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Modeling and Implementation of a New Negotiation Decision Support System for Conflict Resolution Under Uncertainty

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Abstract

The dynamic nature of negotiation and decision-making necessitates a flexible negotiation decision support system (NDSS) that can systematically investigate real-world strategic conflicts with a wide range of preference information with uncertainty. However, most of the existing decision support system, such as GMCR II under the framework of the graph model for conflict resolution (GMCR), is only capable of handling simple preference. This research develops a new NDSS based on an algebraic representation to implement modeling and analysis for GMCR. More specifically, an algebraic approach using option prioritizing to generate unknown preference is proposed and then implemented to build an efficient and flexible modeling and analysis system. This new system may provide decision makers with valuable strategic insights for negotiations and decisions, especially in dynamic conflict environments. The procedure of applying the proposed theory and system is demonstrated using a South Sudan conflict with unknown preference involving the third-party intervention.

Keywords Negotiation decision support system · Graph model for conflict resolution · Option prioritizing · Uncertain preference

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1 Introduction

Preference plays an important role in modeling and resolving real-world strategic conflicts. However, deriving precise preference information is rather difficult given the dynamic nature of negotiation and decision-making. For example, when facing a dispute over a polluted underground aquifer, an environmental-oriented local government may prefer that an industrial enterprise minimizes pollution of the underground aquifer supplying water to the region; while an economical-oriented local government may consider supporting a polluting enterprise due to the urgent need for jobs. Either of these two tendencies may describe the local government's possible preference, but the actual preference remains uncertain.

The graph model for conflict resolution (GMCR) was originally proposed by Kilgour et al. (1987) to provide a convenient and effective means to model and analyze a strategic conflict. In 2004, Li et al. (2004) extended GMCR to include uncertain preference in terms of the logical representation. Despite the simplicity and descriptivity, the logical representation is highly unfriendly to software developments and technical adoptions. This situation was eased later by the matrix representation for conflict resolution (MRCR) proposed in Xu et al. (2009) considering simple preferences and Xu et al. (2011) with uncertain preferences. Nevertheless, existing literature on uncertain preferences focused on the analysis stage without addressing the process of generating such preferences practically.

Under the framework of GMCR, three methods can be used to derive the preference: direct ranking, option weighting, and option prioritizing proposed by Fang et al. (2003a). Let m and h express the number of feasible states and the number of options, respectively. The number of states, $m = 2^h$, can be extremely large compared to h. Therefore, it is more efficient to apply the two option-based approaches, i.e., option weighting or option prioritizing, than obtaining the direct ranking of all feasible states. Particularly, the option prioritizing technique is based on the option statements that directly reflect decision makers' (DMs') preference attitudes when they negotiate in a conflict. Hou et al. (2015) and Yu et al. (2016) respectively presented methods for the three-level preference and for the unknown preference elicitation based on option prioritization. They looked into the uncertainty of the same statement level in the uncertain connectives, yet neglected the uncertain situation arising from different statement levels. They also failed to point out the layer number of statements in the conditional connective, which may cause unnecessary misunderstandings. Furthermore, the descriptive representation of option prioritizing for unknown preferences in that work makes coding difficult to implement their proposed method.

This paper constructs an algebraic approach built upon the option prioritizing technique to generate uncertain preferences that may directly reflect DMs' strategy, strategic weight and strategic attitudes during their negotiation, intervention, and decision. Compared with the method proposed by Yu et al. (2016), our approach is more effective and convenient for computer implementation and technology adaption. Moreover, because of the inherent link between modeling and analysis, the proposed algebraic approach, together with the matrix representation for stability calculation under uncertainty, establishes an integrated paradigm of algebraic expression to formulate and investigate conflicts in practice. For practical computational assistance, an effective decision support system (DSS) is necessitated.

Written in Visual C++, GMCR II was a DSS developed for analyzing conflicts with simple preference by employing the logical definitions of GMCR (Fang et al. 2003a, 2003b). GMCR II can generate preferences using the option prioritizing approach, but is difficult to modify or adjust to unknown preferences due to the nature of logical representations. Jiang et al. (2015) presented a DSS, MRCRDSS used for many areas (Xu et al. 2011), based on MRCR. That system is capable of handling unknown preference, yet requires direct input of unknown preference over states for stability calculation. That is to say, MRCRDSS lacks an efficient modeling module to directly derive unknown preferences. A new DSS for GMCR, called GMCR+ within a hybrid system combining logical and matrix representations together was designed by Kinsara et al. (2015). GMCR+ is also suitable for simple preference conflicts. Furthermore, the most widely used development architectures of DSSs are usually based on Microsoft Foundation Classes (MFC), for example, GMCR II and MRCRDSS. However, MFC can be extremely cumbersome, leading to low development efficiency. The User Interface (UI) function of MFC is neither sufficient nor user-friendly.

To implement the proposed integrative algebraic system, a comprehensive web-based negotiation decision support system (NDSS) is designed in this paper for conflict resolution under preference uncertainty. For the sake of simplicity, usability, and function completeness, the Browser/Server (B/S) structure is adopted, where the browser and server are respectively constructed with ASP.NET and SQL server. Storing core functions in the server and allowing user date exchange via the browser, this structure can simplify the system as well as decrease the costs of further development and system maintenance. In more detail, ASP.NET is a technical standard that separates the front code and back code. The separation greatly reduces the coupling degree of software components. ASP.NET also provides a higher level of control for the developers to facilitate the programming process. Meanwhile, the SQL server ensures data security, storage, and recovery, and is easy to be integrated with many other server software. Compared with the GMCR II MFC-based system, the newly developed NDSS is advantageous in providing better user accessibility and more concise result presentation. The modeling module of GMCR II for simple preference is implemented; the preference module, specifically the option prioritizing approach, for simple preference is extended to uncertain preference illustrated by preference tree; and the analysis module for calculating stabilities with both simple and unknown preferences is re-developed.

The remainder of the paper is organized as follows. Sections 2 and 3 respectively provides modeling procedure to generate simple and unknown preferences based on option prioritizing. Section 4 contains a case study of the South Sudan conflict with unknown preferences involving the third party intervention and demonstrates how to employ the new NDSS to resolve the conflict. Finally, concluding remarks and possible future directions are presented in Sect. 5.

2 Graph Model in Option Form for Simple Preference

Three preference structures, simple preference, strength of preference, and unknown preference have been proposed and were introduced into GMCR (Fang et al. 1993; Hamouda et al. 2004; Li et al. 2004). However, the three kinds of preferences were generated using direct ranking on states, which are only suitable for some small conflict cases. The ordinal preference, cardinal preference and relative preference are often used by DMs to present their favorite based on their information resources. The ordinal preference is to rank states from most to least with ties allowed, the cardinal preference denotes to assign a real number to a state. The graph model in this research only needs the relative preference information for each DM. The preference produced using option prioritization in this research is ordinal (Fang et al. 2003a).

2.1 States in Option Form

In a conflict situation, a DM may face various courses of actions which are called the DM's options that directly reflect the DM's attitudes and strategies in the conflict. This paper is based on option statements, so we introduce some important concepts related to options as follows. Let N denote a DM set with n DMs and $O_i = \{o_{i1}, \dots, o_{ij}, \dots, o_{ik_i}\}$ is the option set of DM i, where o_{ij} is DM i's j^{th} option. Then, the set of all options with the size h in a conflict is $O = \bigcup_{i \in N} O_i$ in which index i indicates which DM controls the options.

A state is formed when each DM has selected a specific strategy. In other words, for each option the DM controlling the option has decided whether or not he or she will choose it. The formal definition of a state based on options is as follows (Hou et al. 2015; Xu et al. 2018).

Definition 1 *(State in Option Form)* Let N denote a DM set. $O = \bigcup_{i \in N} O_i$ is the set of all options in a conflict in which $O_i = \{o_{i1}, \dots, o_{ij}, \dots, o_{ik_i}\}$ is DM i's option set for $i = 1, 2, \dots, n$. State s may be defined as the h-dimensional column vector $f_s = [g^s(O_1), \dots, g^s(O_i), \dots, g^s(O_n)]^T$ and $g^s(O_i) = [g^s(o_{i1}), \dots, g^s(o_{ik_i})]^T$ in which

$$g^s(o_{ij}) = \begin{cases} 1 & \text{if DM } i \text{ selects option } o_{ij}, \text{for } i = 1, 2, \dots, n, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, f_s, the h-dimensional column vector is used to express state s that denotes all DMs' strategy combination. If m is the number of states, then $m = 2^h$. In fact, the number of feasible states is less than m because some infeasible states are removed.

2.2 Simple Preference Ranked Using Option Statement Prioritization

If states are ranked an order to produce DMs' preferences, we call that the way is a direct ranking approach. The direct ranking approach over states is suitable only for analysis stage for some small conflicts. The ranking approach based on "option" called option prioritization that facilitates preference generated efficiently. The preference ranking approach of option prioritization in GMCR was designed based on the "preference tree" method originally suggested by Fraser and Hipel (1988). In order to understand the option prioritization clearly, the option statement is defined first (Fang et al. 2003a; Hou et al. 2015; Xu et al. 2018).

Definition 2 *(Option Statement)* Each DM's option statements consist of options of all DMs with logical connectives as shown in Table 1 in which "−" , "&" , and "|" stand for nonconditional logical relations "*not*", "*and*", and "*or*", respectively, as well as conditional relationships between two nonconditional statements, "*IF*" and "*IFF*".

In option prioritizing, option statements are ranked different weights according to each DM's strategy interest. Each option statement is assigned a truth value, either True (T) or False (F), at a particular state. The relative importance of option statements is reflected by its position (or level) in the list: the higher an option statement is put in the list, the more important in determining the DM's preferences.

For simple preference, DM i's preference between any two states is determined using the option statements $\Omega_1, \Omega_2, \dots, \Omega_k$ in the order of priority in which k is the number of option statements for DM i. Let S denote the state set. State $s \in S$ is preferred to state $q \in S (s \neq q)$ for DM i if and only if there exists j, $1 \leq j \leq k$, such that

$$\begin{aligned}
&\Omega_1(s) = \Omega_1(q)\\
&\Omega_2(s) = \Omega_2(q)\\
&\dots\dots\\
&\Omega_{j-1}(s) = \Omega_{j-1}(q)\\
&\Omega_j(s) = T \ and \ \Omega_j(q) = F.
\end{aligned}$$

State s is assigned a "score" $\Psi(s)$ by DM i according to its truth values when the statements are employed. $\Psi_j(s)$ is defined by

Table 1 Truth-value for simple preference connectives

A	B	−A	A & B	A|B	B *IF* A	B *IFF* A
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

$$\Psi_j(s) = \begin{cases} 2^{k-j} & \textit{if } \Omega_j(s) = T \text{ for DM i,} \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

and $\Psi(s) = \sum_{j=1}^{k} \Psi_j(s)$ in which $1 \leq j \leq k$ is the level of the option statement $\Omega_j(s)$ locates. For the first level ($j = 1$), the value or weight of the option statement $\Psi_j(s) = 2^{k-1}$ is the largest when $\Omega_j(s)$ is selected by DM i as "T". This idea determining simple preference is extended to produce unknown preference using option statement prioritization in this research.

2.3 Graph Model in Option Form

The graph model was defined based on states in previous conflict situations (Fang et al. 1993; Xu et al. 2018) that cannot directly reflect the processes of conflicts and negotiations. The option form of a graph model is formally defined as follows.

Definition 3 (*Graph Model in Option Form*) A graph model GD in option form is usually written as $GD = \langle N, \{O_i\}_{i \in N}, S, \{\Psi^i\}_{i \in N}, \{G_i\}_{i \in N} \rangle$, where

- $N = \{1, 2, \ldots, n\}$ is a non-empty set of DMs;
- for each DM $i \in N$, O_i is the non-empty option set;
- $S = \{s_1, s_2, \ldots, s_m\}$ defined in options (see Definition 1) is a non-empty set of feasible states and m is the number of states;
- for each DM $i \in N$, Ψ^i represents i's simple preference, where $\Psi^i(s_k) \geq \Psi^i(s_t)$ means that DM i prefers s_k to s_t or indifferent between s_k and s_t;
- for each DM $i \in N$, G_i denotes DM i's direct graph that may present movements among states in S.

3 Generate Unknown Preference Based on Option Statement Prioritization

The theories of generating simple preference and strength of preference based on option prioritization were developed (Fang et al. 2003a; Hou et al. 2015) and implemented into the new NDSS. The theory of generating unknown preference is proposed in this paper using option prioritization that is very efficient for modeling complex conflicts. This approach extends and improves the method designed by Yu et al. (2016) and is implemented into the new NDSS. The modeling and implementation of the new NDSS for conflict resolution within uncertainty are discussed as follows.

3.1 Introduction to the New NDSS for Unknown Preference

How to develop a unique representation of conflict resolution that is easy to code and adapt to new procedures? How to design a comprehensive negotiation decision support system for conflict analysis to include modeling and analysis functions? These are essential motivations to develop an integrated algebraic approach for GMCR. Many researches have been developed for the analysis stage within a logical system for simple preference (Fang et al. 1993) in GMCR II and the analysis stage within an algebraic system in MRCRDSS (Xu et al. 2018) including unknown preference. Until now, no DSS may analyze a conflict within a uncertain environment including modeling and analysis stages. The new NDSS has many functions that may model, analyze, and draw graph model and preference tree.

Figure 1 is the interface of the NDSS designed for unknown preference. The theory of analysis process is based matrix representation of conflict resolution with unknown preference (Xu et al. 2011, 2018). The GMCR II does not contain the function to draw a graph, but the NDSS may draw the colour direct graph in which different colour denotes different DM. The output functions of NDSS can be summarized as follows.

- Output a direct graph after inputting DMs, each DM's options and some additional information to describe DMs' movements among the feasible states;
- Output preference tree and unknown preference;
- Output 16 stabilities within unknown preference (Li et al. 2004; Xu et al. 2011);
- Output evolutionary conflict process using the color pathes to present DMs' negotiation procedures.

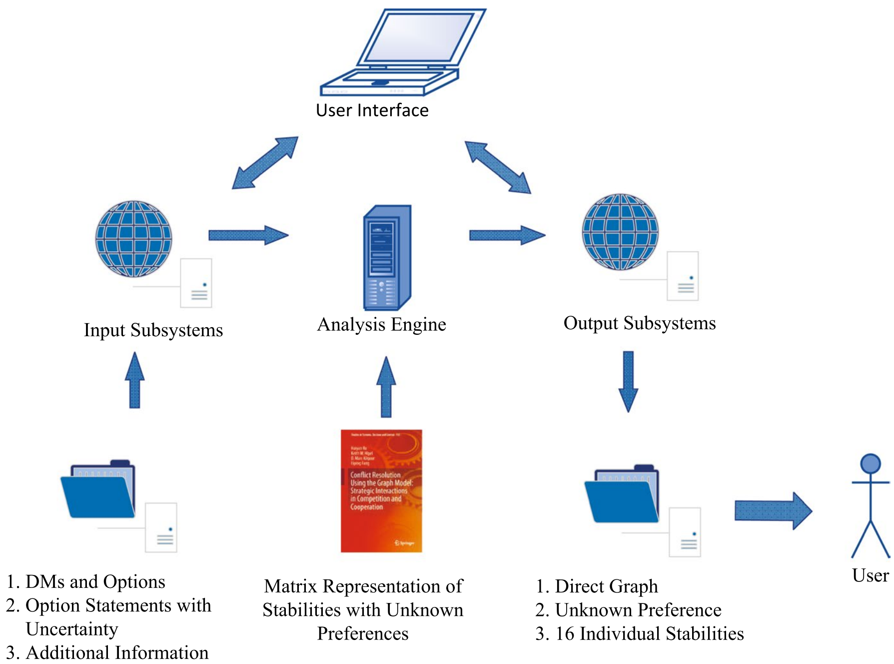

Fig. 1 Interface of the NDSS for unknown preference

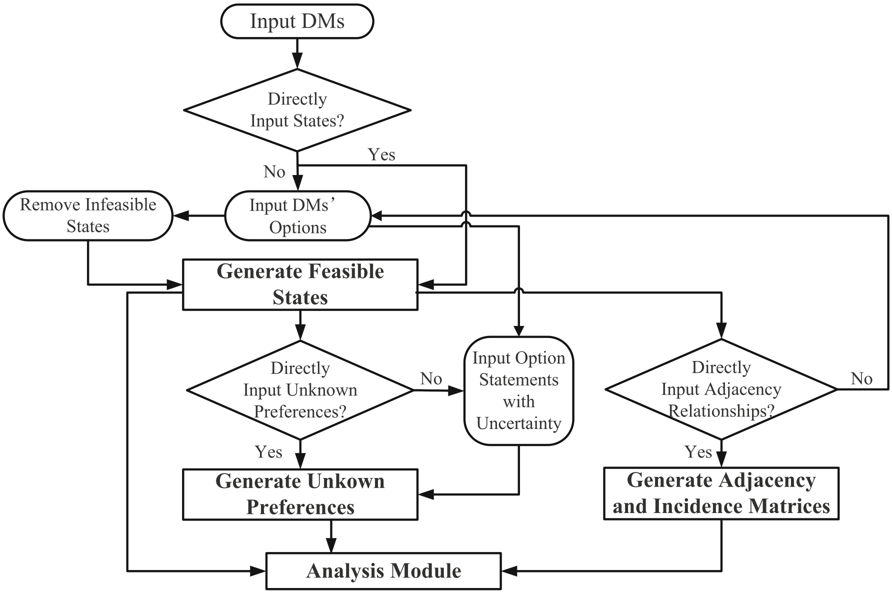

Fig. 2 Subsystem of input in NDSS

Here, Fig. 2 (updated from (Xu et al. 2018)) depicts the input system for modeling stage in NDSS, including the analysis module. The modeling module contains directly inputting unknown preference that are often employed by theoretical analysts for finding equilibria of conflicts. The modeling stage to produce preference based on option prioritization will be discussed in Sect. 3.2.

To incorporate preference uncertainty into the graph model methodology, Li et al. (2004) proposed DM i's a triple of relations $\{\succ_i, \sim_i, U_i\}$ on S, where $s \succ_i q$ indicates the strict preference, $s \sim_i q$ indicates the indifference, and sU_iq means that DM i may prefer state s to state q, may prefer q to s, or may be indifferent between s and q. It is assumed that the preference relations of each DM $i \in N$ have the following properties:

(i) $\succ_i$ is asymmetric.
(ii) $\sim_i$ is reflexive and symmetric.
(iii) U_i is symmetric.
(iv) $\{\succ_i, \sim_i, U_i\}$ is strongly complete.

Property (iv) implies that, for any $s, t \in S$, exactly one of the following statements is true: $s \succ_i t$, $t \succ_i s$, $s \sim_i t$, or $s\ U_i\ t$. In fact, unknown preference, U, is intransitive. It means that although $s\ U_i\ q$ and $q\ U_i\ k$, DM i's preference between s and k may be certain.

With regard to the simple preference, the option prioritizing technique based on the "preference tree" method was proposed by Fraser and Hipel (1988). The detailed introduction of option prioritization can be understood and learned in Fang et al.

(2003a). Most of the unknown preference representation was employed for the direct ranking method (Li et al. 2004). But it is complicated to model preferences by pairwise comparison over states with regard to direct ranking technique for unknown preference. The ranking approach based on option prioritization for unknown preference is introduced in the following section.

3.2 Theory Design of Generating Unknown Preference

Under the simple preference, suppose that a DM's option statements are $Q = \{\Omega_1, \Omega_2, \dots, \Omega_k\}$ using option prioritization in a conflict for $k \geq 1$. The statement Ω_j in Q is only one preference statement at the j^{th} level for $0 < j \leq k$. In some situations, nevertheless, DM's preference statement is uncertain. In other words, the preference statements at the j^{th} level may be more than one. The following is the definition that the representation method to express the uncertainty of the j^{th} preference statement for DM i.

Definition 4 Assume that there are two different option statements Ω_j^1 and Ω_j^2 at the j^{th} level for $0 < j \leq k$. The relation between the two diverse statements at the j^{th} level is unknown for the focal DM. Here, a new logical connective "**U**" is defined to express the unknown relation between the two statements. Then, $\Omega_j^1 \mathbf{U}\ \Omega_j^2$ indicates that the DM is uncertain which one of Ω_j^1 and Ω_j^2 is his choice for the j^{th} option statement.

Notably, the symbol "**U**" can be applied to link two or more preference statements at the same level. For instance, there might be three possible statements at the j^{th} level for the Q, $\Omega_j = \{\Omega_j^1, \Omega_j^2, \Omega_j^3\}$, which can be represented by $\Omega_j^1\ \mathbf{U}\ \Omega_j^2\ \mathbf{U}\ \Omega_j^3$. In addition, note that "**U**" is totally different from "U" for uncertain preference among states (Li et al. 2004). "**U**" is used to connect two or more option statements at the same level, while "U", as a preference connective, is employed to connect two or more feasible states in a conflict.

In some situations, there might have two or more unknown relations in the set Q. The option statement $\Omega_{j+l}^1 \mathbf{U}\ \Omega_{j+l}^2$ at the $(j + l)^{th}$ level, for example, might follow $\Omega_j^1 \mathbf{U}\ \Omega_j^2$ at the j^{th} level. In this case, there would be $2^2 = 4$ potential combinations for the statement set Q. Therefore, $Q = \{Q_1, Q_2, Q_3, Q_4\}$ in which

$$Q_1 = \{\Omega_1, \dots, \Omega_j^1, \dots, \Omega_{j+l}^1, \dots, \Omega_k\};\ Q_2 = \{\Omega_1, \dots, \Omega_j^1, \dots, \Omega_{j+l}^2, \dots, \Omega_k\};$$
$$Q_3 = \{\Omega_1, \dots, \Omega_j^2, \dots, \Omega_{j+l}^1, \dots, \Omega_k\};\ Q_4 = \{\Omega_1, \dots, \Omega_j^2, \dots, \Omega_{j+l}^2, \dots, \Omega_k\}.$$

Note that it is not allowed for the same option statement emerging at some levels in the set Q twice or more. If Ω_{j+l}^1 is identical with Ω_j^1, Q_1 is an infeasible or impossible option statement set, so $Q = \{Q_2, Q_3, Q_4\}$.

As for $\Omega_j^1\ \mathbf{U}\ \Omega_j^2$ and $\Omega_{j+l}^1\ \mathbf{U}\ \Omega_{j+l}^2$, in some cases, Ω_j^1 or Ω_j^2 is the condition of the occurrence of Ω_{j+l}^1 or Ω_{j+l}^2. For instance, Ω_{j+l}^2 might arise only when Ω_j^1 occurs.

Table 2 Uncertain connectives for option statement

Symbol	Nomenclature	Example	Explanation
U	Uncertain relation	$\Omega_j^1\mathbf{U}\Omega_j^2$	There are two statements at j^{th} level, Ω_j^1 and Ω_j^2, but the DM is not uncertain which one to choose
Λ	Conditional relation	$\Omega_{j+l}\Lambda\Omega_j$	For statements Ω_{j+l} at $(j+l)^{th}$ level and Ω_j at j^{th} level prior $(j+l)^{th}$, if Ω_j occurs, then Ω_{j+l} exists
@	Postfix of conditional relation	$\Omega^1\Lambda\Omega^2@l$	Only when Ω^2 occurs at the l level, Ω^1 exists

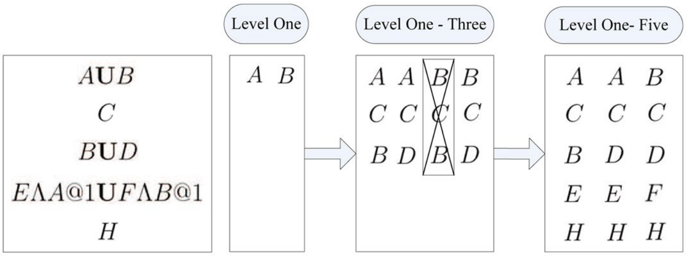

Fig. 3 Convert uncertain statement representation to certain statement situation

Then, Q_4 is infeasible because Ω_j^1 is the condition for Ω_{j+l}^2 appearance in Q_4. We can conclude that $Q = \{Q_1, Q_2, Q_3\}$. The condition relation for uncertain statements is defined as follows.

Definition 5 Providing that Ω_{j+l}^1 or Ω_{j+l}^2 might occur only in the option statement set when Ω_j^1 or Ω_j^2 exists at the j^{th} level, respectively, then the condition relation can be expressed using two logical connectives "Λ" and "@", that is $\Omega_{j+l}^1\Lambda\Omega_j^1@j\mathbf{U}\Omega_{j+l}^2\Lambda\Omega_j^2@j$.

The symbol "Λ" in Definition 5 indicates that the statement before "Λ" can exist only when the statement after "Λ" occurs. The number after "@" is the located level of the statement before "@". This notation "@" is very important because the identical statements may occur twice or more at different statement levels. If the exact level hasn't been pointed out, the statement set Q may be different. Yu et al. (2016) haven't considered the situation. Uncertain statement connectives contain seven logical relations. The five relations for simple preference presented in Table 1 and the others are explained in Table 2.

For example, the set of option statements with uncertainty is $Q = \{A\mathbf{U}B, C, B\mathbf{U}D, E\Lambda A@1\mathbf{U}F\Lambda B@1, H\}$ in which A, B, C, D, E, F, and H are certain option statements. Hence, after removing an infeasible statement set, we can get the three feasible statement sets without uncertainty $Q = \{Q_1, Q_2, Q_3\}$ with $Q_1 = \{A, C, B, E, H\}$, $Q_2 = \{A, C, D, E, H\}$, and $Q_3 = \{B, C, D, F, H\}$. The process to convert the uncertain statements in the unknown preference to the certain statements in the simple preference is presented in Fig. 3.

From the above discussions, the statements including uncertainty may be converted to the certain statement representation in the proposed NDSS using the seven logical connectives "−" , "&" ,"|", "*IF*", "*IFF*", "**U**", and "Λ" with a special notation "@".

If the option statement Ω_j is uncertain including $\Omega_j = \{\Omega_j^1, \Omega_j^2, \dots, \Omega_j^l\}$, $l > 1$, and the probability of selecting one of them is $p_{j1}, p_{j2}, \dots, p_{jl}$, respectively and $\sum_{t=1}^{l} p_{jt} = 1$. Then, $\Psi_j^t(s)$ gives the score for state s based upon statement Ω_j^t, $1 \le t \le l$. It can be calculated by

$$\Psi_j^t(s) = \begin{cases} 2^{k-j} & \text{if } \Omega_j^t(s) = T, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Hence, $\Psi_j(s)$ can be computed by

$$\Psi_j(s) = \sum_{t=1}^{l} p_{jt} \Psi_j^t(s) \tag{3}$$

and $\Psi(s) = \sum_{j=1}^{k} \Psi_j(s)$. Notice that the preference generated using the option prioritization are ordinal.

In fact, option statements with uncertainty are converted to a set of option statements with certain preference first as shown in Fig. 3. Then, the NDSS generates a group of preferences having inconsistence that means some preferences with uncertainty.

3.3 Model Construction of Generating Unknown Preference

Through employing the three connectives related to unknown preference and calculating the scores of all feasible states in a conflict based on Eq. (3), then, we can obtain the unknown preference by comparing the scores of any two states. We use a small case to describe the procedure.

For instance, the option statement set of a DM in a conflict is $Q = \{A\mathbf{U}B, C, B\mathbf{U}D, E\Lambda A@1\mathbf{U}F\Lambda B@1, H\}$, whereby, A, B, C, D, E, F, H are option statements. From the structure of Q, we can see that the relation between two option statements A and B at the first level is uncertain and both two are likely to occur. Let $p_{11}, p_{12}, p_{31}, p_{32}, p_{41}, p_{42}$ denote the possibility of occurrence for A, B, D, E and F, respectively. Here, the six variables with value 0 or 1 and $p_{11} + p_{12} = 1, p_{31} + p_{32} = 1, p_{41} + p_{42} = 1$.

Notably, the first and third levels in Q are $A\mathbf{U}B$ and $B\mathbf{U}D$, therefore, $p_{12} + p_{31} \le 1$ owing to that the statement B at the third level cannot happen while statement B at the first level in Q occurs, namely the same statement can occur once at most.

Table 3 Statement and probability for $Q = \{A\mathbf{U}B, C, B\mathbf{U}D, E\Lambda A@1\mathbf{U}F\Lambda B@1, H\}$

Statement	Probability	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	
$A\mathbf{U}B$	$A(p_{11})$	T	T	T	T	F	F	F	F	T	T	T	T	2^4
	$B(p_{12})$	F	F	F	F	T	T	T	T	F	F	F	F	
C	C	T	T	T	T	T	T	F	F	F	F	F	F	2^3
$B\mathbf{U}D$	$B(p_{31})$	F	F	F	F	T	T	T	T	F	F	F	F	2^2
	$D(p_{32})$	T	T	T	F	F	F	T	T	T	F	F	F	
$E\Lambda A@1\mathbf{U}$	$E(p_{41})$	T	T	F	F	T	T	F	F	T	T	F	F	2^1
$F\Lambda B@1$	$F(p_{42})$	F	F	T	T	F	F	T	T	F	F	T	T	
H	H	T	F	T	F	T	F	T	F	T	F	T	F	2^0

Table 4 Score for information in Table 3

State	Score	State	Score
s_1	$16p_{11} + 4p_{32} + 2p_{41} + 9$	s_7	$16p_{12} + 2p_{42} + 5$
s_2	$16p_{11} + 4p_{32} + 2p_{41} + 8$	s_8	$16p_{12} + 2p_{42} + 4$
s_3	$16p_{11} + 4p_{32} + 2p_{42} + 9$	s_9	$16p_{11} + 4p_{32} + 2p_{41} + 1$
s_4	$16p_{11} + 2p_{42} + 8$	s_{10}	$16p_{11} + 2p_{41}$
s_5	$16p_{12} + 4p_{31} + 2p_{41} + 9$	s_{11}	$16p_{11} + 2p_{42} + 1$
s_6	$16p_{12} + 4p_{31} + 2p_{41} + 8$	s_{12}	$16p_{11} + 2p_{42}$

Besides, with regard to $E\Lambda A@1\mathbf{U}F\Lambda B@1$, we have $p_{41} = 0 \text{ } if \text{ } p_{11} = 0$, and $p_{42} = 0 \text{ } if \text{ } p_{12} = 0$.

Suppose the conflict has twelve states as shown in Table 3. "T" or "F" represents that option statement takes a truth value, either True (T) or False (F), at a particular state. The score of all states can be calculated based on Eq. (3) and the computing result is shown in Table 4. Then, we can obtain the unknown preference over the twelve states in Table 3 by calculating the difference between the two states. The details are presented as follows.

For states s, q among the twelve states in Table 3, the scores of them are $\Psi(s)$ and $\Psi(q)$, respectively, then the difference of $\Psi(s)$ and $\Psi(q)$ is expressed as $D(s, q)$, namely $D(s,q) = \Psi(s) - \Psi(q)$. Therefore, the maximum and minimum values of $D(s, q)$ which is a function of the six variables $p_{11}, p_{12}, p_{31}, p_{32}, p_{41}$, and p_{42} can be calculated under the conditions: $p_{11} + p_{12} = 1, p_{31} + p_{32} = 1, p_{41} + p_{42} = 1, p_{12} + p_{31} \leq 1, p_{41} = 0 \text{ } if \text{ } p_{11} = 0, p_{42} = 0$ $if \text{ } p_{12} = 0$, $p_{11}, p_{12}, p_{31}, p_{32}, p_{41}, p_{42} = \{0, 1\}$. Therefore, the maximum and minimum values of $D(s, q)$, namely $D_{max}(s, q)$ and $D_{min}(s, q)$, can be obtained by the following two 0-1 linear programming.

$$\begin{array}{ll}\underset{s,q\in S}{\text{maximize}} & \mathrm{D_{max}(s,\,q)} = \Psi(\mathrm{s}) - \Psi(\mathrm{q}) \\ \text{subject to} & p_{11}+p_{12}=1, p_{31}+p_{32}=1, p_{41}+p_{42}=1; \\ & p_{12}+p_{31}\le 1; \\ & p_{41}=0 \textit{ if } p_{11}=0, p_{42}=0 \textit{ if } p_{12}=0; \\ & p_{11},p_{12},p_{31},p_{32},p_{41},p_{42}=\{0,1\}.\end{array} \tag{4}$$

$$\begin{array}{ll}\underset{s,q\in S}{\text{minimize}} & \mathrm{D_{min}(s,\,q)} = \Psi(\mathrm{s}) - \Psi(\mathrm{q}) \\ \text{subject to} & p_{11}+p_{12}=1, p_{31}+p_{32}=1, p_{41}+p_{42}=1; \\ & p_{12}+p_{31}\le 1; \\ & p_{41}=0 \textit{ if } p_{11}=0, p_{42}=0 \textit{ if } p_{12}=0; \\ & p_{11},p_{12},p_{31},p_{32},p_{41},p_{42}=\{0,1\}.\end{array} \tag{5}$$

Then the preference ranking of the two states s and q can be determined by the following model.

Model A For $s, q \in S$, their scores are $\Psi(s)$ and $\Psi(q)$, respectively, and the difference of $\Psi(s)$ and $\Psi(q)$ is $D(s, q)$ which is a function of some variables valuing 0 or 1. The maximum and minimum values of $D(s, q)$ are $D_{max}(s, q)$ and $D_{min}(s, q)$, respectively, then the preference ranking of state s and q is

$$\begin{cases} s \succ q & \text{if } D_{min}(s,q) > 0, \\ s \sim q & \text{if } D_{max}(s,q) = D_{min}(s,q) = 0, \\ sUq & \text{if } D_{min}(s,q) \le 0 \le D_{max}(s,q) \text{ and } D_{min}(s,q) \ne D_{max}(s,q), \\ q \succ s & \text{if } D_{max}(s,q) < 0. \end{cases} \tag{6}$$

In Table 4, the difference of $\Psi(s_1)$ and $\Psi(s_2)$ is

$$D(s_1,s_2) = 16p_{11} + 4p_{32} + 2p_{41} + 9 - 16p_{11} + 4p_{32} + 2p_{41} + 8 = 1,$$

then, we can conclude that $D_{max}(s_1,s_2) = D_{min}(s_1,s_2) = 1$. Therefore, the preference ranking of s_1 and s_2 is $s_1 \succ s_2$ based on the Eq. (6) of Model A. As to s_1 and s_3, $D(s_1,s_3) = 2p_{41} - 2p_{42}$, hence $D_{max}(s_1,s_3) = 2$ and $D_{min}(s_1,s_3) = -2$ through Eqs. (4) and (5), so s_1Us_3. In the same way, the unknown preference over the twelve states in Table 3 is, ultimately,

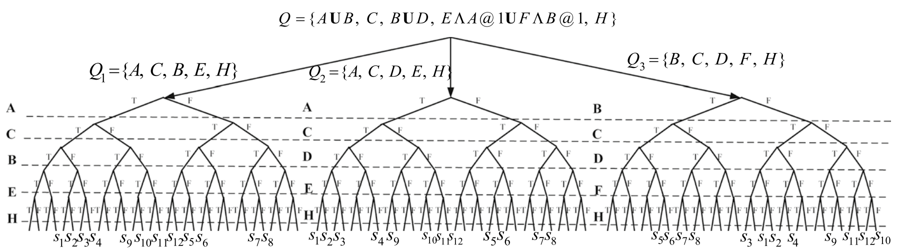

Fig. 4 Tree representation for statements $Q = \{A\mathbf{U}B, C, B\mathbf{U}D, E\Lambda A@1\mathbf{U}F\Lambda B@1, H\}$

$$\{s_1 \succ s_2 \succ s_9\}\ U\ \{s_3 \succ s_5\}\ U\ \{s_4 \succ s_{11} \succ s_{12}\}\ U\ \{s_7 \succ s_8\}\ U\ s_6\ U\ s_{10}.$$

To display the ranking procedure intuitively, the tree presentation of statement structure in option prioritizing for simple preference suggested by Fang et al. (2003a) is extended to present for unknown preference in this research, and show the structure of the extended tree with the small case presented in Fig. 4.

As for $Q = \{A\mathbf{U}B, C, B\mathbf{U}D, E\Lambda A@1\mathbf{U}F\Lambda B@1, H\}$, hence $Q = \{Q_1, Q_2, Q_3\}$, whereby, $Q_1 = \{A, C, B, E, H\}$, $Q_2 = \{A, C, D, E, H\}$, and $Q_3 = \{B, C, D, F, H\}$. In Fig. 4, the top of tree shows the statement set Q above the three branches, Q_1, Q_2, and Q_3, which illustrate three possible statement sets for Q. The form of tree under the any branches is unanimous with the tree presentation of statement structure for the simple preference.

The unknown preference can be obtained through the tree presented in Fig. 4. For instance, $s_1 \succ s_2 \succ s_9$ in Q_1, Q_2, and Q_3, so $s_1 \succ s_2 \succ s_9$ in Q. As to $s_{10} \succ s_6$ in Q_1 and Q_2, and $s_6 \succ s_{10}$ in Q_3, hence we can conclude that $s_6\ U\ s_{10}$ in Q. In the same way, the preference can be calculated ultimately.

Through the above ranking procedure for the unknown preference, we can get an inference as follows.

Corollary 1 *If the statement set Q for a DM does not involve uncertainty, in the situation, the option prioritizing for the unknown preference is consistent with the option prioritizing for the simple preference, and the unknown preference reduces to the simple preference.*

4 Application Using New NDSS: South Sudan Conflict

To demonstrate the process and the applicability of our NDSS, a real-world dispute, namely the South Sudan Conflict (SSC), is employed for a detailed investigation, with a special emphasis on the uncertain nature of the situation.

4.1 Background of the Conflict in South Sudan

We first start with a thorough description of the background, from both the dispute and a possible third party aspects, based on which a mathematical analysis of this conflict is conducted for relevant insights.

In 2011, the new Republic of South Sudan gained independence from the Republic of Sudan through a referendum. However, the independence did not eliminate various disagreements existing in the country, especially the racial issue. To be particular, the president of South Sudan, Salva Kiir Mayardit, and the Vice-President, Riek Machar Teny Dhurgon, respectively belong to the two most powerful races of South Sudan's many ethnic groups. Thus the conflict between these two leaders is deemed to be unavoidable. This conflict has escalated into a multi-sided civil war in

South Sudan, which led to violent attacks, arbitrary detentions, enforced disappearances, even brutal killings.

As an outside neutral party of the conflict, China has found itself immediately affected by the conflict. China currently imports approximately 70% of Sudan and South Sudan's oil (although the oil production has fallen by at least 20% since the fighting began). Its leading national oil company, China National Petroleum Corporation (CNPC), is the most heavily invested foreign company in South Sudan's oil sector. In addition to the economic strike, the Chinese government has been highly concerned about the safety of over 300 Chinese workers from South Sudan's oilfields. The apprehension in regard to the broader regional stability has also been increasing as more than 31,000 South Sudanese having rushed into neighboring states after the deadly clashes.

Given the close connection with this young nation, the Chinese government may act directly in helping resolve SSC. Although one of the key principles of China's foreign policy is not to interfere with the internal affairs of other countries, a mediation role as a neutral third party would not break that rule especially when both sides of the conflict request it.

4.2 Modeling the Conflict in South Sudan Using the NDSS

$N = \{1, 2, 3\} = \{President, Vice - President, China\}$ is the set of three DMs. To use the South Sudan Conflict with unknown preference as an example to demonstrate how the new NDSS works, one can adhere to the following steps:

- Input three DMs and their options presented in Table 5 into the NDSS to generate the feasible states and graph models shown in Figs. 5, 6, 7;
- Input President's uncertain statements in Table 6 to build President's preference tree in Fig. 8 based on the decomposition of uncertain statements described in Fig. 3 and generate unknown preference $\{s_1 \succ s_4\}U\{s_2 \succ s_5\}U\{s_3 \succ s_6\}$ $U\{s_7 \succ s_{10}\}U\{s_8 \succ s_{11}\}U\{s_9 \succ s_{12}\}$ using NDSS based on the theory foundation of the Model A;
- Input Vice-President and China's certain statements in Table 9 to obtain their preferences in Table 10;
- Output the stabilities of the South Sudan Conflict under unknown preference presented in Fig. 9 using the NDSS.

4.2.1 Build the States and Graph Models for the South Sudan Conflict

Now, three DMs are involved in this conflict: President (DM1), Vice-Precident (DM2) and China (DM3). Their corresponding options and detailed descriptions are listed in Table 5. Note that China may or may not be a mediator between the other two parties.

According to the modeling procedure, a set of possible states can be generated using the new NDSS. Figure 5 shows a screen shot from the NDSS, listing the set of 12 states. In fact, there exist 2^4 states in the South Sudan Conflict, but the infeasible

Table 5 DM's options and descriptions in the South Sudan Conflict

DM	Option		Description
President	1. Negotiate	Y	The President is willing to negotiate with the Vice-President toward a new transitional government
		N	The President does not enjoy the governance of the country together with the Vice-President
	2. Escalate	Y	The President keeps on fighting with the vice-president
		N	The President terminates the war with the Vice-President
Vice-President	3. Resist	Y	The Vice-President against the government and go to a civil war
		N	The Vice-President stops the war against the government
China	4. Act	Y	China takes the role of mediation and reconciliation between the President and the Vice-President
		N	Do not intervene

President	Negotiate	0	1	0	0	1	0	0	1	0	0	1	0
	Escalate	0	0	1	0	0	1	0	0	1	0	0	1
Vice-President	Resist	0	0	0	1	1	1	0	0	0	1	1	1
China	Act	0	0	0	0	0	0	1	1	1	1	1	1
		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12

Fig. 5 Generating feasible states for the South Sudan conflict

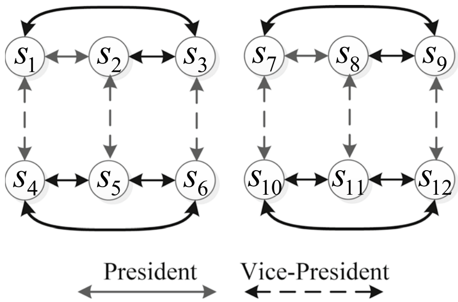

Fig. 6 Integrated graph model for both president and vice-president

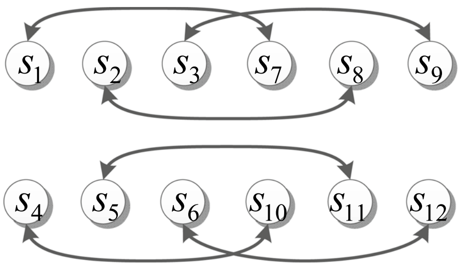

Fig. 7 Graph model for China

Table 6 President's statements and explanations

Statement	Explanation
−2**U** 2	President is uncertain about whether to escalate the war
−3	Don't wish vice-president would resist against him
4 *Λ* -2@1 **U** -4 *Λ* 2@1	President would prefer China intervene or does not intervene the war if President prefer to not escalate or escalate the war
1 *Λ* -2@1 **U** -1 *Λ* 2@1	President would prefer to negotiate or not negotiate with the Vice-President if President prefer to not escalate or escalate the war

states are removed using the system. The movements among the states can be represented using the graph models of the three DMs in Figs. 6 and 7.

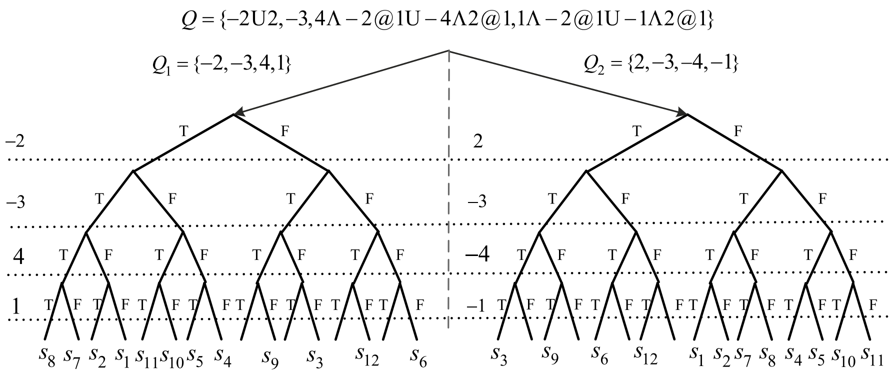

Fig. 8 Preference tree of president with uncertain statement set

4.2.2 Generate DMs' Preference Information for the South Sudan Conflict

After a careful examination of the conflict situation given above, the option prioritization for unknown preference is developed for each DM. More specifically, for the President, a critical DM in the conflict, the most important preference is about the war, but it is not sure whether to escalate or to stop the war. So we consider the preference statement with uncertainty by **U** as shown in Table 6.

The President's preference statement set with uncertainty is Q = {− 2 **U** 2, − 3, 4 *Λ* − 2@1 **U** − 4 *Λ* 2@1, 1 *Λ* − 2@1 **U** − 1 *Λ* 2@1}. It is a special case of the example presented in Fig. 3. According to the decomposition rule of uncertain statements described in Fig. 3, we can get two feasible statement sets without uncertainty $Q = \{Q_1, Q_2\}$ with $Q_1 = \{-2, -3, 4, 1\}$ and $Q_2 = \{2, -3, -4, -1\}$. The preference tree contains the two preference statement sets are presented in Fig. 8. The NDSS may output two preference trees of the President based on statement sets Q_1 and Q_2, respectively, which are the left tree and the right one in Fig. 8.

Let p_{11}, p_{12}, p_{31}, p_{32}, p_{41}, p_{42} denote the possibility of occurrence for the six statements: "− 2", "2", "4", "− 4", "1" and "− 1". Given the relationships of these six statements, we have $p_{11} + p_{12} = 1$, $p_{31} + p_{32} = 1$, $p_{41} + p_{42} = 1$. Besides, consider

Table 7 Q = {−2**U**2, −3, 4 *Λ* −2@1 **U** −4 *Λ* 2@1, 1 *Λ* −2@1 **U** −1 *Λ* 2@1}

		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	
−2 **U**2	$-2(p_{11})$	T	T	F	T	T	F	T	T	F	T	T	F	2^3
	$2(p_{12})$	F	F	T	F	F	T	F	F	T	F	F	T	
−3	−3	T	T	T	F	F	F	T	T	T	F	F	F	2^2
4 *Λ* -2@1 **U**	$4(p_{31})$	F	F	F	F	F	F	T	T	T	T	T	T	2^1
−4 *Λ* 2@1	$-4(p_{32})$	T	T	T	T	T	T	F	F	F	F	F	F	
1 *Λ* -2@1 **U**	$1(p_{41})$	F	T	F	F	T	F	F	T	F	F	T	F	2^0
−1 *Λ* 2@1	$-1(p_{42})$	T	F	T	T	F	T	T	F	T	T	F	T	

Table 8 Scores for Table 7

State	Score	State	Score
s_1	$8p_{11}+2p_{32}+p_{42}+4$	s_7	$8p_{11}+2p_{31}+p_{42}+4$
s_2	$8p_{11}+2p_{32}+p_{41}+4$	s_8	$8p_{11}+2p_{31}+p_{41}+4$
s_3	$8p_{12}+2p_{32}+p_{42}+4$	s_9	$8p_{12}+2p_{31}+p_{42}+4$
s_4	$8p_{11}+2p_{32}+p_{42}$	s_{10}	$8p_{11}+2p_{31}+p_{42}$
s_5	$8p_{11}+2p_{32}+p_{41}$	s_{11}	$8p_{11}+2p_{31}+p_{41}$
s_6	$8p_{12}+2p_{32}+p_{42}$	s_{12}	$8p_{12}+2p_{31}+p_{42}$

"– 2" and "2" as the premise of the occurrence of statements "4", "1" and statements "– 4", "– 1". We can conclude that $p_{31}=0$, $p_{41}=0$ if $p_{11}=0$, and $p_{32}=0$, $p_{42}=0$ if $p_{12}=0$.

The operational process of the option prioritization approach for the President's uncertain statement set can be seen through the score calculation in Tables 7 and 8 based on the theory foundation developed in Sect. 3. According to the Model A in Sect. 3, the difference of any two states s and q among the twelve feasible states, $D(s, q)$, can be calculated; and the maximum and minimum values of $D(s, q)$, $D_{max}(s,q)$ and $D_{min}(s,q)$ can be obtained. Similarly, for the Vice-President and China, their preference statements have no uncertainty and are described in Table 9. Then, the preference rankings of these two parties can be calculated using the option prioritization for simple preference. After inputting DMs, options and their statements with uncertainty or certainty, the NDSS will generate the President's

Table 9 Preference statement and the illustration of vice-president and China

DM	Statement	Explanation
Vice-President	−3 IFF 1	Terminates the war against the government if and only if the President accept to form a new transitional government
	3 IF 2	Keeps on the reactionary behavior if the President continues fighting
	1	Wants the President to negotiate
	4	Wishes China intervene as a third party
	−2	Does not want the President to fight
China	−3	The Vice-President terminates the war against the government
	1	Want the President to negotiate with the Vice President and form a new transitional government
	4	Intervene as a third party
	−2	Would not like the President to fight with the Vice-President

Table 10 The preferences of the three DMs in the South Sudan conflict

DM	Preference over States
President	$\{s_1 \succ s_4\}U\{s_2 \succ s_5\}U\{s_3 \succ s_6\}U\{s_7 \succ s_{10}\}U\{s_8 \succ s_{11}\}U\{s_9 \succ s_{12}\}$
Vice-President	$s_8 \succ s_2 \succ s_{10} \succ s_{12} \succ s_4 \succ s_6 \succ s_{11} \succ s_5 \succ s_7 \succ s_1 \succ s_9 \succ s_3$
China	$s_8 \succ s_2 \succ s_7 \succ s_9 \succ s_1 \succ s_3 \succ s_{11} \succ s_5 \succ s_{10} \succ s_{12} \succ s_4 \succ s_6$

unknown preference and the Vice-president and China's simple preferences shown in Table 10.

4.3 Analyzing Stabilities for the South Sudan Conflict with Uncertainty Using the NDSS

Based on unknown preference, Li et al. (2004) proposed Nash, GMR, SMR, and SEQ stabilities (or solution concepts) with indexes *a*, *b*, *c*, and *d*, according to whether the focal DM would move to a state with unknown preference and whether the focal DM would be sanctioned by a responding move to a state including unknown preference, relative to the status quo. According to the definitions of Nash, GMR, SMR, and SEQ (Fang et al. 1993), there exist 16 situations including uncertainty. Therefore, Nash, GMR, SMR, and SEQ were extended to address 16 stabilities using indexes *a*, *b*, *c*, and *d*. The 16 stabilities may be used to analyze the diversity of possible risk profiles in face of uncertainty. The NDSS has the analysis

state		Nash				GMR				SMR				SEQ			
		President	Vice-President	China	Eq	President	Vice-President	China	Eq	President	Vice-President	China	Eq	President	Vice-President	China	Eq
S1	a							1				1				1	
	b	1				1		1		1		1		1		1	
	c					1		1		1		1		1		1	
	d	1				1		1		1		1		1		1	
S2	a		1				1	1			1	1			1	1	
	b	1	1			1	1	1	1	1	1	1	1	1	1	1	1
	c		1			1	1	1	1	1	1	1	1	1	1	1	1
	d	1	1			1	1	1	1	1	1	1	1	1	1	1	1
S3	a							1				1				1	
	b	1				1		1		1		1		1		1	
	c					1		1		1		1		1		1	
	d	1				1		1		1		1		1		1	
S4	a		1				1				1				1		
	b	1	1			1	1			1	1			1	1		
	c		1			1	1			1	1			1	1		
	d	1	1			1	1			1	1			1	1		
S5	a						1	1				1			1	1	
	b	1				1	1	1	1	1		1		1	1	1	1
	c					1	1	1	1	1		1		1	1	1	1
	d	1				1	1	1	1	1		1		1	1	1	1
S6	a		1				1				1				1		
	b	1	1			1	1			1	1			1	1		
	c		1			1	1			1	1			1	1		
	d	1	1			1	1			1	1			1	1		
S7	a			1				1				1				1	
	b	1		1		1		1		1		1		1		1	
	c			1		1		1		1		1				1	
	d	1		1		1		1		1		1		1		1	
S8	a		1	1			1	1			1	1			1	1	
	b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	c		1	1		1	1	1	1	1	1	1	1	1	1	1	1
	d	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S9	a			1				1				1				1	
	b	1		1		1		1		1		1		1		1	
	c			1		1		1		1		1				1	
	d	1		1		1		1		1		1		1		1	
S10	a		1	1			1	1			1	1			1	1	
	b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	c		1	1		1	1	1	1	1	1	1	1		1	1	
	d	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S11	a			1			1	1				1			1	1	
	b	1		1		1	1	1	1	1		1		1	1	1	1
	c			1		1	1	1	1	1		1			1	1	
	d	1		1		1	1	1	1	1		1		1	1	1	1
S12	a		1	1			1	1			1	1			1	1	
	b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	c		1	1		1	1	1	1	1	1	1	1		1	1	
	d	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Fig. 9 Output of the stability results in the South Sudan conflict with uncertainty

Table 11 Equilibrium results in the South Sudan conflict with uncertainty

State		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}
a	Nash												
	GMR												
	SMR												
	SEQ												
b	Nash								√		√		√
	GMR		√			√			√		√	√	√
	SMR		√						√		√		√
	SEQ		√			√			√		√	√	√
c	Nash												
	GMR		√			√			√		√	√	√
	SMR		√						√		√		√
	SEQ		√			√			√				
d	Nash								√		√		√
	GMR		√			√			√		√	√	√
	SMR		√						√		√		√
	SEQ		√			√			√		√	√	√

function to calculate the 16 stabilities for unknown preference based on MRCR (Xu et al. 2009). The stability results for the South Sudan Conflict are output using the NDSS (see Fig. 9).

Solution concepts indexed *a* shows the stability for the most aggressive DMs. For the stabilities indexed b, uncertainty in preferences is not considered by a DM. The stabilities indexed c incorporate a mixed attitude toward the risk associated with states of unknown preference. Finally, the solution concepts indexed d represent stabilities for the most conservative DMs. An equilibrium s means that s is stable for some stability with an index for all DMs. Therefore, the equilibria of the South Sudan Conflict are summarized in Table 11.

From Table 11, except under extension *a*, states s_2, s_5, s_8, s_{10}, s_{11}, and s_{12} are the equilibria in the South Sudan Conflict. s_8, s_{10}, and s_{12}, nevertheless, are most likely the resolutions because they are the states satisfying all of the four solution concepts under extension *b* and *d*. State s_{12} cannot resolve the conflict because China, as the third party, must make more effort and see how to change and modify state s_{12} to finally reach s_8. State s_8 corresponds to the scenario that the President wants to negotiate with the Vice-President and does not want the situation to escalate, the Vice-President says 'yes' to resistance, under the powerful intervention of China. This information is momentous to China as a third party: China can try to lead the conflict to a better direction, i.e., state s_8, through influencing the President.

The equilibrium results obtained under extension *c* show a pretty pass currently confronted by the President: when it keeps on fighting, the Vice-President may have powerful force to resist it; thereupon then, the game is not worth the candle. If it chooses to negotiate, the victory in the war with the small loss makes it

Table 12 Evolution of the conflict

DM	Option											
President	1. Negotiate	N		N		N		N		N	$\longrightarrow$	Y
	2. Escalate	N		N	$\longrightarrow$	Y		Y	$\longrightarrow$	N		N
Vice-President	3. Resist	N	$\longrightarrow$	Y		Y		Y		Y	$\rightarrow$	N
China	4. Act	N		N		N	$\longrightarrow$	Y		Y		Y
States number		s_1		s_4		s_6		s_{12}		s_{10}		s_8

restless while vice-president has weak force. This significant feature purports that the conflict will probably hold unsettled into the future until China intervened in the conflict.

From Table 12, it can be seen that the conflict is in the stalemated stage. The President doesn't want to negotiate with the Vice-President. The President expects the situation to escalate, but the Vice-President says "yes" to select resistance. China as the third party has not intervened in deed. State s_6 is the status quo of South Sudan conflict. The conflict evolutionary process is described in Table 12 in which state s_1 to s_4, then to s_6 before China's intervention. After the third party mediation involved, the conflict from s_6 to s_{12}. State s_{12} can reach equilibrium state s_8 through s_{10} with the help from China by influence the President. Following this strategic decision, the conflict can be solved satisfactorily.

5 Conclusion and Future Work

This research develops a new NDSS based on the matrix representation to implement modeling and analysis for GMCR. To be particular, an algebraic approach using option prioritizing to generate unknown preference is proposed and then implemented to build an efficient and flexible modeling and analysis system. The Browser/Server structure is adopted to avoid many issues in MFC-based systems. The NDSS can practically provide decision makers with valuable strategic insights for negotiations and decisions, especially in dynamic conflict environments. A case study of the real-world South Sudan conflict with unknown preferences is performed for possible resolutions when the third-party intervention is involved. The workability and competence of the system are demonstrated through screen shots for major steps of the conflict analysis taken from our developed NDSS.

The future work is to extend this approach to include a hybrid preference combining unknown preference and strength preference (Hamouda et al. 2004) together to model and analyze complicated conflicts. And the hybrid structure is added into the new NDSS.

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