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Exploring the Ordinal Classifications of Failure Modes in the Reliability Management: An Optimization-Based Consensus Model with Bounded Confidences

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Abstract

Failure mode and effect analysis (FMEA) is a system activity that identifies, evaluates and eliminates potential failure modes (FMs) in a system/process to enhance the quality and reliability of a product. In order to improve the implementation efficiency of FMEA, this study proposes a consensus-based FMEA method to derive the ordinal classifications of FMs, in which the FMEA team members employ linguistic distribution to convey their preferences. In the proposed FMEA method, a multi-stage consensus optimization model with bounded confidences is designed to help the FMEA team reach a consensus. In the consensus reaching process, a maximum consensus optimization model based on bounded confidences is provided to obtain the adjustment suggestions by maximizing the level of consensus among the FMEA team. If the predetermined level of consensus cannot be reached, the adjustment suggestions obtained by the maximum consensus optimization model are adopted to guide the preference-modification of the FMEA team members. Otherwise, a two-stage consensus optimization model based on bounded confidences is designed to derive the adjustment suggestions for the preference-modification of the FMEA team members. Finally, a case study of marine diesel engine crankcase explosion, a sensitivity analysis and a comparative analysis are proposed to illustrate the feasibility and effectiveness of the proposed FMEA method.

Keywords Failure mode and effect analysis · Ordinal classification · Linguistic distribution · Consensus model · Bounded confidences

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1 Introduction

Failure mode and effect analysis (FMEA) is a reliability management method based on a multi-disciplinary team, used to analyze systems/processes to identify potential failure modes (FMs), their causes, and their possible consequences on system/process performance (Liu et al. 2016b). FMEA has received extensive attention and recognition since it was first implemented in the US aerospace industry in the 1960s (Huang et al. 2020). Nowadays, FMEA is considered an effective tool for managing safety and reliability issues, and it has been widely used in various industries, such as engineering (Yang and Wang 2015), climate (Shrestha et al. 2015), manufacturing (Baghery et al. 2018), water resources management (Ardeshirtanha and Sharafati 2020), healthcare (Faiella et al. 2018), and oil and gas (Catelani et al. 2018) etc. A typical FMEA includes five key steps, that is, Preparation, Identification, Prioritization, Risk Reduction and Reassessment. Among these steps, the prioritization of FMs is the most fundamental and influential because the following corrective actions are based on this (Bozdag et al. 2015).

The risk priority number (RPN) is utilized by the traditional FMEA to determine the prioritization of FMs. RPN is calculated based on the product of the following risk factors: severity (S), occurrence (O) and detection (D). Generally, all FMs are assessed on a 10-point qualitative scale based on the three risk factors (Pillay and Wang 2003), where a higher score means a higher risk. A series of recommended actions are taken to mitigate the risk of FMs with high RPN values. Although the conventional RPN method is proven to be effective in the early prevention of risks, it still has many drawbacks, such as the evaluation of FMs and the prioritization of FMs (Liu et al. 2016b; Wang et al. 2019). The drawbacks of the RPN method have motivated researchers to develop multiple streams of approaches for improving the application of FMEA process. For example, fuzzy set theory (Bozdag et al. 2015), cloud model theory (Liu et al. 2017) and rough set theory (Wang et al. 2018) are proposed to evaluate the FMs respect to risk factors in FMEA. Compared with precise numbers, these evaluation methods can better capture the real perception of FMEA team members and enable FMEA team members to deal with information insufficient and professional restrictions in the actual FMEA process. Moreover, numerous effectiveness methods have been utilized to prioritize FMs. For instance, ELECTRE-based ranking approach (Liu et al. 2016b), improved TODIM approach (Huang et al. 2017) and grey relational projection method (Liu et al. 2014) are proposed to obtain a complete risk ranking of FMs. These ranking methods have enhanced the application of FMEA. Although traditional and improved FMEA methods have made great progress, the FMEA in the real world still needs to fill in some gaps: (i) Given the large number of FMs, it is a great burden for the FMEA team members to obtain an accurate and complete risk ranking; (ii) Although the complete risk ranking of FMs has been obtained, further analysis is needed to determine which FM sequences require corrective actions; (iii) The previous method seldom considered the issue of consensus among the FMEA team members, which may lead to conflicts between members.

Compared with the complete risk ranking of FMs, the merits of assigning the FMs into several ordinal classes (such as low, medium, and high) are proposed in recent literatures (Certa et al. 2017; Lolli et al. 2015): (i) Several ordinal classes of FMs make it easier for the FMEA team members to understand and visualize a large number of FMs; (ii) Several ordinal classes of FMs allow the FMEA team members to quickly access or analyze a large number of FMs to take effective actions. To our best knowledge, only a few methods for classifying FMs have been proposed. Considering the benefits of assigning the FMs into ordinal classes in the practical applications of FMEA, relevant studies are insufficient.

The process of prioritizing FMs is quite complicated and is usually carried out by a multi-disciplinary team composed of decision makers from different departments (Bozdag et al. 2015). Due to different backgrounds and interests, the preferences of the FMEA team members are very different. The existing ranking methods (Liu et al. 2019; Wang et al. 2019) and sorting methods (Certa et al. 2017; Lolli et al. 2015) only focus on aggregating individual preferences into collective preference and further generating a group solution, without considering the consensus issue among the FMEA team members (Bozdag et al. 2015; Liu et al. 2016a; Wang et al. 2019). The obtained group solution is difficult to be recognized by most FMEA members, which leads to a series of negative effects, such as conflicts among FMEA members and negative treatment of the group solution. In practice, deriving a consensual group solution is essential in FMEA because: (i) Obtaining a consensual group solution requires communication and understanding between FMEA members, which helps to create a harmonious working atmosphere; and (ii) The consensual group solution can be widely accepted and recognized, which enables the group solution to be implemented smoothly. The benefits of consensual group solution analyzed above are presented in detail by Susskind et al. (1999). Consensus models are used in group decision making (GDM) to eliminate conflicts between decision makers and help decision makers reach a consensus, thereby obtaining a widely recognized consensual group solution (Li et al. 2020; Zhang et al. 2019). Consensus models are widely applied in various fields (Liu et al. 2020; Yu et al. 2020). Despite the great practical value of the consensus models in GDM, few literatures aim to integrate consensus models into FMEA process to improve its application.

Due to the complexity of FMEA, it is natural and convenient for the FMEA team to apply multiple linguistic terms to convey their assessments of FMs respect to the three risk factors. Moreover, the possibility and importance of each linguistic term may be different. Therefore, the linguistic distribution is very suitable to be used by the FMEA team members to express their risk assessments of FMs. Linguistic distribution enables decision makers to assign different probabilities to different linguistic terms by incorporating distribution information into linguistic terms (Zhang et al. 2014). Wu et al. (2021) provided a comprehensive perspective on the development of distributed linguistic representations and showed that linguistic distribution is a powerful tool for modeling uncertainty. Linguistic distribution is widely recognized and used in various applications (Chen et al. 2020; Ju et al. 2020; Zhang et al. 2020b). On one hand, linguistic distribution can preserve the original assessment information of FMEA team members by means of multiple linguistic terms. On the other hand, linguistic distribution facilitates FMEA team members' efforts to handle sophisticated situation by considering the probabilities of linguistic terms. However, to our knowledge, linguistic distribution is seldom applied in FMEA process.

Motivated by the challenge of remedying the above-mentioned insufficient research on existing FMEA methods and inspired by the consensus models (Xiao et al. 2020a; Zha et al. 2019), this study proposes a FMEA method based on a multi-stage consensus optimization model with bounded confidences to derive the ordinal classifications of FMs. The main contributions of this study are summarized as follows:

1. This study uses linguistic distribution to model the vague assessments of FMs with respect to each risk factors provided by the FMEA team members.
2. This study proposes a multi-stage consensus model to help the FMEA team reach a consensus while considering the bounded confidences of the FMEA team members.
3. This study helps the FMEA team members to classify FMs into several ordinal classes to deal with a large number of FMs.

Following this, a case study on the marine diesel engine crankcase explosion is proposed to demonstrate the application of the consensus-based FMEA. A sensitivity analysis and a comparative analysis are offered to verify the effectiveness of the proposed approach.

The rest of the research arrangements are as follows. Section 2 reviews the improved FMEA methods and the consensus models in GDM. Section 3 introduces the two-tuple linguistic model and the linguistic distribution assessment. Section 4 presents the general FMEA and establishes a consensus-based FMEA to facilitate its resolution. Section 5 proposes a multi-stage consensus optimization model with bounded confidences. A case study regarding the problem of crankcase explosion of marine diesel engines is presented in Sect. 6. A detailed sensitivity analysis and comparative analysis are presented in Sect. 7. Finally, Sect. 8 offers conclusions and future research directions.

2 Literature Review

Here, we introduce the improved FMEA approach and the consensus models in GDM based on their relevance to the subject of this study.

(1) FMs evaluation in FMEA

Information insufficient, professional limitations, and the inherent vagueness of human judgement make it difficult for the FMEA team to evaluate the FMs with precise numbers. Therefore, numerous integrated FMEA approaches have been provided to address the risk assessment information of FMs in FMEA. Bowles and Peláez (1995) pioneered the use of fuzzy set theory to evaluate FMs in FMEA. Followed by Bowles and Peláez (1995), many fuzzy set theory based methods have been presented to address the uncertain evaluation information offered by the FMEA team (Jee et al. 2015; Pillay and Wang 2003; Yang et al. 2008). Examples include triangular fuzzy set (Bhuvanesh Kumar and Parameshwaran 2018), intuitionistic fuzzy set (Liu et al. 2019), interval-valued fuzzy set (Baghery et al. 2018) and trapezoidal fuzzy set (Liu et al. 2012). Besides, the linguistic assessment approaches are widely applied by the FMEA team to convey their preferences for FMs. Liu et al. (2016b) and Ko et al. (2013) utilized a two-tuple linguistic computational method to address the assessment information of the FMs in FMEA. Li et al. (2017) used linguistic terms based method to model the evaluation information offered by the FMEA team. Other methods used by the FMEA team to model uncertain assessment information include cloud model theory (Liu et al. 2017; Wang et al. 2020), evidential reasoning approach (Chin et al. 2009; Liu et al. 2013) and rough set theory (Song et al. 2014). Particularly, Huang et al. (2017) utilized linguistic distributions to handle the FMEA team members' preferences for FMs. As an effective tool to model uncertainty of assessment information, linguistic distributions are seldom applied in FMEA with the exception of Huang et al. (2017).

(2) Prioritization of FMs in FMEA

Many literatures have been presented to remedy the drawbacks of conventional RPN method in the process of deriving the prioritization of FMs in FMEA. For instance, Liu et al. (2014) utilized grey relational projection method to derive the ranking of FMs. Pillay and Wang (2003) and Yang et al. (2008) constructed a fuzzy rule based method to generate the risk prioritization of FMs in FMEA. Additional methods presented to derive the ranking of FMs can be found in Bhuvanesh Kumar and Parameshwaran (2018), Jee et al. (2015), Zhou et al. (2016). In essence, the process of deriving the prioritization of FMs in FMEA is a multiple criteria GDM (Das Adhikary et al. 2014). Therefore, various multiple criteria GDM approaches have been proposed to derive the prioritization of FMs in FMEA. Examples include VIKOR approach (Wang et al. 2018), improved TODIM methods (Huang et al. 2017; Wang et al. 2019), TOPSIS methods (Kutlu and Ekmekçioğlu 2012; Song et al. 2013), extended gained and lost dominance score method (Wang et al. 2020), and PROMETHEE method (Liu et al. 2017). Although these ranking methods have greatly improved the implementation efficiency of FMEA, there are still some shortcomings that need to be improved in the real-world FMEA process (Certa et al. 2017; Lolli et al. 2015). Some sorting methods have been proposed to remedy the shortcomings of the ranking methods in FMEA. Certa et al. (2017) proposed an ELECTRE TRI-based approach to assign FMs to several ordinal classes based on the performance of FMs respect to multiple risk factors. Further, Lolli et al. (2015) utilized FlowSort-group decision support systems for sorting FMs into several ordinal classes by considering multiple decision makers. Although the methods proposed in Certa et al. (2017), Lolli et al. (2015) are quite useful, they still have some limitations: (i) The consensus issue among the multiple decision makers cannot guaranteed in Lolli et al.

(2015); (ii) The two methods proposed in Certa et al. (2017), Lolli et al. (2015) require a large number of parameters, such as upper reference profiles of classes, cutting level, indifference and the preference thresholds; and (iii) The two methods proposed in Certa et al. (2017), Lolli et al. (2015) obtain the classifications by comparing the FMs with the determined reference profiles or limiting profiles, and thus cannot guarantee the number of FMs in each category. Hence, it is necessary to continue the related research on sorting methods to improve its performance in FMEA.

(3) Consensus models in GDM

The FMs prioritization process requires a multidisciplinary team with swarm intelligence owing to its complexity, which is actually a GDM problem. In GDM, the original preferences of decision makers usually vary greatly due to their different backgrounds. Hence, reaching a consensus in GDM is essential to eliminate conflicts between decision makers and promote the smooth implementation of GDM. Given the benefits of consensus in GDM, many scholars devote themselves to the research of consensus models to assist decision makers in reaching a consensus (Cheng et al. 2021; Herrera-Viedma et al. 2018; Xu et al. 2020). For instance, Altuzarra et al. (2010) promoted the consensus among decision makers in AHP-GDM by a Bayesian-based approach. Chao et al. (2021) constructed a consensus model to facilitate decision makers with non-cooperative behaviors in large-scale GDM reach a consensus. Preference-modifications are required by decision makers in the consensus process of GDM, which means consuming resources. Considering the limited resources and cost in the consensus building process, Ben-Arieh and Easton (2007) helped decision makers in multi-criteria GDM reach a consensus by a minimum cost consensus model. Zhang et al. (2020a) proposed feedback mechanisms with maximum fuzzy consensus and minimum consensus cost in GDM considering private interest of the moderator. Further, Xu et al. (2020) investigated the minimum consensus cost in GDM under the influence of non-cooperative behaviors and decision rules. Additional minimum adjustment consensus models can be found in Xiao et al. (2020a, b). Recently, some scholars studied the willingness of decision makers to accept adjustment suggestions in the consensus process and showed that decision makers are willing to update their preferences only when the modifications are within a certain level of confidence (Dong et al. 2018). Followed by Dong et al. (2018), Zhang et al. (2021) established a minimum adjustment consensus model based on bounded confidences for multi-criteria GDM. Zha et al. (2019) constructed a consensus-building process based on a feedback mechanism with bounded confidences in large-scale GDM. Zhang et al. (2020c) proposed a consensus reaching algorithm based on the bounded confidences and leadership of decision makers in the social network GDM. Although the consensus models have received widespread attention and recognition, and have made achievements in many fields (Liu et al. 2020; Yu et al. 2020), the consensus models are rarely used to help the FMEA members eliminate conflicts and obtain consensual group solution in the process of deriving the prioritization of FMs in FMEA.

3 Preliminaries

This section begins with an introduction to the widely used two-tuple linguistic model. Furthermore, the linguistic distribution is introduced in detail.

(1) Two-tuple linguistic model

Let $S = \{s_t | t = 0, 1, \dots, g\}$ be a linguistic term set with the granularity $g + 1$, where $s_t(t = 0, 1, \dots, g)$ denotes a linguistic term. Generally, the following two basic requirements should be satisfied by the linguistic term set S:

(i) It is ordered $s_i \le s_j \Leftrightarrow i \le j$,

(ii) There is such an inverse function $neg(s_t) = s_{g-t}$.

Naturally, decision makers prefer to utilize linguistic terms rather than precise numerical values to convey their preferences. To address the linguistic evaluation information, Herrera and Martinez (2000) constructed a famous linguistic computing model, called the two-tuple linguistic model.

Definition 1 (Herrera and Martinez 2000) Let S be as given above. The linguistic two-tuple representing information equivalent to $\beta \in [0, g]$ can be generated by the following function:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5), \tag{1}$$

$$\Delta(\beta) = (s_t, \alpha), with \begin{cases} s_t, & t = round(\beta) \\ \alpha = \beta - t, & \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

where $round(\cdot)$ is a rounding operator.

The set of linguistic two-tuples is expressed as $\overline{S} = \{(s_t, \alpha) | s_t \in S, \alpha \in [-0.5, 0.5)\}$. It is obvious that Δ is a one-to-one mapping function. Hence, there is an inverse function of Δ, such that: $\Delta^{-1} : \overline{S} \rightarrow [0, g]$ with $\Delta^{-1}((s_t, \alpha)) = t + \alpha$. As an effective tool to deal with uncertainty, the two-tuple linguistic model is widely used in GDM. In addition, many extended linguistic computing models based on the two-tuple linguistic model have been proposed, such as linguistic distribution.

(2) Linguistic distsribution assessment

In GDM, a single linguistic term (for example, weak or strong) may not fully represent the perception of decision makers. Decision makers often hesitate between several linguistic terms when providing assessments. Moreover, the possibility or importance of each linguistic term may be different. To this end, Zhang et al. (2014) proposed linguistic distribution to increase the flexibility of the expression of linguistic assessment information by introducing distribution information into linguistic terms. A detailed description of the linguistic distribution presented in the Refs. (Wu et al. 2018; Zhang et al. 2014) is given as follows.

Definition 2 (Wu et al. 2018; Zhang et al. 2014) Let $S = \{s_t | t = 0, 1, \ldots, g\}$ be as given above. Let $d = \{(s_t, p_t) | t = 0, 1, \ldots, g\}$, where $s_t \in S$, $p_t \in [0, 1]$ is the symbolic proportion of s_t, and $\sum_{t=0}^{g} p_t = 1$. Then, d is called a linguistic distribution assessment of S.

Definition 3 (Wu et al. 2018; Zhang et al. 2014) Let $d = \{(s_t, p_t) | t = 0, 1, \ldots, g\}$ be as given above. The expectation of d is computed as follows:

$$E(d) = \Delta\left(\sum_{t=0}^{g} p_t \times NS(s_t)\right) \tag{3}$$

where $NS(s_t) = t$ is the numerical scale of $s_t \in S$.

For a detailed introduction to the numerical scale, please refer to (Dong et al. 2009). Clearly, $E(d) \in \overline{S}$. Moreover, we have that $\Delta^{-1}(E(d)) = \sum_{t=0}^{g} p_t \times NS(s_t)$.

A computational model for linguistic distribution is presented as follows. Let $d_z = \{(s_t, p_t^z) | t = 0, 1, \ldots, g\}$ and $d_u = \{(s_t, p_t^u) | t = 0, 1, \ldots, g\}$ be any two linguistic distributions over S. Then,

(i) A comparison operator: (a) If $E(d_z) < E(d_u)$, then d_z is smaller than d_u; (b) If $E(d_z) = E(d_u)$, then d_z and d_u represent the same assessment information.

(ii) The distance function: The distance between d_z and d_u is defined as follows:

$$d(d_z, d_u) = \frac{|\Delta^{-1}(E(d_z)) - \Delta^{-1}(E(d_u))|}{g} \tag{4}$$

Obviously, we have $d(d_z, d_u) \in [0, 1]$. Besides, a larger $d(d_z, d_u)$ value indicates a greater distance between d_z and d_u.

4 FMEA and Its Resolution Framework

The general FMEA is formally presented in this section. Further, a consensus-based FMEA is developed to facilitate the FMEA team members obtain the consensual classifications of FMs.

4.1 Problem Formulation

Let $DM = \{DM_1, DM_2, \ldots, DM_q\}$ be a set of FMEA team members. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_q)^T$ be the weight vector over DM, where $\lambda_k \in [0, 1]$ is the weight of DM_k and $\sum_{k=1}^{q} \lambda_k = 1$. Let $FM = \{FM_1, FM_2, \ldots, FM_m\}$ be a set of FMs and $RF = \{RF_1, RF_2, \ldots, RF_n\}$ be a set of risk factors. Let $W = (w_1, w_2, \ldots, w_n)^T$ be the weight vector over RF, where $w_j \in [0, 1]$ is the weight of RF_j and $\sum_{j=1}^{n} w_j = 1$. Let $V^k = (v_{ij}^k)_{m\times n}$ $(k = 1, 2, \ldots, q)$ be the linguistic distribution assessment matrix (LDAM) provided by DM_k, where $v_{ij}^k = \{(s_t, p_{ij,t}^k) | t = 0, 1, \ldots, g\}$ denotes the risk degree of FM_i over RF_j.

In this study, the FMEA classifies the FMs into several ordinal classes based on the LDAMs $\{V^1, V^2, \ldots, V^q\}$ provided by the FMEA team members. The FMs

$FM = \{FM_1, FM_2, \ldots, FM_m\}$ are classified into r ($r \geq 2$) ordinal classes, denoted as C_1, C_2, … C_r. The risk of FMs in Category C_i is higher than that of FMs in Category C_j when $i > j$. Let h_l ($l = 1, 2, \ldots, r$) be the number of FMs in Category C_l. For simplification, let $Q = \{1, 2, \ldots, q\}$, $M = \{1, 2, \ldots, m\}$, $N = \{1, 2, \ldots, n\}$, $G = \{0, 1, 2, \ldots, g\}$, and $R = \{1, 2, \ldots, r\}$.

4.2 Resolution Framework

As we mentioned above, the existing literature mainly focuses on the complete ranking of FMs without considering the consensus issue among the FMEA team members. In order to remedy the disadvantages of the existing FMEA methods and improve the implementation efficiency of FMEA, this study proposes a FMEA method based on a consensus model to derive the consensual collective classifications of FMs, as shown in Fig. 1.

The proposed consensus-based FMEA contains two key processes.

(1) Consensus measure

In practice, a complete agreement between the FMEA team members is usually not necessary, and it is also difficult to achieve. It is necessary to measure the degree of consensus among the FMEA team members. Two methods are widely

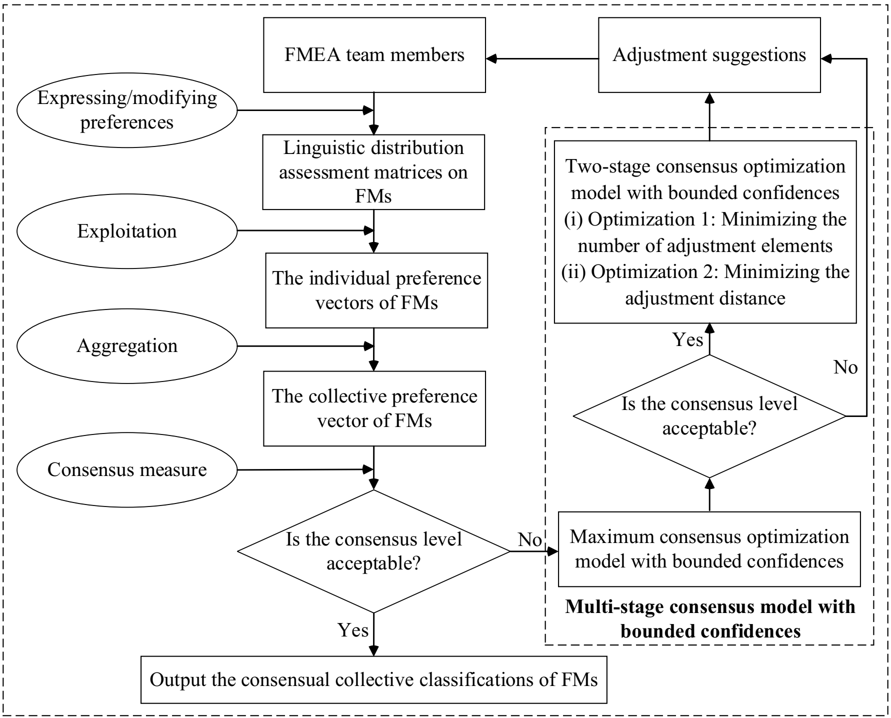

Fig. 1 Consensus-based FMEA

used to measure the consensus of decision makers (Chiclana et al. 2013; Herrera-Viedma et al. 2014): One method is to calculate the distances between the individual preferences and group preference, and the other method is to calculate the distances between the preferences of every two decision makers. Obviously, the previous consensus measurement method based on the complete ranking is not suitable for the FMEA based on the ordinal classes. Hence, a consensus measurement method for the FMEA team based on the ordinal classes is proposed below.

Let $V^k = (v_{ij}^k)_{m\times n}$ $(k \in Q)$, where $v_{ij}^k = \{(s_t, p_{ij,t}^k) | t = 0, 1, \ldots, g\}$, and $W = (w_1, w_2, \ldots, w_n)^T$ be as given above. Let $PV^k = (pv_1^k, pv_2^k, \ldots, pv_m^k)^T (k \in Q)$ be the individual preference vector derived from V^k. Then, pv_i^k $(k \in Q, i \in M)$ can be computed as follows:

$$pv_i^k = \sum_{j=1}^{n} \frac{w_j \times \Delta^{-1}(E(v_{ij}^k))}{g} \tag{5}$$

Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_q)^T$ be as given above. Let $PV^c = (pv_1^c, pv_2^c, \ldots, pv_m^c)^T$ be the collective preference vector derived from $\{PV^1, PV^2, \ldots, PV^q\}$. Then, pv_i^c $(i \in M)$ can be obtained as follows:

$$pv_i^c = \sum_{k=1}^{q} \lambda_k \times pv_i^k \tag{6}$$

Definition 4 Let h_l $(l \in R)$ and $PV = (pv_1, pv_2, \ldots, pv_m)^T$ be as given above. Let $O = (o_1, o_2, \ldots, o_m)^T$ be the classifications of FMs. Then, o_i $(i \in M)$ can be obtained in the following way:

$$o_i = \begin{cases} 1, \text{ if } pv_i \text{ is } jth \text{ largest value in } PV \text{ and } h_2 + \cdots + h_r + 1 \le j \le h_1 + \cdots + h_r \\ l, \text{ if } pv_i \text{ is } jth \text{ largest value in } PV \text{ and } h_{l+1} + \cdots + h_r + 1 \le j \le h_l + \cdots + h_r \\ r, \text{ if } pv_i \text{ is } jth \text{ largest value in } PV \text{ and } j \le m - (h_1 + \cdots + h_{r-1}) \end{cases} \tag{7}$$

Example 1 Let $FM = \{FM_1, FM_2, \ldots, FM_8\}$ be eight FMs that need to be classified into four ordinal classes: C_1, C_2, C_3 and C_4. Let $h_1 = 1$, $h_2 = 2$, $h_3 = 3$ and $h_4 = 2$. Moreover, the preference vector of the eight FMs is $PV = (0.44, 0.29, 0.5, 0.19, 0.56, 0.3, 0.2, 0.62)^T$.

Then, we have pv_1 as the $4th$ largest value in PV and $h_4 + 1 \le 4 \le h_3 + h_4$. Hence, we have $o_1 = 3$. Similarly, we obtain $O = (3, 2, 3, 1, 4, 3, 2, 4)^T$. Based on O, we have $C_1 = \{FM_4\}$, $C_2 = \{FM_2, FM_7\}$, $C_3 = \{FM_1, FM_3, FM_6\}$ and $C_4 = \{FM_5, FM_8\}$.

Let $O^k = (o_1^k, o_2^k, \ldots, o_m^k)^T$ $(k \in Q)$ and $O^c = (o_1^c, o_2^c, \ldots, o_m^c)^T$ be the ordinal risk classes of FMs derived from $PV^k = (pv_1^k, pv_2^k, \ldots, pv_m^k)^T$ and $PV^c = (pv_1^c, pv_2^c, \ldots, pv_m^c)^T$ respectively.

Definition 5 (Xiao et al. 2020b) Let $O^k = (o_1^k, o_2^k, \ldots, o_m^k)^T$ $(k \in Q)$ and $O^c = (o_1^c, o_2^c, \ldots, o_m^c)^T$ be as given before. Then, the level of consensus on DM_k based on his/her ordinal classifications of FMs can be obtained as follows:

$$CL\{PV^k\} = 1 - \sum_{i=1}^{m} \frac{|o_i^k - o_i^c|}{m(r-1)} \tag{8}$$

The level of consensus on $\{DM_1, \ldots, DM_q\}$ based on their ordinal classifications of FMs can be obtained as follows:

$$\begin{aligned} CL\{PV^1, \ldots, PV^q\} &= \frac{1}{q} \sum_{k=1}^{q} CL\{PV^k\} \\ &= 1 - \sum_{k=1}^{q} \sum_{i=1}^{m} \frac{|o_i^k - o_i^c|}{qm(r-1)} \end{aligned} \tag{9}$$

The larger the value of $CL\{PV^1, \ldots, PV^q\}$, the higher the level of consensus among the FMEA team members. $CL\{PV^1, \ldots, PV^q\} = 1$ means that the FMEA team members have reached a complete agreement. In particular, let $\alpha \in [0, 1]$, we consider the consensus level among the FMEA team is acceptable if $CL\{PV^1, \ldots, PV^q\} \geq \alpha$.

(2) Consensus model based on bounded confidences

A multi-stage consensus model considering the bounded confidences among the FMEA team is proposed to facilitate the FMEA team reach a consensus. In the consensus reaching process, a maximum consensus optimization model (MCOM) based on bounded confidences is advised to derive the optimal LDAMs by maximizing the consensus level among the FMEA team. If the predetermined level of consensus cannot be reached, the optimal LDAMs obtained by the MCOM are used as adjustment suggestions to guide the preference-modification of the FMEA team members. Otherwise, a two-stage consensus optimization model (TSCOM) based on bounded confidences is designed to derive the optimal LDAMs as adjustment suggestions for the preference-modification of the FMEA team members. Especially, the TSCOM contains two optimization processes: (i) Minimizing the number of adjustment elements between the original LDAMs and adjusted LDAMs; and (ii) Minimizing the adjustment distance between the original LDAMs and adjusted LDAMs.

The consensus model based on bounded confidences is described in detail in Sect. 5.

The consensus-based FMEA is described as follows.

First, the FMEA team members provide LDAMs on FMs. Further, the individual and collective preference vectors of FMs are obtained based on the LDAMs. If the consensus level among the FMEA team is acceptable, then the consensual collective classifications of FMs are obtained based on the collective preference vector of FMs. Otherwise, the MCOM is constructed to maximize the consensus level among the FMEA team. If the consensus level obtained by the MCOM is acceptable, then the TSCOM is designed to generate adjustment suggestions for the FMEA team to modify their original LDAMs. Otherwise, the adjustment suggestions obtained by the MCOM are used for the FMEA team to modify their original LDAMs. Repeat the above process until the consensual collective classifications of FMs are obtained.

5 Consensus model based on bounded confidences

A multi-stage consensus model based on bounded confidences is designed to facilitate the FMEA team members eliminate conflicts and reach a consensus. The multi-stage consensus model generates the optimal LDAMs as adjustment suggestions to guide the preference-modification of the FMEA team members.

Because the original preferences provided by the FMEA team vary widely, they usually need to modify the original preferences to reach a consensus. Naturally, the FMEA team members hope to preserve their original preferences as much as possible. Hence, the preference-modification process is a difficult compromise process. A key question is how can the FMEA team be willing to modify their original preferences? Recently, some literatures showed that the FMEA team members are willing to update their preferences only when the modifications are within a certain range, that is, the FMEA team members have bounded confidences (Dong et al. 2018; Zha et al. 2019; Zhang et al. 2021). Let $V^k = (v_{ij}^k)_{m\times n}$ $(k \in Q)$ be as given above, where $v_{ij}^k = \{(s_t, p_{ij,t}^k) | t = 0, 1, \ldots, g\}$. Let $\overline{V^k} = (\overline{v_{ij}^k})_{m\times n}$ be the adjusted LDAM associated with V^k, where $\overline{v_{ij}^k} = \{(s_t, \overline{p_{ij,t}^k}) | t = 0, 1, \ldots, g\}$. Let $U = \{u_1, u_2, \ldots, u_q\}$ be the bounded confidences associated with DM, where $u_k \in [0, 1]\,(k \in Q)$ be the bounded confidence of DM_k. The FMEA team member DM_k is willing to change v_{ij}^k to $\overline{v_{ij}^k}$, if $d(v_{ij}^k, \overline{v_{ij}^k}) \le u_k$; otherwise DM_k rejects the modification. Therefore, $\overline{V^k} = (\overline{v_{ij}^k})_{m\times n}$ should satisfy the following condition:

$$d(v_{ij}^k, \overline{v_{ij}^k}) = \frac{|\Delta^{-1}(E(v_{ij}^k)) - \Delta^{-1}(E(\overline{v_{ij}^k}))|}{g} \le u_k \tag{10}$$

(1) Maximum consensus optimization model with bounded confidences

A higher level of consensus among the FMEA team means a higher quality of FMEA implementation. Therefore, the level of consensus among the FMEA team is expected to be as large as possible with the predetermined bounded confidences. Based on this idea, we construct the MCOM as follows:

$$\max(1-\sum_{k=1}^{q}\sum_{i=1}^{m}\frac{|\overline{o_i^k}-\overline{o_i^c}|}{qm(r-1)})$$

$$s.t.\begin{cases}\overline{o_i^k}=\begin{cases}1,\ \text{if}\ \overline{pv_i^k}\ \text{is}\ jth\ \text{largest value in}\ \overline{PV^k}\ \text{and}\ h_2+\cdots+h_r+1\le j\le h_1+\cdots+h_r\\ l,\ \text{if}\ \overline{pv_i^k}\ \text{is}\ jth\ \text{largest value in}\ \overline{PV^k}\ \text{and}\ h_{l+1}+\cdots+h_r+1\le j\le h_l+\cdots+h_r\\ r,\ \text{if}\ \overline{pv_i^k}\ \text{is}\ jth\ \text{largest value in}\ \overline{PV^k}\ \text{and}\ j\le m-(h_1+\cdots+h_{r-1})\end{cases} & (a)\\ \overline{o_i^c}=\begin{cases}1,\ \text{if}\ \overline{pv_i^c}\ \text{is}\ jth\ \text{largest value in}\ \overline{PV^c}\ \text{and}\ h_2+\cdots+h_r+1\le j\le h_1+\cdots+h_r\\ l,\ \text{if}\ \overline{pv_i^c}\ \text{is}\ jth\ \text{largest value in}\ \overline{PV^c}\ \text{and}\ h_{l+1}+\cdots+h_r+1\le j\le h_l+\cdots+h_r\\ r,\ \text{if}\ \overline{pv_i^c}\ \text{is}\ jth\ \text{largest value in}\ \overline{PV^c}\ \text{and}\ j\le m-(h_1+\cdots+h_{r-1})\end{cases} & (b)\\ \overline{pv_i^k}=\sum_{j=1}^{n}\frac{w_j\times\Delta^{-1}(E(\overline{v_{ij}^k}))}{g},\ k\in Q, i\in M & (c)\\ \overline{pv_i^c}=\sum_{k=1}^{q}\lambda_k\times\overline{pv_i^k},\ i\in M & (d)\\ \frac{|\Delta^{-1}(E(v_{ij}^k))-\Delta^{-1}(E(\overline{v_{ij}^k}))|}{g}\le u_k,\ k\in Q, i\in M, j\in N & (e)\end{cases}\tag{11}$$

In model (11), $\overline{V^k}=(\overline{v_{ij}^k})_{m\times n}$ $(k\in Q)$, $\overline{PV^c}=(\overline{pv_1^c},\ldots,\overline{pv_m^c})^T$, $\overline{PV^k}=(\overline{pv_1^k},\ldots,\overline{pv_m^k})^T$ $(k\in Q)$, $\overline{O^k}=(\overline{o_1^k},\ldots,\overline{o_m^k})^T$ $(k\in Q)$, and $\overline{O^c}=(\overline{o_1^c},\ldots,\overline{o_m^c})^T$ are decision variables. The objective function of model (11) aims to obtain the maximum consensus level of $\{\overline{V^1},\ldots,\overline{V^q}\}$. Constraint (a) is employed to derive the individual classifications of FMs $\overline{O^k}=(\overline{o_1^k},\ldots,\overline{o_m^k})^T$ $(k\in Q)$ based on $\overline{PV^k}$. Constraint (b) is used to generate the collective classifications of FMs $\overline{O^c}=(\overline{o_1^c},\ldots,\overline{o_m^c})^T$ based on $\overline{PV^c}$. Constraints (c) and (d) are adopted to derive the individual preference vector $\overline{PV^k}(k\in Q)$ and collective preference vector $\overline{PV^c}$, respectively. Constraint (e) is utilized to ensure that the adjustment of the FMEA team member DM_k $(k\in Q)$ is made with the specific bounded confidence u_k.

Solving model (11) to generate the optimal LDAM $\overline{V^{k,*}}=(\overline{v_{ij}^{k,*}})_{m\times n}$ $(k\in Q)$ associated with $V^k=(v_{ij}^k)_{m\times n}$. Moreover, we can obtain the consensus level of $\{\overline{V^{1,*}},\ldots,\overline{V^{q,*}}\}$, which is denoted as MCL^*. MCL^* is the maximum consensus level among the FMEA team members. $MCL^*<\alpha$ means that the predefined consensus level among the FMEA team cannot be obtained with the given bounded confidences. Then, the FMEA team member DM_k $(k\in Q)$ is advised to revise v_{ij}^k $(k\in Q, i\in M, j\in N)$ into $\overline{v_{ij}^{k,*}}$ in order to increase the consensus among the FMEA team as much as possible. If $MCL^*\ge\alpha$, then the TSCOM is proposed to obtain the optimal LDAMs as the adjustment suggestions for the process of preference-modification.

Model (11) is difficult to solve because it is a nonlinear programming model. Thus, we introduce Lemma 1 and Theorem 1 to facilitate the resolution of model (11), which are presented in the following.

Lemma 1 (Xiao et al. 2020b) *Let* h_l $(l\in R)$, $PV^c=(pv_1^c,pv_2^c,\ldots,pv_m^c)^T$, *and* $PV^k=(pv_1^k,pv_2^k,\ldots,pv_m^k)^T$ $(k\in Q)$ *be as given above. Let* $\theta^c=\{\theta_1^c,\theta_2^c,\ldots,\theta_r^c\}$

and $\theta^k = \{\theta^k_1, \theta^k_2, \ldots, \theta^k_r\}$ $(k \in Q)$ *be a set of parameters, where* $0 \le \theta_p < \theta_{p+1} \le 1$ $(p = 1, 2, \ldots, r-1)$. *Then,* o^c_i $(i \in M)$ *and* o^k_i $(k \in Q, i \in M)$ *can be obtained as follows.*

$$\begin{cases} o^k_i = \sum_{p=1}^{r} y^k_{ip} \\ o^c_i = \sum_{p=1}^{r} y^c_{ip} \end{cases} \tag{12}$$

where $y^k_{ip}, y^c_{ip} \in \{0, 1\}$ *can be determined as follows.*

$$\begin{cases} pv^k_i - \theta^k_p < y^k_{ip} \\ y^k_{ip} - 1 < pv^k_i - \theta^k_p \end{cases} \tag{13}$$

And

$$\begin{cases} pv^c_i - \theta^c_p < y^c_{ip} \\ y^c_{ip} - 1 < pv^c_i - \theta^c_p \end{cases} \tag{14}$$

Meanwhile, y^k_{ip} *and* y^c_{ip} *satisfy the following condition.*

$$\begin{cases} \sum_{i=1}^{m} y^k_{ip} = h_p + \cdots + h_r \\ \sum_{i=1}^{m} y^c_{ip} = h_p + \cdots + h_r \end{cases} \tag{15}$$

Theorem 1 *Let* a^k_i $(k \in Q, i \in M)$ *and* b^k_{ij} $(k \in Q, i \in M, j \in N)$ *be a set of non-negative variables. Then, the following linear programming model can be equivalently transformed from model* (11):

$$\max\left(1-\sum_{k=1}^{q}\sum_{i=1}^{m}\frac{a_i^k}{qm(r-1)}\right)$$

$$s.t\begin{cases}\overline{o_i^k}-\overline{o_i^c}\le a_i^k, \quad k\in Q, i\in M \quad (a)\\ -\overline{o_i^k}+\overline{o_i^c}\le a_i^k,\ k\in Q,\quad i\in M \quad (b)\\ \overline{pv_i^k}=\sum_{j=1}^{n}\frac{w_j\times\Delta^{-1}(E(\overline{v_{ij}^k}))}{g}, \quad k\in Q, i\in M \quad (c)\\ \overline{pv_i^c}=\sum_{k=1}^{q}\lambda_k\times\overline{pv_i^k}, \quad i\in M \quad (d)\\ b_{ij}^k/g\le u_k,\ k\in Q,\quad i\in M, j\in N \quad (e)\\ \Delta^{-1}(E(v_{ij}^k))-\Delta^{-1}(E(\overline{v_{ij}^k}))\le b_{ij}^k, \quad k\in Q, i\in M, j\in N \quad (f)\\ -\Delta^{-1}(E(v_{ij}^k))+\Delta^{-1}(E(\overline{v_{ij}^k}))\le b_{ij}^k, \quad k\in Q, i\in M, j\in N \quad (g)\\ \overline{o_i^k}=\sum_{p=1}^{r}y_{ip}^k, \quad k\in Q, i\in M \quad (h)\\ \overline{o_i^c}=\sum_{p=1}^{r}y_{ip}^c, \quad i\in M \quad (i)\\ \overline{pv_i^k}-\theta_p^k<y_{ip}^k, \quad k\in Q, i\in M, p\in R \quad (j)\\ y_{ip}^k-1<\overline{pv_i^k}-\theta_p^k,\ k\in Q,\quad i\in M, p\in R \quad (k)\\ \overline{pv_i^c}-\theta_p^c<y_{ip}^c, \quad i\in M, p\in R \quad (l)\\ y_{ip}^c-1<\overline{pv_i^c}-\theta_p^c, \quad i\in M, p\in R \quad (m)\\ \sum_{i=1}^{m}y_{ip}^k=h_p+\cdots+h_r, \quad k\in Q, p\in R \quad (n)\\ \sum_{i=1}^{m}y_{ip}^c=h_p+\cdots+h_r,\ p\in R \quad (o)\\ \theta_p^k,\theta_p^c\in[0,1], \quad k\in Q, p\in R \quad (p)\end{cases} \tag{16}$$

Proof In model (16), we have $|\overline{o_i^k}-\overline{o_i^c}|\le a_i^k$ based on the constraints (*a*) and (*b*). Model (16) can obtain its optimal objective function value only when $|\overline{o_i^k}-\overline{o_i^c}|=a_i^k$. Moreover, constraints (*f*) and (*g*) guarantee that $|\Delta^{-1}(E(v_{ij}^k))-\Delta^{-1}(E(\overline{v_{ij}^k}))|\le b_{ij}^k$. Thus, $|\Delta^{-1}(E(v_{ij}^k))-\Delta^{-1}(E(\overline{v_{ij}^k}))|/g\le b_{ij}^k/g\le u_k$ can be guaranteed. Constraints (*h*) and (*p*) can be generated from Lemma 1. This is complete the proof of Theorem 1. □

Model (16) can be easily solved by MATLAB and CPLEX. The optimal solution of model (11) can be obtained according to Theorem 1.

(2) Two-stage consensus optimization model based on bounded confidences

(i) Optimization 1: Minimizing the number of adjustment elements

The preference-modification process is a process of resource and cost consumption. With the limited costs and resources, the FMEA team members may hope that the number of adjustment elements is minimal in the preference-modification process.

Let $V^k = (v_{ij}^k)_{m\times n}$ $(k \in Q)$ and $\overline{V^k} = (\overline{v_{ij}^k})_{m\times n}$ $(k \in Q)$ be as given before, where $v_{ij}^k = \{(s_t, p_{ij,t}^k) | t = 0, 1, \ldots, g\}$ and $\overline{v_{ij}^k} = \{(s_t, \overline{p_{ij,t}^k}) | t = 0, 1, \ldots, g\}$. Let x_{ij}^k $(k \in Q, i \in M, j \in N)$ be a 0–1 variable. If $v_{ij}^k \neq \overline{v_{ij}^k}$, that is, the FMEA team member DM_k changes the risk degree of FM_i over RF_j, then $x_{ij}^k = 1$. If $v_{ij}^k = \overline{v_{ij}^k}$, that is, the risk degree of FM_i over RF_j provided by the FMEA team member DM_k remains unchanged, then $x_{ij}^k = 0$. Then x_{ij}^k can be obtained as follows:

$$x_{ij}^k = \begin{cases} 0, \text{ if } \overline{p_{ij,t}^k} = p_{ij,t}^k \ \forall t = 0, 1, \ldots, g \\ 1, \ otherwise \end{cases} \tag{17}$$

The total number of the adjustment elements is expected to be minimal, i.e.,

$$\min \sum_{k=1}^{q} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}^k \tag{18}$$

Obviously, the consensus degree among $\{\overline{V^1}, \ldots, \overline{V^q}\}$ should be acceptable, i.e., $CL\{\overline{V^1}, \ldots, \overline{V^q}\} \geq \alpha$. Then,

$$CL\{\overline{V^1}, \ldots, \overline{V^q}\} = 1 - \sum_{k=1}^{q} \sum_{i=1}^{m} \frac{|\overline{o_i^k} - \overline{o_i^c}|}{qm(r-1)} \geq \alpha \tag{19}$$

Based on above analysis, the minimum adjustment element consensus model based on bounded confidences is developed as follows:

$$\begin{aligned} & \min \sum_{k=1}^{q} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}^k \\ & s.t. \begin{cases} x_{ij}^k = \begin{cases} 0, \text{ if } \overline{p_{ij,t}^k} = p_{ij,t}^k \ \forall t = 0, 1, \ldots, g \\ 1, \text{ otherwise} \end{cases}, \quad k \in Q, i \in M, j \in N \\ 1 - \sum_{k=1}^{q} \sum_{i=1}^{m} \frac{|\overline{o_i^k} - \overline{o_i^c}|}{qm(r-1)} \geq \alpha \\ \text{All other constrints in model (11)} \end{cases} \end{aligned} \tag{20}$$

Theorem 2 *The following model can be equivalently transformed from model* (20):

$$\min \sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}^{k}$$
$$s.t.\begin{cases} \sum_{t=0}^{g} |\overline{p_{ij,t}^{k}} - p_{ij,t}^{k}| \leq M \times x_{ij}^{k}, k \in Q, i \in M, j \in N \\ x_{ij}^{k} \in \{0,1\}, k \in Q, i \in M, j \in N \\ 1 - \sum_{k=1}^{q}\sum_{i=1}^{m} \frac{|\overline{o_i^k} - \overline{o_i^c}|}{qm(r-1)} \geq \alpha \\ \text{All other constrints in model (11)} \end{cases} \tag{21}$$

where M is a large enough number.

Proof The proof is obvious and we omit it for space limitation. □

Model (21) can be solved with reference to model (11). The optimal solution of model (20) can be obtained according to Theorem 2.

(ii) Optimization 2: Minimizing the adjustment distance

Let $V^k = (v_{ij}^k)_{m\times n}$ $(k \in Q)$ and $\overline{V^k} = (\overline{v_{ij}^k})_{m\times n}$ $(k \in Q)$ be as given above. In the preference-modification process, it is naturally that the FMEA team members hope that the total adjustment distance is minimal. That is,

$$\min \sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n} d(v_{ij}^k, \overline{v_{ij}^k}) \tag{22}$$

Equation (22) can be written as follows:

$$\min \sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{|\Delta^{-1}(E(v_{ij}^k)) - \Delta^{-1}(E(\overline{v_{ij}^k}))|}{g} \tag{23}$$

Let X^* be the optimal objective function value of model (20). Then, the minimum adjustment distance consensus model with bounded confidences can be obtained as follows:

$$\min \sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{|\Delta^{-1}(E(v_{ij}^k)) - \Delta^{-1}(E(\overline{v_{ij}^k}))|}{g}$$
$$s.t\begin{cases} \sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}^{k} = X^* \\ \text{All other constrints in model (20)} \end{cases} \tag{24}$$

Model (24) can be solved with reference to model (20). Solving model (24) to obtain the optimal LDAM $\ddot{V}^{k,*} = (\ddot{v}_{ij}^{k,*})_{m\times n}$ $(k \in Q)$ associated with $V^k = (v_{ij}^k)_{m\times n}$.

Then, the FMEA team member DM_k $(k \in Q)$ is advised to change v_{ij}^{k} $(k \in Q, i \in M, j \in N)$ into $\ddot{v}_{ij}^{k,*}$ when offering new LDAMs.

Note: The proposed TSCOM first minimizes the number of adjustment elements and then minimizes the adjustment distance. If their order changes, then the result may change. By changing the optimization order of TSCOM, that is, minimizing the adjustment distance first and then minimizing the number of adjustment elements, a new two-stage consensus optimization model is obtained, denoted as TSCOM-II. Let X^* be as given above. Let M^* be the minimum adjustment distance obtained from TSCOM. Let M_{II}^* and X_{II}^* be the minimum adjustment distance and the minimum number of adjustment elements obtained from TSCOM-II, respectively. Then, we have $X^* \leq X_{II}^*$ and $M_{II}^* \leq M^*$. The FMEA team members can flexibly choose TSCOM or TSCOM-II according to their needs. Due to limited space, only TSCOM is presented in this study.

(3) Algorithm for the consensus-based FMEA

We present an Algorithm (i.e., Algorithm I) to introduce in detail the steps of deriving the consensual collective classifications of FMs by using the consensus-based FMEA.

Algorithm I

Input: The LDAMs $\{V^1, V^2, \ldots, V^q\}$, the weight vector over DM $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_q)^T$, the weight vector over RF $W = (w_1, w_2, \ldots, w_n)^T$, the bounded confidences $U = \{u_1, u_2, \ldots, u_q\}$, h_l $(l \in R)$, and the consensus threshold α

Output: The consensual collective classifications of FMs, i.e., $C_1, C_2, \ldots C_r$

Step 1: Let $z = 0$, and $V^{k,z} = V^k$ $(k \in Q)$

Step 2: Applying Eqs. (5) and (6) to derive the individual preference vectors $\{PV^{1,z}, \ldots, PV^{q,z}\}$ and collective preference vector $PV^{c,z}$, respectively

Step 3: Utilizing Eqs. (7) and (9) to derive the consensus level among $\{PV^{1,z}, \ldots, PV^{q,z}\}$, i.e., $CL\{PV^{1,z}, \ldots, PV^{q,z}\}$. If $CL\{PV^{1,z}, \ldots, PV^{q,z}\} \geq \alpha$, then proceed to Step 6. Otherwise, proceed to the next step

Step 4: Applying model (11) to generate the optimal LDAM $\overline{V^{k,z,*}} = (\overline{v}_{ij}^{k,z,*})_{m\times n}$ $(k \in Q)$, and the maximum consensus level MCL^*

If $MCL^* < \alpha$, then the adjusted LDAM $V^{k,z+1} = (v_{ij}^{k,z+1})_{m\times n}$ $(k \in Q)$ is constructed as follows: $v_{ij}^{k,z+1} = v_{ij}^{k,z,*}$. Let $z = z + 1$, and proceed to Step 2

If $MCL^* \geq \alpha$, proceed to the next step

Step 5: Using models (20) and (24) to derive the optimal LDAM $\ddot{V}^{k,z,*} = (\ddot{v}_{ij}^{k,z,*})_{m\times n}$ $(k \in Q)$. Then, the adjusted LDAM $V^{k,z+1} = (v_{ij}^{k,z+1})_{m\times n}$ $(k \in Q)$ is constructed as follows: $v_{ij}^{k,z+1} = \ddot{v}_{ij}^{k,z,*}$

Let $z = z + 1$, and proceed to Step 2

Step 6: The consensual collective classifications of FMs, i.e., $C_1, C_2, \ldots C_r$, can be obtained based on $PV^{c,z}$. Output $C_1, C_2, \ldots C_r$

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6 Case study

This section presents the application of the consensus-based FMEA to the problem of crankcase explosion in marine diesel engines, which is borrowed from Wang et al. (2019). The explosion of the crankcase of the marine diesel engines has caused fatal damage to the crew members onboard and ship structure, and therefore it has attracted widespread attention from marine engineers and engine manufacturers. The ship operators and marine engine manufacturers have made great efforts to prevent crankcase explosion. Equipment-based solutions to prevent crankcase explosions, including flame arrester, crankcase relief valves, oil mist detector, etc. Despite these innovative marine technologies, crankcase explosions still occur frequently, which greatly bothers the ship operators and marine engine manufacturers. Thus, it is necessary to apply the FMEA-based method to prevent marine diesel engine crankcase explosions by translating operational evidences and feedbacks to preventive measures (Wang et al. 2019). Wang et al. (2019) proposed a FMEA method to prevent marine diesel engine crankcase explosion, in which the FMEA team uses 9-member linguistic terms to evaluate FMs to obtain a complete priority orders of FMs from the highest to the least risky. However, the issue of whether the FMEA team members reach a consensus has not been considered by the method proposed by Wang et al. (2019). Therefore, the proposed method is used to further improve the implementation efficiency in preventing marine diesel engine crankcase explosion.

In this section, six FMs that are most likely to cause crankcase explosion are determined according to the accident investigations and functional requirements. The six FMs and their detailed information are shown in Table 1. A four-member cross-functional FMEA team, i.e., $DM = \{DM_1, DM_2, DM_3, DM_4\}$, is required to sort the six FMs into three ordinal risk classifications based on the lowest to highest risk level (i.e., low, medium, and high), represented by C_1, C_2, and C_3, respectively. Let $h_1 = 2$, $h_2 = 2$, and $h_3 = 2$. Let the weight vector of the four FMEA team members be $\lambda = (0.23, 0.3, 0.27, 0.2)^T$. D, O and S are the risk factors adopted to assess the six FMs, and their weight vector is $W = (0.33, 0.45, 0.22)^T$. The proposed consensus-based FMEA method is adopted to prevent marine diesel engine crankcase explosion, as shown below.

First, the FMEA team members utilize a seven-grade linguistic term set S to model their preferences for FMs based on the three risk factors, as shown below:

$$S = \{s_0 = Very\,Low, s_1 = Low, s_2 = Moderately\,Low,$$
$$s_3 = Moderate, s_4 = Moderately\,High, s_5 = High,\ s_6 = Very\,High\}$$

Let the consensus threshold be $\alpha = 0.8$. The FMEA team members generate the four individual LDAMs $V^k = (v_{ij}^k)_{6\times3}$ $(k = 1, 2, 3, 4)$ on the six FMs based on the three risk factors, as shown in Tables 2, 3, 4 and 5.

(1) Consensus measure

First, Eqs. (5) and (6) are adopted to generate the individual preference vectors $\{PV^1, PV^2, PV^3, PV^4\}$ and collective preference vector PV^c, respectively. The obtained $\{PV^1, PV^2, PV^3, PV^4\}$ and PV^c are shown in the following: $PV^1 = (1.59, 4.36, 2.61, 2.93, 1.51, 4.11)^T$; $PV^2 = (3.09, 3.84, 3.91, 3.42, 2.59, 1.5)^T$;$PV^3 = (1.39, 5.04, 1.82, 3.98, 3.98, 4.02)^T$;$PV^4 = (4.82, 2.24, 0.69, 4.23, 3.24, 1.45)^T$;$PV^c = (2.63, 3.96, 2.41, 3.62, 2.85, 2.77)^T$.

Table 1 FMs, their component locations, their causes, and their effects

Component	FMs	Failure causes	Failure effects
Oil mist detector	Inoperable (FM_1)	Oil mist detector calibration error	Unable to detect oil mist in the crankcase
Piston	Hole on the top of the piston (FM_2)	The fuel valve drips oil	The combustion gas is transmitted to the crankcase
Piston ring	Adhere to the groove (FM_3)	Deposits	The gap is too large, blowing fire
Stuffing box	Not functioning correctly (FM_4)	Incorrect spring installed in piston rod stuffing box	Transfer combustion gas from the combustion chamber to the crankcase
Fuel valve	Open the fuel valve early (FM_5)	Service pressure is too light	Timing issues, poor atomization, temperature variations and power balance
Engine performance monitoring system	Not fully functional (FM_6)	Check the electronic card irregularly	Do not understand the abnormal conditions during combustion

Table 2 The LDAM provided by DM_1

	O	S	D
FM_1	$\{(s_1, 0.3), (s_2, 0.7)\}$	$\{(s_1, 0.34), (s_2, 0.66)\}$	$\{(s_1, 0.7), (s_2, 0.3)\}$
FM_2	$\{(s_4, 0.5), (s_5, 0.5)\}$	$\{(s_3, 0.45), (s_4, 0.55)\}$	$\{(s_5, 0.2), (s_6, 0.8)\}$
FM_3	$\{(s_2, 0.3), (s_3, 0.4), (s_4, 0.3)\}$	$\{(s_1, 0.6), (s_2, 0.4)\}$	$\{(s_4, 0.5), (s_5, 0.5)\}$
FM_4	$\{(s_1, 0.6), (s_2, 0.4)\}$	$\{(s_4, 0.3), (s_5, 0.7)\}$	$\{(s_1, 0.4), (s_2, 0.6)\}$
FM_5	$\{(s_0, 0.2), (s_1, 0.8)\}$	$\{(s_1, 0.5), (s_2, 0.3), (s_3, 0.2)\}$	$\{(s_2, 0.8), (s_3, 0.2)\}$
FM_6	$\{(s_4, 0.45), (s_5, 0.55)\}$	$\{(s_5, 0.5), (s_6, 0.5)\}$	$\{(s_0, 0.4), (s_1, 0.6)\}$

Table 3 The LDAM provided by DM_2

	O	S	D
FM_1	$\{(s_3, 0.3), (s_4, 0.7)\}$	$\{(s_1, 0.34), (s_2, 0.66)\}$	$\{(s_5, 0.88), (s_6, 0.12)\}$
FM_2	$\{(s_0, 0.5), (s_1, 0.5)\}$	$\{(s_5, 0.6), (s_6, 0.4)\}$	$\{(s_5, 0.33), (s_6, 0.67)\}$
FM_3	$\{(s_2, 0.55), (s_3, 0.45)\}$	$\{(s_5, 0.4), (s_6, 0.6)\}$	$\{(s_2, 0.34), (s_3, 0.66)\}$
FM_4	$\{(s_5, 0.72), (s_6, 0.28)\}$	$\{(s_3, 0.46), (s_4, 0.54)\}$	$\{(s_0, 0.6), (s_1, 0.4)\}$
FM_5	$\{(s_1, 0.44), (s_2, 0.56)\}$	$\{(s_2, 0.15), (s_3, 0.85)\}$	$\{(s_3, 0.4), (s_4, 0.6)\}$
FM_6	$\{(s_2, 0.5), (s_3, 0.5)\}$	$\{(s_0, 0.34), (s_1, 0.66)\}$	$\{(s_1, 0.3), (s_2, 0.7)\}$

Table 4 The LDAM provided by DM_3

	O	S	D
FM_1	$\{(s_2, 0.6), (s_3, 0.4)\}$	$\{(s_0, 0.66), (s_1, 0.34)\}$	$\{(s_2, 1)\}$
FM_2	$\{(s_5, 0.5), (s_6, 0.5)\}$	$\{(s_4, 0.55), (s_5, 0.45)\}$	$\{(s_5, 0.45), (s_6, 0.55)\}$
FM_3	$\{(s_0, 0.7), (s_1, 0.3)\}$	$\{(s_2, 0.78), (s_3, 0.22)\}$	$\{(s_3, 0.7), (s_4, 0.3)\}$
FM_4	$\{(s_3, 0.44), (s_4, 0.56)\}$	$\{(s_6, 1)\}$	$\{(s_0, 0.54), (s_1, 0.46)\}$
FM_5	$\{(s_1, 0.44), (s_2, 0.56)\}$	$\{(s_6, 1)\}$	$\{(s_3, 0.5), (s_4, 0.5)\}$
FM_6	$\{(s_4, 0.64), (s_5, 0.36)\}$	$\{(s_4, 0.44), (s_5, 0.56)\}$	$\{(s_2, 0.6), (s_3, 0.4)\}$

Table 5 The LDAM provided by DM_4

	O	S	D
FM_1	$\{(s_6, 1)\}$	$\{(s_4, 0.3), (s_5, 0.55), (s_6, 0.15)\}$	$\{(s_3, 1)\}$
FM_2	$\{(s_2, 0.45), (s_3, 0.55)\}$	$\{(s_2, 0.7), (s_3, 0.3)\}$	$\{(s_1, 0.34), (s_2, 0.66)\}$
FM_3	$\{(s_0, 0.4), (s_1, 0.6)\}$	$\{(s_1, 1)\}$	$\{(s_0, 0.8), (s_1, 0.2)\}$
FM_4	$\{(s_5, 0.5), (s_6, 0.5)\}$	$\{(s_2, 0.33), (s_3, 0.67)\}$	$\{(s_5, 0.5), (s_6, 0.5)\}$
FM_5	$\{(s_1, 0.26), (s_2, 0.74)\}$	$\{(s_4, 0.88), (s_5, 0.12)\}$	$\{(s_3, 0.3), (s_4, 0.7)\}$
FM_6	$\{(s_3, 0.55), (s_4, 0.45)\}$	$\{(s_0, 0.45), (s_1, 0.55)\}$	$\{(s_0, 0.7), (s_1, 0.3)\}$

Further, $CL\{PV^1, PV^2, PV^3, PV^4\} = 0.67$ is obtained by putting $\{PV^1, PV^2, PV^3, PV^4\}$ and PV^c into Eqs. (7) and (9). The consensus level among the FMEA team is unacceptable because $CL\{PV^1, PV^2, PV^3, PV^4\} < 0.8$. This means that there are large differences between $\{V^1, V^2, V^3, V^4\}$, which is not conducive to the implementation of the FMEA process. Therefore, the proposed multi-stage consensus model based on bounded confidences is utilized to assist the FMEA team members in deriving the consensual collective classifications of FMs in the following step.

(2) Multi-stage consensus model based on bounded confidences

First, the MCOM (i.e., model (11)) is applied to derive the maximum consensus level MCL^* and the optimal LDAMs $\{\overline{V^{1,*}}, \overline{V^{2,*}}, \overline{V^{3,*}}, \overline{V^{4,*}}\}$. Let the bounded confidences be $U = \{0.1, 0.08, 0.08, 0.08\}$. Taking U, $\{V^1, V^2, V^3, V^4\}$, $h_1 = 2$, $h_2 = 2$, $h_3 = 2$, $W = (0.33, 0.45, 0.22)^T$, and $\lambda = (0.23, 0.3, 0.27, 0.2)^T$ into model (11), we can obtain:

$$\max\left(1 - \frac{|\overline{o_1^1} - \overline{o_1^c}| + |\overline{o_2^1} - \overline{o_2^c}| + \cdots + |\overline{o_6^4} - \overline{o_6^c}|}{4 \times 6 \times 2}\right)$$
$$s.t.\begin{cases} \overline{pv_1^1} = \frac{0.33\times\Delta^{-1}(E(\overline{v_{11}^1}))+0.45\times\Delta^{-1}(E(\overline{v_{12}^1}))+0.22\times\Delta^{-1}(E(\overline{v_{13}^1}))}{6} \\ \cdots \\ \overline{pv_6^4} = \frac{0.33\times\Delta^{-1}(E(\overline{v_{61}^4}))+0.45\times\Delta^{-1}(E(\overline{v_{62}^4}))+0.22\times\Delta^{-1}(E(\overline{v_{63}^4}))}{6} \\ \overline{pv_1^c} = 0.23 \times \overline{pv_1^1} + 0.3 \times \overline{pv_1^2} + 0.27 \times \overline{pv_1^3} + 0.2 \times \overline{pv_1^4} \\ \cdots \\ \overline{pv_6^c} = 0.23 \times \overline{pv_6^1} + 0.3 \times \overline{pv_6^2} + 0.27 \times \overline{pv_6^3} + 0.2 \times \overline{pv_6^4} \\ \frac{|\Delta^{-1}(E(v_{11}^1))-\Delta^{-1}(E(\overline{v_{11}^1}))|}{g} \le 0.1 \\ \cdots \\ \frac{|\Delta^{-1}(E(v_{63}^4))-\Delta^{-1}(E(\overline{v_{63}^4}))|}{g} \le 0.08 \\ \overline{o_1^1} = \begin{cases} 1, \text{ if } \overline{pv_1^1} \text{ is } jth \text{ largest value in } \overline{PV^1} \text{ and } 5 \le j \le 6 \\ 2, \text{ if } \overline{pv_1^1} \text{ is } jth \text{ largest value in } \overline{PV^1} \text{ and } 3 \le j \le 4 \\ 3, \text{ if } \overline{pv_1^1} \text{ is } jth \text{ largest value in } \overline{PV^1} \text{ and } j \le 2 \end{cases} \\ \cdots \\ \overline{o_6^c} = \begin{cases} 1, \text{ if } \overline{pv_6^c} \text{ is } jth \text{ largest value in } \overline{PV^c} \text{ and } 5 \le j \le 6 \\ 2, \text{ if } \overline{pv_6^c} \text{ is } jth \text{ largest value in } \overline{PV^c} \text{ and } 3 \le j \le 4 \\ 3, \text{ if } \overline{pv_6^c} \text{ is } jth \text{ largest value in } \overline{PV^c} \text{ and } j \le 2 \end{cases} \end{cases} \tag{25}$$

Based on Theorem 1, an equivalent linear programming model can be obtained from the above model to facilitate the solution of the above model. By solving the above model, the maximum consensus level $MCL^* = 0.875$ and the optimal LDAMs

$\{\overline{V^{1,*}}, \overline{V^{2,*}}, \overline{V^{3,*}}, \overline{V^{4,*}}\}$ are obtained. The optimal LDAMs $\{\overline{V^{1,*}}, \overline{V^{2,*}}, \overline{V^{3,*}}, \overline{V^{4,*}}\}$ are omitted for space limitation.

Since $MCL^* > \alpha$, the TSCOM is applied to derive the optimal LDAMs as adjustment suggestions for the preference-modification of the FMEA team members. First, the minimum adjustment element consensus model with bounded confidences (i.e., model (20)) is applied to derive the minimum number of adjustment elements. Taking $U, \{V^1, V^2, V^3, V^4\}, h_1 = 2, h_2 = 2, h_3 = 2, \alpha = 0.8, W = (0.33, 0.45, 0.22)^T$, and $\lambda = (0.23, 0.3, 0.27, 0.2)^T$ into model (20), we can obtain:

$$\min x^1_{11} + x^1_{12} + \cdots + x^4_{63}$$
$$s.t.\begin{cases} x^1_{11} = \begin{cases} 0, \text{ if } \overline{p^1_{11,t}} = p^1_{11,t}\ \forall t = 0, 1, \ldots, 6 \\ 1, \ otherwise \end{cases} \\ \cdots \\ x^4_{63} = \begin{cases} 0, \text{ if } \overline{p^4_{63,t}} = p^4_{63,t}\ \forall t = 0, 1, \ldots, 6 \\ 1, \ otherwise \end{cases} \\ 1 - \frac{|\overline{o^1_1} - \overline{o^c_1}| + |\overline{o^1_2} - \overline{o^c_2}| + \cdots + |\overline{o^4_6} - \overline{o^c_6}|}{4 \times 6 \times 2} \geq 0.8 \\ \text{All other constrints in model (25)} \end{cases} \tag{26}$$

Based on Theorems 1 and 2, an equivalent linear programming model can be obtained from the above model to facilitate the solution of the above model. By solving the above model, the minimum number of adjustment elements can be obtained $X^* = 13$.

Furthermore, the minimum adjustment distance consensus model with bounded confidences (i.e., model (24)) is utilized to derive the optimal LDAMs $\{\ddot{V}^{1,*} = (\ddot{v}^{1,*}_{ij})_{6\times3}, \ldots, \ddot{V}^{4,*} = (\ddot{v}^{4,*}_{ij})_{6\times3}\}$. Taking U, $\{V^1, V^2, V^3, V^4\}$, $h_1 = 2$, $h_2 = 2$, $h_3 = 2$, $\alpha = 0.8$, $X^* = 13$, $W = (0.33, 0.45, 0.22)^T$, and $\lambda = (0.23, 0.3, 0.27, 0.2)^T$ into model (24), we can obtain:

$$\min \frac{|\Delta^{-1}(E(v^1_{11})) - \Delta^{-1}(E(\overline{v^1_{11}}))| + |\Delta^{-1}(E(v^1_{12})) - \Delta^{-1}(E(\overline{v^1_{12}}))| + \cdots + |\Delta^{-1}(E(v^4_{63})) - \Delta^{-1}(E(\overline{v^4_{63}}))|}{6}$$
$$s.t\begin{cases} x^1_{11} + x^1_{12} + \cdots + x^4_{63} = 13 \\ \text{All other constrints in model (26)} \end{cases} \tag{27}$$

Based on Theorems 1 and 2, an equivalent linear programming model can be obtained from the above model to facilitate the solution of the above model. By solving the above model, the optimal LDAMs $\{\ddot{V}^{1,*} = (\ddot{v}^{1,*}_{ij})_{6\times3}, \ldots, \ddot{V}^{4,*} = (\ddot{v}^{4,*}_{ij})_{6\times3}\}$ are obtained as adjustment suggestions for the preference-modification of the FMEA team. Let $V^k = (v^k_{ij})_{6\times3}$ $(k = 1, 2, 3, 4)$ be as given above, then $\ddot{V}^{k,*} = (\ddot{v}^{k,*}_{ij})_{6\times3}$ $(k = 1, 2, 3, 4)$ are shown as follows:

$\ddot{V}^{1,*}$: $\ddot{v}^{1,*}_{31} = \{(s_3, 1)\}$, $\ddot{v}^{1,*}_{32} = \{(s_1, 0.6), (s_2, 0.4)\}$, $\ddot{v}^{1,*}_{33} = \{(s_4, 0.5), (s_5, 0.5)\}$, $\ddot{v}^{1,*}_{51} = \{(s_0, 0.2), (s_1, 0.8)\}$, $\ddot{v}^{1,*}_{52} = \{(s_1, 0.3), (s_2, 0.7)\}$, $\ddot{v}^{1,*}_{53} = \{(s_2, 0.8), (s_3, 0.2)\}$, and $\ddot{v}^{1,*}_{ij} = v^1_{ij}$ for $(i,j) \neq (3,1) \wedge (3,2) \wedge (3,3) \wedge (5,1) \wedge (5,2) \wedge (5,3)$.

$\ddot{V}^{2,*}$: $\ddot{v}_{11}^{2,*} = \{(s_3, 0.3), (s_4, 0.7)\}$ $\ddot{v}_{12}^{2,*} = \{(s_1, 0.34), (s_2, 0.66)\}$ $\ddot{v}_{32}^{2,*} = \{(s_5, 0.4), (s_6, 0.6)\}$ $\ddot{v}_{41}^{2,*} = \{(s_5, 0.72), (s_6, 0.28)\}$ $\ddot{v}_{42}^{2,*} = \{(s_3, 0.46), (s_4, 0.54)\}$ $\ddot{v}_{52}^{2,*} = \{(s_2, 0.15), (s_3, 0.85)\}$ and $\ddot{v}_{ij}^{2,*} = v_{ij}^2$ for $(i,j) \neq (1,1) \wedge (1,2) \wedge (3,2) \wedge (4,1) \wedge (4,2) \wedge (5,2)$.

$\ddot{V}^{3,*}$: $\ddot{v}_{41}^{3,*} = \{(s_3, 0.44), (s_4, 0.56)\}$, and $\ddot{v}_{ij}^{3,*} = v_{ij}^3$ for $(i,j) \neq (4,1)$.

$\ddot{V}^{4,*}$: $\ddot{V}^{4,*} = \overline{V^4}$.

Let $\{\overline{V^1} = (\overline{v_{ij}^1})_{6\times3}, \dots, \overline{V^4} = (\overline{v_{ij}^4})_{6\times3}\}$ be the new LDAMs provided by the FMEA team members. When constructing $\overline{V^k} = (\overline{v_{ij}^k})_{6\times3}$ $(k = 1, 2, 3, 4)$, we suggest that $\overline{V^k} = \ddot{V}^{k,*}$. Obviously, the degree of consensus between $\{\overline{V^1}, \overline{V^2}, \overline{V^3}, \overline{V^4}\}$ is acceptable. Further, the individual preference vectors $\{\overline{PV^1}, \overline{PV^2}, \overline{PV^3}, \overline{PV^4}\}$ and collective preference vector $\overline{PV^c}$ associated with $\{\overline{V^1}, \overline{V^2}, \overline{V^3}, \overline{V^4}\}$ are generated by applying Eqs. (5) and (6), which are presented as follows: $\overline{PV^1} = (0.27, 0.73, 0.35, 0.49, 0.35, 0.68)^T$; $\overline{PV^2} = (0.47, 0.64, 0.62, 0.62, 0.47, 0.25)^T$; $\overline{PV^3} = (0.23, 0.84, 0.3, 0.67, 0.66, 0.67)^T$; $\overline{PV^4} = (0.8, 0.37, 0.12, 0.7, 0.54, 0.24)^T$; $\overline{PV^c} = (0.42, 0.66, 0.37, 0.62, 0.51, 0.46)^T$.

According to $\overline{PV^c}$, the consensual collective classifications of FMs can be obtained: $C_1 = \{FM_1, FM_3\}$, $C_2 = \{FM_5, FM_6\}$, and $C_3 = \{FM_2, FM_4\}$. Therefore, FM_2 and FM_4 are the most important FMs and therefore should be given sufficient attention to eliminate risks.

7 Sensitivity and Comparative Analysis

A sensitivity analysis and a comparative analysis are provided to assess the performance of the proposed consensus-based FMEA. The preference-modification is critical to achieving consensus because the original preferences provided by the FMEA team vary widely. Naturally, the FMEA team hope that the number of adjustment elements (AE) between the original LDAMs and adjusted LDAMs and the number of FM adjustments (FMA) between the original LDAMs and adjusted LDAMs are minimal in the consensus reaching process. Hence, AE and FMA are crucial criteria to assess the performance of consensus model. The lower values of AE and FMA indicate a better performance of consensus model.

7.1 Sensitivity Analysis

Here, Simulation method I is constructed to analyze the influence of the parameter values of $U = \{u_1, u_2, \dots, u_q\}$ and α on values of AE and FMA. The calculation process of Simulation method I is to randomly obtain the LDAMs of q FMEA members, and put the generated LDAMs into the MCOM. If the objective function value obtained by the MCOM is acceptable, then put the generated LDAMs into the TSCOM to obtain AE and FMA. Otherwise, regenerate the LDAMs of q FMEA members until the objective function value obtained by the MCOM under the generated LDAMs is acceptable. The following two combination of parameter scenarios are set: (i) $q = 4$, $m = 7$, $n = 3$, $h_1 = 3$, $h_2 = h_3 = 3$, $\lambda = (0.25, 0.25, 0.25, 0.25)^T$,

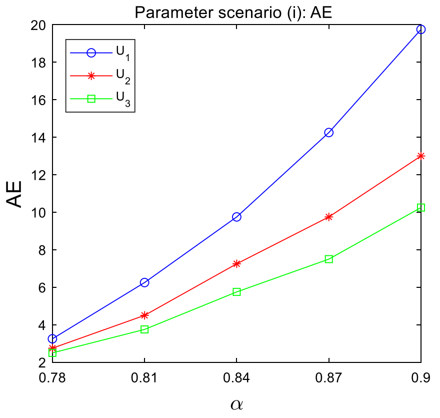

Fig. 2 The average values of AE and FMA under parameter scenario (i)

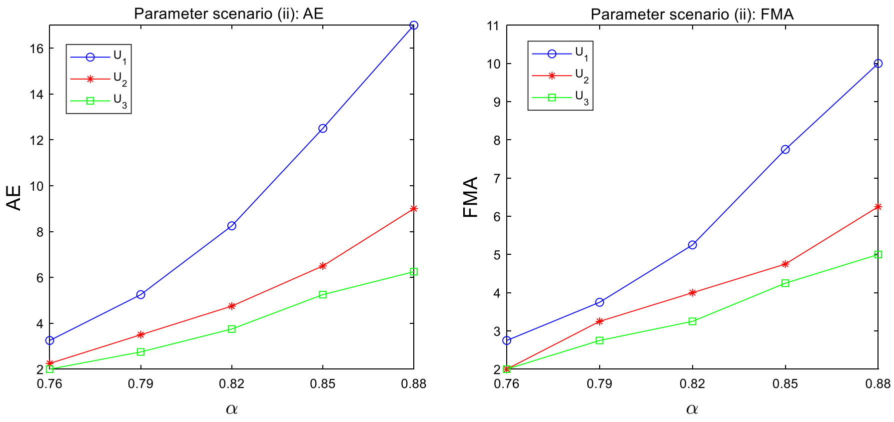

Fig. 3 The average values of AE and FMA under parameter scenario (ii)

$W = (0.3, 0.4, 0.3)^T$, $U_1 = \{0.12, 0.12, 0.12, 0.12\}$, $U_2 = \{0.18, 0.18, 0.18, 0.18\}$, $U_3 = \{0.24, 0.24{,}0.24, 0.24\}$, and $\alpha = \{0.78, 0.81{,}0.84, 0.87, 0.9\}$; (ii) $q = 6$, $m = 5$, $n = 3$, $h_1 = 3$, $h_2 = h_3 = 1$, $\lambda = (0.15, 0.25, 0.2, 0.1, 0.2, 0.1)^T$, $W = (0.4, 0.4, 0.2)^T$, $U_1 = \{0.1, 0.1, 0.1, 0.1, 0.1, 0.1\}$, $U_2 = \{0.2, 0.2{,}0.2, 0.2, 0.2, 0.2\}$, $U_3 = \{0.3, 0.3, 0.3, 0.3, 0.3, 0.3\}$, and $\alpha = \{0.76, 0.79, 0.82, 0.85, 0.88\}$.

Then, Simulation method I is operated 1000 times for parameter scenarios (i) and (ii) to derive the average values of AE and FMA, and the results are plotted in Figs. 2 and 3, respectively.

Figures 2 and 3 show that the values of AE and FMA increase with the increase of α; the value of AE decreases as U increases; in most cases, the value of FMA decreases as U increases; in rare cases, the value of FMA increases as U increases.

7.2 Comparative Analysis

The minimum adjustment distance consensus model (MADCM) has received widespread attention due to its efficiency in preserving the original preferences of decision makers in the process of consensus reaching (Xiao et al. 2020a; Xu et al. 2020; Zhang et al. 2020a). Here, we compare the proposed TSCOM with MADCM to assess the performance of our proposal based on the criteria: AE and FMA. In this study, the MADCM can be obtained by removing x_{ij}^k $(k \in Q, i \in M, j \in N)$ from model (24). Simulation methods I and II are utilized to conduct the comparative analysis. Simulation method I is the one used in Sect. 7.1. The calculation process of Simulation method II is to randomly obtain the LDAMs of q FMEA members, and put the generated LDAMs into the MCOM. If the objective function value obtained by the MCOM is acceptable, then put the generated LDAMs into the MADCM to obtain AE and FMA. Otherwise, regenerate the LDAMs of q FMEA members until the objective function value obtained by the MCOM under the generated LDAMs is acceptable. The following four combination of parameter scenarios are set: (i) $q = 5$, $m = 6$, $n = 3$, $h_1 = h_2 = h_3 = 2$, $\lambda = (0.2, 0.2, 0.2, 0.2, 0.2)^T$, $W = (0.3, 0.4, 0.3)^T$, $U_1 = \{0.1, 0.1, 0.1, 0.1, 0.1\}$, $U_2 = \{0.12, 0.1, 0.12, 0.11, 0.15\}$, $U_3 = \{0.1, 0.15, 0.1, 0.15, 0.15\}$, and $\alpha = \{0.75, 0.8, 0.85\}$; (ii) $q = 5$, $m = 6$, $n = 3$, $h_1 = h_2 = h_3 = 2$, $\lambda = (0.1, 0.3, 0.1, 0.3, 0.2)^T$, $W = (0.4, 0.2, 0.4)^T$, $U_1 = \{0.12, 0.12, 0.12, 0.12, 0.12\}$, $U_2 = \{0.12, 0.15, 0.12, 0.15, 0.12\}$, $U_3 = \{0.15, 0.12, 0.15, 0.15, 0.12\}$, and $\alpha = \{0.76, 0.81, 0.86\}$; (iii) $q = 6$, $m = 6$, $n = 3$, $h_1 = 3$, $h_2 = 2$, $h_1 = 1$, $\lambda = (0.15, 0.15, 0.15, 0.15, 0.2, 0.2)^T$, $W = (0.2, 0.6, 0.2)^T$, $U_1 = \{0.2, 0.2, 0.2, 0.2, 0.2, 0.2\}$, $U_2 = \{0.2, 0.25, 0.2, 0.25, 0.2, 0.25\}$, $U_3 = \{0.25, 0.25, 0.25, 0.2, 0.2, 0.2\}$, and $\alpha = \{0.8, 0.85, 0.9\}$; (iv) $q = 6$, $m = 6$, $n = 3$, $h_1 = 3$, $h_2 = 2$, $h_1 = 1$, $\lambda = (0.2, 0.1, 0.2, 0.1, 0.2, 0.2)^T$, $W = (0.25, 0.35, 0.4)^T$, $U_1 = \{0.18, 0.25, 0.18, 0.25, 0.18, 0.25\}$, $U_2 = \{0.2, 0.23, 0.25, 0.23, 0.2, 0.23\}$, $U_3 = \{0.23, 0.25, 0.2, 0.23, 0.23, 0.2\}$, and $\alpha = \{0.8, 0.85, 0.9\}$.

Then, Simulation methods I and II are operated 1000 times for parameter scenarios (i)—(iv) to derive the average values of AE and FMA, and the results are plotted in Figs. 4, 5, 6 and 7, respectively.

Figures 4, 5, 6 and 7 show that the values of AE and FMA obtained by the TSCOM are smaller than those obtained by MADCM under parameter scenarios (i)—(iv). This means that the proposed TSCOM has higher consensus efficiency compared to MADCM in terms of AE and FMA.

8 Conclusion

This study constructs a consensus-based FMEA that combines the multi-stage consensus model based on bounded confidences and linguistic distribution assessments with FMEA to derive the consensual collective classifications of FMs. In the consensus-based FMEA, the FMEA team utilizes LDAMs to express

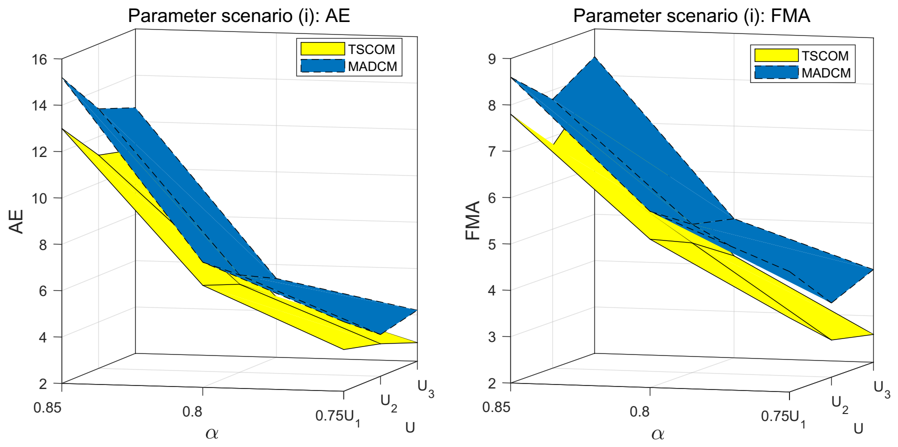

Fig. 4 The average values of AE and FMA obtained by TSCOM and MADCM under parameter scenario (i)

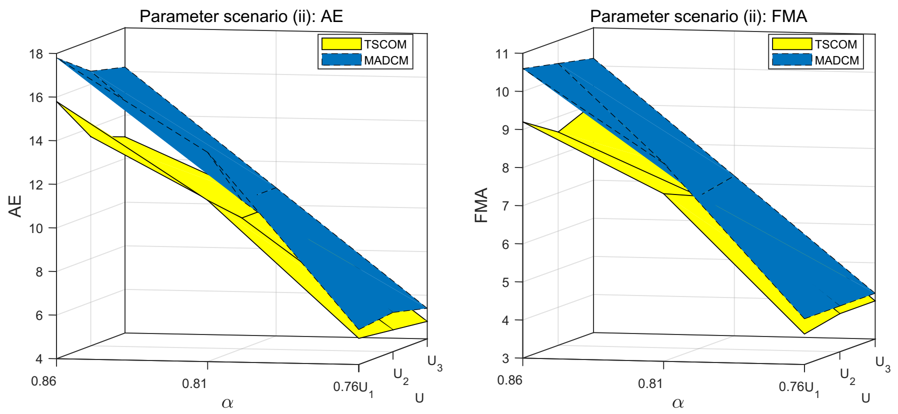

Fig. 5 The average values of AE and FMA obtained by TSCOM and MADCM under parameter scenario (ii)

their preferences for FMs against to risk factors. The MCOM and TSCOM are proposed to help the FMEA team obtain the consensual collective classifications of FMs. A case study regarding the problem of marine diesel engine crankcase explosion is offered to demonstrate the feasibility of the consensus-based FMEA. The results of sensitivity analysis show that the values of U and α have a significant effect on the values of AE and FMA. The results of comparative analysis show that the proposed TSCOM has higher consensus efficiency compared to MADCM in terms of AE and FMA.

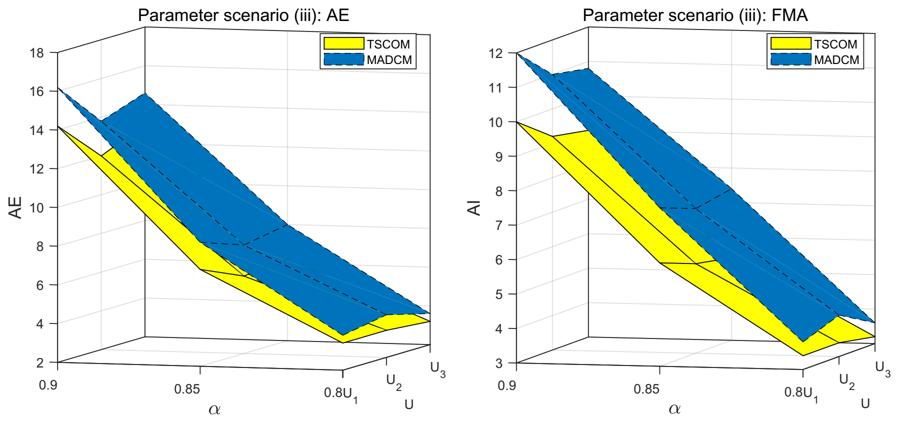

Fig. 6 The average values of AE and FMA obtained by TSCOM and MADCM under parameter scenario (iii)

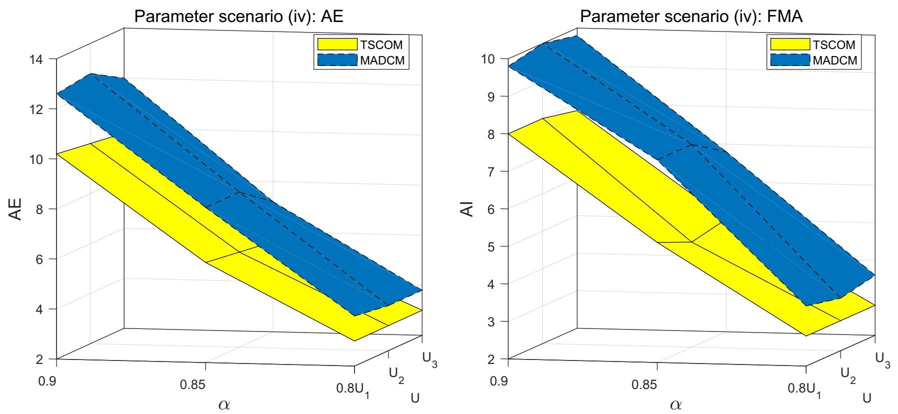

Fig. 7 The average values of AE and FMA obtained by TSCOM and MADCM under parameter scenario (iv)

Moreover, we point out the following research directions for future studies:

1. In reality, the behavior and psychology of decision makers, such as strategic manipulation, non-cooperative behavior, and private interests, have an important influence on consensus reaching (Xu et al. 2020; Zhang et al. 2020a). Therefore, it will be interesting to study the behavior and psychology of the FMEA team in the consensus process.
2. Moreover, several novel GDM methods are developed in recent years, including large-scale GDM methods (Chao et al. 2021; Gou et al. 2021), and flexible linguistic expressions-based GDM method (Wu et al. 2020). It is of great significance to export these new GDM approaches into FMEA to improve the implementation efficiency of FMEA.

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