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A Multi-Criteria and Multi-Agent Framework for supporting complex decision-making processes

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Abstract
A framework for modeling multi-criteria and multi-agent decision making processes as a non-cooperative game including a phase for solving the game by using the concept of equilibrium solution is presented. In the presence of a non-singular solution, the framework includes a phase for refining the solution by the application of a social welfare function. The framework is named Multi-Criteria and Multi-Agent Framework. The framework makes possible the strategic performance in complex decision-making, creating transparency within the process of selecting alternatives that are under evaluation in a multi-criteria perspective by agents with heterogeneous preferences. This paper includes a simulation to demonstrate the applicability of the framework to a complex engineering problem such as the choice of a Wastewater Treatment Plant for a municipality. Convergence of choices of five experts that participated in the simulation was demonstrated by the application of the framework.

Keywords MCDM · Game theory · Utility function · Equilibrium selection problem · Social welfare functions

1 Introduction

Selecting the best alternative among a set of different propositions is often a complex decision-making problem. Methods with the purpose of aiding this process have been proposed with a strong emphasis on deterministic approaches, including the evaluation of economic and/or operational aspects of the alternatives under

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consideration (Padalkar & Gopinath, 2016). Recently, the use of Multi-criteria Decision-Making (MCDM) methods has gained popularity, since they provide means for modeling and solving complex decision-making problems under multiple and conflictive criteria, by adopting the concept of a satisficing solution (Garrido-Baserba, et al., 2016; Pohekar & Ramachandran, 2004; Simon, 1979). However, MCDM methods are mostly designed for the use of a single agent and are not appropriate, *a priori*, to be applied in the scenario of multi-agent decision making processes, which is the scenario of most of the current decision-making processes within organizations.

In the literature, propositions can be found for the use of MCDM methods within a multi-agent scenario through the aggregation of different agents' preferences into a unity, usually using arithmetic or geometric mean.[1] It is noteworthy to cite one of the first uses of such procedure, which can be seen in the paper of Lockett et al. (1987), that used the arithmetic mean for presenting the discrepancies of the Analytical Hierarchy Process (AHP) between individual and group scenarios. Due to the possibility of losing information in scenarios with high heterogeneous preferences (Basak, 1988; Leoneti, 2016), other approaches suggest the adaptation of the classical MCDM methods for their suitable operation within a multi-agent scenario. In this direction, the contributions of Dyer & Forman (1992), Basak & Saaty (1993), Chen (2000), Lai et al. (2002), Shih et al. (2007), Escobar & Moreno-Jimenéz (2007), Arnette et al. (2010), Altuzarra et al. (2010), Sanayei et al. (2010), Dong et al. (2010), Hatami-Marbini and Tavana (2011), Huang et al. (2013), Wu et al. (2018) can be cited. Conversely, although avoiding the use of aggregation of agents' preferences, such adapted MCDM methods still contemplate some aggregation procedures, and therefore, do not allow to fully model the heterogeneity of the agents' preferences for finding a satisficing solution.

In contrast, game theory is a technique designed for modeling the strategic iteration of multiple agents with heterogeneous preferences and to support their choice. The choice that agents can reach through a game theory structure is usually based on the concept of equilibrium solutions (Hipel et al., 2011). According to Hipel and Fang (2005), in the presence of multiple agents with divergent objectives or value systems, game theory would be the most appropriate decision-making technique for better modeling the conflicting aspirations of agents in a fair and sustainable manner.[2] Nevertheless, most game theory models assume a scalar to represent the outcomes (payoffs) of the strategic interactions of the agents. In consequence, the

[1] Huang et al. (2013) present other possible aggregation approaches, including: (i) relative closeness; (ii) separation measure; (iii) fuzzy consensus; (iv) evidential reasoning; and (v) Borda count.

[2] It should be stressed that the development of game theory is mostly related to the analytical analysis of simplified models, which, despite their capacity of explaining reasonably well strategic interaction of agents for specific cases, might be inflexible to be used in other conflict situations. In this context, a notorious contribution is the Graph Model for Conflict Resolution, first appearing in Kilgour et al. (1987) and later in the text of Fang et al. (1993), which is a method for modeling and analyzing strategic conflicts for a broader scope of applications with more flexibility on the principles of game theory. According to Kilgor & Hipel (2005), the basis for all the definitions and all of the analysis of the method is the creation of different alternatives in relation to the *status quo* and their comparison using ordinal preference information provided by multiple-agents for evaluating the possible transition states to reach an agreement.

outcomes are analyzed on a one-dimensional basis, which disregards the potential benefits that could be provided by multi-criteria analysis.

An initial attempt to model multi-criteria problems using the structure of games was introduced by Shapley (1959) and Blackwell (1956). The so-called vector payoff games (or multi-criteria games) contemplates heterogeneous agents and their multiple decision criteria that are modeled by means of vector utility functions (Sasaki, 2018). An example of the application of vector payoff games to a complex environmental problem can be seen in Lejano & Ingram (2012) for the tragedy of commons. Nevertheless, an exchange ratio between each component of the vector should be known by the designer of the game, which makes this approach very difficult to be applied, since the analyst may lack the knowledge or resources to elicit such utility functions, especially when there are numerous criteria involved (Voorneveld et al., 2000). It should be noted that this is the same type of difficulty generally associated to MCDM methods that depends on the elicitation of utility functions, such as Multi Attribute Utility Theory (MAUT). Given that difficulty, the vector payoff games approach had not gained strong and continuous efforts for its applications and for further developments.

Other approaches focused on hybrid methodologies that linked multi-criteria decision-making techniques and game theory, aimed to favor the analysis of different objectives within a game theoretical structure where the heterogeneous preferences of agents could be fully considered without amalgamation. Towards that integration, a remote example can be seen in Szidarovszky and Duckstein (1984) that proposed to use the principles of game theory for solving a multi-objective programming model in which different viewpoints were represented by objective functions. It is noteworthy that the authors offered means for finding a satisficing solution to group multi-criteria problems, rather than seeking an overall optimal solution, as in the multi-objective programming approach. This direction gained notoriety in the literature, from which other examples can be seen, including Chen (Chen, 2000), Madani and Lund (2011), Ke et al. (2012), Wibowo and Deng (2013), Aplak and Sogut (2013), Deng et al. (2014), and Jing et al. (2018). Nevertheless, the modeling of the games in such approaches still depends on specific elicitation procedures that need to be conducted by the analyst, which represents the similar difficulties from approaches such as vector payoff games or MAUT. Furthermore, in the presence of a non-singular solution, i.e. multiple equilibria solutions, a criterion for equilibrium selection is not addressed in the mentioned approaches, which mostly used Nash equilibrium (Nash, 1951) as the equilibrium solution concept for solving the games.

Following the evident relationship of complementarities between multi-criteria and game theory, the aim of this paper is to present a framework for modeling a multi-agent and multi-criteria decision-making problem as a non-cooperative game for considering the strategic interaction among agents based on their distinctive preferences. These preferences are related to the agents' evaluation of the multiple criteria, such as those that pursue environmental, social, and economic aspects of sustainability. In this scenario, the framework allows to find alternatives that satisfy equilibrium conditions for solving the game. Also, in the presence of more than one equilibrium, a refinement to the solution is proposed by the application of a social welfare function. This study contributes to the literature by presenting a framework that, based on the use of utility functions, provides means for the modeling of multi-criteria and multi-agent complex decision-making problems as a non-cooperative game from which a solution that is based on equilibrium concepts can be reached as the social choice. This framework can be used to support the process of selecting alternatives in the presence of multiple heterogeneous agents through the resolution of the problem without the need for amalgamating their preferences. Consequently, the framework makes possible to fully consider each agent with their set of different objectives and value systems, which are represented by a set of criteria and their respective preferences.

The mechanism of the framework is built on the theoretical assumption of expected agent's behavior when trading alternatives within a strategic interaction environment, which is the tendency to reject proposals when realizing that the proposal significantly either favors or disfavors their counterparts (Blount & Bazerman, 1996; Camerer, 2003). The necessary conditions to assume this rational behavior of the agents under this scenario is based on the assumption of Binmore (1988), namely: (i) the fact that it involves a finite and known number of agents, alternatives, and criteria; and (ii) the agents have already dealt with similar situations several times and have got the ability to learn from it. Therefore, the rational behavior assumed is based on the assumption that agents are familiar with: (i) choosing alternatives taking into account the choices of their counterparts; and, if necessary, (ii) trading for alternatives that are not very different between them.

For illustrating the application of the framework herein proposed, the scenario of the choice of a Wastewater Treatment Plant (WWTP) for a municipality has been chosen. Garrido-Baserba, et al. (2016) and Madani and Lund (2011) advocate that engineering problems related to water resources are complex problems that usually involve the interaction of multi-agents with multiple conflictive objectives. In this sense, choosing a WWTP for a municipality can be considered a good illustration for the application of the framework to a complex scenario with multiple agents, which is very common in current societal and technological systems, as described by Hipel and Fang (2005). It should be stressed, however, that the framework can be applied to any other kind of problem involving multiple criteria and multiple agents, which is here also illustrated by means of a numerical example.

Section 2 presents the background of the framework, Sect. 3 presents each step of the framework in a detailed level with their respective references of Sect. 2, Sect. 4 presents a numerical example, and Sect. 5 presents a simulated example of complex multi-criteria group decision-making problem, which is the choice of a WWTP to a municipality. Finally, the comparison of the presented framework with the state of art is presented in Sect. 6 and the final remarks are part of Sect. 7.

2 Technical backgrounds

2.1 The General Structure of a Multi-Criteria and Multi-Agent method

The general structure of multi-criteria methods includes two mathematical objects, namely: (i) a matrix, usually called decision matrix; and (ii) a vector, usually called weighting vector, which is obtained by the application of an elicitation procedure to the agent involved in the decision-making process. The decision matrix $\{x_{ij}\}$ is composed by $i = 1, 2, \ldots, m$ alternatives and $j = 1, 2, \ldots, n$ discrete and/or continuous criteria (attribute or variable) of cost and/or benefit type, which is the basis of the structure of an MCDM problem. For the scenario of multi-agent decision making, the number of weighting vectors is necessarily more than one (equal to the number of the agents) and the number of matrixes might eventually be more than one, depending on the set of criteria that each agent is taking into account (Kilgour & Hipel, 2005; Keeney, 2013). Consequently, in a multi-agent environment, the decision matrixes $\{x_{ij}\}_k$ are conventionally composed by $i = 1, 2, \ldots, m$ alternatives and $j = 1, 2, \ldots, n_k$ criteria, which correspond to the set of criteria of each *k-th* agent.

Franco & Montibeller (2010) state that the creation of such decision matrixes involves several steps, in the following order: (i) the identification of the agent's fundamental values and objectives; (ii) the choice of the criteria for measuring each identified value or objective; and (iii) the characterization of the alternatives, which comprises the performance measurement for each evaluation criteria. For assisting in the proposition of such decision matrixes, Marttunen et al. (2017) present a review of Problem Structure Modeling (PSM) techniques, which includes: (i) Soft Systems Methodology (SSM); (ii) Strategic Assumptions Surfacing and Testing (SAST); and (iii) Strategic Options Development and Analysis (SODA). Other widely known PSM techniques are the Strategic Choice Approach (SCA) and Value-Focused Thinking (VFT).

After the modeling of the decision matrix, it is eventually necessary to apply a standardization process,[3] since criteria generally have different units of measurement and can be either of benefit or cost type (Kosareva et al., 2018; Vafaei et al., 2016). Yoon and Hwang (1995) present two functions for standardizing the decision matrix: (i) linear standardization, and (ii) vector standardization, both keeping the ordinal property from the original criterion. These functions are the most common, others being the logarithmic standardization technique and fuzzification (Kosareva et al., 2018; Vafaei et al., 2016; Leoneti & Gomes, 2021).

Subsequently, the process of measuring the relative preference of each agent over the criteria of the decision matrix, named elicitation, is performed for providing the weight vectors that are used to weight the standardized decision matrix. According to Jia et al. (1998), there are different elicitation techniques created for different purposes. When the ratio scale properties of the agent's judgments need to

[3] This process is usually called "normalization process" in the literature. However, instead of using the term normalization, the term standardization is used here, since the variables do not necessarily need to follow the normal distribution.

be preserved, there are techniques such as the Swing and the Trade-off techniques. Techniques that preserve only the ordinal properties of agents are classified as rank-order techniques, including the Rank-Sum (RS), the Rank-Reciprocal (RR), and the Rank-Ordered Centroid (ROC) techniques. Ahn (2011) complements by describing subjective techniques that determine the weights solely according to the preferential judgments of the decision-maker, which includes the Simple Multi-attribute Rating Technique by Swing (SMART), the revised Simos' procedure, the AHP, and objective techniques in which the weighting vectors are defined by solving mathematical models without any consideration of the agents' preferences, for example, the entropy method.

The standardized decision matrix and the weighting vectors are then operated through the application of an algebraic operator, which can be represented by an MCDM method, for providing different possible outcomes, including: (i) ranking; (ii) classification; or (iii) selection of the alternatives. The general structure of such algebraic operators is that based on pairwise comparison of alternatives,[4] which occurs by the performance comparison of each alternative per criterion. When the performance of a criterion is better for a given alternative (bigger for benefit criteria and lower for cost criteria) than another, the former must be better scored by the pairwise procedure. Some of the most widely used MCDM methods are: (i) AHP; (ii) *Elimination et Choice Traduisant la Realité* (ELECTRE); (iii) Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE); (iv) Technique for Order Preference by Similarity to Ideal Solution (TOPSIS); (v) Measuring Attractiveness by a Categorical Based Evaluation TecHnique (MACBETH); and (vi) Multi-attribute Utility Theory (MAUT) (Leoneti & Pires, 2017).

Here, the standardized decision matrix and the weighting vectors are used as input for the creation of a non-cooperative game by an algebraic operator represented by a utility function, of which the details of its application are described in the next section.

2.2 The General Structure of a Non-Cooperative Game and its Solution Based on Equilibrium Solution Concepts

The non-cooperative game is the branch of game theory that models situations where agents have no guarantee that their counterparts would implement agreements made, which is more adherent to group decision-making scenarios. The general structure of a non-cooperative game can be summarized by the tuple $<K, A_k, \prec_k>$, where K is the number of the agents, A_k is the set of m_k alternatives for each agent $k = 1, \dots, K$, and $\prec_k$ is the preferences' relations of each agent $k = 1, \dots, K$ on all

[4] When all pairs of objects can be compared, pairwise procedures are commonly employed to obtain perceptions of similarities. According to Hair et al. (2006) three procedures are commonly used: (i) objective pair comparison, the most commonly used method for obtaining similarity judgments; (ii) subjective pair comparison, a method that uses objects to aid in the pairwise comparison; and (iii) derived pairwise comparison, which is based on the process of creating measures to be used in the pairwise comparison.

alternatives of the set A_k. The values for the operator $\prec_k$ are given by a utility function $\pi_k : \mathbb{R}^K \rightarrow [0, 1]$, which provides outcomes (payoffs) for each possible strategy that an agent may chose in response to a particular counterpart's action.

The rational principles of such utility functions are those summarized by von Neumann and Morgenstern (1944). The authors extended the interpretation of Bernoulli's classic utility theory by adding the idea of pairwise comparison of alternatives in a strategic interaction environment. According to the rational principles of classic utility theory, agents make their choice aiming to improve the expected utility of goods subjected to the Weber and Fechner Law, which states that the agent's perception is subjective and proportional to a logarithm intensity (Lengwiler, 2009; Stevens, 1986). Von Neumann and Morgenstern (1944), complementarily, suggested that this measure should be relative and subjected to the relationship of each pair of strategies.[5] In this sense, considering two hypothetical alternatives x and y, the operator $\prec_k$ must assume the state $x \prec_k y$, when the alternative y is strictly preferable to the alternative x; $x \preccurlyeq_k y$, when the alternative y is preferable to the alternative x; $x \approx_k y$, when the alternative x is indifferent to the alternative y; $x \succcurlyeq_k y$, when the alternative x is preferable to the alternative y; and $x \succ_k y$, when the alternative x is strictly preferable to the alternative y. It follows that in the case $x \prec_k y$, then $\pi_k(x) < \pi_k(y)$; case $x \preccurlyeq_k y$, $\pi_k(x) \leq \pi_k(y)$; case $x \approx_k y$, $\pi_k(x) = \pi_k(y)$, and so on.

When the chronological order of agents' choices is not considered, the structure for presenting the values of the utility function is a matrix. The matrix form is usually called the strategic form, where no information is provided to any agent with relation to their counterparts' previous choices, which is the scenario of games with incomplete information (Osborne, 1994). The payoff of each agent is given by the arrangement of agent's choice together with their counterparts' choices. Therefore, in the particular case of non-cooperative games in the strategic form, the order of the multi-dimensional matrix is given by the number of arrangements $A_1 \times A_2 \times \ldots \times A_K$. Particularly to the case of multi-agent decision making where it is mandatory that agents share the same set of alternatives (a unique set A containing m elements), the number of arrangements is given by m^K, where m is the number of the alternatives of the set A, and K is the number of agents. Consequently, within the elicitation process for generating the utility functions π_k for each of the K agents, each one of those possible arrangements should be contemplated. These values provide the payoff tables for each agent that composes the non-cooperative game.

Leoneti (2016) proposed a utility function that allows calculating those payoffs for each arrangement in $A_1 \times A_2 \times \ldots \times A_K$ through the pairwise comparison of vector alternatives. This utility function can be seen in the equation

$$\pi_k(x_k, Y) = \varphi_k(x_k, IA_k) \prod_{k \neq p, p=1}^{K} \varphi_k(x_k, y_p) . \varphi_k(y_p, IA_k) \quad (1)$$

[5] One could note that this is similar to the pairwise process that occurs in the MCDM approach, which makes the integration between multicriteria approach and game theory possible.

where π_k is the payoff that the *k-th* agent would obtain in the arrangement of its alternative x_k with the subset of alternatives $Y(y_{p\neq k})$ proposed by all other $K-1$ agents jointly with $p = 1, \ldots, p \neq k, \ldots, K$, being each alternative represented by vectors constituted by the n_k multiple independent benefit criteria, and IA_k (Ideal Alternative) is the utopian alternative that is composed by the best value of each criteria that the agent k could obtain. The pairwise comparison function $\varphi : \mathbb{R}_+^{n_k \times 2} \rightarrow [0,1]$, inspired by Deng (2007), gives the value for the pairwise comparison between these alternatives according to the equation

$$\varphi_k(x_k, x_p) = \left[\frac{\alpha_{x_k x_p}}{\left\| x_p \right\|} \right]^{\delta} . \cos\theta_{x_k x_p}, \text{and } \delta = \begin{cases} 1, & if\,\alpha_{x_k x_p} \leq \left\| x_p \right\| \\ -1, & \text{otherwise} \end{cases} \tag{2}$$

where $\alpha_{x_k x_p} = \left\| x_k \right\| cos\theta_{x_k x_p}$ is the scalar projection of the vector representing the alternative x_k onto the vector representing the alternative x_p, and $\left\| x_p \right\| = \sqrt{\left(x_p^1\right)^2 + \left(x_p^2\right)^2 + \ldots + \left(x_p^{n_k}\right)^2}$ is the norm of the respective vector with n_k components.

For the solution of the non-cooperative game, the application of an equilibrium solution concept to the payoff tables is necessary. Hipel et al. (2011) summarized possible equilibrium solution concepts for non-cooperative games, including: (i) general meta-rationality; (ii) symmetric meta-rationality; (iii) sequential stability; (iv) non-myopic stability; (v) limited-move stability; and (vi) Nash equilibrium. Among them, the most known and used equilibrium solution concept for solving non-cooperative games is the one proposed by Nash (1951). According to Osborne and Rubinstein (1994), an arrangement is a Nash equilibrium if, and only if, $\pi_k\left(s_1^*, s_2^*, \ldots, s_k^*, \ldots, s_{K-1}^*, s_K^*\right) \geq \pi_k\left(s_1^*, s_2^*, \ldots, s_k, \ldots, s_{K-1}^*, s_K^*\right)$, for all $k = 1, 2, \ldots, K$ agents, meaning that no agent's payoff can be increased if they individually choose to move from that arrangement. However, according to Binmore (2007), the Nash equilibrium could not be considered, *a priori*, a good outcome for a social choice, since it may not be unique and/or Pareto efficient. When more than one equilibrium is found in a game, it is necessary to eliminate undesirable equilibria (Maskin, 1999). This process is known as the equilibrium selection problem (Harsanyi & Selten, 1988), which is addressed in the next section.

2.3 The General Structure of the Equilibrium Selection Problem and the Social Welfare Functions

The general structure of an equilibrium selection problem involves a finite set of K agents, a finite set of alternatives A' (with at least three elements), which can be a subset of an original set of alternatives A after a previous filtering phase for eliminating unfeasible social solutions, and a social welfare function $\phi : \mathbb{R}^K \rightarrow \mathbb{R}$, which is used for ranking the set of alternatives A'. Thus, the best ranked equilibrium can be selected as a solution to the social choice. However, Maskin (1999) demonstrated that social welfare functions should be designed under the assumption of monotonicity and no-veto power. It should be noted, therefore, that there is no unique formal approach for equilibrium selection (Mailath, 1998). Consequently, the choice of such social welfare functions, rather than be determined by axiomatic guidance, should be focused on meeting the philosophical and anthropological aspects of the social dilemma (Kamaga, 2018).

A well-known social welfare function is the MaxMin social welfare function, which aims to maximize the minimal utility among the equilibria found. In other words, it aims to find the worst outcome among the equilibria found, and then to use this set of minimal outcomes as a reference for selecting the maximal outcome among them. The MaxMin social welfare function is related to the concept of egalitarian solutions according to Rawls's theory of justice (1971), which provides that the agent with the lowest utility in the group would be in the best possible scenario. It is considered a pessimistic and risk averse approach. Another famous social welfare function is the MaxMax social welfare function (also known as pure utilitarianism), which aims to sum the agents' outcomes per each equilibrium found, and then to use this set of aggregated values as a reference for selecting the equilibrium with the maximum summation among them. The MaxMax social welfare function is commonly related to the utilitarian principle of Bentham's theory of justice (1977), which states that social welfare is achieved when the sum of the group's utility is maximized. In Arrow et al. (2010), it is discussed very common social welfare functions, including: (i) LexiMin; (ii) weighted utilitarianism; (iii) Gini index; and (iv) Nash's bargaining solution. Other developed welfare functions to be applied to the equilibrium selection problem include those proposed by Matsui and Matsuyama (1995) and Kim (1996).

Finally, after the selection of an equilibrium, a consultive procedure can be performed with the agents in order for the group to accept or decline the solution proposed. Such procedure can be performed by the use of qualitative techniques, such as negotiation, or quantitative techniques, such as voting procedures.

3 The Proposed Framework

This section presents the framework to assist multi-agents in evaluating multiple criteria for selecting alternatives by modeling their strategic interaction as a non-cooperative game, in order to find alternatives that satisfy equilibrium solution concepts. In the presence of more than one equilibrium, the framework contemplates the adoption of a social welfare function for the refinement of the solution found.

Let $k = 1, 2, \ldots, K$ denote the K agents involved in a decision-making process and let $i = 1, 2, \ldots, m$ be the alternatives to be chosen. Consider that each agent k will use multiple criteria for their evaluation of m alternatives,[6] hence $j = 1, 2, \ldots, n_k$ are the criteria used by the *k-th* agent. The following steps summarizes the

[6] It is not a necessary condition that each agent has their own set of criteria for evaluation. The situation where the set of criteria is shared with some or all agents for their evaluation and choice is also possible.

implementation of the framework to support multi-criteria and multi-agent decision making processes.

Step 1 Each agent presents their own decision matrix containing the alternatives (the same as the other agents) and criteria (not necessarily the same as the other agents) that will be used within the process of choice.

Step 2 An elicitation procedure is applied to each agent in order to obtain their weighting vectors.

Step 3 Each decision matrix is standardized using a function for standardizing decision matrixes (each cost criterion is also converted to benefit criterion in this process), and weighted, by using the weighting vectors produced by the elicitation procedure, which will generate a standardized and weighted decision matrix for each agent.

Step 4 For each agent of the set of K agents the payoffs are calculated by means of a utility function applied to each of the m^K arrangements formed by the agent's initial alternative and the alternative proposed by each specific counterpart from the sub-set of $K-1$ agents. Finally, the non-cooperative game is composed by K column vectors (one for each of the agents $k = 1, 2, ..., K$) with m^K components, which are the agents' payoff for each of the possible arrangements . The results of this process are the payoff tables.[7]

Step 5 An equilibrium solution concept is applied to the payoff tables for finding the equilibria solutions to the game.

Step 6 If more than one equilibrium is found, a social welfare function is applied for selecting a social choice among the equilibria.

Step 7 Additionally, a negotiation phase or a voting procedure can be carried out for accepting or declining the equilibrium selected.

In order to the better identification of the different techniques that can give support within the framework, the available techniques described in the Sect. 2 are summarized in Table 1.

For illustrating the application of the framework, a numerical example is firstly presented. Subsequently, the choice of a WWTP to a municipality is demonstrated with the participation of five experts.

4 A numerical example

Let us consider a hypothetical group decision situation where three agents $\{P1, P2, P3\}$ are going to decide from a set of non-dominated alternatives $\{A1, A2, A3\}$ the one to be selected as the social choice. Each agent has their own

[7] It can be noted that from all steps of the framework, the more operationally demanding is step 4, in which it is requested the elicitation of the utility functions of all agents in order to calculate their payoffs tables. This is may not be an easy task, even abstracting from communication costs (Maskin, 1999). Here, from the operationalization of the multiple objectives through a pairwise comparison procedure, the framework translates the multi-criteria structure into a non-cooperative game for solving the game by means of an equilibrium solution concept, which is considered the rational solution to strategic interactions.

Table 1 Available techniques for supporting in each respective framework's steps

Step	Action	Aim	Available techniques
1	Structuring the problem	To structure the decision-making problem as a decision matrix for each agent	Soft Systems Methodology (SSM); Strategic Assumptions Surfacing and Testing (SAST); Strategic Options Development and Analysis (SODA); Strategic Choice Approach (SCA); Value-Focused Thinking (VFT)
2	Eliciting the agents' preferences	To obtain the preferences of the agents on the criteria under evaluation and to create a weighting vector from the elicitation process	Swing; Trade-off; Rank-Sum (RS); Rank-Reciprocal (RR); Rank-Ordered Centroid (ROC); Simple Multi-attribute Rating Technique by Swing (SMART); revised Simos' procedure; Analytic Hierarchy Process (AHP)
3	Generating the standardized and weighted decision matrixes	To standardize the decision matrix of each agent and to weight the matrixes by using the weighting vector generated in the previous step	Linear standardization; Vector standardization; Logarithmic standardization; Fuzzification
4	Generating the payoff tables	To apply a utility function to the standardized and weighted decision matrix for associating a payoff to each of the m^K possible arrangements of choices of each agent, which forms the structure of the non-cooperative game	Utility function that translate multi-criteria decision making problems into a non-cooperative game
5	Searching for equilibria	To choose an equilibrium solution concept and to search for such equilibrium in the generated payoff tables	General meta-rationality; Symmetric meta-rationality; Sequential stability; Non-myopic stability; Limited-move stability; Nash equilibrium
6	Selecting an equilibrium as the social choice	To select, among the equilibria found, the one which best fits the social choice for the group	MaxMin; MaxMax; LexiMin; Weighted utilitarianism; Gini index; Nash's bargaining solution; Entropy-norm space
7	Negotiating on the selected equilibrium	To gather group members to negotiate on the acceptance or the rejection of the selected equilibrium	Negotiation; Voting procedures

Table 2 Decision matrixes of the hypothetical situation

Alternatives	P1		P2			P3	
	C1	C2	C3	C4	C5	C2	C6
A1	4	7	30	49	98	7	9
A2	6	2	65	21	55	2	12
A3	8	1	43	72	12	1	18

Table 3 Standardized decision matrixes

Alternatives	P1		P2			P3	
	C1	C2	C3	C4	C5	C2	C6
A1	0.371	0.953	0.359	0.547	0.867	0.953	0.384
A2	0.557	0.272	0.778	0.234	0.487	0.272	0.512
A3	0.743	0.136	0.515	0.804	0.106	0.136	0.768

Table 4 Standardized and weighted decision matrixes

Alternatives	P1		P2			P3	
	C1	C2	C3	C4	C5	C2	C6
A1	0.111	0.667	0.216	0.109	0.173	0.191	0.307
A2	0.167	0.191	0.467	0.047	0.097	0.054	0.410
A3	0.223	0.095	0.309	0.161	0.021	0.027	0.615

set of criteria, being: $P1 : \{C1, C2\}$, $P2 : \{C3, C4, C5\}$, and $P3 : \{C2, C6\}$. For the purpose of making the calculations easier, all criteria were set in $\mathbb{N}^*$ and were of the benefit type (the more the better). It is noteworthy that one criterion, the criterion $C2$, is shared by two agents, $P1$ and $P3$. The next paragraphs illustrate the framework application.

Step 1 Each agent presents their own decision matrix containing the alternatives and criteria. The hypothetical decision matrixes can be seen in Table 2.

Step 2 It is assumed arbitrarily that as the result of an elicitation procedure, the following weighting vectors were obtained: $P1 : \{0.3, 0.7\}$, $P2 : \{0.6, 0.2, 0.2\}$, $P3 : \{0.8, 0.2\}$.

Step 3 Since all criteria are of the benefit type, the conversion of any cost criteria was not necessary. Therefore, all criteria were standardized using a standardization technique, e.g. the vector standardization technique, as can be seen in the equation

$$\left\{s_{ij}\right\}_k = \frac{\left\{x_{ij}\right\}_k}{\sqrt{\sum_{j=1}^{n_k} \left\{x_{ij}\right\}_k^2}} \tag{3}$$

where $\left\{x_{ij}\right\}_k$ is the performance of alternative i for the criterion j in the view of agent k, and $\left\{s_{ij}\right\}_k$ is the correspondent standardized value. The application of the equation generated the standardization decision matrixes presented in Table 3.

Table 5 Payoff tables

Arrangements			π_{P1}	π_{P2}	π_{P3}
A1	A1	A1	0.826	0.137	0.146
A2	A1	A1	0.020	0.088	0.132
A3	A1	A1	0.001	0.113	0.086
A1	A2	A1	0.101	0.104	0.126
A2	A2	A1	0.024	0.234	0.179
A3	A2	A1	0.004	0.138	0.116
A1	A3	A1	0.048	0.098	0.111
A2	A3	A1	0.010	0.120	0.171
A3	A3	A1	0.002	0.158	0.248
A1	A1	A2	0.101	0.104	0.126
A2	A1	A2	0.024	0.234	0.179
A3	A1	A2	0.004	0.138	0.116
A1	A2	A2	0.012	0.080	0.109
A2	A2	A2	0.029	0.626	0.243
A3	A2	A2	0.009	0.168	0.157
A1	A3	A2	0.006	0.075	0.096
A2	A3	A2	0.012	0.321	0.232
A3	A3	A2	0.006	0.192	0.336
A1	A1	A3	0.048	0.098	0.111
A2	A1	A3	0.010	0.120	0.171
A3	A1	A3	0.002	0.158	0.248
A1	A2	A3	0.006	0.075	0.096
A2	A2	A3	0.012	0.321	0.232
A3	A2	A3	0.006	0.192	0.336
A1	A3	A3	0.003	0.070	0.084
A2	A3	A3	0.005	0.165	0.222
A3	A3	A3	0.004	0.220	0.716

Subsequently, the standardized decision matrixes were weighted by using the weighting vectors obtained from the elicitation procedure, which generated the standardized and weighted decision matrixes that can be seen in Table 4.

Step 4 A utility function must be applied to each standardized and weighted decision matrix for generating the payoffs tables which contains the 27 (m^K) possible arrangements associated with the strategic choice of the three agents. It was used the utility function presented in Leoneti (2016) for generating such payoff tables for the agents, which can be seen in Table 5.

Step 5 The Nash equilibrium solution concept was applied to the generated payoff tables. For searching Nash equilibria, the add-in for Microsoft Excel® named Nash Equilibrium Finder was employed, (Sugiyama and Leoneti, 2021), which uses an exhaustive search algorithm for searching all pure Nash equilibria within a non-cooperative game. Table 6 presents the two Nash equilibria found.

Table 6 Nash equilibria found and their respective payoffs

	Nash equilibria			Payoffs		
	P1	P2	P3	P1	P2	P3
NE1	A1	A1	A1	0.826	0.137	0.146
NE2	A2	A2	A2	0.029	0.626	0.243

Step 6 It can be noticed that a set of non-singular solution was found for the game. In this case, the selection of one equilibrium to be adopted as the social choice should be performed by using a social welfare function. Here, the solution provided by both well-known social welfare functions, MinMax and the MaxMax, converged to the equilibrium NE1, which involves a consensus solution on the alternative A1. This solution is the one provided by the application of the framework.

Step 7 Given the nature of the numerical example, here this step is not addressed.

5 A Simulation: The Choice of a WWTP to a Municipality

Five Brazilians experts in the design of WWTP projects for municipalities, all of them civil engineers, were invited to participate in a simulation of the choice of a WWTP for a hypothetical Brazilian municipality with the following characteristics: (i) population of 40,000 inhabitants; (ii) effluent Biochemical Oxygen Demand (BOD) concentration of 305 mg/L; (iii) effluent nitrogen concentration of 30 mg/L; and (iv) effluent flow of 7×10^3 m^3/d. An on-line questionnaire was made available and during the period between December 2019 and January 2020 the experts were invited to participate by means of an email that contained an access link. The next paragraphs detail the steps of the simulation.

Step 1 The decision matrix of the simulated problem was created by using six sustainability indicators for WWTP recurrently found in the literature, namely: (i) implementation cost (IC); (ii) operation and maintenance cost (O&MC) (iii) residual BOD; (iv) residual Nitrogen (N) (v) the required area for implementation (RA); and (vi) sludge production (SP) (Leoneti et al. 2013). Six commonly used alternatives by Brazilian municipality populations of 40 thousand inhabitants were used, according to data from the Brazilian National Sanitation Research (IBGE 2008), namelly: (i) Upflow Anaerobic Sludge Blanket Digestion Reactor (UASB), followed by Aerobic Activated Sludge (WWTP A); (ii) UASB followed by Facultative Lagoon (WWTP B); (iii) UASB followed by High Load Biological Filter (WWTP C); (iv) UASB followed by Aerated Lagoon (WWTP D); (v) UASB followed by Facultative Aerated Lagoon (WWTP E); and (vi) Anaerobic Lagoon followed by Facultative Lagoon (WWTP F). The performances of the alternatives for each criterion were valued by the functions represented in the Appendix section, for which the inputs were the data from the hypothetical municipality. The decision matrix containing the WWTP considered to the hypothetical Brazilian municipality was presented to the experts by means of the on-line questionnaire and can be seen in Table 7.

Table 7 Decision Matrix with the WWTP considered for the hypothetical Brazilian municipality

	IC (R$ thou)	O&MC (R$ thou)	BOD (mg/L)	N (mg/L)	RA (m^2)	SP(L/inhab.year)
WWTP A	7,400	800	7.47	7.49	11,600	97,200
WWTP B	5,300	320	13.59	4.44	124,400	19,500
WWTP C	8,600	700	10.19	7.49	11,600	63,200
WWTP D	6,500	480	30.57	7.77	16,400	23,500
WWTP E	6,000	480	13.59	7.77	19,200	20,200
WWTP F	5,800	320	14.41	4.44	211,600	18,000

Table 8 Weight vectors of each expert

	IC (R$ thou)	O&MC (R$ thou)	BOD (mg/L)	N (mg/L)	RA (m^2)	SP(L/inhab.year)
Expert 1	0.158 (3rd)	0.242 (2nd)	0.061 (5th)	0.028 (6th)	0.408 (1st)	0.103 (4th)
Expert 2	0.408 (1st)	0.242 (2nd)	0.061 (5th)	0.028 (6th)	0.158 (3rd)	0.103 (4th)
Expert 3	0.242 (2nd)	0.158 (3rd)	0.408 (1st)	0.061 (5th)	0.028 (6th)	0.103 (4th)
Expert 4	0.158 (3rd)	0.408 (1st)	0.061 (5th)	0.028 (6th)	0.242 (2nd)	0.103 (4th)
Expert 5	0.158 (3rd)	0.408 (1st)	0.028 (6th)	0.061 (5th)	0.103 (4th)	0.242 (2nd)

Step 2 With basis on the decision matrix, the experts were asked to provide a ranking of its criteria and alternatives. The rankings to the criteria that each expert provided were subsequently used for translating them into weighting vectors by using the procedures of the ROC technique. According to Barron and Barrett (1996) and Ahn (2011) the ROC technique is more accurate than other rank-order techniques and is commonly used within multi-agent applications. The application of the ROC is performed by the means of the equation

$$w(j) = \frac{1}{n}\sum_{l=j}^{n}\frac{1}{l} \tag{4}$$

where n is the number of criteria, and $w(j)$ calculates the value for the weight of the j-th criterion. Table 8 shows the criteria rankings and weighing vectors of the experts.

Step 3 All cost criteria were converted into benefit criteria by using the formula $1/x_{ij}$, making them all of benefit type. The vector standardization function was then used for standardizing all the criteria. Subsequently, the decision matrixes of each expert were generated by weighting the standardized decision matrix using each of the weighting vectors obtained through the elicitation process. Table 9 shows the standardized and weighted decision matrix of each expert.

Step 4 The utility function presented in Leoneti (2016) was applied to each standardized and weighted decision matrix for generating the payoffs to be associated to each of the 216 (m^K) possible arrangements that can result from the strategic

Table 9 Standardized and weighted decision matrixes of each expert

	IC (R$ thou)	O&MC (R$ thou)	BOD (mg/L)	N (mg/L)	RA (m^2)	SP (L/inhab.year)
Expert 1						
WWTP A	0.056	0.054	0.039	0.009	0.241	0.010
WWTP B	0.078	0.134	0.021	0.015	0.022	0.052
WWTP C	0.048	0.061	0.029	0.009	0.241	0.016
WWTP D	0.063	0.089	0.010	0.009	0.170	0.043
WWTP E	0.068	0.089	0.021	0.009	0.145	0.050
WWTP F	0.071	0.134	0.020	0.015	0.013	0.056
Expert 2						
WWTP A	0.143	0.054	0.039	0.009	0.093	0.010
WWTP B	0.200	0.134	0.021	0.015	0.009	0.052
WWTP C	0.123	0.061	0.029	0.009	0.093	0.016
WWTP D	0.163	0.089	0.010	0.009	0.066	0.043
WWTP E	0.177	0.089	0.021	0.009	0.056	0.050
WWTP F	0.183	0.134	0.020	0.015	0.005	0.056
Expert 3						
WWTP A	0.085	0.035	0.260	0.020	0.016	0.010
WWTP B	0.118	0.088	0.143	0.033	0.002	0.052
WWTP C	0.073	0.040	0.190	0.020	0.016	0.016
WWTP D	0.096	0.058	0.063	0.019	0.012	0.043
WWTP E	0.105	0.058	0.143	0.019	0.010	0.050
WWTP F	0.108	0.088	0.135	0.033	0.001	0.056
Expert 4						
WWTP A	0.056	0.090	0.039	0.009	0.142	0.010
WWTP B	0.078	0.226	0.021	0.015	0.013	0.052
WWTP C	0.048	0.103	0.029	0.009	0.142	0.016
WWTP D	0.063	0.151	0.010	0.009	0.101	0.043
WWTP E	0.068	0.151	0.021	0.009	0.086	0.050
WWTP F	0.071	0.226	0.020	0.015	0.008	0.056
Expert 5						
WWTP A	0.056	0.090	0.018	0.020	0.061	0.024
WWTP B	0.078	0.226	0.010	0.033	0.006	0.122
WWTP C	0.048	0.103	0.013	0.020	0.061	0.038
WWTP D	0.063	0.151	0.004	0.019	0.043	0.101
WWTP E	0.068	0.151	0.010	0.019	0.037	0.118
WWTP F	0.071	0.226	0.009	0.033	0.003	0.132

interaction of the five experts. The use of the utility function generates the payoff tables for the experts.

Step 5 The Nash equilibrium solution concept was applied to the generated payoff tables. Table 10 presents the pure Nash equilibria found though the search

Table 10 Pure Nash equilibria found and their respective payoffs

	Nash equilibria					Payoffs				
	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5
NE1	B	B	B	B	B	0.001	0.393	0.110	0.148	0.662
NE2	C	C	C	C	C	0.316	0.070	0.133	0.055	0.016
NE3	D	D	D	D	D	0.034	0.187	0.005	0.091	0.150
NE4	E	E	E	E	E	0.107	0.285	0.083	0.133	0.193

Table 11 Acceptable and preferable alternatives suggested by the experts

	Acceptable WWTPs	Preferable WWTP
Expert 1	D, E, F	F
Expert 2	A, B, C, D, E, F	C
Expert 3	E, F	E
Expert 4	B, D, E	B
Expert 5	B, E, F	F

algorithm provided by NEFinder (Sugiyama and Leoneti, 2021) and the respective agent's payoff for each equilibrium found.

Step 6 Four Nash equilibria were found using pure strategies, all of them composed by consensus solutions. It should be noted that following the Bentham utilitarianism principle, the social outcome would be WWTP "B", since it led the agents to the highest sum of payoffs. However, in this equilibrium a significant unbalanced distribution of payoffs is noticed, which corresponds to the different preferences of the experts. In this sense, the MaxMin criterion was also considered, since whatever the chosen equilibrium is, there will eventually be an unsatisfied expert, which could render problems with alternative implementation given the expected rational behavior of rejecting proposals that either significantly favors or disfavors counterparts. Therefore, following Rawls's theory of justice through the MaxMin criterion, the WWTP "E" would be the one that maximizes the minimum of the payoffs among the equilibria found, being the recommended social outcome.

Step 7 The next section provides the experts' qualitative discussions regarding the result of the simulated case.

6 Discussions

The experts were requested to indicate which alternatives they would eventually accept and which they would suggest as the best alternative to be selected as the WWTP for the hypothetical Brazilian municipality. It is possible to see in Table 11 that only WWTP "E" would be accepted by all experts, which is aligned to the choice provided by the framework. In regards to this alternative, Expert 3 stated that "small municipalities do not have qualified technical staff for the proper operation of UASB reactors […] and the Facultative Aerated Lagoon does not require much operation and could mitigate operational failures that occur at UASB". One can notice that if a plurality voting approach was used, which is a common practice for solving group decision making problems within committees, WWTP "F" would be the winner alternative. However, it would not be an acceptable solution for Expert 4 who stated that "the required area for implementation is an important criterion here". Furthermore, it should be noted that there is no Nash equilibrium formed by this alternative, which could indicate lower chances of commitment to its implementation (Ziotti & Leoneti, 2020).

Table 12 Priorities of experts in the choice of a WWTP

	Social dimension	Environmental dimension	Economic dimension
Expert 1	1st	2nd	3rd
Expert 2	3rd	2nd	1st
Expert 3	2nd	1st	3rd
Expert 4	2nd	1st	3rd
Expert 5	3rd	2nd	1st

The experts also measured the importance of the different dimensions of sustainability covered by the selected criteria, namely: (i) social dimension, by the criterion RA; (ii) environmental dimension, by the criteria residual BOD, residual N, and SP; and (iii) economic dimension, by the criteria IC and O&MC. The ranking that the experts indicated with regards to the priorities of the sustainability dimensions in the process of choosing a WWTP can be seen in Table 12.

Considering the heterogeneous preferences of the agents involved in the selection process of the WWTP to the municipality, it can be assumed that each agent analyzes the alternatives emphasizing one of the three main sustainability dimensions of the problem. In this sense, it reinforces that "WWTP E" could be considered a satisficing solution to this social choice problem, since this alternative would simultaneously meet economic, environmental, and social criteria of sustainability, from the experts' point of view.

From the results it is plausible to assume that the framework aided the group decision process with transparency and high accuracy. However, the shortcoming of a non-singular solution should be emphasized, which is due to the use of equilibrium solution concepts from the theory of non-cooperative games as the pathway for solving the social dilemma. Nash equilibrium is a complex computational problem and would require the support of a software for being applied, especially when the number of arrangements increase. Furthermore, according to Binmore (2007), the Nash equilibrium may not be unique and/or Pareto efficient. In this sense, a post refinement process for excluding undesirable equilibria might be necessary. Therefore, on one hand, the framework has demonstrated to be effective, since it successfully found an satisficing outcome as the social choice, which originated from the solution of an equilibrium selection problem. It should be emphasized that it overcomes the limitation on the hybrid multi-criteria and game theory approaches found in the literature (Dyer & Forman, 1992; Basak & Saaty, 1993; Chen, 2000; Shih et al., 2007; Escobar & Moreno-Jimenéz, 2007; Arnette et al., 2010; Altuzarra et al., 2010; Sanayei et al., 2010; Dong et al., 2010; Hatami-Marbini & Tavana, 2011; Huang et al., 2013; Wu et al., 2018; Lai et al. 2002; Chen, 2000; Madani & Lund, 2011; Ke et al., 2012; Wibowo & Deng, 2013; Aplak & Sogut, 2013; Deng et al., 2014; and Jing et al., 2018). In contrast, it might be not efficient when the number of agents or alternatives increases significantly.

A possibility for circumventing this shortcoming was investigated by Ziotti and Leoneti (2020) who enlarged the analysis of the non-cooperative game in the light of the Nash Program. From the perspective of the Nash Program, the solution of a non-cooperative game would converge to the same solution of a cooperative game if the former is modeled as an enlarged game that would contain the pre-play interaction phase that occurs preliminarily of a cooperative game (Binmore, 1994). By assuming that all possible arrangements within a non-cooperative game that is modeled from a multi-criteria perspective by a utility function would contain all the possible outcomes of a pre-play interaction, the authors proposed to use the Nash bargaining social welfare function for solving the non-cooperative game. The Nash bargaining social welfare function provides a unique rational solution to the bargaining problem, which would be given by the maximization of the equation

$$N\left(\pi_1, \pi_2, \dots, \pi_K\right) = \left(\pi_1 - \xi_1\right)\left(\pi_2 - \xi_2\right) \dots \left(\pi_K - \xi_K\right) \tag{5}$$

where ξ_k are the *status quo* points that represent the outcome that each agent would receive if the negotiation would fail, in this case zero, and π_k are the respective outcomes calculated by the application of the utility function.

Here, the application of the Nash bargaining social welfare function also indicates that "WWTP E" would be the winning alternative, which is in accordance with the solution provided by the framework. In fact, Ziotti and Leoneti (2020) have found that the convergence of the Nash bargaining solution to the Nash equilibrium was high, having converged in about 87% of the simulated scenarios that were evaluated.

7 Conclusions

The Multi-Criteria and Multi-Agent Framework proposed in this paper allows to model the decision process involving multiple conflicting objectives within group decision making, aiming for the searching of satisficing solutions to be understood as a recommendation for action towards meeting different goals and values. The numerical example and the simulation presented show how the framework can support complex decision-making processes, providing the possibility to model and solve the problem where individuals interact their conflictive preferences strategically. By stablishing a link between MCDM and game theory, the solutions of the framework aim to provide the means to see the problem of selecting alternatives from different points of view and to assess different scenarios and agents' preferences, thus contributing to extend the possibilities to perform strategically in the group decision making. This approach could render more efficient decision-making processes for choosing a social outcome that matches the sustainability goals when involving heterogeneous agents with different preferences over the criteria under consideration (environmental, social and economic).

While in MCDM methods the alternatives are simply the labels that represent a set of performance values to each criterion in consideration, in game theory the focus is precisely on the alternatives, which can be assumed as an amalgamation of all diverse types of objectives within a scalar payoff value. The integration proposed here is based on the use of the multi-criteria approach through the pairwise comparison of criteria to generate the payoffs that will be used within the game theory approach. This feature allows the agent to assess eventual concessions to be made in the process of trading alternatives when seeking for a consensus solution in group decision-making, which is in accordance with the expected behavior of rational agents defined in this research (Blount and Bazerman, 1996; Camerer, 2003; Binmore, 1988). Furthermore, the proposed framework contemplates an equilibrium selection phase based on social welfare functions in which the analyst can chose the mathematical formulation that better address different philosophical and anthropological principles.

By means of using utility functions that are generated based on the preferences of agents within pairwise comparisons, it should be noted that the integration proposed here does not require the arbitration by the analyst for defining the payoffs to the agents, as occurs in vector payoff games. For instance, compared to other methods that propose the use of game theory for solving multi-criteria problems, the method proposed in Madani and Lund (2011) requires the creation of a transition matrix, the method presented in Wibowo and Deng (2013) requires intuitionistic assessments for each alternative in a form of an intuitionistic preference relation, and the method of Deng et al. (2014) requires the construction of the utility function by the arbitration given by the analyst based on the evaluation of linguistic terms that range from “very poor” to “very good”. Yet, compared to other MCDM methods adapted for a multi-agent environment, such as the methods proposed in Sanayei et al. (2010) and Dong et al. (2010), which have a good treatment of conflicts, but require the use of linguistic variables and thresholds limits, the framework presented here is cognitively less demanding. Consequently, an advantage of the proposed framework compared to other approaches previously reported is that it only requires the elicitation of the weighting vectors in order to calculate the payoffs, which makes the level of effort required very low while not losing accuracy. Therefore, the proposed framework allows to structure larger games than those with two agents and two alternatives, which are very common in most game theory applications.

Notwithstanding that the use of game theory in group decision-making has been previously reported, the existing literature does not provide a general framework to model group decision making problems that: (i) considers multiple criteria and agents without preferences aggregation; (ii) presents a solution for the interpersonal comparison of the utility among agents, since each payoff table is generated by the individual evaluation of trade-offs between alternatives; (iii) refines the non-singular solution by means of a social welfare function; and (iv) allows shortening the negotiation process. In contrast, the main limitation of the framework is, paradoxically, the use of an equilibrium solution concept as the pathway for finding the solutions for the game. A feature of an equilibrium solution is that it either may not exist, or be more than one, yielding a new problem that is the choice of the best equilibrium for the game solution. Finding equilibrium is a very demanding computational task, although for the case presented here the computing time was very low and other solutions concepts better tractable were presented. Future research could explore the search for other kinds of equilibrium solutions with a lower demand for computational time.

Appendix

The functions in Leoneti et al. (2013) chosen to be applied as criteria in this research were: (i) implementation cost—IC; (ii) operation and maintenance cost—O&MC; (iii) residual Biochemical Oxygen Demand—BOD; (iv) residual Nitrogen—N; (v) the required area for implementation—RA; and (vi) sludge production—SP, which can be seen in Table 13.

Table 13 Functions and values settings for each criteria

Name of the system	Range	Function	Source
Implementation Cost—IC			
Preliminary	30–50 R$/ inhab	40 R$/inhab	Von Sperling, (2006), p 340
High load biological filter	120–150 R$/ inhab	135 R$/inhab	Von Sperling, (2006), p 340
Activated sludge	90–120 R$/ inhab	105 R$/inhab	Von Sperling, (2006), p 340
UASB	30–50 R$/ inhab	40 R$/inhab	Von Sperling, (2006), p 340
Anaerobic lagoon	30–75 R$/ inhab	52.5 R$/inhab	Von Sperling, (2006), p 340
Facultative aerated lagoon	50–90 R$/ inhab	70 R$/inhab	Von Sperling, (2006), p 340
Aerated lagoon	65–100 R$/ inhab	82.5 R$/inhab	Jordão and Pessôa, (2009), p 852
Facultative lagoon	30–75 R$/ inhab	52.5 R$/inhab	Von Sperling, (2006), p 340
Operation and Maintenance Cost—O&MC			
Preliminary	1.5–2.5 R$/inhab	2 R$/inhab	Von Sperling, (2006), p 340
High load biological filter	10–15 R$/inhab	12.5 R$/inhab	Von Sperling, (2006), p 340
Activated sludge	10–20 R$/inhab	15 R$/inhab	Von Sperling, (2006), p 340
UASB	2.5–3.5 R$/inhab	3 R$/inhab	Von Sperling, (2006), p 340
Anaerobic lagoon	2–4 R$/inhab	3 R$/inhab	Von Sperling, (2006), p 340
Facultative aerated lagoon	5–9 R$/inhab	7 R$/inhab	Von Sperling, (2006), p 340
Aerated lagoon	5–9 R$/inhab	7 R$/inhab	Von Sperling, (2006), p 340
Facultative lagoon	2–4 R$/inhab	3 R$/inhab	Von Sperling, (2006), p 340
Residual Biochemical Oxygen Demand—BOD			
Preliminary	30–35% BOD	32.5% BOD	Von Sperling, (2006), p 339
High load biological filter	80–90% BOD	85% BOD	Von Sperling, (2006), p 339
Activated sludge	90–97% BOD	89% BOD	Von Sperling, (2006), p 339
UASB	60–75% BOD	67% BOD	Von Sperling, (2006), p 339
Anaerobic lagoon	50–85% BOD	65% BOD	Metcalf and Eddy, (1991), p 645
Facultative aerated lagoon	75–85% BOD	80% BOD	Von Sperling, (2006), p 339
Aerated lagoon	50–60% BOD	55% BOD	Jordão and Pessôa, (2009), p 797
Facultative lagoon	75–85% BOD	80% BOD	Von Sperling, (2006), p 339
Residual Nitrogen—N			
Preliminary	5–10% N	7.5% N	Metcalf and Eddy, (1991), p 692
High load biological filter	15–50% N	32.5% N	Metcalf and Eddy, (1991), p 170
Activated sludge	15–50% N	32.5% N	Metcalf and Eddy, (1991), p 170
UASB	60% N	60% N	Von Sperling, (2006), p 339
Anaerobic lagoon	60% N	60% N	Von Sperling, (2006), p 339
Facultative aerated lagoon	30% N	30% N	Von Sperling, (2006), p 339

Table 13 (continued)

Name of the system	Range	Function	Source
Aerated lagoon	30% N	30% N	Jordão and Pessôa, (2009), p 797
Facultative lagoon	60% N	60% N	Von Sperling, (2006), p 339
Required Area for Implementation—RA			
Preliminary	0.03–0.05 m^2/inhab	0.04 m^2/inhab	Von Sperling, (2006), p 340
High load biological filter	0.12–0.25 m^2/inhab	0.18 m^2/ inhab	Von Sperling, (2006), p 340
Activated sludge	0.12–0.25 m^2/inhab	0.18 m^2/ inhab	Von Sperling, (2006), p 340
UASB	0.03–0.1 m^2/inhab	0.07 m^2/ inhab	Von Sperling, (2006), p 340
Anaerobic lagoon	1.5–3 m^2/inhab	2.25 m^2/ inhab	Von Sperling, (2006), p 340
Facultative aerated lagoon	0.25–0.5 m^2/inhab	0.37 m^2/ inhab	Von Sperling, (2006), p 340
Aerated lagoon	0.2–0.4 m^2/inhab	0.3 m^2/ inhab	Von Sperling, (2006), p 340
Facultative lagoon	2–4 m^2/inhab	3 m^2/ inhab	Von Sperling, (2006), p 340
Sludge Production—SP			
Preliminary	110–360 L/hab	235 L/hab	Von Sperling, (2006), p 340
High load biological filter	500–1900 L/hab	1200 L/hab	Von Sperling, (2006), p 340
Activated sludge	1100–3000 L/hab	2050 L/hab	Von Sperling, (2006), p 340
UASB	70–220 L/hab	145 L/hab	Von Sperling, (2006), p 340
Anaerobic lagoon	55–160 L/hab	107,5 L/hab	Von Sperling, (2006), p 340
Facultative aerated lagoon	30–220 L/hab	125 L/hab	Von Sperling, (2006), p 340
Aerated lagoon	55–360 L/hab	207,5 L/hab	Von Sperling, (2006), p 340
Facultative lagoon	55–160 L/hab	107,5 L/hab	Von Sperling, (2006), p 340

Source Adapted from Leoneti et al. (2013)

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Declarations

Conflict of interests None.

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