adjustments to address multi-criteria group classification problems. Additionally, they introduced a novel ordinal-based consensus-building model that takes into account DMs' indirect and imprecise heterogeneous preference information (Li et al. 2023b). We aim to consider uncertainty in the consensus decision model using the robust optimization method.

Moderators are required to invest a significant amount of time and resources in persuading DMs to amend their viewpoints. Ben-Arieh and Easton (2007) are the first to propose the concept of minimum consensus cost and design an algorithm to help facilitators reach consensus at minimum cost. Zhang et al. (2011) expand the collective opinion expression by introducing the aggregation operator into the model. And they study the minimum cost consensus model (MCCM) under the weighted averaging operator and ordered weighted averaging operator. Zhang et al. (2021a) propose a minimum consensus cost model with private benefits based on consensus granularity and a maximum expert consensus model. However, in the MCCM model, it is necessary for all DMs to reach a consensus, which imposes a mandatory constraint on the number of DMs who must achieve consensus. In this context, consensus granularity can enhance greater flexibility. Consensus granularity expresses the degree and complexity of consensus. Consensus granularity is highly important in CRP because it not only describes the level of consensus reached, but also allows for flexible control over the number of DMs required to achieve consensus. They innovatively introduce the concept of consensus granularity in GDM, and define fuzzy consensus measure by maximizing the quality of consensus granularity. The use of consensus granularity can enhance flexibility and robustness in GDM

(Qin et al. 2023). Cui et al. (2022) developed a comprehensive system approach for ranking alternative solutions in multi-criteria GDM environments using consensus granularity. Zhang et al. (2017) introduced an adaptive consensus model based on fuzzy consensus granularity.

With the development of GDM, many scholars begin to consider the uncertainty in GDM (Li et al. 2021; Zhang et al. 2022). Cheng et al. (2018) propose the MCCM with asymmetric unit cost by considering the different unit adjustment costs of DMs to raise or lower their opinions. In terms of uncertain opinions, Tan et al. (2018) study consensus models with opportunity constraints. Zhang et al. (2021b) consider robust cost consensus models with interval-valued opinions and uncertain costs. Wei et al. (2022) considered individual opinion uncertainty in three different aggregation operators for robust MCCM. Han et al. (2019) incorporated unit adjustment costs into three uncertain scenarios, confirming that robust optimization enhances the conservatism of the model and, to some extent, mitigates the uncertainty associated with unit adjustment costs.

Unfortunately, despite systematic research on consensus models addressing the MCCM problem in GDM, there are several unresolved issues:

- (1) In existing MCCM studies incorporating consensus granularity, DMs' unit adjustment costs are treated as deterministic. However, due to the uncertainty in the environment, DMs' unit adjustment costs are inherently uncertain.
- (2) In current robust consensus models, consensus is defined to require the agreement of all DMs' opinions. However, for GDM scenarios where consensus from the majority of DMs suffices, mandating consensus from all DMs would undoubtedly consume more time and resources.

In real life, there are many uncertain flexible consensus decision problems. For instance, in negotiations between government and businesses regarding carbon quotas in a green supply chain, the costs incurred by the government for each business are challenging to determine due to varying carbon emission quotas among different enterprises. Under conditions of cost uncertainty, in order to facilitate the determination of carbon emission quotas, the government aims to ensure, to the greatest extent possible, that 80% of the enterprises adhere to a uniform carbon emission standard. Our work is to combine the two and propose a novel flexible consensus model.

In this paper, a robust flexible minimum cost consensus model with uncertain unit adjustment cost is constructed based on fuzzy consensus measure. The main contributions of this paper are summarized as follows:

- (1) This paper proposes a more flexible and versatile fuzzy consensus measure. In traditional CRP processes, consensus among all DMs is often required, which does not align with real-world decision environments and can lead to certain

- losses. The fuzzy consensus measure introduced in this paper addresses this issue by not mandating consensus from all DMs.
- (2) An approach employing robust optimization is introduced to address the uncertainty in DMs' unit adjustment costs. The paper presents the MCFCM based on three classical uncertainty sets: box set, ellipsoid set, and polyhedron set, to tackle uncertainty issues in three distinct scenarios.
 - (3) Through concrete numerical simulations, the paper validates the effectiveness and feasibility of the model, and discusses the impact of various parameters on the model. The results indicate that the proposed MCFCM not only meets consensus requirements but also reduces the risks associated with uncertainty in unit adjustment costs.

The rest of this paper is organized as follows. Section 2 introduces some preliminary information about MCCM and fuzzy consensus measures. Section 3 is divided into three subsections, detailing the construction of a fuzzy consensus measure-based consensus model with uncertain unit adjustment cost under three uncertainty sets. Then, Sect. 4 illustrates the validity of the proposed models by a specific GDM example. Section 5 presents and analyzes the results in detail. Finally, Sect. 6 concludes the paper.

2 Preliminaries

Suppose there are n decision makers $D = \{d_1, d_2, \dots, d_n\}$ in the GDM. Let $O = (o_1, o_2, \dots, o_n)$ represent DMs' initial opinions, $\bar{O} = (\bar{o}_1, \bar{o}_2, \dots, \bar{o}_n)$ refer to their adjustment opinions, \bar{o}^c be the collective opinion, and $C = (c_1, c_2, \dots, c_n)$ represent the set of unit adjustment costs of DMs. The MCCM can be presented as follows (Zhang et al. 2011):

$$\begin{aligned} & \text{Min} \sum_{i=1}^n c_i |\bar{o}_i - o_i| \\ & \text{s.t.} |\bar{o}_i - \bar{o}^c| \leq \sigma, i = 1, 2, \dots, n \\ & \bar{o}^c = \sum_{i=1}^n w_i \bar{o}_i \end{aligned}$$

where σ represent consensus threshold, $w = (w_1, w_2, \dots, w_n)^T$ is a n dimensional weight vector and $\sum_{i=1}^n w_i = 1, w_i \in [0, 1]$.

In MCCM, we found that consensus requires the agreement of all DMs in the first constraint. However, this constraint does not adequately meet our needs in the GDM when consensus is achieved with the agreement of the majority of DMs. Therefore,

we introduce the concept of consensus granules to describe fuzzy consensus measures and establish MCCM around fuzzy consensus measures.

2.1 MCCM Based on Fuzzy Consensus Measure

2.1.1 Fuzzy Consensus Measure

Based on the consensus granules proposed by Zhang et al. (2021a), let ϵ represent the size of the consensus granules, then the consensus granules A , constructed around the collective opinion, are as follows:

$$A = [\bar{o}^c - 0.5\epsilon, \bar{o}^c + 0.5\epsilon]$$

To guarantee the consensus granule $A \subseteq [0, 1]$, we assume that $\epsilon \in [0, 2 \min\{\bar{o}^c, 1 - \bar{o}^c\}]$. We assume that $sp = 1 - \epsilon$ denotes the specificity, $sp \in [1 - 2 \min\{\bar{o}^c, 1 - \bar{o}^c\}, 1]$. And cov is the coverage of A , defined as:

$$cov = \frac{1}{n} \sum_{i=1}^n x_i$$

$$x_i = \begin{cases} 1, & o_i \in A \\ 0, & else \end{cases}$$

Specificity reflects the precision achieved by the consensus granule, different ϵ can construct different consensus granules. The larger the consensus granule, the larger the number of DMs whose opinions are in the consensus granule range. Whereas coverage reflects the number of individual opinions contained in the consensus granule, it is clear that the specificity and coverage criteria are in conflict: the increase in coverage comes at the cost of a decrease in specificity and vice versa. Larger specificity and coverage values indicate a higher quality of the shared consensus granule A .

Based on the specificity and coverage of the consensus granule, we define the fuzzy consensus measure by maximizing the product of the specificity and coverage of the consensus granule A .

$$FC = \max_A sp \cdot cov \tag{1}$$

From the definition of FC , we can obtain the following related properties:

- (1) $FC \leq 1$.
- (2) The larger the FC , the higher the degree of consensus reached by the GDM.

2.1.2 Minimum Cost Fuzzy Consensus Model

Let α be the fuzzy consensus threshold, based on the minimum cost fuzzy consensus model (MCFCM) proposed by Zhang et al. (2021a) as shown below.

$$\begin{aligned} & \min \sum_{i=1}^n c_i |\bar{o}_i - o_i| \\ & \left\{ \begin{array}{l} \frac{1}{n} \sum_{i=1}^n sp \cdot x_i \geq \alpha \\ x_i = \begin{cases} 1, & |\bar{o}_i - \bar{o}^c| \leq 0.5(1 - sp) \\ 0, & |\bar{o}_i - \bar{o}^c| > 0.5(1 - sp) \end{cases}, i = 1, 2, \dots, n \\ \text{s.t.} \left\{ \begin{array}{l} \bar{o}^c = \sum_{i=1}^n w_i \bar{o}_i \\ sp \geq 1 - 2 \min\{\bar{o}^c, 1 - \bar{o}^c\} \\ sp \in [0, 1] \\ \bar{o}_i \in [0, 1], i = 1, 2, \dots, n \end{array} \right. \end{array} \right. \end{aligned}$$

Lemma 1: Let $z_i = sp \cdot x_i$. The constraint $\frac{1}{n} \sum_{i=1}^n sp \cdot x_i \geq \alpha$ in MCFCM can be equivalently transformed to the following condition,

$$\left\{ \begin{array}{l} \frac{1}{n} \sum_{i=1}^n z_i \geq \alpha \\ 0 \leq z_i \leq x_i, i = 1, 2, \dots, n \\ sp + x_i - 1 \leq z_i \leq sp, i = 1, 2, \dots, n \end{array} \right.$$

Lemma 2: The constraint $x_i = \begin{cases} 1, & |\bar{o}_i - \bar{o}^c| \leq 0.5(1 - sp) \\ 0, & |\bar{o}_i - \bar{o}^c| > 0.5(1 - sp) \end{cases}, i = 1, 2, \dots, n$ in MCFCM can be equivalently translated into the following condition:

$$\left\{ \begin{array}{l} |\bar{o}_i - \bar{o}^c| + 0.5(sp - 1) \leq 1 - x_i, i = 1, 2, \dots, n \\ |\bar{o}_i - \bar{o}^c| + 0.5(sp - 1) > -x_i, i = 1, 2, \dots, n \\ x_i \in \{0, 1\}, i = 1, 2, \dots, n \end{array} \right.$$

Let $a_i = |\bar{o}_i - o_i|$ and $b_i = |\bar{o}_i - \bar{o}^c|$ be two sets of auxiliary variables. By Lemma 1 and Lemma 2, MCFCM is equivalently transformed into the following mixed 0–1 linear programming model.

$$\begin{aligned}
 & \min \sum_{i=1}^n c_i a_i \\
 & \left\{ \begin{array}{l}
 \bar{o}_i - o_i \leq a_i, \quad i = 1, 2, \dots, n \\
 -\bar{o}_i + o_i \leq a_i, \quad i = 1, 2, \dots, n \\
 \frac{1}{n} \sum_{i=1}^n z_i \geq \alpha \\
 0 \leq z_i \leq x_i, \quad i = 1, 2, \dots, n \\
 sp + x_i - 1 \leq z_i \leq sp, \quad i = 1, 2, \dots, n \\
 b_i + 0.5(sp - 1) \leq 1 - x_i, \quad i = 1, 2, \dots, n \\
 b_i + 0.5(sp - 1) > -x_i, \quad i = 1, 2, \dots, n \\
 P1 \text{ s.t. } \left\{ \begin{array}{l}
 \bar{o}_i - \bar{o}^c \leq b_i, \quad i = 1, 2, \dots, n \\
 -\bar{o}_i + \bar{o}^c \leq b_i, \quad i = 1, 2, \dots, n \\
 \bar{o}^c = \sum_{i=1}^n w_i \bar{o}_i \\
 sp \geq 1 - 2\bar{o}^c \\
 sp \geq 2\bar{o}^c - 1 \\
 x_i \in \{0, 1\} \\
 \bar{o}_i \in [0, 1] \quad i = 1, 2, \dots, n \\
 sp \in [0, 1]
 \end{array} \right.
 \end{array} \right. \tag{2}
 \end{aligned}$$

where $\bar{o}_i, x_i, a_i, b_i, z_i$ and sp are decision variables and \bar{o}^c is an intermediate variable. In this paper, we consider uncertainty in P1. We construct a novel robust minimal consensus model by applying robust optimization methods, which is more closer to the real decision making environment.

3 Model Construction

3.1 Uncertain Unit Adjustment Costs for Uncertain Set

In this paper, we consider the unit adjustment cost to be uncertain. We use the classical robust optimization method to construct the uncertainty set U about the unit adjustment cost, and let \tilde{c} represent the uncertain parameter of the unit adjustment cost.

$$U = \{ \tilde{c} \in R_+^n \mid \tilde{c} = c + Q\tau, \tau \in \Omega \} \tag{3}$$

where $c = (c_1, c_2, \dots, c_n)^T$ denotes the unit adjustment cost reference point of the DM, Q is the $n \times m$ -coefficient matrix and Ω denotes the non-empty convex set.

Let $a = (a_1, a_2, \dots, a_n)^T$, based on the uncertain set of unit adjustment costs, we propose a minimum cost fuzzy consensus model with uncertainty.

$$\begin{aligned}
 & \min \sup_{\tilde{c} \in U} \tilde{c}^T a \\
 & \text{s.t.} \left\{ \begin{array}{l}
 \bar{o}_i - o_i \leq a_i, \quad i = 1, 2, \dots, n \\
 -\bar{o}_i + o_i \leq a_i, \quad i = 1, 2, \dots, n \\
 \frac{1}{n} \sum_{i=1}^n z_i \geq \alpha \\
 0 \leq z_i \leq x_i, \quad i = 1, 2, \dots, n \\
 sp + x_i - 1 \leq z_i \leq sp, \quad i = 1, 2, \dots, n \\
 b_i + 0.5(sp - 1) \leq 1 - x_i, \quad i = 1, 2, \dots, n \\
 b_i + 0.5(sp - 1) > -x_i, \quad i = 1, 2, \dots, n \\
 \bar{o}_i - \bar{o}^c \leq b_i, \quad i = 1, 2, \dots, n \\
 -\bar{o}_i + \bar{o}^c \leq b_i, \quad i = 1, 2, \dots, n \\
 \bar{o}^c = \sum_{i=1}^n w_i \bar{o}_i \\
 sp \geq 1 - 2\bar{o}^c \\
 sp \geq 2\bar{o}^c - 1 \\
 x_i \in \{0, 1\}, \quad i = 1, 2, \dots, n \\
 \bar{o}_i \in [0, 1] \quad i = 1, 2, \dots, n \\
 sp \in [0, 1]
 \end{array} \right. \tag{4}
 \end{aligned}$$

we denote the model (4) as robust minimum cost fuzzy consensus model (RMCFCM).

The model (4) cannot be applied directly because the objective function contains min-sup operations. The choice of the convex set Ω is directly related to the computational cost of the model. Next, we will consider three worst-case RMCFCM problems for uncertain sets. The first one we consider to compute RMCFCM when the uncertain set is the Box set; the second one considers to compute RMCFCM when the uncertain set is the Ellipsoid set; and the third one considers computing RMCFCM when the uncertain set is the Polyhedral set. To mitigate the risks associated with uncertain unit adjustment costs and make the models more realistically effective, we have chosen three uncertainty sets. These three uncertainty sets are selected to address the uncertainty in unit adjustment costs in different scenarios, and their distinctions are as follows:

- (1) In the Box Uncertainty Set, parameters are assumed to lie within a given rectangular or hypercubic region, with upper and lower bounds for each dimension. This set offers low parameter flexibility, allowing parameters to vary only within uniform ranges.
- (2) The Ellipsoid Uncertainty Set assumes parameter variations within an ellipsoid, with the size and shape of the ellipsoid representing the degree of uncertainty.

The Ellipsoid Uncertainty Set permits parameter variations in different directions and magnitudes within the ellipsoid, providing greater flexibility.

- (3) The Polyhedron Uncertainty Set allows parameters to fluctuate within a polygon or polyhedron, which can be defined by a finite number of constraints or inequalities. This set offers the highest representation flexibility, with the degree of uncertainty represented by the shape and size of the polygon or polyhedron.

3.2 RMCFCM with The Box Set

The uncertainty set with the Box set is expressed as follows,

$$U = \{\tilde{c} \geq 0 | c^- \leq \tilde{c} \leq c^+\}$$

where c^- and c^+ are the upper bound and lower bound of \tilde{c} , respectively.

Theorem 1: *Considering uncertain unit adjustment costs with Box sets, the uncertainty in model (4) is given by the equation, then model (4) is equivalent to model (5), as follows.*

$$\begin{aligned}
 & \min \sum_{i=1}^n c_i^+ a_i \\
 P2 \text{ s.t. } & \left\{ \begin{aligned}
 & \bar{o}_i - o_i \leq a_i, \quad i = 1, 2, \dots, n \\
 & -\bar{o}_i + o_i \leq a_i, \quad i = 1, 2, \dots, n \\
 & \frac{1}{n} \sum_{i=1}^n z_i \geq \alpha \\
 & 0 \leq z_i \leq x_i, \quad i = 1, 2, \dots, n \\
 & sp + x_i - 1 \leq z_i \leq sp, \quad i = 1, 2, \dots, n \\
 & b_i + 0.5(sp - 1) \leq 1 - x_i, \quad i = 1, 2, \dots, n \\
 & b_i + 0.5(sp - 1) > -x_i, \quad i = 1, 2, \dots, n \\
 & \bar{o}_i - \bar{o}^c \leq b_i, \quad i = 1, 2, \dots, n \\
 & -\bar{o}_i + \bar{o}^c \leq b_i, \quad i = 1, 2, \dots, n \\
 & \bar{o}^c = \sum_{i=1}^n w_i \bar{o}_i \\
 & sp \geq 1 - 2\bar{o}^c \\
 & sp \geq 2\bar{o}^c - 1 \\
 & x_i \in \{0, 1\}, \quad i = 1, 2, \dots, n \\
 & \bar{o}_i \in [0, 1], \quad i = 1, 2, \dots, n \\
 & sp \in [0, 1]
 \end{aligned} \right. \tag{5}
 \end{aligned}$$

The Proof of Theorem 1 is in the Appendix A.

3.3 RMCFCM with Ellipsoidal Set

Theorem 2 Let $\Omega = \{\tau \in \mathbb{R}^m \mid \|\tau\|_2 \leq 1\}$ be a non-empty set. Suppose there exists $\hat{\tau}$, such that $c + Q\hat{\tau} \geq 0$ and $\|\hat{\tau}\|_2 \leq 1$, Model (4) is equivalent to:

$$\begin{aligned}
 & \min \quad c^T s + \|Q^T s\|_2 \\
 P3 \quad & \text{s.t.} \quad \begin{cases} a - s \leq 0 \\ \bar{o}_i - o_i \leq a_i, & i = 1, 2, \dots, n \\ -\bar{o}_i + o_i \leq a_i, & i = 1, 2, \dots, n \\ \frac{1}{n} \sum_{i=1}^n z_i \geq \alpha \\ 0 \leq z_i \leq x_i, & i = 1, 2, \dots, n \\ sp + x_i - 1 \leq z_i \leq sp, & i = 1, 2, \dots, n \\ b_i + 0.5(sp - 1) \leq 1 - x_i, & i = 1, 2, \dots, n \\ b_i + 0.5(sp - 1) > -x_i, & i = 1, 2, \dots, n \\ \bar{o}_i - \bar{o}^c \leq b_i, & i = 1, 2, \dots, n \\ -\bar{o}_i + \bar{o}^c \leq b_i, & i = 1, 2, \dots, n \\ \bar{o}^c = \sum_{i=1}^n w_i \bar{o}_i \\ sp \geq 1 - 2\bar{o}^c \\ sp \geq 2\bar{o}^c - 1 \\ x_i \in \{0, 1\}, & i = 1, 2, \dots, n \\ \bar{o}_i \in [0, 1] & i = 1, 2, \dots, n \\ sp \in [0, 1], s \in_n^+ \end{cases} \tag{6}
 \end{aligned}$$

where $s = a + \lambda$, while λ denotes the Lagrange multiplier associated with the inequality $-c - Q\tau \leq 0$.

The Proof of Theorem 2 is in the Appendix B.

3.4 RMCFCM with Polyhedral Set

Theorem 3 Let $\Omega = \{\tau \in \mathbb{R}^m \mid Y\tau = y, K\tau \geq k\}$ be a non-empty set and model (4) is equivalent to

$$\begin{aligned}
 \min \quad & c^T s - y^T v - k^T m \\
 \text{P4 s.t.} \quad & \left\{ \begin{array}{l}
 Q^T s + Y^T v + K^T m = 0 \\
 a - s \leq 0 \\
 \bar{o}_i - o_i \leq a_i, \quad i = 1, 2, \dots, n \\
 -\bar{o}_i + o_i \leq a_i, \quad i = 1, 2, \dots, n \\
 \frac{1}{n} \sum_{i=1}^n z_i \geq \alpha \\
 0 \leq z_i \leq x_i, \quad i = 1, 2, \dots, n \\
 sp + x_i - 1 \leq z_i \leq sp, \quad i = 1, 2, \dots, n \\
 b_i + 0.5(sp - 1) \leq 1 - x_i, \quad i = 1, 2, \dots, n \\
 b_i + 0.5(sp - 1) > -x_i, \quad i = 1, 2, \dots, n \\
 \bar{o}_i - \bar{o}^c \leq b_i, \quad i = 1, 2, \dots, n \\
 -\bar{o}_i + \bar{o}^c \leq b_i, \quad i = 1, 2, \dots, n \\
 \bar{o}^c = \sum_{i=1}^n w_i \bar{o}_i \\
 sp \geq 1 - 2\bar{o}^c \\
 sp \geq 2\bar{o}^c - 1 \\
 x_i \in \{0, 1\}, \quad i = 1, 2, \dots, n \\
 \bar{o}_i \in [0, 1] \quad i = 1, 2, \dots, n \\
 sp \in [0, 1] \\
 s \in \mathbb{R}_+^n, v \in \mathbb{R}^p, m \in \mathbb{R}_+^q
 \end{array} \right. \quad (7)
 \end{aligned}$$

The Proof of Theorem 3 is in the Appendix C.

Since the Slater condition is true, the pairwise optimality of the original problem and the dual problem described above can be achieved (Ben-Tal et al. 2009).

4 Case Studies

In this section, we verify the feasibility and validity of the model with an application. All the code is written on a laptop (Intel i7 CPU and 12 GB RAM) using Python calls to the gurobipy library.

We consider a simple GDM problem with five DMs and a moderator, which has been used in Labella et al. (2020). In a procurement negotiation, suppose that there are five teachers (DMs) d_1, d_2, d_3, d_4 and d_5 from a high school and one seller (moderator) from a software company that sells an instructional software to the high school. Conditions that require some consensus among the five teachers (i.e., $\alpha \geq 0.8$). In the negotiation, the five teachers are given weights $w_1 = 0.375, w_2 = 0.250, w_3 = 0.1875, w_4 = 0.0625, w_5 = 0.125$. The initial opinions of the five teachers are $o_1 = 0.05, o_2 = 0.10, o_3 = 0.25, o_4 = 0.30, o_5 = 0.60$. To successfully sell the software, the seller spends some time (cost) to convince the five teachers to adjust their opinions, where the unit cost (i.e., the number of hours required to convince the teacher to adjust his/her opinion from 0 to 1) is difficult for the seller to determine the unit adjustment cost of the five teachers due to the

uncertainty and incompleteness of information collection in the DM’s thinking. The decision problem for the seller is how to determine the optimal modification to guide the five teachers to a given level of consensus ($FC \geq \alpha$).

Because $\tilde{c} = c + Q\tau$, we assume that $c = (c_1, c_2, c_3, c_4, c_5)^T = (5, 3, 10, 9, 12)^T$, Q obeys a normal distribution $Q \sim N(0, 0.01^2)$.

$$Q = \begin{pmatrix} -0.467 & 0.0257 & -0.1693 & -0.0415 & -0.0625 \\ -0.1625 & -0.0974 & -0.0449 & 0.1912 & -0.1169 \\ -0.1965 & -0.1146 & -0.0084 & -0.0391 & 0.0393 \\ 0.2605 & 0.0548 & -0.1992 & 0.0409 & 0.1302 \\ 0.0972 & 0.1565 & 0.0841 & -0.1142 & 0.0594 \end{pmatrix}$$

For the sake of generality, assume that all elements of the matrix Y, Z and the vector y, z are generated randomly at $[-1, 1]$, $c^+ = 1.2 * c$.

$$Y = \begin{pmatrix} -0.0497 & -0.6863 & 0.5692 & -0.5048 & -0.1837 \\ 0.0368 & 0.9996 & -0.3949 & 0.0079 & 0.2814 \\ 0.8620 & -0.1840 & -0.7509 & -0.0529 & 0.8994 \\ -0.7182 & 0.7028 & -0.2721 & 0.0382 & -0.5219 \\ -0.9393 & -0.0291 & 0.2728 & -0.9588 & -0.7647 \end{pmatrix}$$

$$K = \begin{pmatrix} .1960 & -0.2894 & -0.4320 & 0.3057 & 0.5964 \\ 0.6172 & -0.5750 & -0.8847 & 0.6564 & 0.2857 \\ 0.2709 & 0.2119 & -0.4294 & 0.6185 & -0.4966 \\ -0.7142 & -0.9139 & -0.1119 & -0.9439 & 0.4193 \\ -0.5052 & 0.6159 & -0.1539 & 0.5675 & 0.6221 \end{pmatrix}$$

$y = (-0.0838 - 0.2347 - 0.0018 \ 0.83980.4803), k = (0.6617 - 0.4679 - 0.0268 \ 0.7018 - 0.1302)$
 Assuming a given consensus threshold $\alpha = 0.8$, by solving for P1, P2, P3, and P4, we obtain the results shown in Table 1 and 2:

By solving P1,P2,P3, and P4 four models, we get the minimum consensus cost of the four models are 4.14, 4.97, 4.25, and 4.21, respectively; the consensus opinions of the four models are 0.35, 0.35, 0.28, and 0.2, respectively. Tables 1 and 2 show more data details for the four models of P1, P2, P3, and P4. Compared to the results for P1, RMCFCM incur higher costs. This reflects the advantages of a robust model. When dealing with uncertainties, coordinators can allocate a larger budget, ensuring that DMs can reach consensus even in the worst-case scenario and obtain better solutions.

Table 1 Results for P1, P2, P3 and P4

$\alpha = 0.8$	Mc	cov	sp	FC	$\bar{\sigma}^c$
P1	4.14	1	0.8	0.8	0.35
P2	4.97	1	0.8	0.8	0.35
P3	4.25	1	0.8	0.8	0.28
P4	4.21	1	0.8	0.8	0.2

Table 2 Individual adjustment opinions for P1, P2, P3, and P4

$\alpha = 0.8$	\bar{o}_1	\bar{o}_2	\bar{o}_3	\bar{o}_4	\bar{o}_5
P1	0.31	0.45	0.25	0.3	0.45
P2	0.31	0.45	0.25	0.3	0.45
P3	0.19	0.38	0.25	0.3	0.38
P4	0.1	0.2375	0.25	0.3	0.3

5 Model Analysis

In the previous subsection, we applied the developed model to an example to verify that the results of the minimum cost fuzzy consensus model based on robust optimization are more conservative than the minimum cost fuzzy consensus model. However, the case is only the result obtained for the case of fuzzy consensus threshold $\alpha = 0.8$. It is necessary to continue the analysis for what the result of fuzzy consensus threshold is in other cases. Moreover, the specificity and coverage of all four models in the case are the same, which does not show well the difference of the models in this respect. Next, we will perform sensitivity analysis and comparative analysis of the models in this subsection to further analyze the characteristics of the models.

5.1 Impact of Fuzzy Consensus Thresholds α on Models

The fuzzy consensus threshold represents the minimum standard of fuzzy consensus measure FC , that is, the fuzzy consensus threshold reflects the quality of the degree of consensus reached by GDM. Meanwhile, the fuzzy consensus measure is the product of specificity and coverage, while the specificity reflects the precision achieved by the consensus granule and the coverage reflects the number of individual opinions contained in the consensus granule. In other words, the higher the fuzzy consensus threshold, the more the number of DMs reaching consensus and the closer the adjusted opinion of consensus makers (\bar{o}_i) is to the consensus opinion (\bar{o}^c). Therefore, it is necessary to study the effect of the fuzzy consensus threshold on the results of the model. First, we let the consensus threshold vary from 0.7 to 0.95 in steps of 0.05, and solve the model with different models for different values of α . Finally, we obtain the minimum consensus cost (Mc), coverage (cov), specificity (sp), fuzzy consensus measure (FC) and consensus opinion (\bar{o}^c) for each model, as shown in Table 3. The adjustment opinions of DMs for each model are shown in Table 4.

5.1.1 Impact of Fuzzy Consensus Thresholds α on Model Results

Table 3 shows the detailed data of different models under different fuzzy consensus thresholds. The variation of the minimum cost under different fuzzy consensus thresholds is shown in Fig. 1, as the fuzzy consensus threshold increases, the minimum cost increases with it. This is also in line with the reality that as the

Table 3 Impact of fuzzy consensus thresholds α on model results

α	Model	Mc	cov	sp	FC	$\bar{\sigma}^c$
$\alpha = 0.7$	P1	1.12	0.8	0.875	0.7	0.27
	P2	1.34	0.8	0.875	0.7	0.27
	P3	1.18	0.8	0.875	0.7	0.27
	P4	1.24	0.8	0.875	0.7	0.27
$\alpha = 0.75$	P1	2.95	0.8	0.9375	0.75	0.28
	P2	3.54	0.8	0.9375	0.75	0.28
	P3	3.01	0.8	0.9375	0.75	0.28
	P4	3.06	0.8	0.9375	0.75	0.28
$\alpha = 0.8$	P1	4.14	1	0.8	0.8	0.35
	P2	4.97	1	0.8	0.8	0.35
	P3	4.25	1	0.8	0.8	0.28
	P4	4.21	1	0.8	0.8	0.2
$\alpha = 0.85$	P1	4.51	1	0.85	0.85	0.325
	P2	5.41	1	0.85	0.85	0.325
	P3	4.60	1	0.85	0.85	0.29
	P4	4.57	1	0.85	0.85	0.225
$\alpha = 0.9$	P1	4.875	1	0.9	0.9	0.3
	P2	5.85	1	0.9	0.9	0.3
	P3	4.94	1	0.9	0.9	0.3
	P4	4.93	1	0.9	0.9	0.25
$\alpha = 0.95$	P1	5.24	1	0.95	0.95	0.275
	P2	6.29	1	0.95	0.95	0.275
	P3	5.296	1	0.95	0.95	0.275
	P4	5.301	1	0.95	0.95	0.275

fuzzy consensus threshold increases, the higher the quality of fuzzy consensus is reached and the cost required increases. In Fig. 1, we can easily find that the minimum consensus cost of P1 is lower than the values of P2, P3 and P4 regardless of the consensus threshold α . Also, the magnitude of the minimum cost of P3 and P4 is uncertain under different fuzzy consensus thresholds, i.e., P3 may be more conservative or P4 may be more conservative under different fuzzy consensus thresholds. When considering uncertain factors, the box set provides the highest improvement in model robustness, followed by the ellipsoid set, while the polyhedron set performs the worst. From Fig. 2, it can be observed that as the fuzzy consensus threshold changes, both coverage and specificity also vary. When the fuzzy consensus threshold is low, the coverage is not equal to 1, indicating that there are DMs who have not reached consensus. In other words, coordinators can adjust the size of the fuzzy consensus threshold to control the number of DMs reaching consensus within CRP. This also reflects the model's greater adaptability.

Table 4 Impact of fuzzy consensus thresholds α on adjustment opinions

Model	α	\bar{o}_1	\bar{o}_2	\bar{o}_3	\bar{o}_4	\bar{o}_5
P2	0.7	0.21	0.21	0.25	0.3	0.6
	0.75	0.25	0.25	0.25	0.3	0.475
	0.8	0.31	0.45	0.25	0.3	0.45
	0.85	0.29	0.4	0.25	0.3	0.4
	0.9	0.275	0.35	0.25	0.3	0.35
	0.95	0.26	0.3	0.25	0.3	0.3
P3	0.7	0.21	0.21	0.25	0.3	0.6
	0.75	0.25	0.25	0.25	0.3	0.475
	0.8	0.19	0.38	0.25	0.3	0.38
	0.85	0.24	0.37	0.25	0.3	0.37
	0.9	0.275	0.35	0.25	0.3	0.35
	0.95	0.26	0.3	0.25	0.3	0.3
P4	0.7	0.21	0.21	0.25	0.3	0.6
	0.75	0.25	0.25	0.25	0.3	0.475
	0.8	0.1	0.2375	0.25	0.3	0.3
	0.85	0.15	0.2625	0.25	0.3	0.3
	0.9	0.2	0.29	0.25	0.3	0.3
	0.95	0.26	0.3	0.25	0.3	0.3

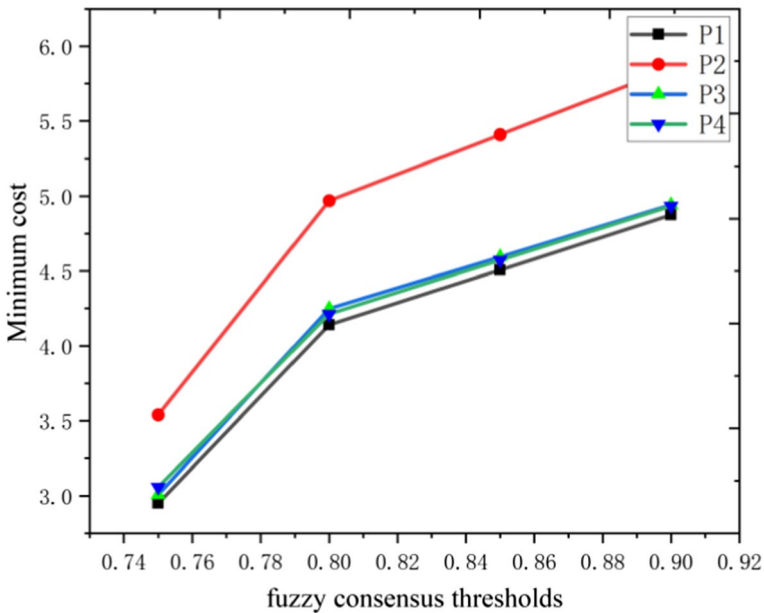


Fig. 1 Minimum cost for different fuzzy consensus thresholds

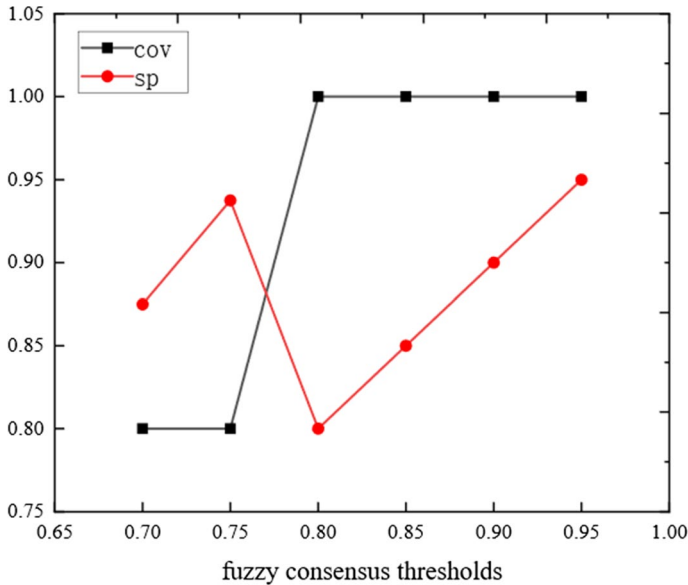


Fig. 2 cov and sp for different fuzzy consensus thresholds

5.1.2 Impact of Fuzzy Consensus Thresholds on Adjustment Opinions

With different fuzzy consensus thresholds, the adjustment opinions of DMs change. As an example, the uncertainty set is the ellipsoidal uncertainty set of P4. Five teachers participated in the evaluation of the instructional software. Figure 3 shows the effect of the change of fuzzy consensus threshold on the adjustment opinions. We found that when the fuzzy consensus threshold is set to 0.7 and 0.75, there is no consensus reached on the adjustment opinions of Teacher 5. When the fuzzy consensus threshold is set to 0.8, the adjustment degree of Teacher 1's opinion is relatively small. From Fig. 3, it can be observed that as the fuzzy consensus threshold increases, the adjustment opinions of teachers gradually tend to converge to a consensus. In the case of this study, when the fuzzy consensus threshold is relatively low, there is no need to spend a significant amount of time to reach a consensus on Teacher 5's opinion, thus reducing costs. As the fuzzy consensus threshold increases, we can reduce the degree of adjustment for Teacher 1's opinion while increasing the degree of adjustment for Teacher 5's opinion, thereby satisfying the requirements of the fuzzy consensus threshold with lower costs for the coordinator.

5.2 Impact of Unit Adjustment Cost Reference Point c on models

The unit adjustment cost reference point c can not only affect the cost required to adjust the DM's opinion, but also have an impact on the value of the DM's adjustment opinion. Therefore, it is essential to study the effect of changes in the

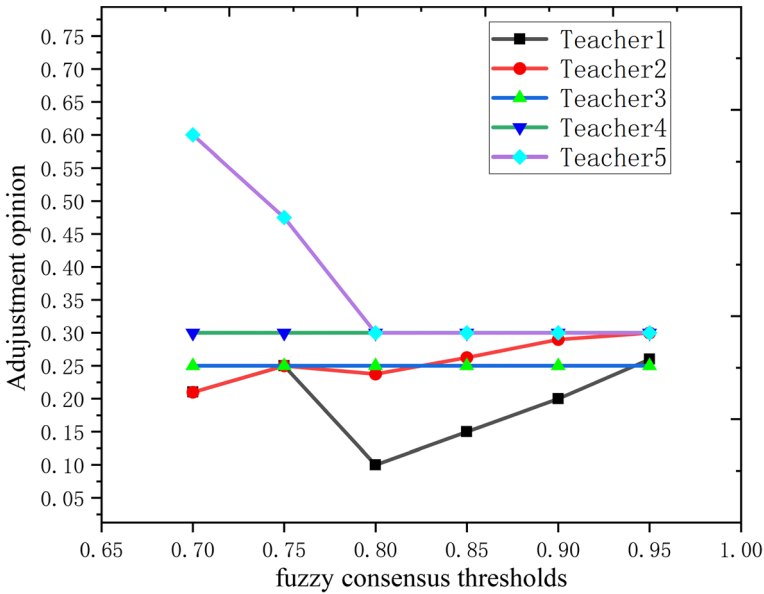


Fig. 3 Adjustment opinion for different fuzzy consensus thresholds

unit adjustment cost reference point on the model. Let c_i change from -0.4 to $+0.4$ in steps of 0.2 for each occurrence of Δc . Solve for P2, P3 and P4 in $\alpha = 0.8$. Finally, we obtain the minimum consensus cost (Mc), coverage (cov), specificity (sp), fuzzy consensus measure (FC) and consensus opinion ($\bar{\sigma}^c$) for each model, as shown in Table 5. The adjustment opinions of DMs for each model are shown in Table 6.

5.2.1 Impact of Unit Adjustment Cost Reference Point c on Model Results

The effect of the change in unit adjustment cost reference point on the minimum cost is shown in Fig. 4, and we can easily find that as Δc increases, the minimum cost increases as well. This is in line with the reality that as the unit adjustment cost reference point increases, the cost required to adjust the DM's opinion also increases, which leads to an increase in the minimum cost. The results for the box uncertainty set are the most conservative of these. As shown in Fig. 5, the change in unit adjustment cost reference points has no change on the consensus opinion of P2, while the consensus opinion of P3 shows a decreasing trend, and the consensus opinion of P4 first decreases and then stabilizes.

5.2.2 Impact of Unit Adjustment Cost Reference Point c on Adjusted Opinions

As the reference point for unit adjustment cost varies, the DM's opinion of adjustment changes. As an example, the uncertainty set is the ellipsoidal uncertainty set of

Table 5 Impact of unit adjustment cost reference point c on model results

Model	Δc	Mc	cov	sp	FC	$\bar{\sigma}^c$
P2	-0.4	4.67	1	0.8	0.8	0.35
	-0.2	4.82	1	0.8	0.8	0.35
	0	4.97	1	0.8	0.8	0.35
	+0.2	5.12	1	0.8	0.8	0.35
	+0.4	5.27	1	0.8	0.8	0.35
P3	-0.4	3.95	1	0.8	0.8	0.35
	-0.2	4.1	1	0.8	0.8	0.35
	0	4.25	1	0.8	0.8	0.28
	+0.2	4.37	1	0.8	0.8	0.25
	+0.4	4.48	1	0.8	0.8	0.23
P4	-0.4	4	1	0.8	0.8	0.2625
	-0.2	4.12	1	0.8	0.8	0.2
	0	4.21	1	0.8	0.8	0.2
	+0.2	4.31	1	0.8	0.8	0.2
	+0.4	4.41	1	0.8	0.8	0.2

Table 6 Impact of unit adjustment cost reference point c on adjustment opinions

Model	Δc	\bar{o}_1	\bar{o}_2	\bar{o}_3	\bar{o}_4	\bar{o}_5
P2	-0.4	0.31	0.45	0.25	0.3	0.45
	-0.2	0.31	0.45	0.25	0.3	0.45
	0	0.31	0.45	0.25	0.3	0.45
	+0.2	0.31	0.45	0.25	0.3	0.45
	+0.4	0.31	0.45	0.25	0.3	0.45
P3	-0.4	0.308	0.45	0.25	0.3	0.45
	-0.2	0.308	0.45	0.25	0.3	0.45
	0	0.19	0.38	0.25	0.3	0.38
	+0.2	0.15	0.34	0.25	0.3	0.35
	+0.4	0.13	0.30	0.25	0.3	0.33
P4	-0.4	0.16	0.36	0.25	0.3	0.36
	-0.2	0.1	0.24	0.25	0.3	0.3
	0	0.1	0.24	0.25	0.3	0.3
	+0.2	0.1	0.24	0.25	0.3	0.3
	+0.4	0.1	0.24	0.25	0.3	0.3

P3. Five faculty members participated in the evaluation of the instructional software. Figure 6 shows the effect of the change in unit adjustment cost reference points on the adjustment opinions. As shown in Fig. 6, Teacher 3 and Teacher 4 were unaffected by the change in c because Teacher 3 and Teacher 4’s opinions were always in agreement with the facilitator. In contrast, Teacher 1, Teacher 2, and Teacher 5 were strongly affected by changes in c . This is because the consensus opinion of P3 kept changing with c , while the initial opinions of Teacher 1, Teacher 2, and Teacher

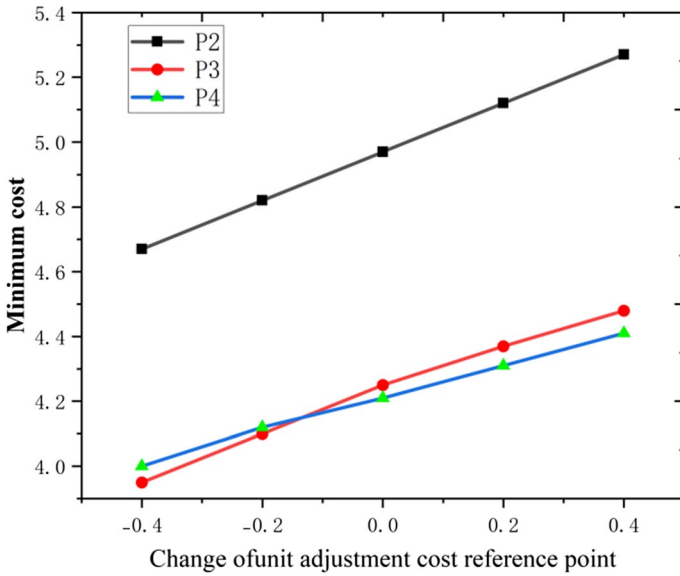


Fig. 4 Minimum cost for different Δc

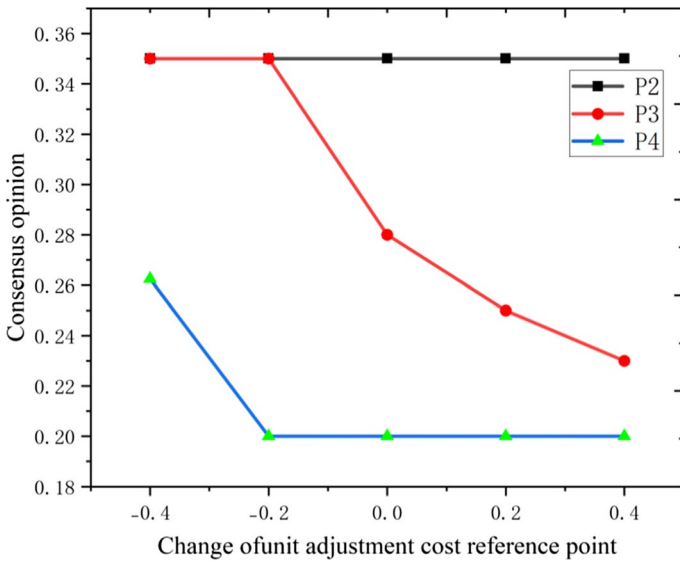


Fig. 5 Consensus opinion for different Δc

5 could not reach a consensus opinion. That is, as c keeps changing, Teacher 1, Teacher 2, and Teacher 5 keep adjusting his initial opinion to reach a consensus opinion. When the reference point for unit adjustment costs changes, coordinators

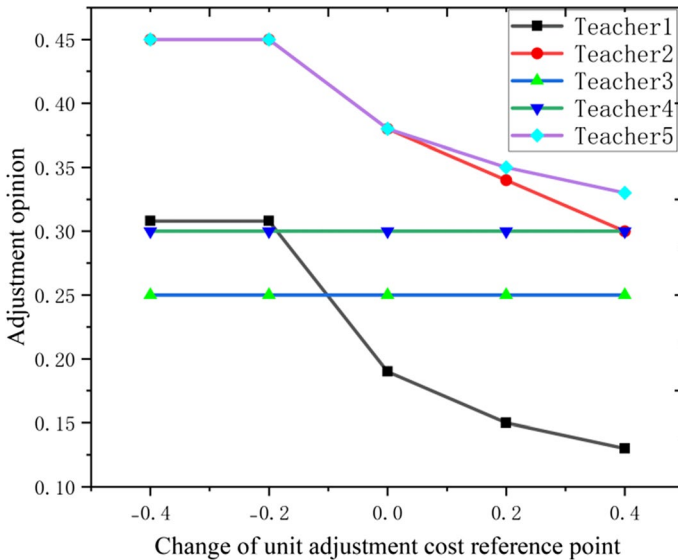


Fig. 6 Adjustment opinion for different Δc

need to focus on adjusting the opinions of Teachers 1, Teacher 2 and Teacher 5 in order to meet consensus requirements with minimal costs.

5.3 Model Comparison

In this subsection, we will compare the minimum cost fuzzy consensus model with the minimum cost consensus model, the minimum cost fuzzy consensus model based on robust optimization with the minimum cost consensus model based on robust optimization to further illustrate the superiority of the models. We abbreviate the minimum cost consensus model, the minimum cost consensus model with box set, the minimum cost consensus model with ellipsoidal set, and the minimum cost consensus model with polyhedral set as MCCM, BMCCM, EMCCM, and PMCCM. since there is a relationship $\alpha = 1 - 2\sigma$ between the fuzzy consensus threshold α and the consensus threshold σ when the coverage is 1. Therefore, next, we will analyze for α equal to 0.8, 0.7, and σ equal to 0.1 and 0.15, respectively.

The detailed data for P1, P2, P3, and P4 fuzzy consensus thresholds equal to 0.8 are shown in Tables 3 and 4. By comparing the results with those in Table 7, it is found that. P1, P2, P3, and P4 are the same as the optimal solutions and optimal values of MCCM, BMCCM, EMCCM and PMCCM. This is also consistent with Zhang et al. (2021a) who propose that the least-cost consensus model is a variant of the least-cost fuzzy consensus model when the coverage is equal to 1. On the other hand, under certain conditions, the minimum cost consensus model is equivalent to the minimum cost fuzzy consensus model.

Table 7 The results of the models for $\sigma = 0.1$

Model	Mc	$\bar{\sigma}^c$	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	$\bar{\sigma}_4$	$\bar{\sigma}_5$
MCCM	4.14	0.35	0.31	0.45	0.25	0.3	0.45
BMCCM	4.97	0.35	0.31	0.45	0.25	0.3	0.45
EMCCM	4.25	0.28	0.19	0.38	0.25	0.3	0.38
PMCCM	4.21	0.2	0.1	0.2375	0.25	0.3	0.3

The detailed data for P1, P2, P3, and P4 with fuzzy consensus threshold equal to 0.8 are shown in Tables 3 and 4. By comparing with the results in Table 8, it is found that P1, P2, P3, and P4 are different from the optimal solutions and optimal values of MCCM, BMCCM, EMCCM and PMCCM. P1, P2, P3, and P4 have much less minimum cost than MCCM, BMCCM, EMCCM and PMCCM. This is because in the case of $\alpha = 0.7$, P1, P2, P3, and P4 there exists a DM who satisfies the condition without reaching consensus, while MCCM, BMCCM, EMCCM and PMCCM require each DM to reach consensus. Therefore, MCCM, BMCCM, EMCCM, and PMCCM are more expensive. When the condition exists that only a majority of DMs need to reach consensus, the results of P1, P2, P3, and P4 will be superior to MCCM, BMCCM, EMCCM and PMCCM. This also illustrates the superiority and flexibility of the model.

6 Conclusion

In this paper, uncertainty is mainly considered for the minimum cost fuzzy consensus model. Since MCFCM is a decision and choice made under advance access to information, which is difficult to achieve in the real world. However, due to the uncertainty of information and the complexity of acquiring information, it is difficult to determine the unit adjustment cost, and the decision made based on MCFCM may not achieve the expected results. Therefore, we re-model the MCFCM using the theory of robust optimization. Firstly, we construct MCFCM with uncertain unit adjustment cost in the Box set. Then, we consider the uncertainty of unit adjustment cost in the Ellipsoid set under minimum cost consensus and propose MCFCM in the Ellipsoid set. And we consider MCFCM with unit adjustment cost determined in the Polyhedral set. Finally, we conduct numerical experiments on the proposed method with the classical GDM problem to verify the validity and feasibility of the model. Also, we obtain some interesting conclusions.

Table 8 The results of the models for $\sigma = 0.15$

Model	Mc	$\bar{\sigma}^c$	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	$\bar{\sigma}_4$	$\bar{\sigma}_5$
MCCM	3.41	0.4	0.342	0.55	0.25	0.3	0.55
BMCCM	4.09	0.4	0.342	0.55	0.25	0.3	0.55
EMCCM	3.76	0.21	0.06	0.304	0.25	0.3	0.36
PMCCM	3.43	0.2	0.05	0.2875	0.25	0.3	0.35

- (1) The change of unit adjustment cost will cause the change of consensus cost, resulting in the decision result of the moderator may be invalid. In contrast, the minimum cost fuzzy consensus model based on robust optimization considers the uncertainty of the unit adjustment cost, and its results are more conservative and face less risk.
- (2) Comparing the P2, P3 and P4 models, the results obtained by the P2 model are too conservative, and the results obtained by the P3 and P4 models are closer to the MCFCM. From the moderator's perspective, the P3 or P4 model can be regarded as a better strategy to solve the uncertainty of the unit adjustment cost, which can eliminate the adverse effects of the uncertainty of the unit adjustment cost to a larger extent and reduce the conservativeness of the model.
- (3) Compared with the MCCM, the minimum cost fuzzy consensus model based on robust optimization not only takes into account the uncertainty of unit adjustment cost but also does not require all DMs to reach a consensus, which is more flexible and more realistic.
- (4) Through the comparison of our proposed model with MCCM models, our model demonstrates greater adaptability in situations where consensus among all DMs is not required. When consensus among the majority of DMs is sufficient in CRP, the coordinator can selectively encourage some DMs to reach consensus, thus reducing the cost within CRP.
- (5) Through the comparison of our proposed model with traditional MCFCM models, our model incurs higher costs, and the model's results are more conservative. When faced with uncertainty in unit adjustment costs, the coordinator can opt to increase costs appropriately to mitigate risks.

In GDM, the relationships among DMs are of utmost importance for the consensus outcome. Future research should focus on studying DMs' trust relationships (Zha et al. 2023a) and their acceptance levels regarding consensus during the group consensus process (Zha et al. 2022). Another interesting direction is to further explore the impact of fairness in the CRP on costs, making the consensus model more realistic (Gong et al. 2023). Furthermore, with the decision problem more complex and comprehensive, exploring how to account for decision-making under uncertainty in large-scale GDM and addressing aggregation discrepancies in failure mode and effect analysis GDM (Zha et al. 2023b) under uncertain environments are intriguing research directions.

Appendix A: Proof of Theorem 1

Proof: We first consider the maximization problem of the objective function in (4).

$$\begin{aligned} & \max_{\tilde{c} \in U} \sum_{i=1}^n \tilde{c}_i a_i \\ & s.t. c^- \leq \tilde{c} \leq c^+ \end{aligned}$$

Because $a_i \geq 0$, to maximize the objective value of the problem in (4), each unit of adjustment cost should take the maximum value, so $\max_{\tilde{c} \in U} \sum_{i=1}^n \tilde{c}_i a_i = \sum_{i=1}^n c_i^+ a_i$. The proof is complete.

Appendix B: Proof of Theorem 2

Proof: $\sup_{\tilde{c} \in U} \tilde{c}^T a$ in (4) can be written as

$$\begin{aligned} & \max_{\tau \in \mathbb{R}^m} c^T a + \tau^T Q^T a \\ & \text{s.t. } c + Q\tau \geq 0 \\ & \|\tau\|_2 \leq 1 \end{aligned} \tag{8}$$

Turn (8) into a problem where the minimization and constraint are less than or equal to

$$\begin{aligned} & \min_{\tau \in \mathbb{R}^m} -c^T a - \tau^T Q^T a \\ & \text{s.t. } -c - Q\tau \leq 0 \\ & \|\tau\|_2 - 1 \leq 0 \end{aligned} \tag{9}$$

Let $\lambda (\lambda \in \mathbb{R}_+^n)$ and $v (v \in \mathbb{R}_+)$ as Lagrange multipliers, where λ is related to the inequality $-c - Q\tau \leq 0$ and v is related to the inequality $\|\tau\|_2 - 1 \leq 0$. So the Lagrangian function of (9) is

$$\begin{aligned} & \min_{\tau \in \mathbb{R}^m} -c^T a - \tau^T Q^T a \\ & \text{s.t. } -c - Q\tau \leq 0 \\ & \|\tau\|_2 - 1 \leq 0 \end{aligned} \tag{10}$$

The Lagrangian dual function formed by the Lagrangian function of (10) is

$$g(\lambda, v) = -c^T(a + \lambda) - v + \min_{\tau} \{-(a + \lambda)^T Q\tau + v\|\tau\|_2\} \tag{11}$$

Let $a + \lambda = s$ and $\min_{\tau} \{-(a + \lambda)^T Q\tau + v\|\tau\|_2\}$ divided by v , (11) can be transformed into

$$g(\lambda, v) = -c^T s - v - \frac{1}{v} \max_{\tau} \left\{ \frac{s^T Q}{v} \tau - \|\tau\|_2 \right\} \tag{12}$$

Since the conjugate function of x is

$$f^*(y) = \max_{x \in \text{dom}f} \{y^T x - f(x)\} \tag{13}$$

Thus the conjugate of $\|\tau\|_2$ can be written as

$$\max_{\tau \in \mathbb{R}_+^m} \{y^T \tau - \|\tau\|_2\} \tag{14}$$

Since $\max_{\tau \in \mathbb{R}_+^m} \{y^T \tau - \|\tau\|_2\}$ is equal to

$$\max_{\tau \in \mathbb{R}_+^m} \{y^T \tau - \|\tau\|_2\} = \begin{cases} 0, & \|y\|_2 \leq 1 \\ \infty, & \text{otherwise} \end{cases} \tag{15}$$

So let $y = \frac{s^T Q}{v}$, we can obtain the dual problem of (10) as follows

$$\begin{aligned} & \max_{\lambda, v} -c^T s - v \\ & \text{s.t.} \left\| Q^T s \right\|_2 \leq v \\ & a + \lambda = s \\ & \lambda \in \mathbb{R}_+^n, v \in \mathbb{R}_+ \end{aligned} \tag{16}$$

Since $\lambda \in \mathbb{R}_+^n$, converting (16) to a minimization problem, it can be written as

$$\begin{aligned} & \min_s c^T s + \left\| Q^T s \right\|_2 \\ & \text{s.t.} a - s \leq 0 \\ & s \in \mathbb{R}_+^n \end{aligned} \tag{17}$$

By replacing $\sup_{\tilde{z} \in U} \tilde{c}^T a$ in (4) with (17), we derive (6). The proof is complete.

Appendix C: Proof of Theorem 3

Proof: $\sup_{\tilde{z} \in U} \tilde{c}^T a$ in (4) can be written as

$$\begin{aligned} & \max_{\tau \in \mathbb{R}^m} c^T a + \tau^T Q^T a \\ & \text{s.t.} c + Q\tau \geq 0 \\ & Y\tau = y \\ & K\tau \geq k \end{aligned} \tag{18}$$

Turning (18) into a minimization and constraint is a less-than-equal problem.

$$\begin{aligned} & \min_{\tau \in \mathbb{R}^m} -c^T a - \tau^T Q^T a \\ & \text{s.t.} -c - Q\tau \leq 0 \\ & y - Y\tau = 0 \\ & k - K\tau \leq 0 \end{aligned} \tag{19}$$

Let $\lambda (\lambda \in \mathbb{R}_+^n)$, $v (v \in \mathbb{R}^p)$ and $m (m \in \mathbb{R}_+^q)$ asb Lagrange multipliers, where λ is related to the inequality $-c - Q\tau \leq 0$, v is related to the equation $y - Y\tau = 0$, and m is related to the inequality $k - K\tau \leq 0$. So the Lagrangian function of (19) is

$$L(\lambda, v, m, \tau) = -c^T a - \tau^T Q^T a - \lambda^T (c + Q\tau) + v^T (y - Y\tau) + m^T (k - K\tau) \tag{20}$$

The Lagrangian dual function formed by the Lagrangian function of (20) is

$$g(\lambda, v, m) = -c^T (\lambda + a) + y^T v + k^T m + \min_{\tau \in \mathbb{R}^n} \{ -((\lambda + a)^T Q\tau + Y^T v\tau + K^T m\tau) \} \tag{21}$$

Let $a + \lambda = s$, (21) can be transformed into

$$g(\lambda, v, k) = -c^T s + y^T v + k^T m + \min_{\tau \in \mathbb{R}^n} \{ -(Q^T s + Y^T v + K^T m)\tau \} \tag{22}$$

Since $\min_{\tau \in \mathbb{R}^n} \{ -(Q^T s + Y^T v + K^T m)\tau \}$ is a linear function on τ , it follows that

$$\min_{\tau \in \mathbb{R}^n} \{ -(Q^T s + Y^T v + K^T m)\tau \} = \begin{cases} 0, & Q^T s + Y^T v + K^T m = 0 \\ -\infty, & \text{otherwise} \end{cases} \tag{23}$$

We can obtain the pairwise problem of (20) as follows

$$\begin{aligned} & \max_{\lambda, v, m, s} -c^T s + y^T v + k^T m \\ & s.t. Q^T s + Y^T v + K^T m = 0 \\ & a + \lambda = s \\ & s \in \mathbb{R}_n^+, \lambda \in \mathbb{R}_n^+, v \in \mathbb{R}^p, m \in \mathbb{R}_+^q \end{aligned} \tag{24}$$

Since $\lambda \in \mathbb{R}_n^+$, converting (24) to a minimization problem, it can be written as

$$\begin{aligned} & \min_{v, m, s} c^T s - y^T v - k^T m \\ & s.t. Q^T s + Y^T v + K^T m = 0 \\ & a - s \leq 0 \\ & s \in \mathbb{R}_n^+, v \in \mathbb{R}^p, m \in \mathbb{R}_+^q \end{aligned} \tag{25}$$

By replacing $\sup_{\tilde{c} \in U} \tilde{c}^T a$ in (4) with (25), we derive (7). The proof is complete.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The authors have no relevant financial or non-financial interests to disclose. No funding is received for conducting this study.

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