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Consensus Building for Uncertain Large-Scale Group Decision-Making Based on the Clustering Algorithm and Robust Discrete Optimization

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Abstract
Consensus reaching processes (CRPs) including the feedback adjustment mechanism generally require extended periods of time to bridge the opinion gap among decision makers. Therefore, minimum cost consensus (MCC) problems with known adjustment costs have been widely reported. However, the exact unit adjustment costs are difficult to obtain through practical CRPs. To solve these problems, this paper proposes a novel CRP framework for uncertain large-scale group decision-making based on robust discrete optimization. First, an enhanced iterative self-organizing data analysis technique algorithm is provided to dynamically cluster decision makers together in small subgroups under interval opinions. Second, to establish the optimization-based consensus rules in the feedback process, an MCC integer optimization model is established to minimize the total consensus costs in consensus reaching. Furthermore, with the indeterminate unit adjusting costs, a robust discrete MCC optimization model is constructed, which can control the degree of conservatism of the optimal consensus opinion and compute the optimal modified opinions of decision makers. Finally, a case study and comparative analysis indicate the effectiveness and superiority of the proposed CRP method and that the robust discrete MCC model has stronger robustness in the uncertain decision environment.

Keywords Consensus reaching process · Large-scale group decision making · ISODATA · Robust optimization

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1 Introduction

The increasing complexity of decision problems makes it difficult for a single decision maker (DM) to comprehensively assess all of the relevant aspects of some practical problems. Therefore, the multiple-attribute group decision making problem has received increasing attention in recent years (Liu and Wang 2020; Liu et al. 2019; Wang et al. 2020; Zhang et al. 2018a), which utilizes the collective wisdom of multiple DMs to select the best alternative(s). Traditional decision problems involve a small number of DMs (e.g., three to five people). With the development of technology, such as social media and e-democracy, however, the number of DMs with different skills and backgrounds is increasing (Li et al. 2019; Lu et al. 2020; Xiao et al. 2020),and multiple-attribute large-scale group decision-making (LSGDM) problems are constantly emerging (Shi et al. 2018; Xu et al. 2018; Zha et al. 2019; Zhang et al. 2020b).

LSGDM problems are generally characterized by the following four features (Tang and Liao 2021; Wu et al. 2019a; Zhang et al. 2018b):(a) The number of DMs is more than;(b)Heterogeneous preference representation structures are used to express DMs'opinions;(c)Reaching a unanimous agreement among the DMs is difficult but necessary; and(d)The degree of uncertainty and complexity in the decision-making process is increased. It is a vital step to decrease the dimensionality of large-scale DMs to address the complexity of LSGDM problems (Wu et al. 2019a). Clearly, an effective method is to cluster the DMs into several smaller subgroups to reduce the number of DMs to a more tractable level. (Wu and Liu 2016) reduced the complexity of multiple-attribute LSGDM by an interval type-2 fuzzy equivalence clustering analysis. (Wu and Xu 2018) reduced the dimensions of DMs based on the k-means clustering method and used CPR to change the clustering results. (Zha et al. 2019) developed a large-scale consensus model with a bounded confidence-based feedback mechanism to classify DMs into different clusters based on a bounded confidence-based optimization approach.(Du et al. 2020) proposed a trust-similarity analysis-based clustering method to manage the clustering operation in LSGDM problems under a social network context. (Liu et al. 2021) introduced a probability k-means clustering algorithm to segment DMs with similar features into different subgroups. Clustering algorithms in machine learning are a kind of unsupervised learning and their main characteristic is that the number of clustering centres must be determined in advance. However, it is difficult to decide the number of clustering centres according to the limited information.

It is critical to obtain a collective solution that is a unanimous agreement of DMs in some practical group decision-making problems (Cheng et al. 2016).One of the major aims of group decision-making is consensus, which facilitates the subsequent implementation of the solution (Herrera-Viedma et al. 2014).Therefore, CPRs are widely used to help DMs reach a consensus on the solution. (Xu et al. 2018) proposed a two-stage CRP method and applied it to select earthquake shelters. (Shi et al. 2018) developed a uninorm-based comprehensive behavior classification CRP model for LSGDM. (Zha et al. 2019) proposed a CRP model with a bounded confidence-based feedback mechanism to promote the consensus level among DMs with bounded confidences.(Li et al. 2021) introduced a consensus model to manage the non-cooperative behaviors of DMs and implemented a dynamic weight punishment mechanism for non-cooperative DMs in LSGDM problems.

Generally, CRPs including consensus measurement and feedback adjustment processes, are dynamic and interactive, where DMs discuss and modify their opinions coordinated by a moderator. A significant part of CRPs is the design of an efficient feedback adjustment process (Zhang et al. 2019). Costs, such as money, time, and reconciliation efforts are also inevitable in the feedback process. Obviously, it is preferable that the costs should be as low as possible. Therefore, the implementation of optimization-based consensus rules has become a new rule of opinion-modification of DMs in the feedback process (Zhang et al. 2019).Optimization-based consensus rules were first proposed by Ben-Arieh (Ben-Arieh and Easton 2007).Subsequently, (Zhang et al. 2011) investigated the MCC problems to take the aggregation operators into account.(Liu et al. 2012) extended the MCC model into the fuzzy GDM. Additionally, (Gong et al. 2015) reported primal–dual models of MCC problems and investigated their economic significance. Furthermore, (Li et al. 2017b) proposed the MCC model with the uncertain interval costs. (Cheng et al. 2018) constructed MCC models with directional constraints based on goal programming theory. (Labella et al. 2020) proposed a comprehensive MCC model that considers the distance to global opinion and consensus degree whose optimal agreed solution was used to evaluate CRPs. (Zhang et al. 2021b) constructed a social trust driven minimum adjustments consensus model for social network group decision-making and proposed a consensus maximum optimization model.

Although previous studies of CRPs mentioned above are very effective tools to reach a consensus among DMs, there are still some challenges to further promote consensus achievement in the real-world LSGDM problems.

(1) Some clustering algorithms are widely used to cluster DMs into subgroups based on the information of DMs, such as preference or social relations. When applying the cluster methods, the number of cluster centres should be predetermined (Liu et al. 2021). And the application of clustering algorithms in LSGDM problems also need predetermine the number of cluster centres (Du et al. 2020; Zha et al. 2019). However, it is difficult to predetermine the number of clustering centres according to the limited information and the number of clustering centres cannot be adjusted during the clustering process. In the existing methods of clustering DMs, there is still no solution to the problem of dynamically adjusting the number of clustering centres. Therefore, it is necessary to propose some methods to solve this problem.

(2) The existing CRPs of LSGDM with interval opinion have not yet considered the optimization-based consensus rules in the feedback process. Using the identification rule (IR) and direction rule (DR) in the feedback process, the consensus costs can only be calculated passively after each round in iterations (Zhang et al. 2019). Resource consumption, such as time, money etc., is indispensable

in CRPs to modify DMs' opinions. Especially, since LSGDM involves more DMs, it is particularly important to take the consensus cost into CPRs. Therefore, establishing the MCC models of the optimization-based consensus rules for uncertain LSGDM is economically significant.

(3) For the existing MCC models, fixed DM costs are often considered (Cheng et al. 2018). However, DM costs are very likely to change due to the behaviors or psychologies of DMs. In real-world LSGDM problems, obtaining accurate unit costs of DMs is almost impossible. It is natural to assume that the unit adjusting costs of DMs are uncertain. Some studies have discussed the uncertain adjustment costs in the MCC model for the traditional group decision making problems (Li et al. 2017b), but few studies consider the uncertainty of unit adjustment costs in LSGDM problems. As the number of DMs increases, uncertain adjustment costs of DMs may have a greatly influence on the total consensus cost. To comprehensively explore the optimization-based consensus rules of the feedback process, it is also very useful to construct the MCC model under uncertainty unit adjustment costs of DMs in uncertain LSGDM.

Based on the above analysis of issues, the objective of this paper is to propose a novel CPR for uncertain LSGDM problems based on robust discrete optimization. Specifically, to solve the first problem mentioned above, this paper developed an enhanced ISODATA clustering method to divide DMs into small subgroups, which dynamically adjusts the clustering results. Then, considering the costs of CRPs, an MCC integer optimization model of LSGDM with interval opinions is constructed. Following this, a robust discrete MCC model is proposed to investigate the influences of the uncertain unit adjusting costs of DMs. The conservative degree of the optimal consensus opinion can be adjusted by the relevant parameter in this model. Finally, this model becomes the optimization-based consensus rule in the proposed CRP.

The rest of the paper is organized as follows. In Sect. 2, some basic concepts and LSGDM problems are introduced. Section 3 proposes a novel CRP framework for uncertain LSGDM. Section 4 develops an enhanced ISODATA. Section 5 establishes an MCC integer model of LSGDM with the determined unit adjustment costs of DMs and further proposes a robust discrete MCC model of uncertain LSGDM. Following this, in Sect. 6, a case study, a comparison analysis, and a simulation are presented to indicate the feasibility of the proposed method. Finally, the conclusion and expectation of future research are shown in Sect. 7.

2 Preliminaries

In this section, we first introduce interval linguistic opinions. Then, the uncertain LSGDM problems are described. Finally, the minimum adjustment consensus model is presented.

2.1 Interval Opinions

In the complex decision-making environment, DMs generally use the linguistic way to express their opinions, which can be described by either single determined linguistic terms or uncertain linguistic variables, which more precisely reflects their ideas and opinions (Dong et al. 2015; Wu et al. 2019b). Many studies introduced the basic symbols and operation rules of language variables (Herrera et al. 2008; Zadeh 1975).Let $S = \{s_\alpha | \alpha = 1, 2, ..., g\}$ be a finite linguistic term set of odd cardinality values and s_α refer to a possible value for a linguistic variable. For example, a linguistic scale can be defined as follows:

$$
\begin{aligned}
S = \{&s_1 = extremely\,poor, s_2 = very\,poor, s_3 = poor,\\
&s_4 = slightly\,poor, s_5 = fair, s_6 = slightly\,good,\\
&s_7 = good, s_8 = very\,good, s_9 = extremely\,good\}
\end{aligned}
$$

Linguistic variables are generally transformed to numerical scales. Several linguistic computational models have been developed to address linguistic variables in recent years (Dong et al. 2009; Massanet et al. 2014). The 2-tuple linguistic representation model is the most widely used method (Morente-Molinera et al. 2015). It defines a transformation function between linguistic 2-tuples and numerical scales. Since the 2-tuple linguistic computational model is a symbolic computation approach, the linguistic terms can be represented by integer values.

Instead of using only one crisp-determined linguistic term to express DM opinions, DMs are inclined to use interval linguistic variables to express their ideas more accurately (Herrera et al. 2008). An interval linguistic variable is denoted $\tilde{s} = \left[s_L, s_U\right]$, where $s_L, s_U \in S$ are the lower bound and upper bound of the interval, respectively. By transformation, interval linguistic variables are represented by interval numbers, which have been shown to be applicable (Wu et al. 2019b). For example, an interval linguistic variable $\left[s_6, s_7\right]$ refers to the rating value being "between slightly good and good". Transformation represents the interval number $[6, 7]$. The interval numbers are used to represent the opinions of DMs in this paper. Part of the operations for interval arithmetic can be defined as follows:

Definition 2 (see (Moore et al. 2009)):For two positive interval numbers $\tilde{a} = [a_L, a_U]$ and $\tilde{b} = [b_L, b_U]$, the addition and multiplication operations between $\tilde{a}$ and $\tilde{b}$ are defined as follows:

$$\tilde{a} + \tilde{b} = [a_L + b_L, a_U + b_U] \tag{1}$$

$$\tilde{a} \times \tilde{b} = [a_L \times b_L, a_U \times b_U] \tag{2}$$

Definition 3 (see (Xu 2004)): Let $\left\{\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n\right\}$ be a set of positive interval set numbers where $\tilde{a}_i = \left[a_{iL}, a_{iU}\right]$.The IWA operator is defined as.

$$IWAA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \left[\sum_{i=1}^{n} \lambda_i a_{iL}, \sum_{i=1}^{n} \lambda_i a_{iU} \right] \tag{3}$$

where $\lambda = [\lambda_1, \lambda_2, ..., \lambda_n]^T$ is the associated weight vector such that $\lambda_i \geq 0, \sum_{i=1}^{n} \lambda_i = 1$.

2.2 Problems Description

Due to the complexity of LSGDM problems and the habits of human thinking and reasoning, DMs prefer to express their preferred information in linguistic variables (Zhang et al. 2021a). An uncertain multi-attribute LSGDM problem can be defined as a situation where a large number of DMs use the interval linguistic opinions to evaluate multiple attributes of a set of alternatives to make decisions**.** The uncertainty refers to it that the DM's opinions are uncertain in this paper. Furthermore, the unit adjustment costs of DMs are also viewed as uncertain in CRPs. To describe the problems, the following notations are used.

For clarity, we denote $M = \{1, 2, ..., m\}$,$N = \{1, 2, ..., n\}$ and $H = \{1, 2, ..., h\}(h \geq 20)$.Let $A = \left\{a_1, a_2, ..., a_m\right\}$ be a set of attributes. Let $\omega = (\omega_1, \omega_2, ..., \omega_m)^T$ be the attribute weight vector, where $\omega_i \geq 0, \sum_{i \in M} \omega_i = 1$.$X = \left\{x_1, x_2, ..., x_n\right\}$ is a discrete finite set of potential alternatives, and $E = \left\{e_1, e_2, ..., e_h\right\}$ is a group of DMs. When the number of DMs is more than or equal to 20,it can be viewed as a large scale GDM problem (Dong et al. 2018). Each DM $e_i (i \in H)$ is assigned a weight $\pi_i (i \in H)$ based on their importance such that $\pi_i \geq 0, \sum_{i \in H} \pi_i = 1$.

In the real decision-making scenarios, the DM's opinions could be uncertain, and interval linguistic variables are used to represent rating values to tackle uncertainty. For an alternative $x_j \in X(j \in N)$ and attribute $a_j \in A(j \in M)$, DM $e_k \in E(k \in H)$ provides his/her opinions by a matrix $\tilde{D}^k = (\tilde{d}_{ij}^k)_{n \times m}, k \in H$,where $\tilde{d}_{ij}^k = \left[d_{ijL}^k, d_{ijU}^k\right]$ denotes the rating value on attribute a_j of alternative x_j.DMs modify their opinions after a moderator guides and supervises in CRPs. Let $\tilde{R}^k = (\tilde{r}_{ij}^k)_{n \times m}, k \in H$ be a revised matrix for DM e_k and $c = (c_1, c_2, ..., c_h)^T$ be the unit adjusting cost vector of DMs.

Remark Many papers have studied the weights of attributes. However, weighting attributes is beyond the range of this study. Thus, the maximum entropy principle is used and all attributes are assumed to have equal weights.

2.3 The Minimum Adjustment Consensus Model

In the feedback process of CRPs, resource consumption is indispensable. Obviously, it is preferable that the costs of the CRPs should be as low as possible. In the last decade, the feedback mechanism with minimum cost or adjustment model has been widely developed. Zhang et. al reviewed these models and proposed future challenges of them (Zhang et al. 2020a). Ben-Arieh et al. first proposed the MCC problem to obtain a group consensus opinion at the lowest possible cost (Ben-Arieh and Easton 2007).

The concepts of unit adjustment costs of DMs and consensus level introduced by Ben-Arieh and Easton. Furthermore, they constructed the MCC model with linear cost, which is not presented in the form of an optimization model. Let $o_i, i \in H$ denote the original opinion of DMs, $\overline{o}_i, i \in H$ be the revised opinions and let $\overline{O}$ be the consensus opinion. Then, Zhang et al. presented the optimization model of original MCC as follows (Zhang et al. 2011):

$$\begin{aligned} &\min \sum_{i=1}^{h} c_i\left|\overline{o}_i - o_i\right| \\ &s.t.\left|\overline{o}_i - \overline{o}\right| \le \varepsilon \end{aligned} \tag{4}$$

where ε is the maximum deviation between the original opinions and revised opinions.

In model (4), the distance is used to measure the consensus level. And several methods also can measure the consensus level. When the DM's unit adjustment cost vector is the unit vector, the MCC problems become the minimum adjustment consensus (MAC) problems (Zhang et al. 2020a).From a practical perspective, the revised opinions of DMs are restricted to the original scale in this model and the consensus level for each DMs regarding his/her adjusted opinions should exceed the consensus threshold. Wu et.al developed the MAC model under the following interval opinions (Wu et al. 2019b):

$$\begin{aligned} &\min \sum_{k=1}^{h} d(\tilde{D}^k - \tilde{R}^k) \\ &s.t.\begin{cases} \min\limits_{k} NCI(\tilde{R}^k) \ge \overline{GCI}, \forall k \\ r_{ijL}^k \le r_{ijU}^k, \forall i,j,k \\ \\ r_{ijL}^k, r_{ijU}^k \in domain_S, \forall i,j,k \end{cases} \end{aligned} \tag{5}$$

where $d(\cdot)$ is the distance between the original opinions and revised, NCI is the individual DM's consensus index and $\overline{GCI}$ is the consensus threshold.

3 Framework of CRPs of LSGDM with Uncertain Unit Adjustment Costs

In this section, we propose a novel CRP method for uncertain LSGDM problems. Specifically, a robust discrete MCC optimization model is used to compute the optimal modified opinions of DMs and the consensus opinions in the feedback adjustment process, and then the moderator guides DMs to revise their opinions by referring to the optimal solution of the robust discrete MCC optimization model. Therefore, an interactive CRP with the robust discrete MCC optimization model is developed.

Typically, the process of group decision making consists of selection process and consensus process. The purpose of the selection process is to aggregate DM's opinions over alternatives into a collective opinion, and the consensus process aims to improve the consensus level among DMs. The proposed consensus framework also includes the above two processes. With traditional group decision making problems, a few DMs usually participate in the decision. However, when involving a mass number of DMs, the decision process becomes more complex. Clustering large-scale DMs has become a crucial process for resolving the complexity of LSGDM problems (Wu et al. 2019a).

Therefore, the clustering process is indispensable for the proposed CRP framework.

For an uncertain LSGDM problem, large-scale DMs participate in the discussion. First, an enhanced ISODATA clustering algorithm is provided to cluster a large number of DMs into some small subgroups, and then assign weights according to the size of the subgroups. Furthermore, an aggregation operator is used to calculate the collective opinions of the different subgroups. Finally, to reach a consensus among DMs, the CRP is activated. The details of the framework are described in Fig. 1.

3.1 (a) The Clustering Process

In the cluster step, an enhanced ISODATA clustering algorithm is proposed in Sect. 4. In recent years, the k-means algorithm has often been used to cluster DMs into subgroups to reduce the dimensionality of large-scale DMs (Liu et al. 2021; Wu

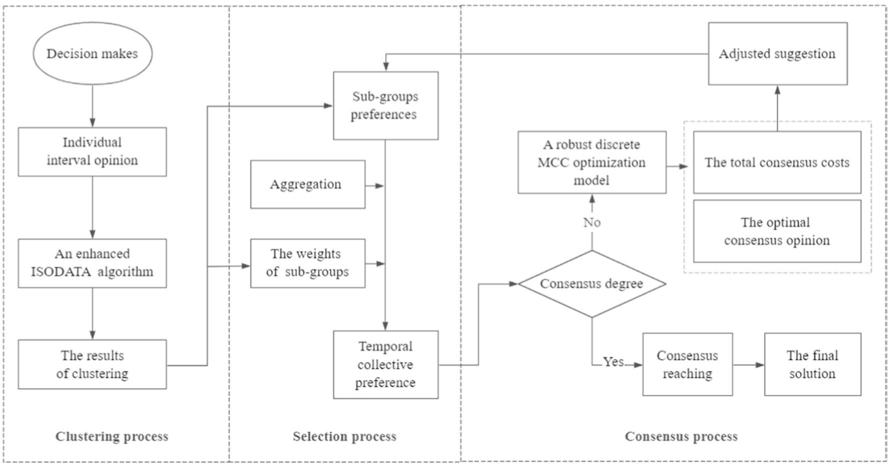

Fig. 1 The overall CRP framework of the uncertain LSGDM problems

and Xu 2018). Clustering algorithms are a kind of unsupervised machine learning, so it is necessary to set the clustering coefficient in advance. However, the number of clustering centres is hardly determined. Based on the k-means clustering algorithm, ISODATA can add two algorithms of merging and splitting to the clustering process to delete or add clustering centres, which can dynamically adjust the number of clusters.

We denote $Q = \{1, 2, ..., N_c\}$. Let $G = \left\{G_1, G_2, ..., G_{N_c}\right\}$ be a set of clusters. N_c is the number of clustering centres. A large-scale DM can be divided into $G_\tau(\tau \in Q)$ cluster. Naturally, the larger scale groups should be assigned larger weights. After the clustering process, the weights of clusters are obtained. We denote λ_τ as the weight of clusters. Without loss of generality, it is defined as (Zhang et al. 2018b):

$$\lambda_\tau = \left|n_\tau\right|^2 \Big/ \sum_{\tau \in Q} \left|n_\tau\right|^2 \tag{6}$$

where n_τ is the number of DMs in cluster G_τ.

The same weight is assigned to DMs in the same cluster in that their opinions are similar. Therefore, the weight π_k of DM e_k in the cluster G_τ is calculated as:

$$\pi_k = 1 \big/ \left|n_\tau\right| \tag{7}$$

3.2 (b) Selection Process

In the selection step, let us first obtain the k-th DM's interval opinion matrix $\tilde{D}^k = (\tilde{d}^k_{ij})_{n\times m}$.Then, the IWAA aggregation operator is used to calculate the collective matrix $\tilde{R}^\tau_c = (\tilde{r}^\tau_c)_{n\times m}$ and $\tilde{r}^\tau_{c,ij} = [r^\tau_{c,ijL}, r^\tau_{c,ijU}]$ of cluster G_τ, $\tau \in Q$.According to the weight π_k of DMs obtained by the clustering process, the collective opinion matrix of cluster G_τ is calculated as follows:

$$\tilde{r}^\tau_{c,ij} = \left[\sum_{\tau\in Q} \sum_{e^{(k)}\in G_\tau} \pi_k d^{\tau(k)}_{ijL}, \sum_{\tau\in Q} \sum_{e^{(k)}\in G_\tau} \pi_k d^{\tau(k)}_{ijU}\right] \tag{8}$$

Based on the weights $\lambda_l, l \in Q$ of the cluster and IWAA aggregation operator, the collective matrix $\tilde{R}_c = (\tilde{r}_{c,ij})_{n\times m}$, and $\tilde{r}_{c,ij} = [r_{c,ijL}, r_{c,ijU}]$ is defined as follows:

$$\tilde{r}_{c,ij} = \left[\sum_{\tau\in Q} \lambda_\tau r^\tau_{c,ijL}, \sum_{\tau\in Q} \lambda_\tau r^\tau_{c,ijU}\right] \tag{9}$$

3.3 (c) Consensus Process

Generally, the consensus process is to obtain a collective solution that meets the predetermined consensus level threshold. The CRP proposed in this study consists of two parts: a consensus measure and feedback adjustment.

The consensus measures are part and parcel (Wu et al. 2019b), when confirming the consensus level of a group of DMs. The distance functions are the basis of defining consensus measures. The Manhattan distance is generally used. Therefore, we define the consensus measures based on the Manhattan distance between uncertain interval variables. The Manhattan distance between interval numbers is given as follows:

Definition 4 Let $\tilde{a}_1 = [a_{1L}, a_{1U}]$ and $\tilde{a}_2 = [a_{2L}, a_{2U}]$ be interval numbers. The Manhattan distance between $\tilde{a}_1$ and $\tilde{a}_2$ is defined as follows:

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{2}\left(\left|a_{1L} - a_{2L}\right| + \left|a_{1U} - a_{2U}\right|\right) \tag{10}$$

The distance between two interval decision matrixes can be obtained according to the distance function (Xu et al. 2014).

Definition 5 (see (Xu et al. 2014)): Let $\tilde{F}_1 = (\tilde{f}_{1ij})_{n\times m}$ and $\tilde{F}_2 = (\tilde{f}_{2,ij})_{n\times m}$ be two interval matrixes, where $\tilde{f}_{1ij} = [f_{1ijL}, f_{1ijU}]$ and $\tilde{f}_{2ij} = [f_{2ijL}, f_{2ijU}]$.The Manhattan distance between them is defined as follows:

$$d(\tilde{F}_1, \tilde{F}_2) = \frac{1}{2nm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\left|f_{1ijL} - f_{2ijL}\right| + \left|f_{1ijU} - f_{2ijU}\right|\right) \tag{11}$$

For clarity, let $NCI(G_\tau)$ be the cluster consensus index, which measures the similarity between the collective opinion matrixes of clusters and the collective opinion matrix of the group. Note that the consensus index takes value in the interval [0, 1]. The value 1 indicates full consensus, and the higher the value, the higher the degree of consensus. The NCI for the cluster G_τ is defined as follow:

$$NCI(G^\tau) = 1 - \frac{1}{2nm}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\Psi}\left(\left|r^{\tau}_{c,ijL} - r_{c,ijL}\right| - \left|r^{\tau}_{c,ijU} - r_{c,ijU}\right|\right) \tag{12}$$

where $\Psi = g - 1$ and g is the number of linguistic term sets S.

The group consensus index GCI refers to the unanimous degree of the whole group defined as:

$$GCI = \min_{\tau} NCI(G_\tau) \tag{13}$$

By Eq. (12), we can obtain the degree of consensus of the subgroups. Using Eq. (13), the degree of consensus of group is obtained. When the group consensus index is below the consensus threshold, it is necessary to enter the feedback process to modify the DMs' opinions. The subgroup with the lowest degree of consensus is selected to modify opinions. The consensus opinion and the revised opinions of the DMs are calculated according to the proposed model under the uncertain costs. Then, the degree of consensus of the group is calculated again, and if it is lower than the consensus threshold, the opinions of DMs are further modified until the consensus threshold is reached.

4 Enhanced ISODATA Clustering Algorithm

Clustering has been widely applied in large-scale group decision-making problems, which divides large-scale DMs into different subgroups to reduce the complexity of the problems (Xu et al. 2019).The most commonly used clustering algorithm is the k-means algorithm ((Lu et al. 2020; Wu and Xu 2018),which minimizes the distance between points and cluster centres (Kanungo et al. 2002). It can classify data with unknown label. The k-means algorithm includes the following steps: (1) Randomly select the initial clustering centres; (2) Calculate the distance between the data and the clustering centre and divide the point into the closest cluster; (3) Recalculate the clustering centre of each cluster; and (4) Repeat steps 2 and 3 until the clustering centres remain the same.

Based on the k-means algorithm, the k-medoids algorithm, the k-means + + algorithm and ISODATA algorithm are developed to optimize the process of k-means clustering. The k-means + + algorithm optimizes the selection of the initial clustering centres by calculating the distance between clustering centres and the roulette method (Arthur and Vassilvitskii 2007) and the ISODATA algorithm is developed to dynamically adjust the number of cluster centres by merging and splitting algorithms (Ahmad and Sufahani 2013; Li et al. 2017a). However, the ISODATA algorithm randomly selects the clustering centres. Therefore, inspired by the k-means + + algorithm, an enhanced ISODATA clustering algorithm is proposed to cluster large-scale DMs into subgroups under the interval opinions in LSGDM. The flow chart of the enhanced ISODATA clustering algorithm is shown in Fig. 2.

For LSGDM problems, DMs $e_k, k \in H$ evaluate multiple attributes $a_j, j \in M$ of a set of alternatives $x_i, i \in N$ to make decisions by a matrix $\tilde{D}^k = (\tilde{d}_{ij}^k)_{n \times m}$, where $\tilde{d}_{ij}^k = \left[d_{ijL}^k, d_{ijU}^k\right]$. The goal of clustering part of the enhanced ISODATA algorithm is minimize the distance between the DMs' opinion matrix $\tilde{D}^k, k \in H$ and the clustering centres matrix $\tilde{V}^\tau = (v_{ij}^\tau)_{n \times m}, \tau \in Q$. Based on the Manhattan distance, the model is constructed:

$$\min \frac{1}{2nm} \sum_{\tau \in Q} \sum_{e_k \in C_l} \sum_{i=1}^{n} \sum_{j=1}^{m} \left| d_{ijL}^{\tau(k)} - v_{ijL}^{\tau(k)} \right| + \left| d_{ijU}^{\tau(k)} - v_{ijU}^{\tau(k)} \right| \tag{14}$$

The enhanced ISODATA involves the following main steps:

(1) Set parameters. The description of parameters in the enhanced ISODATA clustering algorithm is shown in Table 1. Note that if the number of DMs is less than

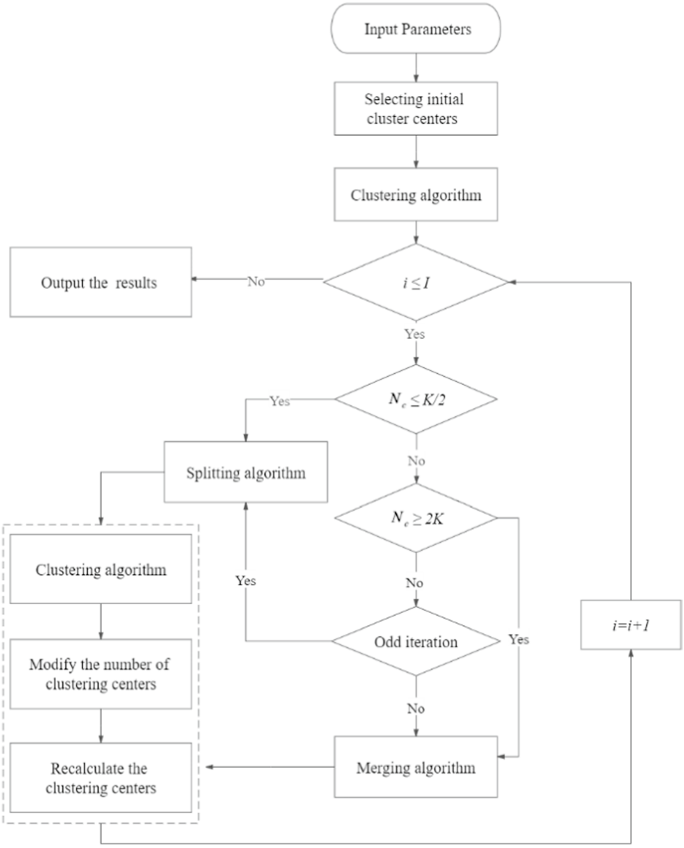

Fig. 2 Flow chart of the enhanced ISODATA clustering algorithm

Table 1 Nomenclature

Notations	Description
K	Expected number of clustering centres
N_I	The initial number of clustering centres
I	The maximum number of interiors
θ_s	The maximum standard deviation of a distance distribution in a cluster
θ_c	The minimum distance between two clustering centres
θ_n	The minimum number of DMs in a cluster
C_r	Clustering results
N_c	The number of clustering centres
$\tilde{V}$	Clustering centres matrix

θ_n, it will not be regarded as a cluster. Then, obtain the initial clustering centres by the selection algorithm.

(2) Calculate Manhattan distances between DMs' opinion matrixes and clustering centres matrixes as their similarity. Then, based on the index of similarity, clustering algorithm is used to divide DMs into subgroups.
(3) Utilize merging or aggregate algorithms to dynamically adjust the clusters according to the associated parameters and conditions.
(4) Repeat the iteration until the maximum number of iterations is reached

The above steps of the enhanced ISODATA have four key subroutines. First, the selection algorithm is used to generate the initial clustering centres. Second, the splitting algorithm is applied to add the new clusters to prevent too many DMs in a cluster. Third, the merging algorithm is adopted to reduce the number of clustering centres to protect the diversity of clusters. Finally, according to the k-means algorithm, the clustering results are obtained. The detailed design of the key subroutines of the enhanced ISODATA clustering algorithm is introduced as follows.

4.1 (a) Selection Algorithm

We design a novel initial point selection scheme to maintain the diversity and dispersion of the clustering centres, which effectively accelerates the convergence rate of clustering algorithm (Arthur and Vassilvitskii 2007). In general, when the distances between clustering centres are greater, the clustering centres are more dispersed and clustering algorithms will converge faster. If the clustering centres are too similar, the number of iterations to find the real clustering centre will increase. Therefore, initial clustering centre selection is a significant process.

Specifically, the selection of initial cluster centres is mainly according to the Manhattan distances between the clustering centre and the potential centre. The proposed selection scheme involves the roulette method and random number, which can ensure the diversity and dispersion of clustering centres. Assuming that the first cluster centre matrix is randomly selected and denoted $\tilde{V}^{\tau} = (v_{ij}^{\tau})_{n\times m}$, $\tau \in Q$, then the selected probability of the next clustering centre is based on the following formula:

$$p\left(e_k\right) = \frac{d(\tilde{V}^{(\tau)}, \tilde{D}^{(k)})}{\sum_{k=1}^{h} d(\tilde{V}^{(\tau)}, \tilde{D}^{(k)})}, e_k \in E \tag{15}$$

where d_M is the Manhattan distance calculated by Eq. (11).

This process is presented in Algorithm 1.

Algorithm 1. The selection of initial clustering centres

Input: The DMs' opinion matrix $\tilde{D}^k = (\tilde{d}_{ij}^k)_{n\times m}, k \in H, \theta_n$

Output: $N_c, C_r, \tilde{V}^\tau = (v_{ij}^\tau)_{n\times m}, \tau \in Q$

Step 1: Randomly generate the initial number of cluster centres N_I and randomly select the first clustering point among the DM's opinion matrix $\tilde{D}^k = (\tilde{d}_{ij}^k)_{n\times m}$ as $\tilde{V}^{(1)} = (\tilde{v}_{ij}^{(1)})_{n\times m}$.

Step 2: Using Eq. (13), calculate the distance of all the remaining opinion matrixes to the clustering centre matrix $\tilde{V}^{(1)}$ and compute the selected probability by Eq. (17).

Step 3: Randomly generate a random number $\mu \in (0,1)$. When the selected probability $p(e_k)$ exceeds the number μ for the first time, the corresponding opinion matrix is the next cluster centre, and this clustering centre cannot be the same as the already determined clustering centres. Then, return to the step 2 until the number of clustering centres equals N_I.

Step 4: According to the clustering centres N_I, the cluster algorithm is used to obtain the clustering results C_r of DMs.

Step 5: When the number of DMs in a cluster is less than θ_n, delete this cluster and modify the number of cluster centres as N_c. If $N_c = 0$,then return to step 1.

Step 6: Output the clustering results R_c, the number of clustering centres N_c and the clustering centres matrix $\tilde{V}^\tau = (v_{ij}^\tau)_{n\times m}, \tau \in Q$.

Step 7: End.

4.2 (b) Clustering Algorithm

The DMs are clustered into small subgroups based on the similarity of their opinion matrixes. It can be calculated by the Manhattan distance from Eq. (12). The general process for DM clustering in Algorithm 2.

Algorithm 2 Clustering Large-scale DMs

Input: The opinion matrixes $\tilde{D}^k(k \in H)$,and the clustering centres $\tilde{V}^\tau = (v_{ij}^\tau)_{n \times m}, \tau \in Q$

Output: The results of clustering C_r

Step 1: According to Eq. (14), calculate the Manhattan distance between the DMs $e_k, k \in H$ and the clustering centres $\tilde{V}^\tau = (v_{ij}^\tau)_{n \times m}, \tau \in Q$ as the similarity χ_τ^k.

Step 2: Denote $\{\delta_{(1)}, \delta_{(2)}, \ldots, \delta_{(N_c)}\}$ as the permutation of $\{1,2,\ldots,N_c\}$ .Order the similarity of the DM's opinions and the clustering centres from smallest to largest: $\{\chi_{\delta(1)}^k, \chi_{\delta(2)}^k, \ldots, \chi_{\delta(N_c)}^k\}$ where $\chi_{\delta(1)}^k \le \chi_{\delta(2)}^k \le \cdots \le \chi_{\delta(N_c)}^k$.

Step 3: The DMs belong into a cluster centre with the greatest similarity between them.

Step 4: Output the results of clustering C_r.

Step 5: End.

After clustering, the number of clusters needs to be checked. If the number does not exceed θ_n, this cluster will be deleted, the DMs of this cluster will be divided into another subgroup according to their similarity, and the number of clusters will be modified. Then, optimize the clustering centres as following:

$$\tilde{V}^{(\tau)} = \frac{1}{n_\tau} \sum_{e_k \in G_\tau} \tilde{D}^{\tau(k)} \tag{16}$$

where n_τ is the number of DMs in G_τ.

4.3 (c) Merging Algorithm

When the number of clustering centres is large, it is necessary to check the distance between clustering centres. Let D be the distance between clustering centres set.The pairwise inter-cluster distances between all distinct pairs of clustering centres are as follow:

$$D_{ij} = \left| \tilde{V}^{(i)} - \tilde{V}^{(j)} \right|, i = 1, 2, \ldots, N_c - 1, j = 1, 2, \ldots, N_c \tag{17}$$

According to these distances D_{ij} and θ_c, it is determined which cluster centres need to be merged. Combining the two clustering centres $\tilde{V}_m^{(i)}$ and $\tilde{V}_m^{(j)}$, the new clustering centre is calculated as follows:

$$\tilde{V}^{(*)} = \frac{1}{(n_i + n_j)} \times (n_i \tilde{V}_m^{(i)} + n_j \tilde{V}_m^{(j)}) \tag{18}$$

The merging process is shown in Algorithm 3.

Algorithm 3 Merging clustering centres

Input: $N_c, \theta_c, \tilde{V}^\tau = (v_{ij}^\tau)_{n\times m}, \tau \in Q$

Output: $N_c, \tilde{V}^\tau = (v_{ij}^\tau)_{n\times m}, \tau \in Q$

Step 1: Using Eq. (19) yields the Manhattan distances between the clustering centres $D_{ij}, i = 1,2,...,N_c - 1, j = 1,2,...,N_c$ to estimate the similarity of the clustering centres.

Step 2: Denote $\left\{\gamma_{(1)}, \gamma_{(2)}, ..., \gamma_{(N_c)}\right\}$ as the permutation of $\{1,2,...,N_c\}$ .Sort the distance set D in ascending order: $D = \left\{D_{\gamma(1),ij}, D_{\gamma(2),ij}, ..., D_{\gamma(N_c),ij}\right\}$ where $D_{\gamma(1),ij} \le D_{\gamma(2),ij}$ .If $D_{\gamma(1),ij}$ is less than θ_c, it indicates that the clustering centres need to merged. Then, go on to step 3; otherwise go on to step 4.

Step 3: Based on the Eq. (20), compute the new clustering centre and $N_c = N_c - 1$.Then, delete the minimized similarity from set D and return to step 2.

Step 4: Output the updated number of clustering centres N_c and clustering centres $\tilde{V}^{(\tau)}, \tau = 1,2,...N_c$.

Step 5: End.

4.4 (d) Splitting Algorithm

An excessive number of DMs in a cluster on account of fewer clustering centres. In this case, clustering centres need to be added. Use the standard deviation or the mean distance within and between clusters to decide whether the clustering centres need to be split. The standard deviation is computed by as follows:

$$\sigma_{ij}^\tau = \sqrt{\frac{1}{n_\tau} \sum_{\tau \in Q} \left\| \tilde{d}_{ij}^{\tau(k)} - \tilde{v}_{ij}^\tau \right\|_2}, i \in M, j \in N \tag{19}$$

The mean distance within clusters $\overline{D}^\tau, \tau \in Q$ is calculated as follows:

$$\overline{D}^\tau = \frac{1}{n_\tau} \sum_{e^k \in G_\tau} \left| \tilde{D}^{\tau(k)} - \tilde{V}^\tau \right| \tag{20}$$

Based on the mean distance within clusters, the overall the mean distance between clusters is computed as following:

$$\overline{D} = \frac{1}{N_c} \sum_{\tau \in Q} n_\tau \overline{D}_\tau \tag{21}$$

The splitting process is shown in Algorithm 4.

Algorithm 4 Splitting clustering centres

Input: $\theta_n, \theta_s, K, N_c, \tilde{V}^{(\tau)}, \tau = 1,2,...N_c$

Output: $N_c, \tilde{V}^{(\tau)}, \tau = 1,2,...N_c$

Step 1: Using the Eq. (21) to compute the standard deviation of each cluster.

Step 2: Calculate the mean distance for each cluster and the overall mean distance based on Eq. (22) and Eq. (23).

Step 3: If $\sigma_{\max}^{\tau} = \max\left\{\sigma_{11}^{\tau}, \sigma_{12}^{\tau}, ..., \sigma_{nm}^{\tau}\right\}$ is greater than θ_s and one of the following two conditions is satisfied, then, go to step 4; otherwise, go to step 5. The first split condition is that N_c is less than $K/2$. The second is that the mean distance of cluster $\bar{D}^{\tau}$ is greater than the overall mean distance $\bar{D}$ and the number of DMs of cluster N_c is greater than $2(\theta_n + 1)$ .The clustering centre $\tilde{V}^{\tau}$ that satisfies the condition is split.

Step 4: Set a random number $\vartheta \in (0,1)$. Replace $\sigma_{\max}^{\tau}$ with $\pm\vartheta * \sigma_{\max}^{\tau}$ to create the two clustering centres.

Step 5: Output the updated the number of the clustering centres N_c and clustering centres $\tilde{V}^{(\tau)}, \tau = 1,2,...N_c$.

Step 6: End.

5 The Robust Discrete Optimization MCC Model for LSGDM

In this section, the deterministic MCC model with uncertain LSGDM is first described. Then, based on the uncertain unit adjusting costs of DMs, the robust discrete optimization MCC model for uncertain LSGDM is proposed.

5.1 The Deterministic MCC Model for LSGDM

If the current *NCI* among subgroups is unacceptable,*GCI* is below the predefined consensus threshold. This indicates that DMs' opinions have not yet reached a predetermined level of consensus. To reach consensus, a CRP must be activated so that subgroup DMs have the opportunity to discuss and change their opinions guided and supervised by a moderator. Inevitably, CRPs involved in the feedback process take a great deal of time and effort; for example, they involve repeated surveys, conversations, and interviews, all of which can noticeably escalate costs.

In the feedback part of the CRP, it is necessary to guide DMs to modify opinions according to certain rules. The first kind of consensus rule is IR and DR, the second type is optimization-based consensus rule. Using IR and DR of the feedback process in the CRP, the consensus costs can only be calculated passively after each round in iterations. From an economic viewpoint, it is necessary to establish an optimization-based rule based on consensus cost for LSGDM problems with interval opinions.

To establish the model, suppose that the unit adjusting costs of DMs are certain. The objective is to minimize the total costs for modifying opinions of DMs and revised opinions are measured by the Manhattan distance. In addition, the modified opinions of DMs are restricted to the original rating scale and the linguistic scale has been transformed to a corresponding integer scale, which is denoted as $domain_S$.For a subgroup G_τ whose DMs are required to adjust opinions, we denote $H_\tau = \{1, 2, ...n_\tau\}$.Let $E_\tau = \{e_1, e_2, ..., e_{n_\tau}\}$ be DMs in cluster G_τ.Then, the deterministic MCC model of uncertain multi-attribute LSGDM for subgroup $G_\tau, \tau \in Q$ is constructed as follows:

$$\min \frac{1}{2nm} \sum_{k \in H_t} \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\left| d_{ijL}^{\tau(k)} - r_{ijL}^{\tau(k)} \right| + \left| d_{ijU}^{\tau(k)} - r_{ijU}^{\tau(k)} \right| \right) \cdot c^{\tau(k)}$$

$$s.t. \begin{cases} \frac{1}{2nm} \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{1}{\Psi} \left(\left| r_{c,ijL}^{\tau} - r_{c,ijL} \right| + \left| r_{c,ijU}^{\tau} - r_{ijU} \right| \right) \leq \theta \\ r_{c,ijL}^{\tau} = \sum_{k=1}^{n_\tau} \pi^{\tau(k)} \cdot r_{ijL}^{\tau(k)}, \forall i, j \\ r_{c,ijU}^{\tau} = \sum_{k=1}^{n_\tau} \pi^{\tau(k)} \cdot r_{ijU}^{\tau(k)}, \forall i, j \\ r_{ijL}^{\tau(k)} \leq r_{ijU}^{\tau(k)}, \forall k \in H_\tau, i, j \\ r_{ijL}^{\tau(k)}, r_{ijU}^{\tau(k)} \in domain_S, \forall k \in H_\tau, i, j \end{cases} \tag{22}$$

where $\tilde{D}^{\tau(k)}$ and $\tilde{R}^{\tau(k)}$ are the original opinions and revised opinions of DM e_k of the cluster G_τ, respectively.θ is the deviation of opinion before and after modification. The decision variables in model (22) are $r_{ijL}^{\tau(k)}$ and $r_{ijU}^{\tau(k)}$, for $i \in N, j \in M, e^k \in G_\tau$. Note that $r_{c,ijL}^{\tau}$ and $r_{c,ijU}^{\tau}$ are computed by $r_{ijL}^{\tau(k)}$ and $r_{ijU}^{\tau(k)}$, respectively.

Since model (22) is difficult to solve, it is further preferable to transform it to a mixed integer programming (MIP) problem.

Theorem 1 *Model (22) is equivalent to integer linear programming (23).*

$$\min \frac{1}{2nm}\sum_{k=1}^{N_\tau}\sum_{i=1}^{n}\sum_{j=1}^{m} z_{ij}^{\tau(k)} \cdot c^{\tau(k)}$$
$$s.t.\begin{cases} \frac{1}{2nm}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\Psi}(t_{ijL}^{\tau}+t_{ijU}^{\tau}) \le \theta \\ r_{c,ijL}^{\tau}-r_{c,ijL} \le t_{ijL}^{\tau}, \forall i,j \\ -r_{c,ijL}^{\tau}+r_{c,ijL} \le t_{ijL}^{\tau}, \forall i,j \\ r_{c,ijU}^{\tau}-r_{c,ijU} \le t_{ijU}^{\tau}, \forall i,j \\ -r_{c,ijU}^{\tau}+r_{c,ijU} \le t_{ijU}^{\tau}, \forall i,j \\ d_{ijL}^{\tau(k)}-r_{ijL}^{\tau(k)} \le y_{ijL}^{\tau(k)}, \forall k \in H_\tau, i,j \\ -d_{ijL}^{\tau(k)}+r_{ijL}^{\tau(k)} \le y_{ijL}^{\tau(k)}, \forall k \in H_\tau, i,j \\ d_{ijU}^{\tau(k)}-r_{ijU}^{\tau(k)} \le y_{ijU}^{\tau(k)}, \forall k \in H_\tau, i,j \\ -d_{ijU}^{\tau(k)}+r_{ijU}^{\tau(k)} \le y_{ijU}^{\tau(k)}, \forall k \in H_\tau i,j \\ y_{ijL}^{\tau(k)}+y_{ijU}^{\tau(k)} = z_{ij}^{\tau(k)}, \forall k \in H_\tau i,j \\ r_{c,ijL}^{\tau} = \sum_{k=1}^{n_h}\pi^{\tau(k)}\cdot r_{ijL}^{\tau(k)}, \forall i,j \\ r_{c,ijU}^{\tau} = \sum_{k=1}^{n_h}\pi^{\tau(k)}\cdot r_{ijU}^{\tau(k)}, \forall i,j \\ r_{ijL}^{\tau(k)} \le r_{ijU}^{\tau(k)}, \forall k \in H_\tau, i,j \\ r_{ijL}^{\tau(k)}, r_{ijU}^{\tau(k)} \in domain_S, \forall k \in H_\tau, i,j \end{cases} \tag{23}$$

Proof On the basis of the property $|x| = \max\{x, -x\}$,let $t_{ijL}^{\tau} = \left|r_{c,ijL}^{\tau} - r_{c,ijL}\right|, t_{ijU}^{\tau} = \left|r_{c,ijU}^{\tau} - r_{c,ijU}\right|$.Then, we have $r_{c,ijL}^{\tau} - r_{c,ijL} \le t_{ijL}^{\tau}, -r_{c,ijL}^{\tau} + r_{c,ijL} \le t_{ijL}^{\tau}$ and $r_{c,ijU}^{\tau} - r_{c,ijU} \le t_{ijU}^{\tau}, -r_{c,ijU}^{\tau} + r_{c,ijU} \le t_{ijU}^{\tau}$. Thus, the second to fifth constraints are added to model (23). Similarly, let $y_{ijL}^{\tau} = \left|d_{ijL}^{\tau(k)} - r_{ijL}^{\tau(k)}\right|, y_{ijU}^{\tau} = \left|d_{ijU}^{\tau(k)} - r_{ijU}^{\tau(k)}\right|.d_{ijL}^{\tau(k)} - r_{ijL}^{\tau(k)} \le y_{ijL}^{\tau(k)}, -d_{ijL}^{\tau(k)} + r_{ijL}^{\tau(k)} \le y_{ijL}^{\tau(k)}$,and $d_{ijU}^{\tau(k)} - r_{ijU}^{\tau(k)} \le y_{ijU}^{\tau(k)}$, $d_{ijU}^{\tau(k)} - r_{ijU}^{\tau(k)} \le y_{ijU}^{\tau(k)}$ are obtained. The sixth to ninth constraints are pulsed to model (23). To subsequently model, we add the equivalent constraint $y_{ijL}^{\tau(k)} + y_{ijU}^{\tau(k)} = z_{ij}^{\tau(k)}$.Therefore, the objective function for model (22) is transformed to the objective function for model (23). This completes the proof of Theorem 1.

5.2 The Robust Discrete MCC Model for LSGDM

The robust discrete optimization method is an effective and popular tool for dealing with data uncertainty in mathematical programming models. In the extant MCC models, the unit adjustment cost of each DM is assumed to be exactly known. In real-world LSGDM, however, it is difficult for the moderator to obtain the exact unit adjustment costs of DMs. To address the uncertain unit adjusting costs of the DM problem, we propose a robust discrete MCC model for uncertain LSGDM under indeterminate costs, which can adjust the degree of conservatism of the solution.

Assuming that adjusting costs of DMs $\tilde{c}_k \in [c_k - \hat{c}_k, c_k + \hat{c}_k], \forall k \in H_h$ is independent, symmetric, and by a bound random variable, but their distribution is unknown, where c_k is the nominal value of uncertain adjusting unit cost of DM e_k, $\hat{c}_k$ is the perturbation value of uncertain adjusting unit cost of DM e_k.For robustness of model (23) purposes, the parameter Γ is introduced that takes values in the interval $[0, |J|]$ and is not necessarily integers, where $J = \{k|\hat{c}_k \geq 0\}$. Up to $\lfloor\Gamma\rfloor$ of the coefficient is allowed to change. The parameter J is a set of cost coefficients affected by uncertainty. According to a symmetric distribution with a mean equal to the nominal value c_k in the interval $[c_k - \hat{c}_k, c_k + \hat{c}_k]$,$\tilde{c}_k, k \in J$ independently takes values. Therefore, the role of the parameter Γ is the protection level for the constraint. The robust discrete MCC model of uncertain LSGDM problems is defined as:

$$\min Z\ s.t. \begin{cases} \frac{1}{2nm}\sum_{k=1}^{n_\tau}\sum_{i=1}^{n}\sum_{j=1}^{m} z_{ij}^{\tau(k)} \cdot c^{\tau(k)} + \max\limits_{\substack{\{\bar{S}\cup\{t\}|\bar{S}\subseteq J,\\ |\bar{S}|\leq\lfloor\Gamma\rfloor, t\in J\setminus\bar{S}\}}} \{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k\in\bar{S}} \hat{c}^{\tau(k)}\left|z_{ij}^{\tau(k)}\right| + (\Gamma - \lfloor\Gamma\rfloor)\cdot \hat{c}^{\tau(t)}\left|z_{ij}^{\tau(t)}\right|\} \leq Z \\ \frac{1}{2nm}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\Psi}(t_{ijL}^{\tau} + t_{ijU}^{\tau}) \leq \theta \\ r_{c,ijL}^{\tau} - r_{c,ijL} \leq t_{ijL}^{\tau}, \forall i,j \\ -r_{c,ijL}^{\tau} + r_{c,ijL} \leq t_{ijL}^{\tau}, \forall i,j \\ r_{c,ijU}^{\tau} - r_{c,ijU} \leq t_{ijU}^{\tau}, \forall i,j \\ -r_{c,ijU}^{\tau} + r_{c,ijU} \leq t_{ijU}^{\tau}, \forall i,j \\ d_{ijL}^{\tau(k)} - r_{ijL}^{\tau(k)} \leq y_{ijL}^{\tau(k)}, \forall k \in N_\tau, i,j \\ -d_{ijL}^{\tau(k)} + r_{ijL}^{\tau(k)} \leq y_{ijL}^{\tau(k)}, \forall k \in N_\tau, i,j \\ d_{ijU}^{\tau(k)} - r_{ijU}^{\tau(k)} \leq y_{ijU}^{\tau(k)}, \forall k \in N_\tau, i,j \\ -d_{ijU}^{\tau(k)} + r_{ijU}^{\tau(k)} \leq y_{ijU}^{\tau(k)}, \forall k \in N_\tau, i,j \\ y_{ijL}^{\tau(k)} + y_{ijU}^{\tau(k)} = z_{ij}^{\tau(k)}, \forall k \in N_\tau, i,j \\ r_{c,ijL}^{\tau} = \sum_{k=1}^{n_\tau}\pi^{\tau(k)}\cdot r_{ijL}^{\tau(k)}, \forall i,j \\ r_{c,ijU}^{\tau} = \sum_{k=1}^{n_\tau}\pi^{\tau(k)}\cdot r_{ijU}^{\tau(k)}, \forall i,j \\ r_{ijL}^{\tau(k)} \leq r_{ijU}^{\tau(k)}, \forall k \in N_\tau, i,j \\ r_{ijL}^{\tau(k)}, r_{ijU}^{\tau(k)} \in domain_S, \forall k \in N_\tau, i,j \end{cases} \tag{24}$$

To solve model (24), it is necessary to transform it to an MIP formulation that is easily obtains the optimal solution.

Theorem 2 *Model* (24) *has an equivalent MIP model as follows*:

$$\min \frac{1}{2nm}(\sum_{k=1}^{n_\tau}\sum_{i=1}^{n}\sum_{j=1}^{m} z_{ij}^{\tau(k)} \cdot c^{\tau(k)}) + v \cdot \Gamma + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k\in J} p_{ij}^{\tau(k)}$$

$$s.t.\begin{cases} \frac{1}{2nm}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\Psi}(t_{ijL}^{\tau}+t_{ijU}^{\tau}) \le \theta \\ r_{c,ijL}^{\tau} - r_{c,ijL} \le t_{ijL}^{\tau}, \forall i,j \\ -r_{c,ijL}^{\tau} + r_{c,ijL} \le t_{ijL}^{\tau}, \forall i,j \\ r_{c,ijU}^{\tau} - r_{c,ijU} \le t_{ijU}^{\tau}, \forall i,j \\ -r_{c,ijU}^{\tau} + r_{c,ijU} \le t_{ijU}^{\tau}, \forall i,j \\ d_{ijL}^{\tau(k)} - r_{ijL}^{\tau(k)} \le y_{ijL}^{\tau(k)}, \forall k \in H_\tau, i,j \\ -d_{ijL}^{\tau(k)} + r_{ijL}^{\tau(k)} \le y_{ijL}^{\tau(k)}, \forall k \in H_\tau, i,j \\ d_{ijU}^{\tau(k)} - r_{ijU}^{\tau(k)} \le y_{ijU}^{\tau(k)}, \forall k \in H_\tau, i,j \\ -d_{ijU}^{\tau(k)} + r_{ijU}^{\tau(k)} \le y_{ijU}^{\tau(k)}, \forall k \in H_\tau, i,j \\ y_{ijL}^{\tau(k)} + y_{ijU}^{\tau(k)} = z_{ij}^{\tau(k)}, \forall k \in H_\tau, i,j \\ v + p_{ij}^{\tau(k)} \ge \hat{c}^{\tau(k)} \cdot u_{ij}^{\tau(k)}, \forall k \in J, i,j \\ p_{ij}^{\tau(k)} \ge 0, \forall k \in J, i,j \\ u_{ij}^{\tau(k)} \ge 0, \forall k \in J, i,j \\ -u_{ij}^{\tau(k)} \le z_{ij}^{\tau(k)} \le u_{ij}^{\tau(k)}, \forall k \in J, i,j \\ v \ge 0 \\ r_{c,ijL}^{\tau} = \sum_{k=1}^{n_\tau} \pi^{\tau(k)} \cdot r_{ijL}^{\tau(k)}, \forall i,j \\ r_{c,ijU}^{\tau} = \sum_{k=1}^{n_\tau} \pi^{\tau(k)} \cdot r_{ijU}^{\tau(k)}, \forall i,j \\ r_{ijL}^{\tau(k)} \le r_{ijU}^{\tau(k)}, \forall k \in H_\tau, i,j \\ r_{ijL}^{\tau(k)}, r_{ijU}^{\tau(k)} \in domain_S, \forall k \in H_\tau, i,j \end{cases} \tag{25}$$

Proof Let

$$\beta(z_{ij}^{\tau(k)*}, \Gamma) = \max_{\substack{\{\bar{S}\cup\{t\}|\bar{S}\subseteq J,\\ |\bar{S}|\le\lfloor\Gamma\rfloor, t\in J\setminus\bar{S}\}}} \{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k\in\bar{S}} \hat{c}^{\tau(k)}\left|z_{ij}^{\tau(k)*}\right| + (\Gamma - \lfloor\Gamma\rfloor)\cdot \hat{c}^{\tau(t)}\left|z_{ij}^{\tau(t)*}\right|\} \tag{26}$$

This equals to:

$$\beta(z_{ij}^{\tau(k)*},\Gamma)=\max\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k\in J}\hat{c}^{\tau(k)}\left|z_{ij}^{\tau(k)*}\right|\cdot v_k$$
$$s.t.\begin{cases}\sum\limits_{k\in J}v_k\leq\Gamma\\ 0\leq v_k\leq 1,\forall k\in J\end{cases}\tag{27}$$

Obviously, the optimal solution value of model (27) consists of $\lfloor\Gamma\rfloor$ variables at 1 and one variable at $\Gamma-\lfloor\Gamma\rfloor$. It is equivalent to the $\{\overline{S}\cup\{t\}|\overline{S}\subseteq j,\left|\overline{S}\right|\leq\lfloor\Gamma\rfloor,t\in J\backslash\overline{S}\}$ with corresponding function $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}\sum\limits_{k\in S}\hat{c}^{\tau(k)}\left|z_{ij}^{\tau(k)*}\right|+(\Gamma-\lfloor\Gamma\rfloor)\cdot\hat{c}^{\tau(t)}\left|z_{ij}^{\tau(t)*}\right|$.

Consider the dual of model (27),

$$\min\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k\in J}p_{ij}^{\tau(k)}+v\cdot\Gamma$$
$$s.t.\begin{cases}v+p_{ij}^{\tau(k)}\geq\hat{c}^{\tau(k)}\cdot\left|z_{ij}^{\tau(k)}\right|,\forall k\in J,i,j\\ p_{ij}^{\tau(k)}\geq 0,\forall k\in J,i,j\\ v\geq 0\end{cases}\tag{28}$$

Based on strong duality, since model (27) is feasible and bounded for $\Gamma\in[0,|J|]$, the dual model (28) is also feasible and bounded, and their objective values are the same.

Therefore, $\beta(z_{ij}^{\tau(k)*},\Gamma)$ is equal to the objective function value of model (28).

6 Numerical and Simulations Analysis

In this section, a practical case is presented to demonstrate the effectiveness of the proposed consensus model. Then, a comparison analysis with other existing methods is performed to indicate the superiority of the proposed CRP method. Furthermore, we design simulation experiments to discuss the performance of an enhanced ISODATA, the influences of the uncertain unit adjusting cost of each DM and the parameter of the protection level for indeterminacy in the robust discrete MCC model.

6.1 Case study

Sudden cardiac death (SCD) is a sudden death caused by various cardiac conditions. The patient suddenly develops cardiac arrest and other manifestations. During out-of-hospital cardiac arrest (OHCA), survival declines by 7–10% for every minute without bystander intervention (Larsen et al. 1993). Early cardiopulmonary resuscitation (CPRS) and automated external defibrillators (AEDs) can greatly improve the chances of survival of a person with OHCA. If a patient with

OHCA is defibrillated within 3 min, survival can be as high as 70%.Public access defibrillation (PAD) programs refer to collocating AEDs and training CPRS in public places. People with OHCA are defibrillation and treated with AED by the first eyewitness before emergency personnel arrive in public places to improve survival. The number of AEDs in China is small, and the gap with developed countries is large. According to the current literature, there are 700 AEDs (Ringh et al. 2018) in the United States and 276 AEDs (Tsukigase et al. 2017) in Japan per 100,000 people, while in China, there are only 17.5 AEDs per 100,000 people, in Shenzhen, 13 in Haikou, 11 in Pudong New Area of Shanghai and 5 in Hangzhou.

In recent years, China has been gradually promoting China-PAD (C-PAD) programs. With the guidance of the government, the Shanghai Red Cross installed 800 AEDs in Pudong New District by the end of 2018. Beijing, the capital of China has approximately 600 AEDs and plans to install more AEDs on public transport.

Based on the background of the above Beijing AED plan, we formulated four alternatives and five attributes and simulated the LSGDM process using the proposed method to provide a reference for future AEDs layout plans. Given the actual situation, the Beijing Municipal Government convened 30 experts from different fields, including medical, engineering and public transportation to form a large group of DMs, denoted as $E = \left\{e_1, e_2, \ldots, e_{30}\right\}$.The DMs were required to rate four alternatives that combinations of subway lines and select the optimal one to first install AEDs using the linguistic term set S,$S = \{s_1 = extremely\,poor, s_2 = very\,poor, s_3 = poor, s_4 = slightly.$

$poor, s_5 == fair, s_6 = slightly\,good, s_7 = good, s_8 = very\,good, s_9 = extremely\,good\}$.Each of the alternatives is evaluated for attributes $a_j, j = 1, 2, 3, 4, 5$, where a_1 is that which promotes the effect of AEDs after placing in these lines, a_2 is the subsequent costs of maintenance of the AEDs, a_3 is the difficulty in technical maintenance, a_4 is the social impact and a_5 is the conditions to support the facilities. After a pre-evaluation of the AEDs layout plan, the following four alternatives $X = \{x_1, x_2, x_3, x_4\}$ were provided:

x_1: The first alternative is to collocate AEDs at subway lines around approximately 30 universities, specifically, Metro Line 4, Line 5, Line 10, and Line 13. Line 10 also runs through Beijing's Central Business District (CBD).

x_2: According to the 2019 Beijing Traffic Development Annual Report released by the Beijing Transport Development Research Institute, the comfort index of Metro Line 9, Ba Tong Line, Line 8, and Line 1 is higher, which reflects the degree of crowdedness in the metro area; the higher the value of the index, the more crowded it is. The second alternative is to prioritize to collocating AEDs on these lines.

x_3: The third alternative is to focus on Metro Lines 2, Line 13, Line 14, and Beijing Daxing International Airport Express which are surrounded by long distance transport stations such as railway stations and airports where passenger flow will suddenly increase at certain times. Thus, the third alternative is prioritizing the allocation of AEDs in these metro lines.

x_4: In the core of Beijing's CBD, approximately four square kilometres, there are eight subway stations and four Metro lines, including Metro Line 1, Line 6, Line 10, and Line 14. Meanwhile, Metro Line 1, with the largest number of shopping malls is surrounded by 15 shopping malls. This alternative mainly considers the installation of AEDs on the metro lines near the business district.

The DM's opinions are allowed to be interval uncertain linguistic variables in that the real-world decision-making environment is indeterminate. Table 2 shows the 30 DMs opinion matrixes $\tilde{D}^{(k)} = (\tilde{d}_{ij}^{k})_{n\times m}, k \in E$.For convenience, all interval uncertain linguistic variables have been transformed to integers. In the following, the proposed CRP is adopted to help these large-scale DMs reach consensus.

The first step is the clustering process. In this case, we set the parameter values of the proposed enhanced ISODATA to cluster DMs as: $\theta_n = 3$, $\theta_s = 1.2$, $\theta_c = 0.12$,$K = 4$ and $I = 18$. The enhanced ISODATA converges into six categories, so DMs can be divided into 6 subgroups. According to the Eq. (8) and Eq. (9), the weight of clusters and each DM can be calculated. The clustering and weight results are presented in Table 3.

The second step is the selecting process. Using the aggregation operator IWAA and weight information, the group collective interval linguistic matrix $\tilde{R}_c = (\tilde{r}_{ij})_{n\times m}$ can be calculated by Eqs. (8) and (9).We get the following matrix:

$$\tilde{R}_c = \begin{bmatrix} [3.68, 5.06] & [3.32, 4.58] & [3.53, 5.09] & [3.82, 5.12] & [3.90, 5.19] \\ [3.34, 4.80] & [3.32, 4.60] & [3.92, 5.30] & [3.81, 5.17] & [3.63, 5.09] \\ [2.79, 4.13] & [4.02, 5.40] & [4.29, 5.65] & [4.02, 5.42] & [3.59, 4.90] \\ [4.09, 5.41] & [4.23, 5.47] & [3.45, 4.75] & [4.00, 5.26] & [3.80, 5.22] \end{bmatrix}$$

Finally, the last step is the consensus process. We set the parameter values of consensus threshold $\overline{GCI}$ to 0.88. The consensus index $NCI(G_\tau)$ of the subgroup can be calculated using Eq. (12), and they are $NCI(G_1) = 0.844, NCI(G_2) = 0.865$, $NCI(G_3) = 0.888, NCI(G_4) = 0.854, NCI(G_5) = 0.875, NCI(G_6) = 0.856$. Utilizing Eq. (13), we obtain the group consensus index $GCI = 0.844$. Since $GCI \le \overline{GCI}$,the feedback process is activated. Obviously, $NCI(G_1)$ is the lowest among them so subgroup G_1 is chosen to adjust opinions in the first consensus iteration.

The unit adjusting cost of each DM is uncertain. In the first consensus iteration, the nominal values of adjusting the unit cost of DMs are assumed to be $c_4 = 3, c_{11} = 2, c_{14} = 4, c_{17} = 5$ and the perturbation value of the uncertain adjusting unit cost of DMs in G_1 are assumed to be $\hat{c}_4 = 0.5, \hat{c}_{11} = 0.3, \hat{c}_{14} = -0.1, \hat{c}_{17} = -0.2$. In addition, the parameters of model (25) are $\Gamma = 1.5$ and $\theta = 0.11$.Based on model (25), the LINGO solver is used to compute the optimal adjusting opinions of subgroup G_1 and the total cost of consensus. Then, the new consensus indexes are computed for the subgroups after the first consensus iteration is finished, and we obtain $NCI(G_1) = 0.888, NCI(G_2) = 0.865, NCI(G_3) = 0.888, NCI(G_4) = 0.853, NCI(G_2) = 0.878, NCI(G_3) = 0.856$.Using Eq. (13), we obtain the new group consensus index $GCI = 0.853 \le 0.88$. Therefore, the consensus feedback process continues.

Finally, the iteration of the consensus process ended after four times. Each iteration is for subgroup $G_\tau, \tau \in Q$, and the unit adjusting costs of model (25) require different values. Specific parameters of the remaining iterations are described in Table 4.

Table 2 The opinion matrixes of the 30 DMs

$$\tilde{R}^{(1)}=\begin{bmatrix}[6,9]&[4,5]&[2,4]&[6,7]&[2,4]\\ [5,6]&[1,3]&[4,5]&[4,6]&[5,8]\\ [3,5]&[6,7]&[2,4]&[7,9]&[5,6]\\ [2,4]&[5,6]&[1,3]&[6,7]&[1,3]\end{bmatrix}\quad \tilde{R}^{(2)}=\begin{bmatrix}[3,5]&[3,4]&[6,7]&[2,4]&[6,7]\\ [4,5]&[3,4]&[2,3]&[1,3]&[6,8]\\ [2,3]&[3,4]&[2,3]&[3,5]&[6,8]\\ [4,6]&[5,7]&[3,4]&[3,4]&[3,5]\end{bmatrix}\quad \tilde{R}^{(3)}=\begin{bmatrix}[5,6]&[4,6]&[1,3]&[2,3]&[5,6]\\ [2,3]&[2,4]&[5,6]&[4,5]&[4,5]\\ [1,3]&[4,6]&[3,4]&[5,6]&[1,2]\\ [6,7]&[3,5]&[5,6]&[1,2]&[3,5]\end{bmatrix}$$

$$\tilde{R}^{(4)}=\begin{bmatrix}[2,3]&[1,3]&[1,3]&[5,6]&[2,3]\\ [3,5]&[5,6]&[7,8]&[3,4]&[5,8]\\ [3,4]&[7,8]&[6,9]&[3,5]&[3,4]\\ [2,3]&[5,6]&[7,8]&[5,6]&[2,3]\end{bmatrix}\quad \tilde{R}^{(5)}=\begin{bmatrix}[3,4]&[4,5]&[4,5]&[1,3]&[5,6]\\ [3,5]&[7,8]&[4,5]&[2,3]&[3,5]\\ [1,3]&[6,8]&[3,5]&[1,3]&[3,4]\\ [6,9]&[6,9]&[6,8]&[3,4]&[3,5]\end{bmatrix}\quad \tilde{R}^{(6)}=\begin{bmatrix}[4,5]&[6,7]&[4,6]&[5,6]&[4,6]\\ [2,4]&[2,4]&[2,4]&[5,6]&[3,5]\\ [2,4]&[4,6]&[7,8]&[4,6]&[6,7]\\ [5,6]&[2,3]&[1,3]&[8,9]&[7,8]\end{bmatrix}$$

$$\tilde{R}^{(7)}=\begin{bmatrix}[1,2]&[4,5]&[3,6]&[1,2]&[3,4]\\ [6,7]&[3,4]&[6,7]&[2,4]&[2,3]\\ [4,5]&[4,6]&[7,8]&[4,5]&[1,2]\\ [4,6]&[5,6]&[3,4]&[5,6]&[4,5]\end{bmatrix}\quad \tilde{R}^{(8)}=\begin{bmatrix}[5,6]&[4,5]&[4,6]&[6,7]&[3,4]\\ [2,3]&[4,6]&[2,3]&[6,7]&[3,5]\\ [1,3]&[2,3]&[2,4]&[2,3]&[7,8]\\ [6,7]&[4,5]&[1,3]&[5,6]&[5,6]\end{bmatrix}\quad \tilde{R}^{(9)}=\begin{bmatrix}[2,4]&[5,6]&[4,6]&[1,3]&[2,3]\\ [8,9]&[3,4]&[5,7]&[1,2]&[1,3]\\ [5,6]&[1,3]&[5,7]&[5,6]&[1,3]\\ [2,3]&[6,8]&[2,3]&[1,2]&[7,8]\end{bmatrix}$$

$$\tilde{R}^{(10)}=\begin{bmatrix}[6,7]&[2,4]&[3,5]&[4,6]&[1,2]\\ [1,3]&[4,5]&[4,5]&[7,8]&[5,6]\\ [2,3]&[3,5]&[2,3]&[6,7]&[8,9]\\ [2,3]&[6,7]&[3,4]&[7,8]&[2,3]\end{bmatrix}\quad \tilde{R}^{(11)}=\begin{bmatrix}[1,3]&[2,3]&[2,4]&[5,6]&[1,3]\\ [4,5]&[5,6]&[6,7]&[3,5]&[6,7]\\ [2,3]&[4,6]&[5,6]&[4,6]&[2,3]\\ [6,8]&[3,4]&[6,7]&[6,7]&[4,7]\end{bmatrix}\quad \tilde{R}^{(12)}=\begin{bmatrix}[3,4]&[3,4]&[4,5]&[3,4]&[5,6]\\ [4,5]&[3,4]&[2,4]&[2,4]&[5,6]\\ [2,4]&[3,5]&[1,3]&[6,7]&[1,3]\\ [5,6]&[7,8]&[4,6]&[3,5]&[3,5]\end{bmatrix}$$

$$\tilde{R}^{(13)}=\begin{bmatrix}[4,5]&[3,5]&[2,3]&[1,3]&[6,7]\\ [6,7]&[4,5]&[5,6]&[4,5]&[5,6]\\ [2,3]&[5,6]&[2,3]&[2,3]&[4,6]\\ [1,3]&[5,6]&[4,5]&[7,8]&[2,4]\end{bmatrix}\quad \tilde{R}^{(14)}=\begin{bmatrix}[2,3]&[2,4]&[4,6]&[5,7]&[2,4]\\ [4,6]&[6,7]&[5,6]&[2,4]&[6,7]\\ [3,5]&[8,9]&[7,8]&[3,4]&[5,6]\\ [2,3]&[4,5]&[2,3]&[5,6]&[3,5]\end{bmatrix}\quad \tilde{R}^{(15)}=\begin{bmatrix}[6,7]&[3,4]&[5,6]&[6,7]&[4,5]\\ [1,3]&[2,4]&[3,6]&[6,7]&[3,4]\\ [2,3]&[5,6]&[1,2]&[1,3]&[6,7]\\ [4,6]&[6,7]&[2,3]&[3,4]&[1,3]\end{bmatrix}$$

$$\tilde{R}^{(16)}=\begin{bmatrix}[5,8]&[3,5]&[6,7]&[5,6]&[3,5]\\ [2,4]&[2,3]&[6,7]&[7,8]&[5,6]\\ [2,4]&[7,8]&[5,6]&[6,7]&[5,7]\\ [7,8]&[5,6]&[3,4]&[4,6]&[4,5]\end{bmatrix}\quad \tilde{R}^{(17)}=\begin{bmatrix}[2,3]&[1,4]&[5,7]&[6,7]&[1,3]\\ [3,4]&[7,8]&[7,8]&[4,5]&[5,6]\\ [4,5]&[6,9]&[7,8]&[5,6]&[5,6]\\ [2,3]&[2,4]&[6,7]&[7,8]&[7,8]\end{bmatrix}\quad \tilde{R}^{(18)}=\begin{bmatrix}[1,3]&[4,5]&[5,6]&[1,3]&[1,3]\\ [7,8]&[6,7]&[4,5]&[3,4]&[2,4]\\ [4,5]&[3,5]&[6,9]&[4,6]&[2,3]\\ [1,3]&[6,7]&[6,7]&[6,7]&[3,4]\end{bmatrix}$$

$$\tilde{R}^{(19)}=\begin{bmatrix}[2,4]&[3,4]&[3,5]&[2,3]&[7,8]\\ [3,4]&[6,7]&[5,6]&[1,2]&[3,4]\\ [1,2]&[6,8]&[8,9]&[5,6]&[1,3]\\ [6,7]&[5,6]&[6,7]&[5,7]&[2,4]\end{bmatrix}\quad \tilde{R}^{(20)}=\begin{bmatrix}[7,8]&[4,5]&[6,7]&[8,9]&[6,7]\\ [1,3]&[2,3]&[4,6]&[8,9]&[3,5]\\ [1,3]&[4,5]&[2,3]&[5,6]&[7,8]\\ [4,5]&[2,3]&[4,6]&[3,5]&[6,7]\end{bmatrix}\quad \tilde{R}^{(21)}=\begin{bmatrix}[4,5]&[4,5]&[1,3]&[6,7]&[4,5]\\ [1,3]&[1,2]&[7,8]&[5,6]&[4,5]\\ [3,5]&[5,6]&[7,8]&[2,4]&[2,3]\\ [7,8]&[2,3]&[6,7]&[2,3]&[3,4]\end{bmatrix}$$

Table 2 (continued)

$$\tilde{R}^{(22)} = \begin{bmatrix} [6,7] & [5,6] & [4,5] & [6,7] & [6,7] \\ [2,3] & [2,3] & [1,3] & [6,7] & [3,4] \\ [2,3] & [2,3] & [1,3] & [4,5] & [6,7] \\ [4,5] & [3,4] & [3,4] & [3,4] & [3,5] \end{bmatrix} \quad \tilde{R}^{(23)} = \begin{bmatrix} [5,6] & [2,3] & [1,2] & [5,6] & [5,6] \\ [1,2] & [1,3] & [2,4] & [1,3] & [5,7] \\ [2,3] & [1,2] & [2,4] & [6,7] & [1,3] \\ [7,8] & [2,4] & [3,4] & [2,3] & [3,4] \end{bmatrix} \quad \tilde{R}^{(24)} = \begin{bmatrix} [3,5] & [3,4] & [3,5] & [2,4] & [6,7] \\ [2,4] & [4,5] & [2,4] & [3,4] & [5,6] \\ [2,3] & [1,2] & [3,4] & [4,5] & [3,4] \\ [7,8] & [6,7] & [3,4] & [5,6] & [3,5] \end{bmatrix}$$

$$\tilde{R}^{(25)} = \begin{bmatrix} [5,6] & [4,5] & [4,5] & [6,7] & [4,5] \\ [1,2] & [1,2] & [5,6] & [6,8] & [3,4] \\ [2,3] & [3,4] & [6,7] & [1,3] & [5,6] \\ [2,3] & [4,5] & [3,4] & [2,4] & [6,7] \end{bmatrix} \quad \tilde{R}^{(26)} = \begin{bmatrix} [5,8] & [3,4] & [2,4] & [5,6] & [3,5] \\ [2,5] & [2,3] & [6,7] & [7,8] & [5,6] \\ [5,6] & [7,8] & [5,6] & [6,7] & [2,3] \\ [2,4] & [5,6] & [3,4] & [5,6] & [3,4] \end{bmatrix} \quad \tilde{R}^{(27)} = \begin{bmatrix} [2,3] & [1,3] & [4,5] & [1,3] & [5,6] \\ [6,8] & [5,6] & [2,4] & [1,3] & [2,4] \\ [5,6] & [7,8] & [7,8] & [5,7] & [1,3] \\ [4,5] & [3,4] & [2,4] & [2,4] & [4,5] \end{bmatrix}$$

$$\tilde{R}^{(28)} = \begin{bmatrix} [6,7] & [4,5] & [1,3] & [5,6] & [6,7] \\ [2,4] & [2,4] & [5,6] & [2,3] & [3,5] \\ [4,5] & [2,3] & [4,5] & [4,6] & [1,2] \\ [6,7] & [4,5] & [6,7] & [2,3] & [3,5] \end{bmatrix} \quad \tilde{R}^{(29)} = \begin{bmatrix} [1,2] & [2,3] & [2,4] & [1,2] & [3,5] \\ [7,8] & [3,5] & [3,4] & [2,4] & [2,3] \\ [6,7] & [4,5] & [6,7] & [5,6] & [1,2] \\ [2,3] & [4,5] & [4,5] & [4,5] & [6,7] \end{bmatrix} \quad \tilde{R}^{(30)} = \begin{bmatrix} [2,3] & [3,4] & [3,4] & [2,3] & [4,5] \\ [5,6] & [4,5] & [2,3] & [2,4] & [3,4] \\ [6,7] & [1,2] & [5,6] & [5,6] & [2,4] \\ [4,5] & [3,4] & [1,3] & [2,4] & [3,4] \end{bmatrix}$$

Table 3 The results of clustering and weight

The cluster label	DMs	λ_l	π_k
C_1	$\{e_4, e_{11}, e_{14}, e_{17}\}$	0.1	0.025
C_2	$\{e_1, e_{10}, e_{13}, e_{26}\}$	0.1	0.025
C_3	$\{e_2, e_5, e_{12}, e_{19}, e_{24}\}$	0.16	0.032
C_4	$\{e_7, e_9, e_{18}, e_{27}, e_{29}, e_{30}\}$	0.23	0.0383
C_5	$\{e_6, e_8, e_{15}, e_{16}, e_{20}, e_{22}, e_{25}\}$	0.31	0.443
C_6	$\{e_3, e_{21}, e_{23}, e_{28}\}$	0.1	0.025

Model (25) is used to calculate the DMs' adjusted opinions and the total consensus costs for each modification. According to the modified opinions, we can calculate the consensus index of subgroups until the predetermined consensus threshold is reached. The consensus index and total cost after each iteration of the consensus process are shown in Table 5.

6.2 Comparative Analysis

In this subsection, the proposed method compares favorably with that of three existing methods and the comparative analysis is carried out in the same case study.

Specifically, we take into account three indexes in the comparison as follows.

(a) Total consensus cost index of the CRP,$C^* = \sum_{i=1}^{\chi} c(i)$, where χ is the number of iterations in the CRP.
(b) Number of modifications index $\delta(\chi)$ in each iteration of the CRP.
(c) The group consensus index GCI_M in each iteration of the CRP.

For brevity, M1, M2, and M3 denote the three existing methods:

Table 4 The parameters of the iterations of consensus process

The nominal value of costs	The perturbation value of costs
$\{c_7, c_9, c_{18}, c_{27}, c_{29}, c_{30}\} = \{2, 4, 3, 1, 3, 6\}$	$\{\hat{c}_7, \hat{c}_9, \hat{c}_{18}, \hat{c}_{27}, \hat{c}_{29}, \hat{c}_{30}\} = \{-0.6, -0.2, 0.3, 0.6, 0.5, -0.6\}$
$\{c_3, c_{21}, c_{23}, c_{28}\} = \{5, 3, 3, 4\}$	$\{\hat{c}_3, \hat{c}_{21}, \hat{c}_{23}, \hat{c}_{28}\} = \{-0.7, 0.2, 0.6, -0.4\}$
$\{c_{10}, c_1, c_{13}, c_{26}\} = \{2, 3, 2, 4\}$	$\{\hat{c}_{10}, \hat{c}_1, \hat{c}_{13}, \hat{c}_{26}\} = \{0.7, -0.3, 0.7, -0.4\}$

Table 5 The consensus index and total cost of the iterations of consensus process

Iteration	*NCI*	*GCI*	The total costs
1	$\{0.888, 0.865, 0.888, 0.853, 0.878, 0.856\}$	0.853	4.03
2	$\{0.889, 0.870, 0.886, 0.884, 0.882, 0.862\}$	0.862	3.83
3	$\{0.889, 0.870, 0.886, 0.885, 0.881, 0.890\}$	0.870	3.20
4	$\{0.888, 0.893, 0.888, 0.886, 0.881, 0.892\}$	0.881	2.55

Table 6 The results of iterations in the CRP of the M1

Iteration	*NCI*	*GCI*	Costs	$\delta(\chi)$
1	$\{0.888, 0.864, 0.887, 0.851, 0.879, 0.858\}$	0.851	3.1	13
2	$\{0.883, 0.867.883, 0.882, 0.888, 0.860\}$	0.860	1.925	24
3	$\{0.883, 0.866, 0.883, 0.883, 0.886, 0.888\}$	0.866	2.150	9
4	$\{0.888, 0.893, 0.888, 0.886, 0.884, 0.892\}$	0.884	1.700	8

Table 7 The results of iterations in the CRP of the M2

Iteration	*NCI*	*GCI*	Costs	$\delta(\chi)$
1	$\{0.878, 0.859, 0.886, 0.854, 0.882, 0.842\}$	0.842	8.45	69
2	$\{0.877, 0.859, 0.887, 0.854, 0.878, 0.888\}$	0.854	4.70	47
3	$\{0.876, 0.857, 0.882, 0.889, 0.887, 0.891\}$	0.857	3.60	76
4	$\{0.876, 0.889, 0.887, 0.883, 0.889, 0.891\}$	0.876	2.05	32

(1) M1: This mothed, proposed by Wu et al., establishes integer optimization consensus models based on the Manhattan distance (Wu et al. 2019b). However, it does not consider the uncertain unit adjusting costs of DMs. By comparison with this method, we can investigate the influences of the uncertain unit adjusting costs of DMs in the CRP. By applying this model to the case, we can obtain a detailed consensus iteration result shown in Table 6.

(2) M2: The k-means clustering algorithm method introduced by Wu and Xu to classify DMs with a possible distribution-based hesitant fuzzy element into subgroups (Wu and Xu 2018). To investigate the influences of the clustering algorithm, we apply this k-means clustering algorithm to the case, and we obtain the results of the cluster that is $G_1 = \{e_2, e_{12}, e_{12}, e_{24}\}$,$G_2 = \{e_3, e_5, e_{19}\}$, $G_3 = \{e_4, e_7, e_9, e_{11}, e_{14}, e_{17}, e_{18}, e_{21}, e_{27} \quad e_{29}\}$, $G_4 = \{e_1, e_8, e_{10}, e_{15}, e_{26}\}$, $G_5 = \{e_6, e_{16}, e_{20}, e_{22}, e_{25}\}$,$G_6 = \{e_{23}, e_{28}, e_{30}\}$. The results of iterations are in the CRP exhibited in Table 7.

(3) M3: The method of CRP named the uninorm-based comprehensive behavior classification model with enhanced efficiency and rationality is proposed by (Shi et al. 2018). Based on a cooperative index and a non-cooperative index, this model updates the weight of DMs in CRPs. We apply this CRP method to investigate the influence of the different CRP methods.

The result of iterations in this CRP is exhibited in Table 8.

We obtain the total consensus cost index of the CRP, the number of modifications index and the group consensus index, which are described in Figs. 3, 4, 5, and 6.

From Figs. 2, 3, 4, 5, and 6, the following observations are summarized.

Table 8 The results of iterations in CRP of the M3

Iteration	*NCI*	*GCI*	Costs	$\delta(\chi)$
1	$\{0.851, 0.863, 0.889, 0.848, 0.879, 0.864\}$	0.848	6.275	38
2	$\{0.853, 0.856, 0.891, 0.853, 0.877, 0.864\}$	0.853	5.300	27
3	$\{0.862, 0.861, 0.889, 0.866, 0.871, 0.877\}$	0.861	5.575	21
4	$\{0.882, 0.875, 0.890, 0.879, 0.882, 0.884\}$	0.875	3.725	35

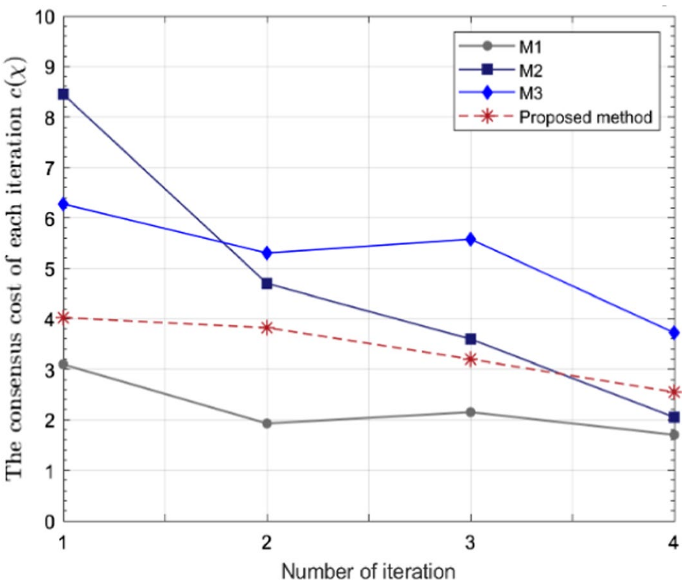

Fig. 3 The consensus cost of each iteration in these different methods

(i) The total consensus costs obtained by the optimization model in the CRP are lower than those of CRP methods. Compared with M1,the total consensus costs increase. The reason is that the proposed method takes into account the uncertainty of the decision-making environment that is characterized by the DMs' uncertain unit adjusting cost. This finding implies that the MCC model based on robust optimization is more realistic and robust. Moreover, by comparing M2 and the proposed method, the total consensus costs of the proposed method are lower than those of M2. This suggests that the clustering method is a key step in LSGDM that influences the costs of the CRP and that the enhanced ISODATA developed in this paper is more effective.

(ii) The total $\delta(\chi)$ value of M1 is lower than that of the proposed method and the corresponding consensus cost is lower than the proposed model. Moreover, the total $\delta(\chi)$ value of M3 is lower than that of the proposed method, however its cost is higher than that of the proposed method. This indicates that when the MCC model is not taken into account, a small amount of modification of DMs'

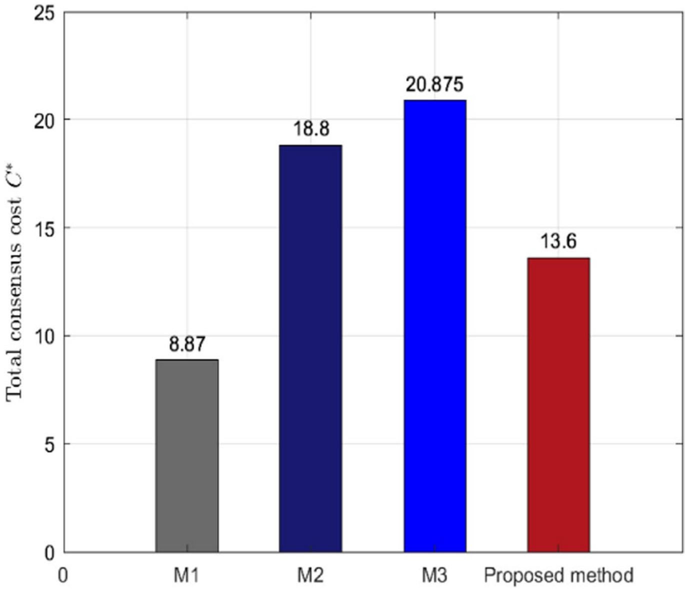

Fig. 4 C^* value in these different methods

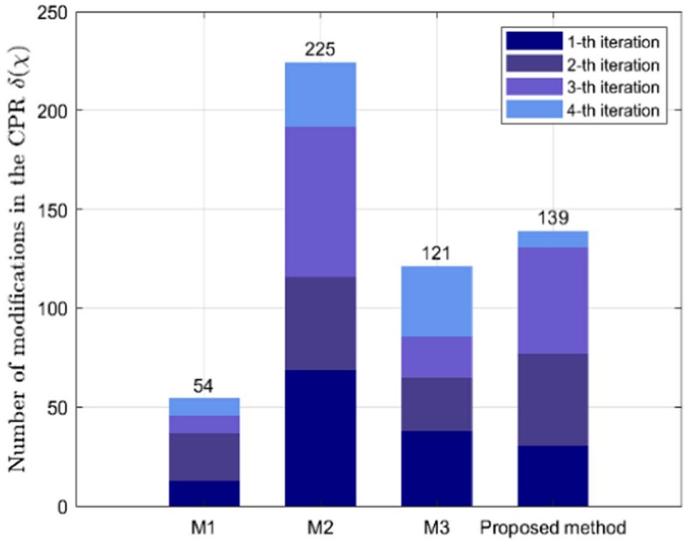

Fig. 5 $\delta(\chi)$ value in these different methods

opinions can also greatly increase the consensus costs. Finally, the total $\delta(\chi)$ value of M2 is lower than that of the proposed method. This indicates that the clustering algorithm affects the number of modifications of DMs' opinions.

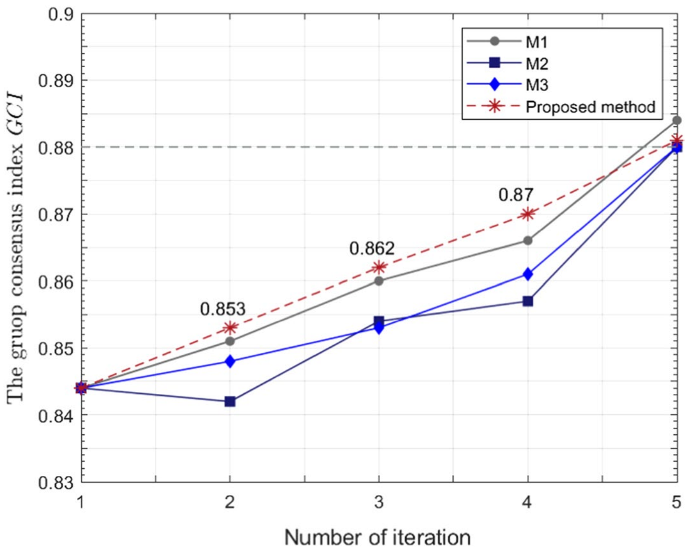

Fig. 6 *GCI* value in these different methods

6.3 Simulation Analysis

In this subsection, we first use the several evaluation indexes to estimate the performance of the enhanced ISODDATA, which are described in Fig. 7.

As the number of iterations increases, the sum of squares due to error (SSE) value is small at the beginning and then sharply rises, then falls, and finally converges. A larger number of cluster points leads to a small SSE value, and the reduction in clustering centres cause an increase in the SSE value. When the number of clustering centres is 6, the SSE value starts to decrease and finally gradually converges and the reduction Calinski-Harabazs (CH) value, indicates that the best number of clustering centres is 6.

To test the convergence performance of the enhanced ISODDATA proposed in this paper, we extended the scope of DMs to 60. The parameters are set as $\theta_n = 5$, $\theta_s = 1.4$, $\theta_c = 0.13$, $K = 4$ and $I = 30$. The iterative process of the clustering algorithm is shown in Fig. 8.

Compared with the original ISODATA algorithm, the proposed algorithm can converge faster. The 60 DMs are clustered into 9 sub-groups.

Second, we conduct a simulation experiments to analyze the influence of the degree of uncertainties in the robust discrete MCC model. We also consider three indexes introduced in Sect. 6.2. Based on the same clustering result, let us set the different uncertain numbers of unit adjusting costs of DMs.(see Table 9).Then, we obtain the three indexes, which are described in Fig. 9.

From the experiment, the following observations are summarized.

(i) With the increase in uncertainty, the method proposed still achieves consensus. Meanwhile, it does not change the number of iterations. However, the

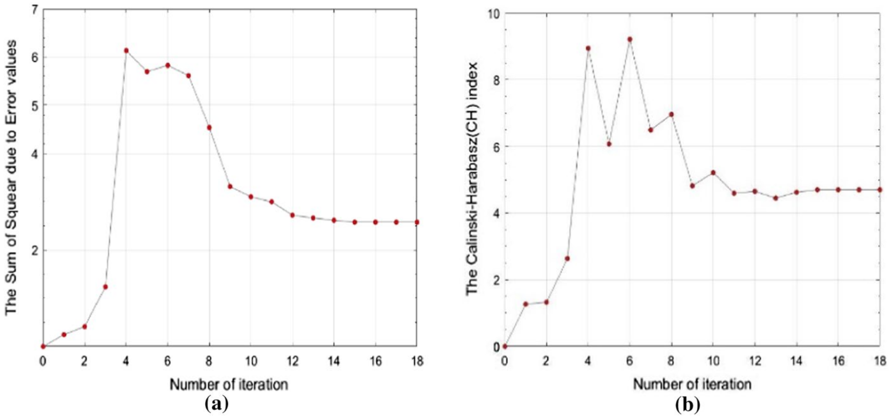

Fig. 7 Performance test for ISODATA. **a** is the SSE value and **b** is the CH value

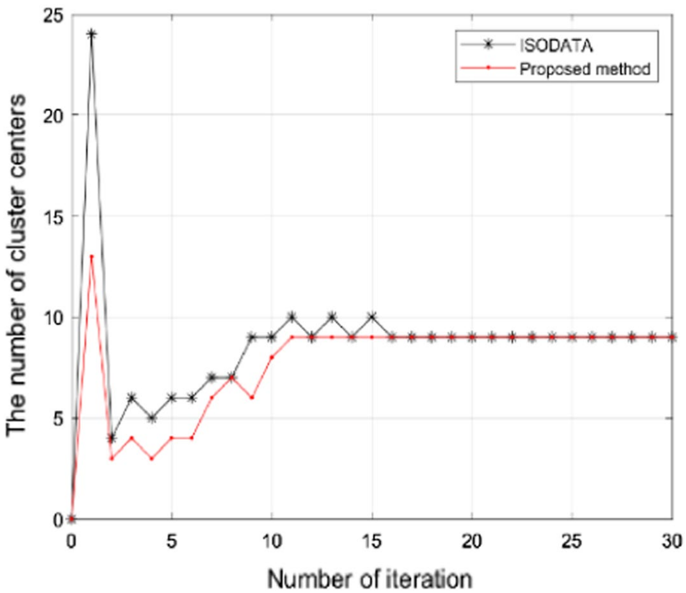

Fig. 8 The iterative process of the ISODATA and proposed method

consensus cost of each iteration rises, and the total consensus cost increases. This finding means that the uncertainty of the decision-making environment causes an increase in the costs of the CRP.

(ii) The increase in uncertainty has little impact on the consensus degree of each iteration of the CRP. However, the number of modifications significantly

Table 9 The different uncertain numbers of unit adjusting costs of DMs

The proportion of uncertain(%)	$\Gamma(\chi)$	The perturbation value of cost
25	$\{0.5, 0.5, 0.5, 0.5\}$	$\{-0.5, 0.3, -0.1, -0.2\}$
		$\{0.6, 0.2, -0.3, -0.6, -0.5, -0.6\}$
		$\{-0.7, 0.2, -0.6, -0.4\}$
		$\{-0.7, 0.3, -0.7, -0.4\}$
50	$\{1.5, 1.5, 1.5, 1.5\}$	$\{0.5, 0.3, -0.1, -0.2\}$
		$\{-0.6, -0.2, 0.3, 0.6, -0.5, -0.6\}$
		$\{-0.7, 0.2, 0.6, -0.4\}$
		$\{0.7, -0.3, 0.7, -0.4\}$
75	$\{2.8, 2.8, 2.8, 2.8\}$	$\{0.5, 0.3, 0.1, -0.2\}$
		$\{0.6, 0.2, 0.3, 0.6, 0.5, -0.6\}$
		$\{0.7, 0.2, 0.6, -0.4\}$
		$\{0.7, 0.3, 0.7, -0.4\}$
100	$\{3.7, 3.7, 3.7, 3.7\}$	$\{0.5, 0.3, 0.1, 0.2\}$
		$\{0.6, 0.2, 0.3, 0.6, 0.5, 0.6\}$
		$\{0.7, 0.2, 0.6, 0.4\}$
		$\{0.7, 0.3, 0.7, 0.4\}$

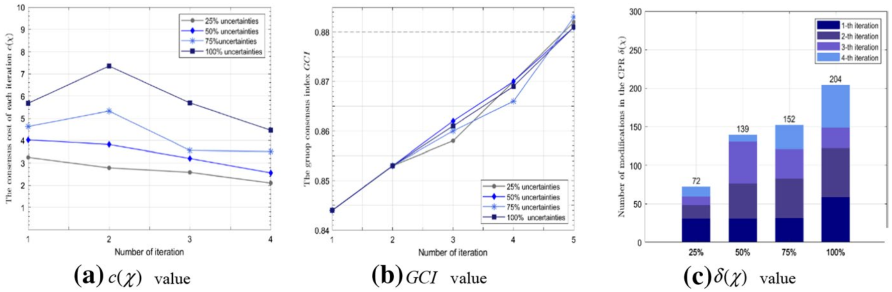

Fig. 9 Simulation experiments under different uncertainties

increased. This indicates that a stronger uncertainty in the decision-making environment will lead to an increase in the variation of DMs' opinions.

A case study proved the effectiveness of the proposed CRP method for uncertain multi-attribute LSGDM problems, and the comparative analysis illustrated that the proposed clustering algorithm can dynamically cluster DMs into subgroups and that the robust discrete MCC model as the optimization-based consensus rules of CRPs, resists the uncertain decision environment and has stronger robustness.

7 Conclusion

The consensus problem is very important in LSGDM. There are still some gaps although CRPs have been thoroughly studied over the last several years. This study researched uncertain LSGDM with interval opinions and proposed a CRP method based on a robust discrete optimization MCC model to address the uncertain unit adjustment cost. Compared with the existing CRPs in LSGDM, the major contributions and innovations of this paper are summarized as follows.

(1) Clustering for large-scale DMs is a crucial step in addressing the complexity of LSGDM problems. Based on the k-means clustering mechanism, an enhanced ISODATA is proposed to dynamically adjust the number of clusters to divide DMs into the subgroups under interval opinions. Meanwhile, this cluster methods improve the quality of initial cluster centre selection inspired by k-means + + which guarantees the diversity and dispersion of cluster centres.

(2) CRPs involving the feedback process take a great deal of time and effort, which can noticeably escalate costs. Therefore, considering the cost in CRPs, the MCC model under the interval opinions of DMs is established. This model aimed to minimize the total cost of modifying the opinions of DMs in uncertain LSGDM problems.

(3) To address the uncertain unit adjusting costs of DMs, we propose a robust discrete MCC model for LSGDM under indeterminate costs. This model is allowed to control the degree of conservatism of the optimal consensus opinion and makes the feedback process of CRPs more economical in uncertain decision-making environments. Meanwhile, a robust discrete counterpart is transformed to solve this model. In numerical experiments, the proposed robust discrete MCC model is more robust under uncertain circumstances.

The proposed method in this paper mainly used a cluster algorithm to reduce the complexity of large-scale DMs and constructed a robust discrete MCC model into a feedback strategy for multi-attribute LSGDM under uncertain situations. However, the mathematical model is not the only model involved in CRPs. The psychological elements of large-scale DMs are significant. Therefore, in future work, we argue that it will be interesting to discuss the influence of the psychological activities of DMs on uncertain LSGDM problems.

Additionally, since the robust discrete counterpart of the MCC model proposed is mixed integer linear programming, the mixed nonlinear integer programming problem is NP-hard when the Euclidean distance consensus measurement is considered in the MCC model. Generally, NP-hard problems take more time to obtain the suboptimal solution not the optimal solution. Thus, it would be interesting to discuss the MCC model with the Euclidean distance consensus measurement. Aggregation operators are used extensively to calculate a consensus opinion based on individual opinions. When a consensus opinion is calculated by aggregation operators, such as weighted average operator or ordered weighted average operator, in our future research, we also try to take the aggregation operators into the MCC model of uncertain LSGDM problems.

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