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The Graph Model for Conflict Resolution: Reflections on Three Decades of Development

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Abstract

The fundamental design and inherent capabilities of the Graph Model for Conflict Resolution (GMCR) to address a rich range of complex real world conflict situations are put into perspective by tracing its historical development over a period spanning more than 30 years, and highlighting great opportunities for meaningful future expansions within an era of artificial intelligence (AI) and intensifying conflict in an over-crowded world. By constructing a sound theoretical foundation for GMCR based upon assumptions reflecting what actually occurs in reality, a fascinating story is narrated on how GMCR was able to expand in bold new directions as well as take advantage of many important legacy decision technologies built within the earlier Metagame Analysis and later Conflict Analysis paradigms. From its predecessors, for instance, GMCR could benefit by the employment of option form put forward within Metagame Analysis for effectively recording a conflict, as well as preference elicitation techniques and solution concepts for defining chess-like behavior when calculating stability of states from the realm of Conflict Analysis. The key ideas outlined in the paper underlying the current and projected capabilities of GMCR include the development of four different ways to handle preference uncertainty in the presence of either transitive or intransitive preferences; a wide range of solution concepts for describing many kinds of human behavior under conflict; unique coalition analysis algorithms for determining if a given decision maker can fare better in a dispute via cooperation; tracing the evolution of a conflict over time; and the matrix formulation of GMCR for computational efficiency when calculating stability and also theoretically expanding GMCR in bold new directions. Inverse engineering is mentioned as an AI extension of GMCR for computationally determining the preferences required by decision makers in order to reach a desirable state, such as a climate change agreement in which all nations significantly cut back on their greenhouse gas emissions. The basic design of a decision support system for permitting researchers and practitioners to readily apply the foregoing and other advancements in GMCR to tough real world controversies is discussed. Although GMCR has been successfully applied to challenging disputes arising in many different fields, a simple climate change negotiation conflict between the US and China

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Extended author information available on the last page of the article

is utilized to explain clearly key concepts mentioned throughout the fascinating historical journey surrounding GMCR.

Keywords Climate change · Conflict Analysis · Decision support system · Graph Model for Conflict Resolution · Metagame Analysis · Preference · Stability

1 Motivation and Conception

A pressing problem of great concern to the authors of this paper was having operational decision technologies readily available to resolve conflicts among competing interest groups over water pollution and utilization as well as other related environmental issues, such as climate change. This motivated them to design the Graph Model for Conflict Resolution (GMCR), as well as an array of associated decision methods to expand its scope, for tackling not only water resources and environmental disputes but conflict arising in a wide range of areas in which humans interact with one another both in purely competitive and cooperative fashions. Although the initial and ongoing research took place within the Conflict Analysis Group in the Department of Systems Design Engineering at the University of Waterloo starting in the 1980s, much progress was accomplished at other universities with which the authors are affiliated as well as at numerous other universities and research institutes around the globe. Moreover, the authors carried out joint research with many doctoral students and visiting researchers, who in turn now have their own research groups whose activities include executing research on the theory and practice of GMCR.

The year 1987 marked the time in history of the first major publication on GMCR (Kilgour et al. 1987), although research on this topic had been initiated a few years earlier. Two key books have been published on GMCR (Fang et al. 1993; Xu et al. 2018a) while numerous journal papers and conference articles have been written by authors from around the world. After a life span of over three decades, the purpose of this article is to put GMCR achievements into perspective, explain the motivation underlying key GMCR concepts, and project what the authors believe to be an ever-expanding universe of GMCR technologies needed for tackling a rich range of challenging disputes facing humanity.

In the next section, the connections of GMCR to the general area of game theory are explained. Subsequently, key developments in Metagame Analysis (Howard 1971) and Conflict Analysis (Fraser and Hipel 1979, 1984) are highlighted along with an explanation of their direct contributions to the design and construction of GMCR, in Sects. 3 and 4, respectively. The sound theoretical foundations of GMCR, which reflect characteristics of real world conflicts, are put forward in Sects. 5 and 6, along with an array of flexible methods built on these solid foundations which form components of the overall GMCR methodology. When one visualizes GMCR from a systems perspective, as is done in Sect. 7, which includes inverse engineering and systems identification, one can appreciate the many theoretical developments required for tackling an ever-expanding realm of tough problems facing society and its natural environment. To permit both practitioners and researchers to apply the many theoretical capabilities of GMCR to challenging real world conflicts, flexible decision support systems are discussed in Sect. 8. In order to illustrate the usefulness and applicability of this rich range of ideas related to GMCR, an intuitive dispute involving climate change negotiations between the US and China over significantly reducing greenhouse gas emissions is employed throughout the paper. Among the many opportunities for meaningful research in GMCR, the authors point out links GMCR could have within an inverse engineering viewpoint to artificial intelligence (AI) in which a computer calculates the vast array of potential preferences decision makers may have to reach a specified desirable result such as having a robust climate, given how they could strategically interact to do so.

2 Game Theory

Conflict, ranging from pure competition to full cooperation, is an integral characteristic of human society and the natural world: family members argue over who is responsible for the completion of household chores, environmentalists attempt to control excessive industrial development, stakeholder groups debate directions for urban expansion, companies vie to increase their market shares within and among nations, regional wars break out over the control of scarce resources, plants in a forest attempt to obtain as much sunlight as possible to ensure their existence, animal species may cooperate to preserve their sources of nourishment and shelter, and societies knowingly emit excessive amounts of greenhouse gases into the atmosphere while nature reacts with devastating effects as the climate dramatically changes. The consequences of conflict can be beneficial, such as having lower prices for products resulting from stiff competition among producers and creating scientific breakthroughs by competing researchers due to the "publish or perish" syndrome. On the other hand, conflict can result in highly negative consequences like destruction of cities due to warfare and exploitation of poorer nations by richer industrialized countries. Because conflict is so ubiquitous and of such great import, a significant amount of research has been carried out over the years to develop formal models for modeling and analyzing conflict in order to better understand conflict and its strategic impacts so that more informed decisions can be made.

Formal approaches for tackling conflict developed within a range of domains are collectively referred to as game theory. Early research in game theory can be attributed to the Italian Gerolamo Cardano of Pavia in the 16th century as well as the famous French mathematicians Pierre de Fermat and Blaise Pascal in the 17th century who created probability theory for modeling parlor games and other games of chance. However, it was the seminal contributions of von Neumann (1928) and especially von Neumann and Morgenstern (1944, 1953) which firmly established game theory as a viable means for rigorously investigating conflict. Many other fine contributions to game theory came from researchers working in fields such as operations research, management sciences and systems engineering, as explained by authors like Hipel et al. (2008b, 2009a, b), Hipel (2009a,b), Kilgour and Eden (2010), Hipel and Bernath Walker (2011), and Xu et al. (2018a, Ch. 2).

A particularly informative means to categorize formal game theoretic techniques is according to the kinds of preferences, as displayed in Fig. 1 (Hipel and Fang 2005). In this genealogy, game theoretic techniques which only require relative preference information appear in the left branch while those dependent upon cardinal preferences for model calibration are listed on the right. An example of relative preferences is, for instance, when one declares that he or she prefers to have coffee over tea or it does not matter since they are equally preferred. In other words, the preference information is relative or nonquantitative, and the amount by which one prefers one item over another is not needed. However, if real numbers as represented by cardinal utility values or monetary units, for example, are utilized to reflect preferences for employment within a game theoretic model, the preferences are said to be cardinal or quantitative. Most of the game theoretic techniques, such as normal form, extensive form, and cooperative game theory under the right classification are part of what is called classical game theory, and were originally introduced by von Neumann and Morgenstern (1944, 1953).

In the left branch of Fig. 1, the methods contained within the dashed area were originally designed within the Conflict Analysis Group in the Department of Systems Design Engineering at the University of Waterloo. Key capabilities which are incorporated into the design of all of the techniques listed on the left for investigating conflict include their ability to handle:

- Relative preference information including both transitive and intransitive preference relationships. Preferences are transitive for a decision maker (DM) in a dispute if whenever scenario *a* is preferred to scenario *b* and *b* to *c*, then *a* is pre-

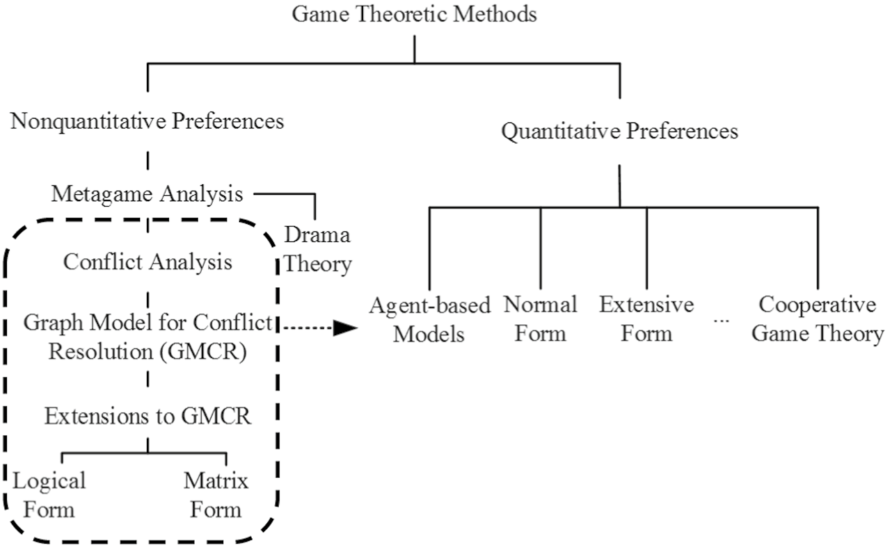


Fig. 1 Genealogy of formal game theoretic techniques

ferred to *c*. However, under intransitive preferences, it may happen that scenario *c* is preferred to or at least as good as scenario *a*.

- Any finite numbers of DMs and options or courses of action which the DMs control in the dispute.
- "Chess-like thinking" to determine stability of a given state in which a DM contemplates whether it is beneficial to take advantage of a unilateral improvement (UI) to move to a preferred state keeping in mind that one or more of the other DMs may move to block or sanction the UIs, and thereby put the focal DM in a worse situation and cause the state to be stable if all UIs by the focal DM can be blocked. Possible ways in which DMs may think under conflict are mathematically captured by what are called solution concepts or stability definitions. If a given state is stable for all DMs according to a particular solution concept, it is called an equilibrium or potential resolution.
- Unilateral moves taken by DMs in any order whatsoever in a dispute or perhaps not at all.

Because of the foregoing and other key properties of the models given in the left categorization in Fig. 1, various authors have stressed the advantages of utilizing these methods, especially GMCR, for realistically studying real world conflict (see, for instance, Zeng et al. (2006), Hipel et al. (2011a), Madani and Hipel (2011), Madani (2013), and Xu et al. (2018a)).

3 Metagame Analysis

3.1 Human Behavior Under Conflict: Strategic Moves and Countermoves

Nigel Howard initiated the sequence of developments shown in the left branch of Fig. 1 via the creation of Metagame Analysis in his 1971 book entitled "Paradoxes of Rationality: Theory of Metagame and Political Behavior" published by the MIT Press (Howard 1971). To explain the motivation underlying Howard's construction of Metagame Analysis, consider the game of Prisoner's Dilemma in normal or tabular form shown in Table 1. This generic conflict describes the basic situation in which one person or organization must decide whether or not to cooperate with another. For illustrative purposes, suppose the interpretation of Prisoner's Dilemma is the situation in which the United States (US) and China are negotiating over whether or not to cooperate with one another by drastically cutting back on greenhouse gas emissions and thereby avoid the severe impacts of climate change in the future. As can be seen in Table 1, the US is the row player which controls the two row strategies of "Cooperate (C)" by greatly reducing the release of greenhouse gases or "Don't Cooperate (D)" by not cutting back and thereby saving money in the short term. China is the column player which also has two strategies: "Cooperate (C)" and "Don't Cooperate (D)". When each player or decision maker (DM) selects a strategy, a state is formed as represented by a cell in the matrix. For instance, when the US chooses D and China selects C, then state DC is formed as indicated in the bottom left cell in the table. For convenience, each cell is also assigned a state

Table 1 Prisoner's Dilemma version of climate change negotiations between the US and China in normal form

		China (DM 2 or Column Player)	
		Cooperate (C)	Don't Cooperate (D)
US (DM 1 or Row Player)	Cooperate (C)	1 CC 3, 3	2 CD 1, 4 R_2
	Don't Cooperate (D)	3 DC 4, 1 R_1	4 DD 2, 2 R_1, R_2

R_1 = rational for US (DM 1)

R_2 = rational for China (DM 2)

number as shown in the top left of each cell. Hence, state DC can also be referred to as state 3. Because each DM controls two strategies, the conflict in Table 1 is called a 2×2 game and is the simplest type of conflict which can occur.

The preferences of the two DMs are represented by the numbers given in each cell where a higher number means more preferred and the first and second entries are the preferences of the US and China, respectively. Notice, for instance, that for state DC, the number 4 means that this is the most preferred state for the US since China will pay for reducing its greenhouse gases while the US saves money in the short term by not doing so. The preferences of 1 for China for state DC indicates that this is its least preferred state. The numbers 3, 3 for state CC or state 1 means that this is the second most preferred state for both the US and China since both nations will be better off in the longer term if they drastically cut greenhouse gas releases.

At the time when Howard was building a theory of metagame, the stability of a state was determined using what is called rational or Nash stability (Nash 1950, 1951). When depicting a game as shown in Table 1, a specific state is defined to be rational for a given DM if the DM has no unilateral improvement (UI) from that state. Consider state DC or 3 in Table 1. On its own, the US can change its strategy from D to C and thereby unilaterally cause the conflict to move from state DC to CC. Because a preference of 4 is greater than 3 for the US for state DC and CC, respectively, this constitutes a unilateral disimprovement, rather than a UI for the US. Accordingly, state 3 is marked with R_1 to point out it is rational or Nash stable for the US. Notice, on the other hand, that when entertaining state DC, China can cause the game to unilaterally move from state DC to DD by altering its strategy selection from C to D. Because state DD is more preferred to DC by China as indicated by the numbers 2 and 1, respectively, the move from state DC to DD constitutes a UI for China. Therefore, state 3 or DC is not rational for China.

After carrying out the Nash stability calculations for each of the three remaining states for each DM, the final results are as shown in Table 1. Because state DD is the only state which is stable for both DMs as indicated by R_1, R_2 in the cell for state 4, this is the only equilibrium or resolution predicted to take place. Because state CC or 1 is more preferred to state DD or 4 by both DMs, the concept of

Nash stability misses what should clearly be another possible equilibrium. Howard (1971, Sect. 2.5) refers to this as the Second Breakdown of Rationality. In his book, he demonstrates the First Breakdown of Rationality using a generic game called Chicken (Howard 1971, Sect. 2.3). Specifically, in the game of Chicken two drivers who are driving their cars towards one another at high speed have the strategy to swerve or not. Rationality fails to predict the state in which both drivers swerve, and thereby act as "Chickens", as a possible equilibrium to this dangerous kind of basic conflict.

To overcome the breakdowns of rationality, Howard recognized that additional stability definitions or solution concepts were required. To accomplish this, Howard (1971) followed a suggestion by von Neumann and Morgenstern (1953) who proposed to investigate any normal-form game one should analyze the metagames founded on it, in which a metagame is the game that would exist if one of the DMs selected its strategy after the others, in knowledge of their choices. By systematically investigating rational stability that could occur in what Howard called the infinite metagame and developing some elegant mathematical theorems, Howard defined two additional kinds of stability that could occur in the basic game: general metarationality (GMR) and symmetric metarationality (SMR). Depending on the situation, the acronyms GMR and SMR are utilized in the text to represent either a noun (ex. general metarationality) or an adjective (ex. general metarational stability).

Under GMR and SMR, one thinks like a chess or go player in terms of the possible consequences of countermoves by an opponent as a result of a potential initial move from a given state. If, after imagining a logical sequence of specified interactions as defined by a certain stability definition the DM under consideration ends up in a worse situation for any UI it is tempted to take, the starting state is stable for that DM for that kind of stability. For example, take a look at state CC in Table 1 from the viewpoint of the US. Notice that the US possesses a UI from state CC to DC since DC is more preferred to CC by this DM. Hence, the US is interested in moving to DC since this move is totally under its control and is an improvement over the current situation. However, if the US thinks about the impacts of its action, it may wish to consider how China may respond. In particular, by examining the second row in Table 1, one can see that China can cause the conflict to progress from state DC to DD by changing its strategy from C to D. Because state DD is less preferred to CC by the US, state CC is GMR stable for the US as its initial UI can be blocked by China. Moreover, notice that if the US tries to escape unilaterally from the sanction levied by China, the conflict will go from state DD to CD. Because CD is also less preferred to CC by the US, state CC is also SMR stable for the US. In fact, using similar arguments for China, one can determine that state CC is also GMR and SMR stable for China. Since CC is stable for both DMs, this state is an equilibrium according to both GMR and SMR stability. Accordingly, these two stability definitions put forward by Howard (1971) overcome the breakdown of rationality for the game of Prisoner's Dilemma.

Because Nash stability is a special case of both GMR and SMR for which no UI exists, if a state is Nash stable it is by definition also GMR and SMR stable. Therefore, state DD in Table 1 is also an equilibrium according to both GMR and SMR stability. Additionally, states DC and CD are stable for the US and China, respectively, but can be shown not to be equilibria.

3.2 Option Form: The Language for Recording, Discussing and Understanding Conflict

The normal or matrix form displayed in Table 1 is a convenient way to write down a conflict when there are only two DMs, especially for the case of a generic 2×2 game. However, this format becomes clumsy to employ when there are more than two DMs. To overcome this problem Howard (1971) came up with a clever solution: the option form of the game displayed in Table 2 for the case of Prisoner's Dilemma. Notice in the left column that one lists each of the DMs followed by the option or options that it controls. For the climate change negotiations, both the US and China have the single option of cooperate. The columns of Ys and Ns represent the four possible states that could occur. For instance, state 2 or CD is the scenario for which Y or yes option 1 is chosen by the US and N or no, option 2 is not selected by China. The normal form notation and state numbers given in Table 2 show the direct connections between the states in Table 2 with those in Table 1. By invoking option 1 in state CD or 2, for example, the US has selected its strategy of cooperate while by not taking its option of cooperate China has chosen the strategy of don't cooperate. Finally, the preferences, shown in the cells in Table 1 as pairs of numbers, can be transferred to an equivalent ranking of states from most to least preferred for each DM as shown in the bottom part of Table 2. An ordering of states, for which ties are allowed, is commonly referred to as a preference ranking for a given DM.

The concepts of only assuming relative preference information, defining flexible solution concepts based on the intuitive idea of moves and countermoves, and designing option form as a smart format for conveniently recording a dispute having any finite number of DMs and options, provided the basic structure for Metagame Analysis to be utilized to model and analyze real-life conflicts. For example, Hipel and Fraser (1980) applied Metagame Analysis to a large-scale international environmental conflict between Canada and the US called the Garrison Diversion Unit dispute, in which their model contained four DMs and a total of nine options. Fraser

Table 2 Climate change negotiations in option form

DMs and Options	States			
US				
1. Cooperate (C_{US})	Y	Y	N	N
China				
2. Cooperate (C_{Ch})	Y	N	Y	N
Normal Form Notation	CC	CD	DC	DD
State Number	1	2	3	4
		Ranking of States		
		US: 3 > 1 > 4 > 2		
		China: 2 > 1 > 4 > 3		

and Hipel (1980) used Metagame Analysis to investigate a water allocation conflict called the Poplar River dispute between Canada and the US, in which their model in option form consisted of three DMs and a total of twelve options.

3.3 Drama Theory

As can be seen in Fig. 1, Metagame Analysis was expanded in two directions. The one path leads to Drama Theory, which was developed by a number of authors (Howard 1994a, b, 1999; Bryant 2003, 2015; Levy et al. 2009a, b). In Drama Theory, the metaphor of a drama is used in conjunction with dilemmas that are confronted and resolved as the drama unfolds. In fact, Nigel Howard presented some of his key ideas on Drama Theory at an international conference called "Decision Making under Conditions of Conflict" held within the Conflict Analysis Group in the Department of Systems Design Engineering at the University of Waterloo, from August 31st to September 2nd, 1992, and hosted by K.W. Hipel and K.J. Radford. Subsequent to the Waterloo conference, two important drama theory papers by Howard (1994a, b) appeared in a special issue of Group Decision and Negotiation having the same title as the conference, for which Hipel and Radford were the Guest Editors (Volume 3, Numbers 2 to 4, 1994). Later, Obeidi and Hipel (2005) employed a practical application over the proposed highly controversial exportation of fresh water in bulk quantities from Canada to illustrate how strategic results found using Drama Theory complement those discovered when utilizing Conflict Analysis and the Graph Model for Conflict Resolution. As depicted on the left in Fig. 1, these two methods are located along the other path emanating from Metagame Analysis and are discussed next.

4 Conflict Analysis

Fraser and Hipel (1979, 1984) purposely designed Conflict Analysis as both an enhancement and expansion of Metagame Analysis for addressing a broader range of conflict situations. They developed sensible methods for taking into account key characteristics of real-life disputes and operationalized them in a truly engineering fashion so they could be readily utilized in practice. Their main contributions to the theory and practice of conflict analysis are now summarized.

4.1 Sequential Stability

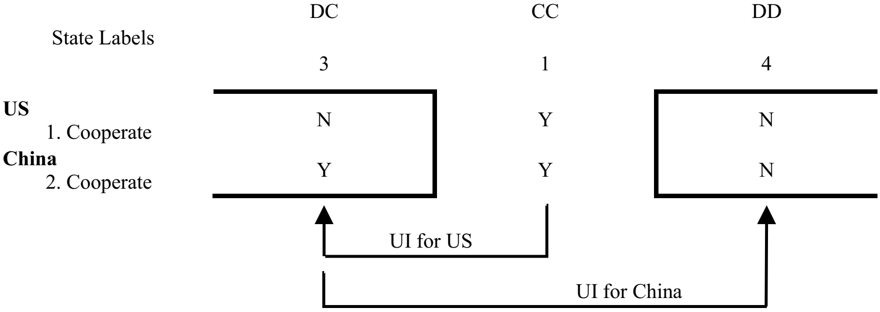
For the solution concepts GMR and SMR, the preferences of the sanctioning DM, or DMs if there are more than two DMs participating in the conflict under study, are not considered when they are trying to block a UI by a focal DM from a particular state. In other words, the DM for whom the state is being checked for stability is thinking in a conservative fashion since it only worries about the existence of a potential sanction and not the preferences of the sanctioning DM even if the sanction goes against the sanctioning DM's interest by being less preferred to the UI.

Because a sanctioning DM may not wish to harm itself in the process of blocking a UI by another DM, Fraser and Hipel (1979, 1984) defined the solution concept called sequential stability (SEQ) based on the concept of credible sanctions. A state is defined to be SEQ for a particular DM if each UI from the starting state can be credibly blocked which means the sanctioning DM only levies its UIs to attempt to stop improvements by its opponent.

As an illustration of a calculation for SEQ, consider state CC or 1 from Table 2 shown in option form in Table 3. Assuming the conflict is now at state CC, the question arises as to whether or not the US will move from it. As can be seen in this table, the US has a UI from state CC or 1 to state DC or 3. Notice that this move is unilateral, since only the US changes its option selection, while China continues to select its option cooperate as indicated by the Y in states 1 and 3. Moreover, this move is an improvement because state 3 is more preferred to 1 by the US as shown in Table 1 and the bottom portion of Table 2. The UI from state 1 to state 3 by the US is marked by the line with an arrowhead connecting these two states in the lower part of Table 3. Under SEQ, one must next ascertain if the possible UI by the US can be credibly blocked by China. As indicated by the arrow at the bottom of Table 3, China can unilaterally move the conflict from state 3 to state 4 or DD. Because, as shown in both Tables 1 and 2, state 4 is more preferred to state 3 by China, this sanction is deemed to be credible. It constitutes a sanction from the US's viewpoint since state 4 is less preferred to state 1 by the US. In this case, there is only one UI for the US from state 1 and since it is credibly blocked, state 1 is SEQ for the US. If this were a larger conflict and there were more than one UI, all UIs would have to be credibly blocked to induce SEQ stability. Additionally, if there were more than two DMs in the conflict, credible sanctions can be generated by UIs based upon UIs among the sanctioning DMs. If at any point in this sanctioning process a state is found that is less or equally preferred to the original state by the focal DM, then the UI is blocked. If all of the UIs from the starting state by the focal DM can be blocked, then the state is said to be SEQ. Finally, since the case of having no UI, which means the state is Nash stable, is a special case of SEQ, then a state which is Nash stable is also SEQ.

In a similar fashion to that done for the US, one can show that state CC or 1 is SEQ for China. Hence, this state in which both countries cooperate by

Table 3 Determining SEQ for state CC or 1 for the US in the climate change negotiations



State Labels	DC 3	CC 1	DD 4
US			
1. Cooperate	N	Y	N
China			
2. Cooperate	Y	Y	N

substantially reducing their greenhouse gas emissions is an SEQ equilibrium. Because the definition of GMR does not take into account preferences of the sanctioning DM, state CC is also a GMR equilibrium as noted earlier. However, the fact that the equilibrium is also SEQ makes it more believable as a possible resolution.

When defining SEQ, Fraser and Hipel (1979, 1984) did this directly in terms of the original conflict and not the infinite metagame as was done by Howard (1971). In fact, all of the stability definitions developed for employment over the years, including GMR and SMR, can be best envisioned and defined by simply contemplating how moves and countermoves could occur among DMs when assessing stability.

In real world conflicts, something always happens even if the "equilibrium" means warfare in a military situation. Therefore, one wishes to have a solution concept that can be guaranteed to predict at least one equilibrium. A particularly attractive feature is that an existence theorem has been proven for SEQ in the sense that at least one SEQ equilibrium will always be found, under the assumption of transitive preferences and moves. In practice, researchers have found that SEQ always predicts reasonable resolutions to a dispute. Even though only one resolution or equilibrium can occur when there are more than one equilibrium, the other resolutions are sensible and explain what else could have happened.

4.2 Tableau Form

As noted earlier, the option form of a conflict is an effective format for representing a dispute of any size, as illustrated in Table 2 for the case of the climate change dispute, which is a very small possible conflict that can occur. For a simple 2×2 conflict, Table 1 shows that the normal form constitutes an informative way to write down small conflicts not having more than two DMs. An intuitively appealing format to employ when carrying out a stability analysis for conflicts having not more than about five DMs and twenty feasible states is the tableau form first proposed by Fraser and Hipel (1979, 1984).

4.3 Simultaneous Sanctioning

If a given state is unstable for two DMs (or for two or more DMs in a conflict containing more than two DMs), stability could be induced by both DMs moving together to a UI which was not blocked sequentially under SEQ. For a given DM, if the calculated state is less or equally preferred than the state under consideration, then that UI is sanctioned. When all UIs are blocked, either sequentially or simultaneously for a given DM, then the state becomes stable according to simultaneous sanctioning and a slash is drawn through the "U" above the state to indicate this. For the case of the climate change conflict in Table 2, this calculation is not carried out since there is no state which is unstable for both DMs. The theoretical definition for simultaneous sanctioning is provided by Fraser and Hipel (1979, 1984).

4.4 Feasible States: Reducing the Size of a Conflict

For the climate change dispute written in option form in Table 2, there are $2^2=4$ states in the conflict, each of which is feasible in the sense that it could take place. The same 4 feasible states are displayed as the four cells within the normal form shown in Table 1. Fortunately, when employing option form, a user only has to supply for each DM the set of options under her or his control, and not the list of states which can be easily determined using a computer program. Unfortunately, one should keep in mind that while a total of 5 options in a conflict model produces only 32 (2^5) possible states, 9 options in a model result in 512 mathematically possible states. Hence, the number of states increases quickly as two to the power of the number of options. Nonetheless, in practice one can significantly reduce the number of states by eliminating ones which are infeasible due to logically impossible combinations of options with respect to the reality of what can occur in the conflict under study.

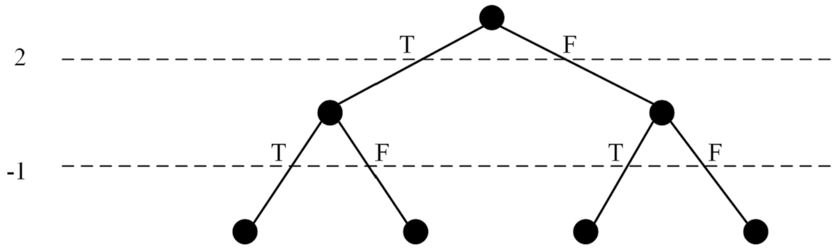
4.5 Preference Elicitation: Overcoming the Toughest Challenge in Conflict Modeling

The most important and often the most difficult step in any decision model is to obtain the values or preferences which are either directly or indirectly required as model input. For the case of a conflict model, one must separately determine the relative preferences for each of the DMs. With respect to the climate change model, the ranking of states for the US and China are provided at the bottom of Table 2. For a small model like this in which there are only two options resulting in four feasible states, one can easily order the states by hand for each of the two DMs. However, for larger disputes in which there are more than 20 feasible states, or perhaps even hundreds of them, one requires an intuitively simple method for preference ranking.

Fraser and Hipel (1988a) defined a clever approach for preference elicitation originally called the preference tree method, but now is usually referred to as option prioritization. One management consultant fondly stated that this approach was "the best thing since sliced bread" developed in Conflict Analysis. A key advantage of conflict analysis and the other techniques listed in the left column of Fig. 1 is that only a relative small amount of information is required to calibrate a given model of an actual dispute. When option form is employed, a user only has to specify a relatively small number of options under the control of each DM from which a computer program can easily generate the states. In a conflict having 6 options, keep in mind only these options must be provided by the user even though they could result in having up to 64 (2^6) feasible states in the model. In reality, a person often expresses his or her basic preferences in terms of what could happen in a dispute using preference statements about options that are hierarchically ranked from most to least important. This fact inspired Fraser and Hipel (1988a) to design the preference tree method for ordering states in which transitivity of preferences is assumed and equally preferred states are allowed.

Consider, for example, the case of the DM US in the climate change model. What is most important for the US is that China cooperates by selecting option 2. Next in importance, the US would not like to choose option 1 in which it cooperates. These two preference statements are shown hierarchically on the left in Fig. 2 where 2 (choose option 2) is written above – 1 (do not select option 1) at two locations (to the left of the graph and on the left in the score calculations at the bottom of Fig. 2). In terms of what this means in the climate change conflict model, take a look at the graph located in the upper part of the figure. Starting from a possible state given as a node at the top of the figure, if option 2 is selected one draws a line slanting to the left from the top node with a T for "true" marked on this branch. This implies that an F for "false" is marked on the branch sloping towards the right from the top node. At the middle level having two nodes, a T or F is appropriately written on the branches emanating from the middle two nodes according to whether or not the preference statement in which option 1 is not chosen (i.e. – 1), which culminates in the four nodes shown as the four leaves at the bottom of the tree. The four leaves are explained in option form or using state numbers just below the tree in which the states are ranked from most preferred on the left to least preferred on the right.

As an illustration, consider the two consecutive branches shown on the left in Fig. 2. When option 2 is true, this means a Y is placed opposite option 2 for China. Next, when option 1 is not taken (indicated as – 1 on the left), this means that an N is written opposite option 1 for the US to create state 3 as shown on the left in the figure. When one uses this interpretation to follow the other paths in



US: 1. Cooperate	N	Y	N	Y
China: 2. Cooperate	Y	Y	N	N
State Numbers	③	①	④	②

Preference Statements	Score Calculations			
2	2^1	2^1	-	-
-1	2^0	-	2^0	-
Total Score	3	2	1	0

Fig. 2 Preference tree for the US (DM 1)

Fig. 2, the ranking of states for the US is 3 > 1 > 4 > 2, which is identical to the ordering furnished at the bottom of Table 2.

Equivalent to drawing the preference tree, one could also employ the scoring method given at the bottom of Fig. 2 in which a higher final number means more preferred. Consider state 4, for instance, given third from the left in Fig. 2. Because the more important preference statement (option 2 is taken) is false, a dash is provided to indicate an entry of zero. Since the second most important preference statement in which option 1 is not taken (i.e. -1) is true, a weight of 2^0 is assigned to this situation. Overall, the total score for state 4 is 1 as indicated at the bottom in Fig. 2. One can see that this scoring method gives exactly the same ranking of states for the US as drawing the preference tree.

Notice in this simple illustration that only two preference statements are required to rank the four states from most to least preferred for the US. The two preference statements needed by China to order the four states using the preference tree method are displayed in Fig. 3. In general, only a relatively small number of prioritized preference statements are required to rank states for a specified DM, where ties are allowed, even in much larger games. Moreover, this option prioritization approach can handle "and" (conjunction), "or" (disjunction), conditional (if situation and if and only if cases (iff)) and any combination of the above. In fact, the option prioritization technique satisfies all of the rules of what is called first order logic (Peng et al. 1997; Hipel et al. 1997; Fang et al. 2003a, b).

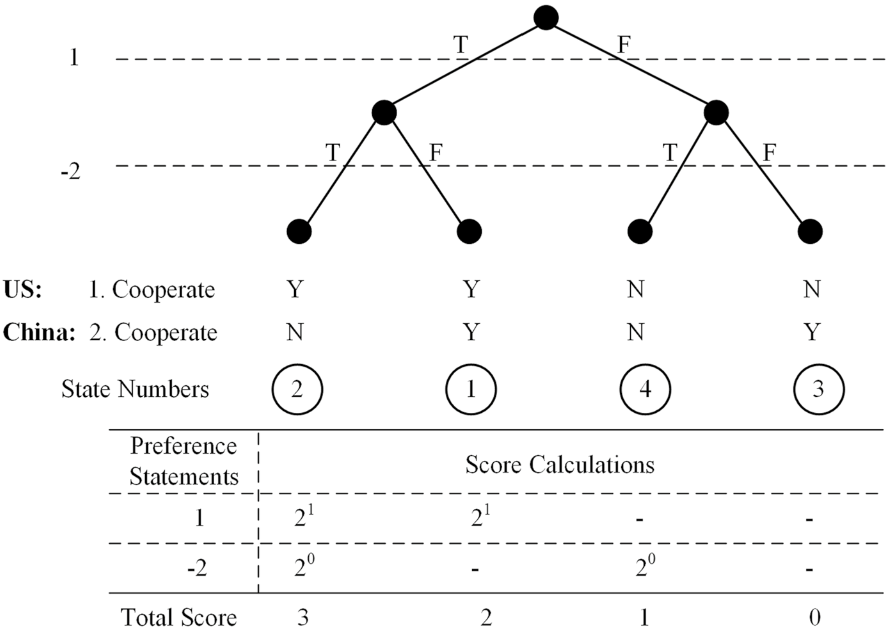


Fig. 3 Preference tree for China (DM 2)

4.6 Coalition Analysis

When carrying out a conflict study, one first wishes to determine how well each DM can fare when acting independently founded on his or her own interests. Subsequently, one can ascertain if a DM can do even better by cooperating with others through joining a coalition, in which all coalition members will somehow be able to improve their situations. Within conflict analysis, a coalition procedure was developed in which the coalition remains in place throughout the duration of the conflict (Kuhn et al. 1983; Meister et al. 1992; Hipel and Meister 1994) while as outlined later in Sect. 5.5 under GMCR another coalition approach is devised whereby coalitions can form and dissipate throughout the evolution of the dispute.

4.7 Hypergames

A hypergame is a dispute in which one or more DMs have misperceptions about the conflict. Mistaken perceptions may exist regarding the preferences of one or more DMs, options, DMs participating in the dispute or any combination of the foregoing. Moreover, because perceptions by one or more DMs can be built upon other DMs' misunderstandings, different levels of misperceptions can take place. By analyzing a hypergame according to the way each DM perceives what is occurring, one can employ Conflict Analysis to execute a hypergame investigation.

The basic idea of a hypergame was first proposed by Bennett (1977, 1980) and Bennett and Dando (1979). However, contributions by Takahashi et al. (1984), Fraser and Hipel (1984) and Wang et al. (1988) made the hypergame methodology fully operational within the realm of Conflict Analysis. Hypergame analysis has also been successfully applied to the Cuban Missile Crisis of 1962 (Fraser and Hipel 1984; Hipel 2011), the nationalization of the Suez Canal by Egyptian President Nasser in 1956 (Shupe et al. 1980), the allied invasion of the Suez Canal later in the same year (Wright et al. 1980), an international trade dispute involving the sale of subway cars to New York city (Stokes et al. 1985), the Falkland Island invasion (Hipel et al. 1988), the Battle of France in 1940 (Fraser and Hipel 1984, Ch. 4), the Normandy invasion of 1944 (Fraser and Hipel 1984, Ch. 3) and an environmental conflict (Fraser and Hipel 1984, Ch. 9).

4.8 Decision Support

A Decision Support System (DSS) is a set of user-friendly computer programs for modeling, analyzing and investigating the output from an exhaustive stability analysis. Fraser and Hipel (1988b) developed a DSS called *DecisionMaker* which could handle Nash and SEQ, as well as certain types of hypergames. Their DSS was utilized by government agencies and management consulting companies. Researchers and practitioners can use a DSS for effectively investigating historical, current and generic conflicts in order to gain strategic insights (see, for instance, Dagnino et al. (1989)). Indepth investigations of actual conflict situations are a good means for testing and refining currently available conflict analysis techniques. However, it also led to the recognition of gaps in the methodology for which significant advances were required in order to handle a richer range of conflict cases for improving both the theory and practice of conflict analysis. This is what led to the development of the next generation of decision technologies, namely the Graph Model for Conflict Resolution, which is now discussed and put into perspective, including the construction of DSSs based on the Graph Model paradigm as put forward in Sect. 8.

5 Graph Model for Conflict Resolution

5.1 Inspirational Design: Function and Form

Like a chess player, DMs involved in a dispute naturally think in terms of moves and countermoves. Accordingly, it is natural to envision a conflict in terms of a graph in which the vertices represent states and the arcs connecting states stand for movement among states unilaterally controlled by the DMs. This intuitive idea led to the development of the Graph Model for Conflict Resolution (GMCR) in which what is called a directed graph for a given DM keeps track of the moves that the DM controls in one step between states (Kilgour et al. 1987).

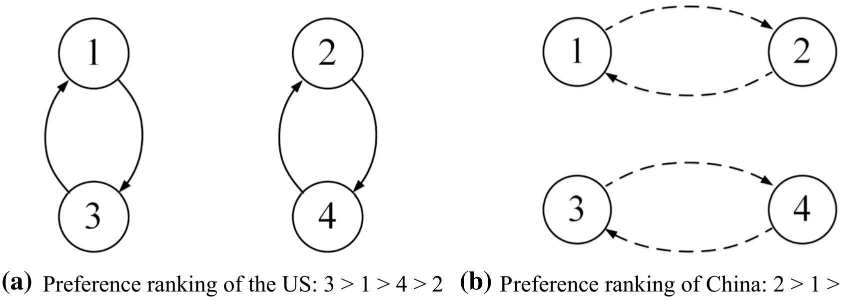
Graphs permit a DM or analyst to use the right part of his or her brain to clearly envision what can dynamically occur in a conflict. Fortunately, when one has a graph to represent a particular situation, the information contained in the graph can be systematically recorded in a matrix or set of matrices. This in turn enables "left brain" computations for calculating stability using what is called the "matrix" formulation (Xu et al. 2009b), explained in Sect. 6, of the "logical" structure of a conflict (Fang et al. 1993) as outlined in Sect. 5. A matrix interpretation of GMCR possesses two key advantages. Firstly, stability calculations can be more efficiently carried out which in turn means the "engine" of a decision support system for GMCR can be designed and programmed using this matrix formulation, as discussed in Sect. 8. Secondly, the matrix formulation of GMCR furnishes a solid theoretical foundation upon which GMCR can be readily expanded to handle other strategic situations as summarized in Sect. 7. By making the mathematics simpler and more pliable, this latter advantage provides the temptation to employ a "lego" approach to research by combining existing developments to build a new structure. Rather, the authors emphasize in Sect. 9 that one should use actual need in the real world to suggest meaningful ways in which to expand and enhance GMCR, such as the development of sensible procedures for coalition analysis to take care of the concept of cooperation among DMs. Moreover, one should ensure that the additional complications in terms of theoretical and practical costs arising over the extension of GMCR are surpassed by the advantages of having an expanded capability to handle a situation such as coalition analysis. Before describing some of the areas in which GMCR has permitted Conflict Analysis to be greatly expanded, ideas underlying GMCR are now briefly explained.

5.2 Definition of a Graph Model

Figure 4a, b shows the directed graphs for the US and China, respectively, for visually capturing the moves that the DMs control in one step. To explain how these graphs are drawn, refer to Table 1 which displays the normal form of this dispute. Notice that each vertex in Fig. 4 containing a specific number stands for exactly the same state as numbered in the top left location of the corresponding cell in Table 1. As can be seen, each state in Fig. 4 is located at the same relative position as in Table 1. An arc containing an arrowhead in Fig. 4 shows the unilateral move controlled by the DM. For example, by fixing China's strategy at Cooperate (C) in the left column in Table 1, the US can unilaterally cause the conflict to go from state 1 (CC) to state 3 (DC) by changing its strategy from C to D. In Fig. 4a, this move is shown by the arc going from state 1 to state 3. Likewise, the US controls the unilateral movement from state 3 (DC) to state 1 (CC) when China is fixed at strategy C in Table 1 by changing its strategy from D in state 3 to C in state 1, which is depicted by the arc from state 3 to state 1 in Fig. 4a. In a similar fashion, one can explain how the US is in charge of the movement from state 2 to state 4 and back again as drawn in Fig. 4a. Additionally, when referring to Table 1, one can draw the unilateral movements in one step under the control of China as is done in Fig. 4b. Notice that China is in charge of the unilateral moves from state 1 to state 2 and back, as well as the unilateral moves from state 3 to state 4 and back.

The final components required to complete the Graph Model in Fig. 4 are the preferences of each DM, which are expressed as numbers in Table 1 (a higher number means more preferred) and as an ordering of states (also called a preference ranking) at the bottom of Table 2. The ordering of states for each DM is also written in Figs. 2 and 3 for the US and China, respectively. The directed graph and preferences for each DM as depicted in Fig. 4 constitute what is called the Graph Model within the overall GMCR methodology. By writing the name of the DM on an arc under the control by that DM, Fig. 4a, b can be combined to create what is labelled as an integrated graph, which is displayed in Fig. 5 for the climate change dispute.

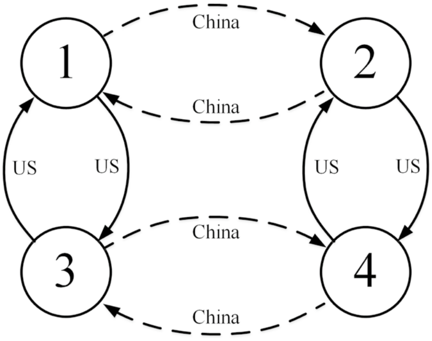
Because of its basic design, the Graph Model possesses a number of inherent advantages. For instance, it can account for what are called irreversible moves. Within Fig. 4, each DM can threaten to move from cooperating to noncooperating



(a) Preference ranking of the US: 3 > 1 > 4 > 2 **(b)** Preference ranking of China: 2 > 1 > 4 > 3

Fig. 4 Directed graph for US (**a**) and China (**b**) in the climate change negotiations dispute

Fig. 5 Integrated graph for the climate change negotiations between the US and China



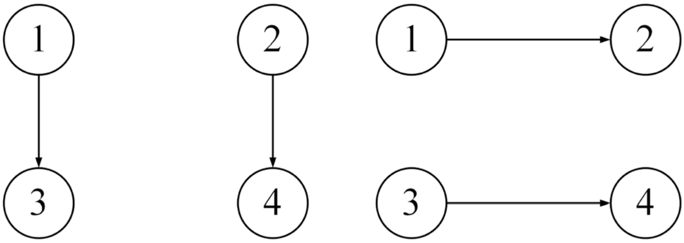
Preference ranking of the US: 3 > 1 > 4 > 2

Preference ranking of China: 2 > 1 > 4 > 3

during negotiations and hence movement is allowed in either direction. However, once a position or situation becomes entrenched or a strong choice is made, a move may become irreversible. This is the situation in Fig. 6a, b in which a given DM cannot reverse its move once it decides to change its strategy selection from cooperating in reducing greenhouse gas emissions to not cooperating, since it may be extremely difficult to reinstate greenhouse gas release policies and associated technologies once they have been cancelled. Figure 6a, b portrays this irreversibility of moves for the US and China, respectively.

Another advantage of the Graph Model is that common moves can be easily accommodated in which two or more DMs involved in a dispute can cause the conflict to go to the same common state by independently choosing certain options under their control. For example, in an all-out nuclear war it does not matter which nation launches the initial nuclear strike, since the final result is essentially the same: nuclear winter and devastation of the planet.

Within a brainstorming session in which a group of stakeholders are trying to ascertain how to sensibly address a pressing conflict situation, a convener could



(a) Preference ranking of the US: 3 > 1 > 4 > 2

(b) Preference ranking of China: 2 > 1 > 4 > 3

Fig. 6 Directed graph for the US (**a**) and China (**b**) containing irreversible moves in the climate change negotiations draw on a blackboard circles or nodes to represent various states and arcs to depict how DMs could move among these states. In practice, one can use option form to generate the states, as depicted in Tables 2 and 3, or normal form, as shown in Table 1, for simple generic disputes.

Definition 1 (*Graph Model*): Theoretically, a Graph Model is a framework defined as (Fang et al. 1993; Xu et al. 2018a):

$$G = \langle N, S, \{A_i, >_i, \sim_i, i \in N\}\rangle, \tag{1}$$

where

- N is a nonempty finite set referred to as the set of DMs,
- S is a nonempty finite set containing the feasible states,
- For each DM $i \in N$, $A_i \subseteq S \times S$ is DM i's set of oriented arcs reflecting the movements controlled in one step by that DM (the symbol × stands for the Cartesian product),
- The symbols $>_i$ and $\sim_i$ reflect the preferences of DM i for any two pair of states s, $q \in S$, for which $s >_i q$ means DM i prefers state s to q and $s \sim_i q$ indicates that DM i equally prefers states s and q or is indifferent between them. Moreover, the preference relationships for each DM abide by the following properties:

(1) $\sim_i$ is reflexive and symmetric. Reflexivity means every state is equally preferred to itself ($\forall s \in S$, $s \sim_i s$) and symmetry indicates for all pairs of states ($\forall$ s, $q \in S$), the order of writing equally preferred states does not matter ($s \sim_i q$ means the same as $q \sim_i s$).
(2) $>_i$ is asymmetric. Therefore, for any pair of states (s, $q \in S$) one prefers one state over the other (ex. $s >_i q$) but not vice versa ($q >_i s$) at the same time.
(3) $\{\sim_i, >_i\}$ is strongly complete which indicates that all pairs of states can be compared according to preference ($\forall$ s, $q \in S$, either $s \sim_i q$ or $s >_i q$ or $q >_i s$).

The most basic type of preference is a preference relationship between two objects, or in the case of the Graph Model between two states. By incorporating this fundamental relationship into the definition of the Graph Model, the Graph Model has the innate capability to account for a rich range of different kinds of preferences. For instance, the Graph Model can handle both transitive and intransitive preferences, as mentioned near the end of Sect. 2 for the conflict methods given on the left branch in Fig. 1. As an example of transitive preferences when comparing beverages, whenever one prefers water to tea and tea to coffee, one should also prefer water to coffee under the transitivity assumption. However, when preferences are intransitive, one may prefer water to tea, tea to coffee but coffee to water. Other preference situations which the Graph Model has been extended to handle are mentioned later.

5.3 Preferences: The Drivers of Conflict

In the physical world, the laws of nature dictate behavior and interactions whether they be physical, chemical, biological or combinations thereof. Within the societal world of humans, it is differences in value systems regarding a particular issue which cause dynamic interactive moves and countermoves to take place among DMs, who can consist of individuals or groups of people, such as nations and industrial enterprises. In the climate change dispute, for example, the differing underlying value systems of the US and China determine their preferences which are reflected in the ranking of states for each DM as provided in Table 2, as well as Figs. 2, 3, 4, 5 and 6. These differences in preference are what can bring about, for instance, the type of chess-like thinking depicted in Table 3 for the case of sequential stability (SEQ) and explained in the corresponding text, as well as the evolution of a conflict over time from a status quo state to a final resolution discussed later. Preferences also form the rationale for DMs to join coalitions: each DM in the coalition will somehow benefit, in terms of preference, or at least not be worse off when he or she is a member of the coalition.

A distinct advantage of assuming relative preference information is that it can be relatively easily obtained in practice. For example, when a friend asks you whether or not you would like to have a cup of coffee or tea to drink, you simply have to respond in a simple way, such as I would prefer to have tea, thank you. You do not have to explain your underlying value system that somehow motivated you to declare that you prefer tea over coffee. Moreover, you do not have to say by how much you prefer tea over coffee, like tea is worth 6.9 utility units to me while coffee is 4.3. The assumption of having relative preference information embedded into the design of a conflict resolution approach like GMCR and the other techniques shown in the left branch of Fig. 1 means that one can more realistically design and construct clever expansions of GMCR, such as coalition analysis, which actually perform well in practice. Additionally, by always assuming pairwise comparisons between any two states as the foundation for preference expansions, one can build highly flexible expansions to GMCR as is explained below for the case of preference uncertainty and degrees of preference. Finally, when cardinal numbers such as utility values or net benefits in dollars are required, as is the situation for most of the methods shown on the right in Fig. 1, these cardinal preferences can be used within the techniques given in the left branch of Fig. 1. This is because an ordering of states is always contained within preferences expressed cardinally, which, of course, are transitive.

Example of DMs and States Using the example of the climate change conflict (see Tables 1 and 2, as well as Figs. 2, 3, 4, 5 and 6), the components of the Graph Model as defined in Eq. (1) are now given.

- The set of DMs is: $N = \{\text{US, China}\} = \{\text{DM 1, DM 2}\}$.
- The set of states consists of: $S = \{\text{CC, CD, DC, DD}\} = \{1, 2, 3, 4\}$.

In Table 1, the set of strategies for the two DMs are US = {C, D} and China = {C, D}. To obtain the set of states, one can use the Cartesian product of these two strategy sets in which all possible combinations of selecting one element from each set are determined as:

$$S = \{C,\ D\} \times \{C,\ D\} = \{CC,\ CD,\ DC,\ DD\}.$$

Additionally, the power set for the option form given in Table 2 can be used to create the set of states in option form as:

$$P(O) = \{YY,\ YN,\ NY,\ NN\}$$

where $O=\{C_{US}, C_{CH}\}$ is the set of options; $P(O)$ reflects the power set of the set of O when using the Y–N notation, which is the set of all subsets of O for which each state in the set of states is written above horizontally in text rather than vertically as in Table 2; Y or N means Yes an option is taken or No it is not selected, respectively; and the first and second entries in a pair are controlled by the US and China, respectively. An alternative notation for reflecting the power set of O is 2^0 which is the set of all mappings (functions) from O to a specified set of two elements which in this case is Y and N.

Notice in the definition for G that preferences are explicitly recorded pairwise in terms of states rather than options or strategies. This is because preferences of a specific DM not only depend on what he or she does within a given pair of states but also what actions her competitors decide to levy. The reader should keep in mind that different approaches can be used to construct states, such as combination of strategies in normal form in Table 1 and the power set of options as is done in Table 2 in option form. When dealing with simple 2×2 conflicts, the normal form of the conflict is quite intuitive for understanding and explaining to others. However, for larger disputes, option form has been shown to work extremely well in practice for display purposes when it is coupled with a sound theoretical foundation based on the GMCR methodology. Given a finite number of DMs with the list of options controlled by each DM shown as the left column in Table 2, one can easily write down the possible feasible states by hand for smaller disputes. For larger conflicts, one can use a computer program to generate the mathematically possible states and subsequently remove the infeasible ones.

Example of Preferences When a dispute is small, one can often record relatively quickly the ranking of states when transitivity is assumed, as is done in Tables 1 and 2 as well as Figs. 2, 3, 4, 5 and 6 for the climate change controversy. When using basic pairwise comparisons to specify preferences, the preferences for the US, for instance, in the climate change dispute are $1 >_{US} 2$, $1 <_{US} 3$, $1 >_{US} 4$, $2 <_{US} 3$, $2 <_{US} 4$, and $3 >_{US} 4$. Because this set of all preference pairs does not violate the concept of transitive preferences, it can be more simply written as: $3 >_{US} 1 >_{US} 4 >_{US} 2$. Likewise, one could write down the pairwise comparison of states regarding preference for China to end up with: $2 >_{CH} 1 >_{CH} 4 >_{CH} 3$.

Option Prioritization Analysts have found in practice that it can be difficult to obtain preference rankings for feasible states, for which ties are allowed, for larger disputes. Accordingly, the option prioritization or preference tree method explained in the Preference Elicitation subsection within Sect. 4 on Conflict Analysis was firstly proposed in 1988 by Fraser and Hipel (1988a). The idea of having a hierarchical list of preference statements expressed in terms of options was refined and extended by Hipel et al. (1997) and Fang et al. (2003a, b) for employment with any preference statements expressed using first order logic. For instance, one of the prioritized preference statements for a specific DM could be written as:

$$\mathrm{O}_3 \text{ and } \mathrm{O}_7 \text{ if } \neg\left(\mathrm{O}_2 \text{ or } \mathrm{O}_6\right)$$

which means that the DM under consideration prefers O_3 and O_7 be selected if O_2 or O_6 is not taken by the DMs controlling them. Under the assumption of transitivity, an algorithm can be used to rank the states for which ties are allowed using a procedure similar to that displayed in Figs. 2 and 3 for the US and China, respectively. As mentioned earlier, the concept of option prioritization closely reflects the way a person may state her preferences in an actual conflict situation. Finally, assuming transitivity, Ke et al. (2012a, b) employed multiple criteria decision analysis to rank states separately for each DM, in which the criteria used to order the states reflect underlying values of that DM.

Preference Uncertainty When employing the GMCR in practice, an analyst or consultant must obtain from a client the DMs, options and relative preferences. Although the required calibration information is relatively small, it may still be difficult to specify the exact preferences of one or more DMs due to uncertainties that could arise such as not having sufficient access to information about some DMs. Hence, four basic approaches to expressing preference uncertainty have been developed within the GMCR paradigm as summarized and explained in Table 4. All of the definitions are formulated using pairwise relationships of states and require that solution concepts or definitions of stability be appropriately revised. An example using the matrix form to record pairwise preference

Table 4 Different kinds of preference uncertainty developed within the GMCR methodology

Types of preferences	Explanations
Unknown (Li et al. 2004; Xu et al. 2011)	When a pairwise comparison for a given DM is not known between states s and q for DM i, it can be recorded as $s\ U_i\ q$
Fuzzy (Al-Mutairi et al. 2008a, b; Hipel et al. 2011b; Bashar et al. 2012, 2014, 2015, 2016, 2018)	When a DM more or less prefers one state over another, this can be formulated using ideas from fuzzy sets developed by Zadeh (1965)
Grey (Kuang et al. 2015a, b, c; Zhao and Xu 2017)	Using grey numbers (Deng 1985) comprised of real numbers, intervals of real number or combinations thereof, one can define grey preferences
Probabilistic (Rêgo and dos Santos 2015, 2018; Silva et al. 2017; Rêgo and Vieira 2019b))	One can state the probability of one state being more preferred than another by a particular DM. One can also entertain upper and lower probabilistic preferences. Probabilistic preferences can be obtained by using probabilistic option prioritizing methods
Combinations	One can employ any combination of the above kinds of preference uncertainty together with crisp preferences

information is shown for the climate change dispute in Sect. 6.2. Beyond the methods given in Table 4, Ben-Haim and Hipel (2002) employ information-gap theory to model preference uncertainty.

Degrees of Preference In the climate change dispute, China may be highly committed to significantly reducing greenhouse gas emissions and thereby greatly prefer states in which it is acting in a cooperative way over ones in which it is not. Accordingly, in the dispute depicted in Table 1, China may: $CC \gg_{CH} CD$ which means China greatly prefers state CC over CD and also $DC \gg_{CH} DD$. At the same time, China prefers state CC over DC as represented by: $CC >_{CH} DC$. Assuming transitivity, China's preferences are:

$CC >_{CH} DC \gg_{CH} CD >_{CH} DD$,

or

$1 >_{CH} 3 \gg_{CH} 2 >_{CH} 4$ when using numbers to represent states. One could further assume that the US preferences remain the same and hence:

$DC >_{US} CC >_{US} DD >_{US} CD$,

or

$3 >_{US} 1 >_{US} 4 >_{US} 2$.

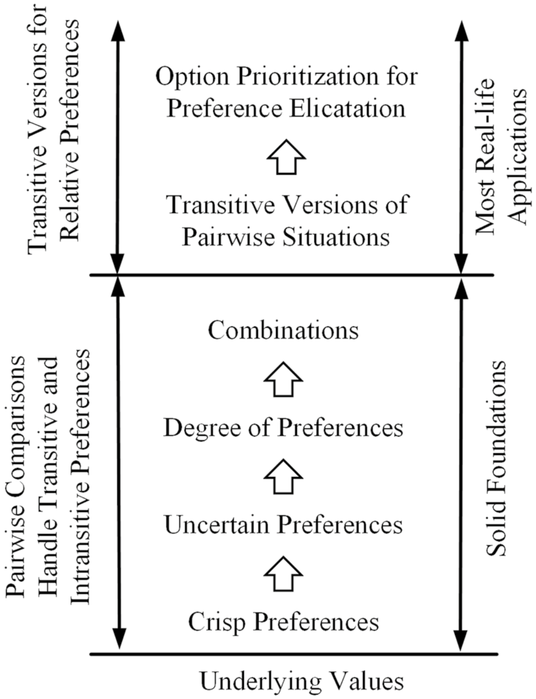
The degree of preference information can be employed when examining moves and countermoves between DMs. Using, for instance, the integrated graph in Fig. 5, notice that China has a strong UI from state 2 to state 1 since $1 \gg_{CH} 2$. However, the US has a UI from state 1 to state 3 because $3 >_{US} 1$. Since $3 \gg_{CH} 2$ by China, its initial UI cannot be credibly blocked by the US.

In what is called two degrees of preference, three pairwise preference relationships exist for each DM: $>$, $\sim$, and $\gg$. If each potential UI by a given DM from a specified state could result in the focal DM being put in a greatly less preferred situation by the sanctioning DMs, then this would result in what is called a strong stability of a given type. If all UIs can be blocked by the sanctioning DMs but at least one can only result in a less preferred situation for the focal DM, this is called a weak stability of a given kind.

Hamouda et al. (2004, 2006) were the first to define the concept of strength of preference to reflect the emotion a DM may feel if it greatly prefers one state over another or greatly less prefers. The question naturally arises as to why not also allow for a preference in which one state is greatly, greatly more preferred than another. Xu et al. (2009a) define the general circumstances in which one can have any finite degree of preference of one state over another. Although preferences of a DM are all defined at the pairwise level, one can see that this concept is filling the gap between relative pairwise preferences to cardinal ones. Finally, because all definitions for degree of preference are defined in a pairwise way, it is valid for employment with transitive and intransitive preferences.

The various kinds of uncertain preferences listed in Table 4 can be combined with strength of preference. For instance, in Chapter 7 of the book by Xu et al. (2018a), as well as in various papers (Xu et al. 2008, 2010a), unknown preference is incorporated into the concept of degree or strength of preference. Figure 7 depicts the connections among the various kinds of preference situations and what has been accomplished within the GMCR paradigm.

Fig. 7 Preference information used in GMCR



5.4 Solution Concepts: Human Behavior Under Conflict

In a dispute, one is wise to contemplate the consequences of making a move before actually doing so. A chess player can think about the possible impacts of moving a chess piece by imagining many moves and countermoves into the future. If the player thinks that he or she may be ultimately better off she will make a specific move. Otherwise, she will think about the consequences of moving another chess piece at this point in time. In a conflict situation, a particular nation may wish to implement tariffs on products being imported from other countries in an attempt to improve its position economically. However, the focal nation must contemplate how the countries may react to ascertain if it will be better off in the long run. If a possible trade war erupts which severely penalizes all of the countries involved, the focal nation may be better off not to implement new tariffs. On the other hand, if through careful analysis the nation initiating the tariffs thinks that this may lead to negotiations which in turn will improve its economic circumstances, then it may be deemed worthwhile to impose tariffs on specified goods and services from other countries.

The foregoing types of possible dynamic interactions among DMs involved in a dispute are defined by what are called solution concepts or stability definitions. Because individuals or groups of individuals, such as nations, may behave differently in a conflict situation, a rich range of solution concepts have been defined within the solid framework of GMCR. The left column in Table 5 provides the names of solution concepts, which have been refined for employment within the

Table 5 Solution concepts defining human interactions in a conflict

Solution concepts	Stability descriptions	Foresight	Knowledge of preferences	Disimprovement	Strategic risk
Nash Stability (*R*) (Nash 1950, 1951)	Focal DM cannot move unilaterally to a preferred state	Low	Own	Never	Ignores risk
General Metarationality (*GMR*) (Howard 1971)	All of focal DM's UIs are sanctioned by subsequent UMs by opponents	Medium	Own	By opponents	Avoids risk: conservative
Symmetric Metarationality (*SMR*) (Howard 1971)	All of focal DM's UIs are sanctioned, even after responses by the focal DM	Medium	Own	By opponents	Avoids risk: conservative
Sequential Stability (*SEQ*) (Fraser and Hipel 1979, 1984)	All of focal DM's UIs are sanctioned by subsequent UIs by opponents	Medium	All	Never	Takes some risks: satisfices
Symmetric Sequential Stability (*SSEQ*) (Rêgo and Vieira 2017b)	All of focal DM's UIs are sanctioned by subsequent UIs by opponents, even after UI responses by the focal DM	Medium	All	Never	Takes some risks: satisfices
Limited-move Stability (L_h) (Kilgour 1984; Zagare 1984; Kilgour et al. 1987)	All DMs are assumed to act optimally after a specified number of state transitions (*h*)	Variable	All	Strategic	Accepts risk; strategizes
Non-myopic Stability (*NM*) (Brams and Wittman 1981; Kilgour 1984, 1985; Kilgour et al. 1987)	Limiting case of limited-move stability as the number of state transitions increases to infinity	High	All	Strategic	Accepts risk; strategizes

GMCR paradigm, along with the original references. The second column from the left explains qualitatively how each solution concept works in practice for determining if a given state is stable for a focal DM, while the four columns on the right qualitatively categorize the solution concepts according to a range of characteristics. Notice, for instance, that Nash stability possesses low foresight, one only has to know the preferences of the DM for whom Nash stability of a state is being determined, the opponents will never invoke disimprovements since no sanctioning is needed for Nash stability, and risk is avoided since stability is caused by the focal DM having no UI to invoke.

One can see in Table 5 that for sequential stability (SEQ) the foresight is classified as being medium since when checking for stability of a state for a particular DM, one only determines if all UIs from the given state by the focal DM can somehow be blocked by UIs levied by its opponents. Therefore, the preference of the DM under consideration must be known and disimprovements by the sanctioning DMs are never considered since only UIs based upon UIs of the sanctioning DMs are employed. Because disimprovements by sanctioning DMs are not taken into account and SEQ possesses medium foresight, some risk is taken and SEQ is satisficing. Additionally, since the situation in which no UIs are present is a special case of SEQ, if a state is Nash stable for a given DM it is also SEQ. Moreover, if a state is stable for all DMs in a dispute for a given solution concept it constitutes an equilibrium or possible resolution for that stability definition. Hence, if the conflict were to arrive at a state which is an equilibrium according to individual stability it would remain there unless an effective coalition were formed or some conflict parameters were to change, perhaps brought about by a third party intervention. Finally, if one has a decision support system available for implementing GMCR, one could calculate stability for every state for each DM for each of the stability definitions in Table 5, which is based upon earlier tables provided by authors such as Hipel et al. (1997).

Beyond Table 5, other solution concepts have also been proposed for employment within GMCR such as Stackelberg stability (Xu et al. 2018b), policy stability (Zeng et al. 2005, 2007), generalized metarational stabilities (Zeng et al. 2006, 2007; Garcia et al. 2017, 2019b), initial state stability (Garcia et al. 2018b), maxmin$_h$ stability (Rêgo and Vieira 2019a), heterogeneous stability (Zhao et al. 2019a, b), as well as consensus and dissent stabilities (Xu et al. 2019). In fact, because of the comprehensive nature of GMCR, as well as the matrix form of GMCR which will be explained in Sect. 6, the number of solution concepts being defined for special situations is exploding. However, the earliest and most widely utilized stability definitions are the top four in Table 5: Nash stability (also referred to as Rational stability), GMR, SMR and SEQ. Because only pairwise comparisons of preference for calculating stability are used with these solution concepts, as well as SSEQ, the top five solution concepts can be employed with both transitive and intransitive preferences. However, this is not the case for limited move and non-myopic stability for which only transitive preferences can be utilized.

To demonstrate the intuitiveness and simplicity of these definitions for human behavior under conflict, theoretical definitions are now furnished for Nash stability and SEQ for the case of having two DMs, as in the climate change conflict. Let

$N=\{i, j\}$ and $s \in S$. In addition, let $R_i^+(s)$ be the set of UIs that DM i has from state s.

Definition 2 (*Nash stability*): State s is Nash stable for DM i, written as $s \in S_i^{Nash}$, *iff* $R_i^+(s)=\varnothing$.

Comment Hence, when a DM cannot improve unilaterally on his or her own in one step, then the DM decides to stay at state s which makes this state stable.

Definition 3 (*Sequential stability (SEQ)*): State s is SEQ stable for DM i, denoted by $s \in S_i^{SEQ}$, *iff* for every $s_1 \in R_i^+(s)$ there exists at least one $s_2 \in R_j^+\left(s_1\right)$ such that $s_2 \leq_i s$.

Comment Therefore, if every UI by DM i can be blocked ($s_2 \leq_i s$) by a credible move by the other DM j ($s_2 \in R_j^+\left(s_1\right)$), then state s is SEQ stable. This sensible solution concept has been demonstrated by numerous applications to function well in practice and to reflect equilibria which actually occur. Moreover, under the assumption of transitivity of preferences and moves, it can be proven theoretically (Fraser and Hipel 1984) that an SEQ equilibrium is always predicted. This existence theorem is extremely important in practice because something always takes place even if it is outright warfare among rival factions.

Definition 2 for Nash stability is also valid for the situation where $|N|>2$, often referred to as the n-DM conflict for which $|N|>2$, since sanctions by other DMs (given as $N-\{i\}$ when using set subtraction) are not taken into account. However, for GMR, SMR, SEQ and SSEQ, one must carefully define the legal sequences of moves that can be made by $N-\{i\}$ both for the case of unilateral moves and unilateral improvements. Subsequently, the definitions can be easily given for the n-DM case by replacing j by $N-\{i\}$, such as can be done in Definition 3 for SEQ.

Example of Stability Calculations As explained in the discussion surrounding Table 1 given in Sect. 3.1, state 4, or DD, is Nash stable for both DM 1 (US) and DM 2 (China). When using Definition 2 for $i=$DM 1=US, notice that $DD \in S_{US}^{Nash}$ because the US has no UI from state DD and hence $R_{US}^+(DD)=\emptyset$. Likewise, for $i=$DM 2=China, $DD \in S_{CH}^{Nash}$ since China has no UI from state DD and therefore $R_{CH}^+(DD)=\emptyset$. Because state DD is Nash stable for both the US and China, it constitutes a Nash equilibrium.

Table 3 portrays why state CC is SEQ stable for DM 1 or the US. One can use Definition 3 to confirm this. Specifically, state $s=$CC (or YY, or 1) is SEQ stable for US $\in N=\{$US, China$\}$ and hence $CC \in S_{US}^{SEQ}$ because the UI to state DC (NY in option form or state 3) which is denoted by $DC \in R_{US}^+(CC)$ can be sanctioned by the UI by China from DC to DD ($DD \in R_{CH}^+(DC)$) which is less preferred by the US to CC (DD $<_{US}$ CC). Since DC is the only UI that the US has from state CC, state CC is SEQ stable for the US. In a similar fashion, one can explain why state CC (YY or 1) is SEQ stable for China. Because state CC is SEQ stable for both DMs, it forms an SEQ equilibrium. The reader should keep in mind that one could use the normal form in Table 1, option form in Table 2, tableau form mentioned in Sect. 4.2, or the graph model depiction in Figs. 4 and 5 to explain why a state is stable according to Nash and SEQ, as well as GMR, SMR and SSEQ stability.

5.5 Coalition Analysis: Faring Better via Cooperation

Suppose that in the climate change dispute between the US and China, the negotiations are stuck in the situation where neither side is decreasing its greenhouse gas emissions. This situation depicted as state DD or state 4 in Table 1 in normal form, state NN in Table 2 in option form and state 4 in Figs. 4, 5 and 6 in graph form. Because neither DM can unilaterally improve on its own from state 4 (see the bottom right cell in Table 1 and the accompanying explanation), state 4 constitutes a Nash equilibrium as indicated in Table 1. Hence, there is no motivation for either the US or China on its own to unilaterally move away from state DD. To explain further, notice when reading from left to right in Table 6a at the top of Table 6 that if the US on its own departs from state 4 to 2 by deciding to cooperate while China does not, the dispute evolves to state 2 which is less preferred to state 4 by the US. If China were then to unilaterally change its option selection to cooperate, the conflict would progress from state 2 to state 1 which is less preferred to state 2 by China. Accordingly, in order for the conflict to progress from state 4 to state 1, which happens to be more preferred to state 4 by both DMs, when moving independently each DM would have to invoke a unilateral disimprovement. As can be seen in Table 6b, a similar strategic situation arises when China moves first from state 4 in attempting to reach state 1: China must make a unilateral disimprovement to state 3 followed by the US selecting a unilateral disimprovement to reach state 1. Clearly, in order for the conflict to progress directly from state 4 to state 1, which as just mentioned is more preferred by both DMs to state 4, the two DMs must agree via effective communication to cooperate with one another to coordinate their behavior for mutual benefit and thereby jointly move from state 4 to state 1 as displayed in Table 6c. This joining of the two DMs in which they share a coalition improvement causes what is called an “equilibrium jump” by Kilgour et al. (2001) to take place. Keep in mind that once state 1 is reached, neither DM individually has the incentive to depart from state 1, since it is SEQ stable for both DMs (see Table 3 for the case of the US) and hence forms an SEQ equilibrium.

This kind of coalitional behavior, which has been observed as a common sociological phenomenon in practice (see for instance Hipel et al. (1993) and Kilgour et al. (2001) for the case of a groundwater contamination dispute), motivated researchers in GMCR to develop a new approach to coalition analysis in which coalitions form at a given state according to whether or not it is advantageous for DMs to do so. Hence, this procedure is quite distinct from the coalition approach described earlier in Sect. 4.6 under the Conflict Analysis framework in which a coalition is expected to form when the DMs in a coalition have similar interests, and the coalition preferences replace the individual preferences of the DMs in the coalition. In fact, this procedure for coalition formation just explained for the climate change negotiations for two DMs constitutes a special case of the general situation in which

Table 6 Evolution of the climate change conflict

	DD		CD		CC
State Labels	4		2		1
US					
1. Cooperate	N	↛	Y		Y
China					
2. Cooperate	N		N	↛	Y

US: 3 > 1 > **4 > 2**

China: **2 > 1** > 4 > 3

6(a)

	DD		DC		CC
State Labels	4		3		1
US					
1. Cooperate	N		N	↛	Y
China					
2. Cooperate	N	↛	Y		Y

US: **3 > 1** > 4 > 2

China: 2 > 1 > **4 > 3**

6(b)

	DD		CC
State Labels	4		1
US			
1. Cooperate	N	→	Y
China			
2. Cooperate	N	→	Y

US: 3 > **1 > 4** > 2

China: 2 > **1 > 4** > 3

6(c)

the two or more DMs are participants in disputes. Suppose, for instance, that a conflict were taking place in which $|N| > 2$ and any coalition $H \subseteq N$ could form for which $|H| \geq 1$. As just explained in Table 6c, the key motivation for a coalition to form, when the conflict has arrived at a given state s, is for a joint coalition unilateral improvement (CUI) to exist for H from that state which is more preferred by every coalition member to state s. For the situation in Table 6c, the CUI from state 4 to 1 cannot be sanctioned by others, since the improvement or equilibrium jump for this illustration consists of the grand coalition, which means all participants in the conflict. Nonetheless, in general, when there are more than two DMs, a CUI could be blocked by others, outside of the coalition, which means that the DMs in the set N

– *H*, who can create a wide range of combinations of coalitions to attempt to block a CUI. To reflect this situation the stability definitions for independent or noncooperative behavior, such as Nash, GMR, SMR and SEQ in Table 5, have been appropriately extended to handle coalitions in cooperative situations, for which independent behavior is a special case.

Kilgour et al. (2001) defined coalitional Nash stability (CNash) for state *s* for a coalition $H \subseteq N$ for the situation in which no CUI exists for *H* from state *s*. If state *s* is CNash stable for all $H \subseteq N$, then this state is referred as being coalitionally Nash stable. Inohara and Hipel (2008a, b) presented coalition stability definitions as extensions of GMR, SMR and SEQ stabilities. To accomplish this, one has to carefully define the unilateral moves and unilateral improvements available to a specific coalition and combinations of coalitions, especially when considering how sanctioning could occur.

Using the example of coalition SEQ stability, a coalition stability definition involving sanctioning is qualitatively defined as follows. State *s* is coalitionally SEQ stable (CSEQ) for a coalition *H* if for all possible CUIs from *s* for *H*, there exists a situation or sanction which is less or equally preferred to *s* by *H* which is caused by legal sequences of UIs or CUIs made by all combinations of possible coalitions that could form in the set $N - H$. A specific state *s* is said to be coalitionally SEQ stable if it is CSEQ for every $H \subseteq N$ for which $|H| \geq 1$. Similar definitions can be made for CGMR and CSMR as well as other stability definitions.

The definitions for coalition stability are framed in a way that makes the stable states found when no coalitions are present to be a subset of the corresponding coalition stability findings. Inohara and Hipel (2008b) established theorems revealing the interrelationships among coalition stabilities. By defining a different way in which sanctioning could take place when determining coalition stability, Xu et al. (2018a, Ch. 8) proposed other related definitions of coalition stability for the four basic types of stability listed at the top of Table 5. Additionally, Xu et al. (2010b, 2018a, Ch. 8) explained how to employ the matrix form to calculate coalition stability and also did this for the case of unknown preference (Li et al. 2004).

Based on the concept of Pareto coalition improvements, Zhu et al. (2018) proposed a new class of coalition stabilities for the four basic noncooperative definitions. To be a Pareto coalition improvement from state *s* for a coalition *H*, the resulting state must be at least equally preferred by all coalition members and more preferred by at least one. Zhu et al. (2018) also defined another way in which sanctioning could occur against UIs, CUIs and Pareto coalition improvements when defining coalition stability, which is different from the approaches provided by Inohara and Hipel (2008a, b) and Xu et al. (2018a, Ch. 8) for the case of UIs and CUIs. Zhao et al. (2019b) developed a coalition analysis approach which considers all possible coalition scenarios with full participation of the sanctioning opponents.

5.6 Evolution of a Conflict over Time: The Power of Graphs

Table 6c displays how the climate change dispute between the US and China could evolve over time if the conflict were to start at state 4 (or also labelled as DD or

(NN)), in which neither country is reducing its greenhouse gas emissions beyond the current situation. As is explained in the previous subsection on coalition analysis, the dispute could follow one of the three possible paths to reach the desirable state in which both nations agree to significantly reduce their greenhouse gas releases. Because the progressions shown in Table 6a, b involves DMs moving to less preferred states, these situations would not occur in practice. However, the joint movement from state 4 to 1 could feasibly take place if the two DMs were to cooperate with one another.

The example in Table 6c nicely illustrates how a dispute could evolve from an initial or status quo state, like state 4, to a desirable state or state of interest such as state 1, in which humanity and the climate systems are beneficiaries. In fact, the unilateral moves which the US and China can make in one step are encapsulated in their directed graphs drawn in Fig. 4a, b, respectively, as well as the integrated graph in Fig. 5. The directed graph for each of the DMs, coupled with its relative preferences, contain the information needed to draw in option form in Table 6 the three ways in which the conflict could progress from state 4 to 1. In general, for larger disputes in which there can be many DMs and options, the information contained in the directed graphs and preferences for all of the DMs could be quite large. Accordingly, algorithms, such as those constructed by Li et al. (2005), are needed to trace the paths that could be followed from a selected status quo to a final state.

Depending on the characteristics of a specific conflict under study and what an analyst wishes to model, situations in which one may wish to have algorithms available to trace paths from a specific status quo to a selected final state include the following:

- Any unilateral moves under the control of the DMs are permitted to find the paths available as DMs interactively move from the initial to the final state, as is done in Tables 6a, b for the case of the climate change conflict;
- Only UIs by the DMs are permitted;
- Some DMs are permitted to invoke any move they control while others can only use UIs;
- Various types of joint movements among DMs are allowed in combination with unilateral moves by DMs;
- Joint coalition improvements are allowed, as shown in Table 6c.

One should keep in mind that determining the possible evolution of a conflict, which is sometimes referred to as status quo analysis, is quite different from carrying out a stability analysis. When executing a stability analysis, one wishes to ascertain if the conflict were at a given state would it be beneficial for a given DM to depart from that state. If the DM could end up being worse off for doing so according to a particular solution concept, then that state is stable according to the particular solution concept being considered. If the state is stable for all of the DMs with respect to the solution concept being entertained then it forms a possible equilibrium. Therefore, if the conflict were to evolve to the state being investigated, the conflict would remain there unless there is an equilibrium jump as depicted for the climate change dispute in Table 6c. Moreover, in practice one can employ a decision support system to analyze each state for stability for each DM for all of the solution concepts listed in Table 5, especially the top four, as explained in Sect. 8. The findings from indepth stability analyses furnish valuable strategic results for understanding and enhancing the decision-making process in complex conflicts. Finally, one can also expeditiously carry out a coalition analysis using a decision support system to ascertain if DMs can fare even better by cooperating.

In contrast to a comprehensive stability analysis, the evolution of conflict vividly displays how a dispute could evolve over time from a status quo to a final equilibrium. In most practical real world applications, the authors have found that it is very informative to provide a table in option form which portrays how the eventual resolution was reached from the starting state. Quite often, the path from the status quo may pass through a number of transition states before arriving at a final equilibrium. In contrast to how states may evolve, Ali et al. (2019b) explain how parameters in a GMCR model may change over time in a process they call the evolutional graph model approach. When these and other advances are combined with the key stability findings from extensive stability analyses, as well as other results found using other conflict analysis methods explained next, a rich range of valuable strategic results can be furnished for better understanding and enhancing decision making under conflict.

5.7 Other Advancements: Psychological Considerations, Power and Transition Time

Prior to describing the matrix formulation of the Graph Model, other important advances originally presented within the "logical" structure of GMCR are mentioned in this subsection. Specifically, three contributions to psychological aspects of conflict resolution are outlined followed by two ways for addressing power in conflict situations and an approach for taking transition time into account.

Psychological Considerations As discussed in Sect. 4.7 under Conflict Analysis, a *hypergame* is a situation in which misunderstandings by one or more DMs in a dispute can occur at different levels. For example, in labor-management negotiations for reaching a new contract agreement within a company, a labor union may try to deceive management into believing that it may prefer to go on strike if management does not sign a generous contract. If management is aware that the labor union is bluffing in its strike threat, since jobs for labor are currently scarce in tough economic times, this forms what is called a second level hypergame: labor thinks management believes its strike threat is genuine but management is not fooled. Aljefri et al. (2017, 2018) have developed a solid theoretical foundation for hypergame analysis within the GMCR paradigm for both the first level (Aljefri et al. 2017) and any level of hypergame (Aljefri et al. 2018) in which different kinds of misunderstandings can occur by one or more DMs involving options, preferences and DMs. This general theory of hypergame, which is built from the option level upwards, includes a DM having misperceptions about itself, such as when a dictator thinks its military forces are more powerful than they are in reality. Furthermore, it includes stability procedures for ascertaining stability results from each DM's viewpoint at any hypergame level as well as overall stability regarding what could actually happen. Finally, Rêgo and Vieira (2017a) develop a concept related to hypergames called interactive unawareness.

People naturally think that attitudes can affect the eventual resolution in a dispute. Hence, positive attitudes among competitors in a trade dispute may result in a better overall agreement for all concerned parties. Within the structure of GMCR, an approach referred to as *attitude analysis* has been developed in which a participant in a dispute can have a positive, negative or neutral attitude towards each of his or her competitors and himself (Inohara et al. 2007; Bernath Walker et al. 2009, 2012a, b; Yousefi et al. 2010a, b, c). Appropriate stability definitions and associated algorithms have been designed for employment with Nash, GMR, SMR and SEQ in the presence of any combination of attitudes among the DMs participating in a particular dispute. Ali et al. (2019a) employ attitude analysis to investigate potential negotiations between Pakistan and India with China as a mediator to demilitarize the Siachen Glacier region in order to decrease environmental damage. Xu et al. (2018c) design an option-oriented attitude analysis which takes into account degree of stabilities.

To handle emotions in a dispute, an approach based on the concept of *perceptual graphs* has been developed (Obeidi et al. 2005, 2009a, b). Emotions can also be modeled within GMCR by utilizing the idea of strength or degree of preference mentioned earlier in Sect. 5.3. For instance, a pacifist greatly prefers the situation in which peace occurs over those in which warfare takes place.

Power The concept of power in a dispute can be expressed by the options a DM controls. For instance, a powerful nation having a large economy, such as the US or China, can have a highly detrimental effect upon a small country when it levies tariffs on products imported from the smaller nation. This relatively strong option of a powerful nation imposing crippling tariffs is greatly less preferred by the smaller country. Power can also be captured in other ways within GMCR. In particular, within what is termed a *hierarchical conflict*, significant progress has been made in which a common DM may be involved in two or more conflicts, for instance, when a central national government interacts separately with each province when providing funding for education or environmental stewardship (He et al. 2013, 2014, 2017a, 2017b, 2018). Hierarchical structures exist in most military organizations, corporations and nations having strong central control such as Japan, France and China. Finally, under a concept called *power asymmetry* a DM taking part in a dispute can influence the preferences of other participants by invoking specific options reflecting the DM's more powerful position (Yu et al. 2015).

State transition time In some situations, the length of time a DM spends on moving from one state to another may affect the evolution paths and results of the conflict. For example, in the Prisoner's Dilemma case, the prisoner who spends less time on deciding to confess may block the other prisoner's move which takes a longer time to occur. This phenomenon is examined in the work by Inohara (2016).

6 Matrix Formulation: A Decision Technology Transformation

As noted previously in Sect. 5.1 under Inspirational Design, the graph model formulation can be readily converted to matrix form as was originally done by Xu et al. (2009b) in their seminal paper on this topic and is explained in detail for many conflict situations in the textbook by Xu et al. (2018a). This permits higher computational efficiency when, for example, calculating a range of stability findings under noncooperative and cooperative behavior. Additionally, the matrix form provides a launch pad for new theoretical advancements in GMCR. For example, a number of advances have been reported in the development of new solution concepts for handling different kinds of strategic thinking, such as heterogeneous stability (Zhao et al. 2019a) and its coalition analysis version (Zhao et al. 2019b) plus consensus and dissent stabilities mentioned in Sect. 5.3. In fact, the first textbook on GMCR by Fang et al. (1993) focuses on a "logical" interpretation of GMCR whereas the latest book by Xu et al. (2018a) a matrix interpretation of GMCR.

6.1 Basic Modeling and Stability Definitions

Before any type of matrix calculations can be carried out using the matrix form of GMCR, appropriate preference and movement matrices must be defined. In particular, the preference matrices P_i^+, P_i^-, $P_i^=$ and $P_i^{-,=}$ as well as unilateral moves and unilateral improvements matrices J_i and J_i^+, are formally defined in Definitions 4 and 5, respectively, (Xu et al. 2009b):

Definition 4 (*Preference matrices*): In a graph model, let P_i^+, P_i^-, $P_i^=$ and $P_i^{-,=}$ be the four $m \times m$ preference matrices for DM i whose entry (s, q) for which $s, q \in S$ is defined as follows:

$$P_i^+(s,q) = \begin{cases} 1, & \text{if } q \succ_i s \\ 0, & \text{otherwise,} \end{cases}$$

$$P_i^-(s,q) = \begin{cases} 1, & \text{if } s \succ_i q \\ 0, & \text{otherwise,} \end{cases}$$

$$P_i^=(s,q) = \begin{cases} 1, & \text{if } q \sim_i s \text{ and } q \neq s \\ 0, & \text{otherwise,} \end{cases}$$

$$P_i^{-,=}(s,q) = \begin{cases} 1 - P_i^+(s,q), & \text{if } q \neq s \\ 0, & \text{otherwise.} \end{cases}$$

For the above definition, it is assumed that $P_i^=(s,s) = P_i^{-,=}(s,s) = 0$.

Definition 5 (*Movement matrices*): In a graph model, let J_i and J_i^+ denote $m \times m$ 0–1 matrices representing the unilateral moves and unilateral improvements of DM i, respectively, as follows:

$$J_i(s,q) = \begin{cases} 1, & \text{if } (s,q) \in A_i \\ 0, & \text{otherwise,} \end{cases}$$

$$J_i^+(s,q) = \begin{cases} 1, & \text{if } J_i(s,q) = 1 \text{ and } q \succ_i s \\ 0, & \text{otherwise.} \end{cases}$$

Theorem 1 (*Nash stability*): *Define*

$$M_i^{Nash} = J_i^+ \cdot E, \tag{2}$$

where E *denotes the* $m \times m$ *matrix with each entry being set to 1. State* $s \in S$ *is Nash stable for DM* i *iff* $M_i^{Nash}(s,s) = 0$.

Definition 6 (*Joint improvement matrix*): In a graph model, the unilateral improvement matrix for H is the $m \times m$ matrix with (s, q) entries:

$$M_H^+(s,q) = \begin{cases} 1, & \text{if } q \in R_H^+(s) \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

where $R_H^+(s)$ is the set of unilateral improvements by H. Note that when there exists only one DM in H, say DM k, then $M_H^+ = J_k^+$.

Theorem 2 (*Sequential stability*): *Define*

$$M_i^{SEQ} = J_i^+ \cdot \left[E - sign\left(M_{N-\{i\}}^+ \cdot \left(P_i^{-,=}\right)^T\right)\right], \tag{4}$$

where $M_{N-\{i\}}^+$ *is the joint improvement matrix for the set of DMs* $N - \{i\}$, *and* $sign(\cdot)$ *is the sign function for which the value of the function equals 1 when the entry is positive, 0 when the entry is zero, and* -1 *when the entry is negative. State* $s \in S$ *is SEQ for DM* i, *denoted by* $s \in S_i^{SEQ}$, *iff* $M_i^{SEQ}(s,s) = 0$.

6.2 Climate Change Negotiations Illustration

To explain how the matrix formulation can be applied in practice, consider the determination of whether or not state 1 (also labelled as CC in normal form in Table 1 and YY in option form in Table 2) is Nash stable and SEQ in the climate change dispute. The "logical" form calculations for Nash stability appear in Table 1, while those for SEQ are shown in Table 3. With respect to the matrix calculations, the following input matrices are needed for the US and China.

Reachable (UM) Matrices:

$$J_{US} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad J_{China} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Preference Relation Matrices:

$$P_{US}^{+} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad P_{China}^{+} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

UI Matrix:

$$J_{US}^{+} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad J_{China}^{+} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix calculation for Nash stability for state 1 (CC) for the US is executed as:

$$M_{US}^{Nash} = J_{US}^{+} \cdot E = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since $M_{US}^{Nash}(1,1) \neq 0$, state 1 is not Nash stable for the US. The determination of Nash stability for state 1 (CC) for China is:

$$M_{China}^{Nash} = J_{China}^{+} \cdot E = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Because $M_{China}^{Nash}(1,1) \neq 0$, state 1 is not Nash stable for China. The matrix computation for SEQ stability for state 1 (CC) for the US is:

$$M_{US}^{SEQ} = J_{US}^{+} \cdot \left[E - sign\left(J_{China}^{+} \cdot \left(P_{US}^{-,=}\right)^{T}\right)\right]$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} - sign\left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}^{T} \right) \right) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Because $M_{US}^{SEQ}(1,1) = 0$, state 1 is SEQ stable for the US. Finally, the determination of SEQ stability for state 1 (CC) for China is as follows:

$$
\begin{aligned}
M_{China}^{SEQ} &= J_{China}^{+} \cdot \left[E - sign\left(J_{US}^{+} \cdot \left(P_{China}^{-,=} \right)^{T} \right) \right] \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} - sign\left(\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^{T} \right) \right) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

Since $M_{China}^{SEQ}(1,1) = 0$, state 1 is SEQ stable for China.

7 System Perspectives for Investigating Conflict Using the Graph Model

7.1 Overview

Figure 8 portrays three systems perspectives in which GMCR can be applied in practice, as suggested by authors such as Hipel et al. (2015) and Kinsara et al. (2015b), reflecting what is often done in the physical sciences. The top flow chart of Fig. 8 depicts a forward investigation using GMCR as explained earlier in this paper. Given the input of a Graph Model in terms of DMs, the options controlled

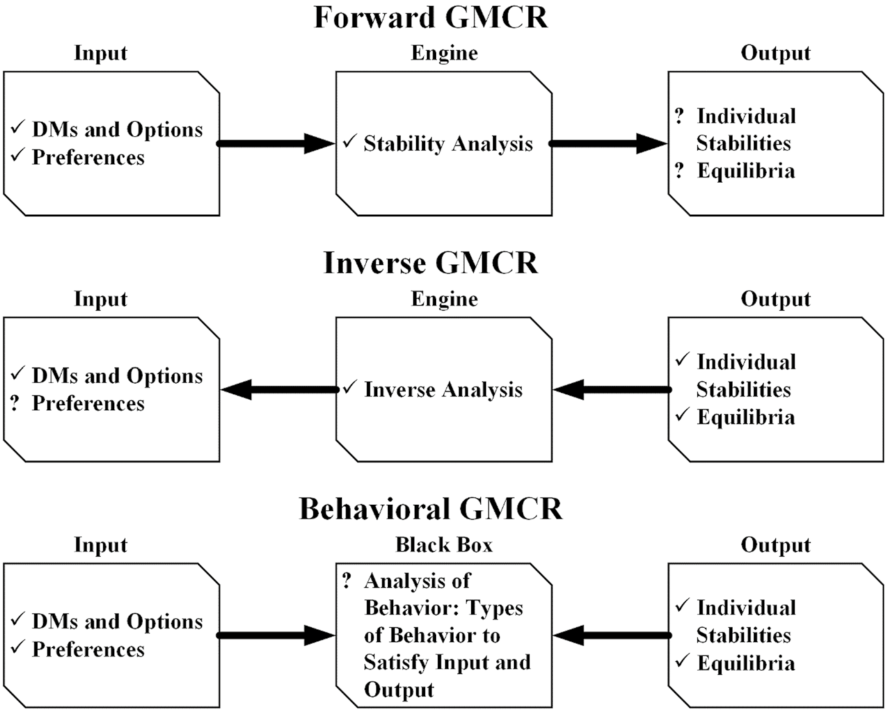


Fig. 8 Three systems perspectives for studying conflict within the GMCR paradigm

by each DM and each DM's relative preferences among the feasible states, a GMCR engine computes as output the stable states for each DM according to a variety of stability definitions, as well as the equilibria and other useful strategic results. Notice that the items which one wishes to determine are marked with question marks in the output while the known input information and calculation procedure in the engine have checkmarks.

In an inverse investigation, depicted as the middle diagram in Fig. 8, one wishes to ascertain the required preferences by DMs, as indicated by the question mark to the left of the preferences in the input, to cause a state to be the desirable equilibrium. For instance, one may wish to determine the needed preferences of a reluctant DM involved in climate change negotiations to arrive at a situation in which it agrees to cutback substantially greenhouse gas emissions. Sakakibara et al. (2002) first suggested that one should study the inverse problem within the framework of GMCR. Kinsara et al. (2015a) designed a pattern analysis algorithm for determining the unknown preferences for the case of two or more DMs. Based upon an option prioritization interpretation, Wu et al. (2019) proposed an optimization procedure as a mediation tool for determining minimal priority adjustment to preference statements to reach a desired state for the case of two DMs. Wang et al. (2018) developed a general analytical approach for doing this using the matrix form of the game. Specifically, their inequality for calculating the preferences within Inverse GMCR for the case of sequential stability (SEQ) is:

$$M^{+}_{N-\{i\}} \cdot \left(\left(P_i^{-,=} \right)^T \cdot e_s \right) \geq \left(J_i^T \cdot e_s \right) \circ \left(\left(P_i^{+} \right)^T \cdot e_s \right) \tag{5}$$

where $M^{+}_{N-\{i\}}$ is the joint improvement matrices for the set of DMs $N-\{i\}$, and the symbol $\circ$ indicates the Hadamard product of two matrices. The term e_s stands for an m-dimensional column vector in which the sth element is 1 and all other entries are 0 when there are m feasible states. Note that when there is only one DM in the set of $N-\{i\}$, say DM k, then $M_{N-\{i\}} = J_k$ and $M^{+}_{N-\{i\}} = J^{+}_k$.

The third kind of systems study which can be carried out is displayed as the flow chart at the bottom of Fig. 8. In this Behavioral GMCR or Engine perspective referred to elsewhere as systems identification, one wishes to find the required behavior by the DMs to cause a known output given the known input. Wang et al. (2019) provide an analytical method for doing this using the matrix form of the conflict.

As mentioned earlier, significant strides have been made in expanding Forward GMCR. However, the doors have just been opened by the fundamental contributions of Wang et al. (2018, 2019) to permit entry by researchers to substantially extend both Inverse and Behavioral GMCR in numerous ways which mirror what has already been achieved within the forward perspective. Both of these recent perspectives could be useful for addressing different aspects of conflict resolution. Consider, for instance, the situation with respect to Inverse GMCR which could be utilized in third party intervention in which a facilitator attempts to encourage DMs in a negotiation to change their preferences in a direction that will bring about a win/win resolution. To appreciate how calculations can be made for an inverse engineering problem using Inequality (5), an illustration is now provided.

7.2 Example of Inverse GMCR in the Climate Change Dispute

Assume that in the climate change dispute China would like to reduce greenhouse gas emissions which indicates it prefers to select strategy C over D. Hence, China prefers state 1 (CC) over state 2 (CD) and state 3 (DC) more than state 4 (DD). This in turn means that China has a UI from state 2 to 1 and another UI from 4 to 3. Therefore, one can substitute the matrices given below into Inequality (5):

$$J_{US} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad J_{China} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad J^{+}_{China} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $J^{+}_{China}(1,2)$ and $J^{+}_{China}(3,4)$ are equal to 1 to indicate that the moves from state 1 to 2 and state 3 to 4 are unilateral improvements for China.

Consider, for example, state 1 as a desirable state for being stable for the US under the solution concept of sequential stability. Substituting the appropriate information into Inequality (5), the required US's preference relations are calculated by solving the following:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \left(\left(P^{-,=}_{US}\right)^{T} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) \geq \left(\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^{T} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) \circ \left(\left(P^{+}_{US}\right)^{T} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

Note that $P^{-,=}_{US}(s,q) = 1 - P^{+}_{US}(s,q)$ if $q \neq s$, and $P^{-,=}_{US}(s,s) = 0$ for $s \in S$ by Definition 4. Therefore, the required preference by the US is given as:

$$P^{+}_{US}(1,3) + P^{+}_{US}(1,4) \leq 1$$

The above calculations mean that states 3 and 4 cannot be simultaneously more preferred than state 1 for the US. Notice that within state 1 (CC) the US is cooperating by selecting strategy C but it does not cooperate (D) in state 3 (DC) or state 4 (DD). When $P^{+}_{US}(1,3) = 0$, which indicates that state 3 is less than or equally preferred to state 1 for the US, state 1 is Nash stable. Hence, state 1 is also SEQ stable for the US. When $P^{+}_{US}(1,3) = 1$, then $P^{+}_{US}(1,4) = 0$, which implies state 3 is more preferred than 1 but state 4 is less than or equally preferred to state 1 for the US. In this case, given that China has a UI from state 3 to 4, the US has a UI from state 1 to 3, but will end up in state 4, which is not more preferred than the starting state 1.

In Artificial Intelligence (AI), also called Machine Intelligence, one wishes to construct machines and associated software to think intelligently like people, such as the ways in which individuals may think in learning and problem solving. As just illustrated, in Inverse GMCR, a computer can be programmed to use Inequality (5) to compute how DMs should think in terms of their preferences in order to bring about a desired resolution to a given dispute. Accordingly, AI has an important role to play within the field of conflict resolution. In fact, within the earlier Expert Systems approaches formulated within AI, Ross et al. (2002) used experience gained by researchers and practitioners examining a wide range of environmental case studies using GMCR to suggest the design of a generic case-based Expert System as an initial design of an environmental graph model. This initial model can be appropriately refined for a specific case study before carrying out indepth stability analyses. If, for instance, an historian wanted to examine a wide range of historical disputes for which abundant information was available, the "data science" component of AI, coupled with some basic concepts from GMCR, could be useful for examining how humanity can learn from its past in order to make more informed decisions now and in the future.

8 Decision Support Systems for the Graph Model

As described in the previous three sections, the GMCR methodology contains an attractive array of linked capabilities for tackling many different aspects of real world conflict which can arise across disciplines. To make these powerful decision technologies available now for addressing actual conflict, decision support systems (DSSs) (Sage 1991) can be constructed. DSSs are in essence user-friendly computer programs of a particular methodology like GMCR for producing insightful findings for aiding a researcher or practitioner in making informed decisions. Because the GMCR methodology possesses a sound theoretical basis and sensible extensions which reflect what can happen in reality, it is especially amenable for implementation as a DSS.

Figure 9 displays a universal design of a DSS for GMCR founded on a proposal put forward by Xu et al. (2018a, Sect. 10.2). From a systems viewpoint, this design reflects the forward perspective portrayed in the top portion of Fig. 8, and it can be readily extended to handle Inverse GMCR and Behavioral GMCR displayed in the middle and lower parts of Fig. 8, respectively. As can be seen on the left in Fig. 9, key input required by the system is the DMs, and each DM's options from which the program can interactively determine the feasible states and state transitions. Option prioritization (see Sects. 4.5 and 5.3) and other methods can be employed to obtain the ordinal preferences for each DM across the feasible states. With this relatively small amount of input information, the DSS can produce stability findings for both purely competitive (Sect. 5.4) and cooperative behavior (Sect. 5.5) for specified types of solution concepts, like the cooperative counterparts of the top five solution concepts in Table 5. Stability results can also be calculated for the different types of preference uncertainty listed in Table 4. To enhance the computational efficiency of the analysis engine, the matrix formulation outlined in Sect. 6 can be utilized.

As indicated on the right in Fig. 9, the stability findings can be organized in a way that enhances a user's interpretation of the strategic situation with respect to the conflict under study. For example, in a larger conflict having many equilibria, one may wish to assemble a set of equilibria containing the selection of certain

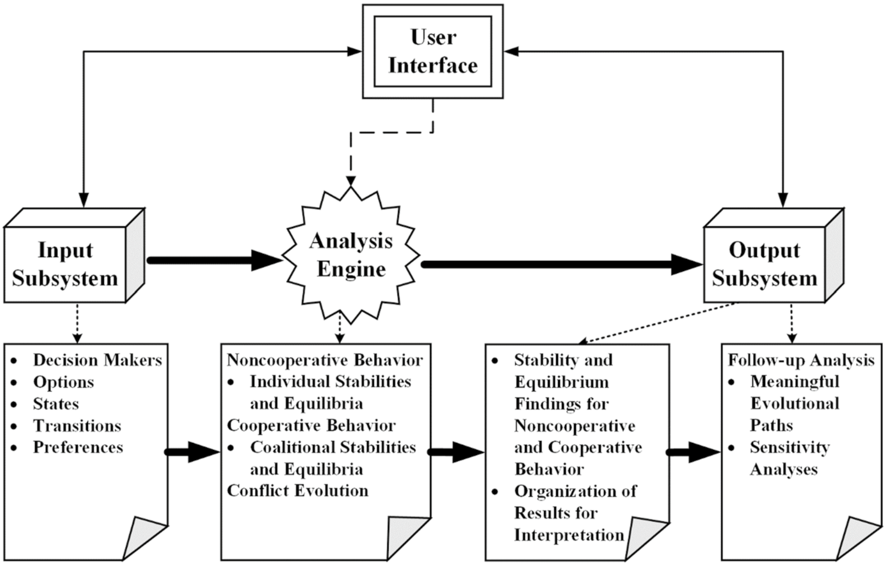


Fig. 9 Structure of a graph model based DSS

options that reflect an overall favorable situation, such as when all of the main nations in the world agree to significantly reduce greenhouse gas emissions in climate change negotiations. Under follow-up analyses, one can trace the evolution of a conflict from a status quo to a final equilibrium, as is done for the US-China negotiations over climate change depicted in Table 6c. Depending on the specific conflict being investigated, a meaningful range of sensitivity analyses can be executed, like ascertaining how small changes in one or more DM's preferences can affect the strategic findings.

A number of DSSs for implementing certain aspects of GMCR have been developed over the years. As a supplement to their 1993 book, Fang et al. (1993) provided a basic analysis engine called GMCR I for calculating stability for all of the solution concepts given in Table 5 except SSEQ. The separately-programmed system referred to as GMCR II (Hipel et al. 1997, 2008a, Fang et al. 2003a, b) can determine stability for the same solution concepts as GMCR I for the case of pure competition but also possesses a very user-friendly input which includes option form (Sect. 3.2) and option prioritization (Sects. 4.5 and 5.3). The system called GMCR+ (Kinsara et al. 2015b, 2018) can compute both individual and coalitional stability for the top four solution concepts in Table 6. The stability computations can be carried out utilizing both the logical (Sect. 5.4) and matrix (Sect. 6) definitions for stability. Within the inverse engineering paradigm, an exhaustive search engine is employed for ascertaining the preferences of the DMs required to produce a desirable final equilibrium. Based on a matrix formulation of GMCR, a DSS system called NUAAGMCR can be used for carrying out stability, coalition, attitude, consensus and status quo analyses (Xu et al. 2018c, 2019).

The next generation of DSSs for implementing the GMCR methodology should follow the fundamental design displayed in Fig. 9 in order to be as flexible, encompassing, computationally efficient, user-friendly and suitable for expansion as possible. As new theoretical breakthroughs regarding GMCR are made in the future, for which many would be associated with Inverse GMCR and Behavioral GMCR depicted in the middle and lower portions of Fig. 8, respectively, the existing GMCR system can be readily expanded to handle these contributions. The authors are convinced that the GMCR methodology will continue to expand in not only expected ways but also in what are now unforeseen directions as unanticipated kinds of conflict arise over time.

9 Expanding Frontiers: Opportunities to Benefit Society

Conflict is ubiquitous within and across virtually every area of human endeavours. In fact, human activities are so massive on a global scale in areas such as agriculture, energy generation, industry, transportation, urban sprawl and land use changes that our indispensable natural systems are under threat of being altered, perhaps in irreversible ways which are extremely harmful to society. For instance, the large-scale release of greenhouse gases in every nation of the world, especially the highly developed and emerging industrial giants, is already causing an observable deterioration in our climate and natural systems, such as having a warmer atmosphere, rising sea levels and extreme weather conditions. The widespread dumping and burial of toxic materials continue to create massive brownfields which pollute our valuable groundwater aquifers which may take decades or perhaps hundreds or even thousands of years to cleanse. As the population of the world continues to expand and human behavior remains unbridled, conflict is bound to expand and intensify, perhaps in unforeseen ways. Therefore, the demand for decision technologies for effectively dealing with a broad range of conflict will certainly dramatically increase as the future unfolds.

Because the world is so highly interconnected within and across its societal and natural systems, an overall system of systems approach is required for designing effective solutions to address tough problems (see, for example, Hipel et al. (2008a, b); Haimes (2016, 2018); Xu et al. (2018a, Ch. 2)). Decision making and governance will have to be integrative in scope and highly adaptive for responding to emerging problems or surprises. Under this systems umbrella, it is of great importance to take into account the value systems and interests of affected stakeholders in a fair and equitable fashion within the plethora of conflicts which will inevitably arise. This is where the GMCR methodology has a key role to play, along with appropriate complementary techniques which involve other aspects of a particular problem under study.

As stressed in Sects. 1 and 5.1, the authors recommend that actual demand or need for tackling different characteristics of real world conflict should be used to guide the direction of GMCR research for expanding its domain of applicability. For example, cooperation, which actually took place in a groundwater contamination dispute that the authors were analyzing, inspired the authors and other colleagues to design the new coalition approach described in Sect. 5.5. In fact, over the years, the authors and their colleagues have investigated a broad range of conflict situations which have inspired their successful expansion of the GMCR methodology in worthwhile directions. As noted in Sect. 1, the authors were particularly motivated by the need for resolving challenging water and environmental problems facing society. A comprehensive list of references regarding application of GMCR to ten key fields such as sustainable development, aquaculture, water resources and energy is provided by Xu et al. (2018a, Table 1.8). These applications provided the demand for capturing key characteristics of conflict situations occurring in reality as well as a means for testing and refining GMCR techniques being constructed.

Bold innovative directions for expanding conflict resolution will no doubt be discovered by researchers and practitioners as they systematically search for strategic insights into disputes that are of direct concern to them. Fortunately, the solid seaworthy design of the basic GMCR vessel permits one to sail into largely uncharted waters for which the development of operational decision technologies for tackling complex challenges in conflict resolution would have been difficult to achieve in the past. Founded on their many years of experience, much of which has been put into perspective in this paper as well as some in earlier articles (Kilgour and Hipel 2005; Hipel et al. 2011a), the authors would like to suggest some areas which currently require opportunities for expansion.

One route for exploration is the design of learning methods for discovering how a DM may be thinking under conflict. Based on the observed actions that a DM invokes in a dispute in which preferences may be largely unknown, initial progress has been made on determining associated preferences (Garcia and Hipel 2017; Garcia et al. 2018a, 2019a). The hypergame situation mentioned in Sect. 5.7 could be expanded to handle learning. In particular, in a hypergame in which misunderstandings exist among DMs for which a surprising result happens, learning methods could be built for determining the correct intentions and preferences of a DM. After learning as much as possible about a particular conflict, the inverse engineering approach mentioned in Sect. 7 and displayed in the middle part of Fig. 8 could be appropriately expanded and employed for determining the preference changes needed by DMs to reach an attractive outcome benefiting society, such as significant cutbacks in greenhouse gas emissions discussed in Sect. 7. This kind of information could be highly informative in a third party intervention in which a facilitator attempts to lead the parties in conflict over a particular issue towards a responsible resolution in which, for example, a polluting company eventually prefers to cut back significantly its generation of wastes by adopting more advanced production technologies so that the surrounding community is not adversely affected. The behavioral engine problem depicted in the lower portion of Fig. 8 could be refined for ascertaining the types of required behavior by DMs to cause a specified beneficial outcome to occur given the DMs' preferences (Wang et al. 2019). In general, as needs arise in practice, many of the techniques developed for the forward systems perspective portrayed in the upper portion of Fig. 8 could be re-engineered for employment within the Inverse and Behavioral GMCR viewpoints, drawn in the middle and lower parts of Fig. 8, respectively.

As noted earlier, the matrix formulation for GMCR mentioned in Sect. 6 can be useful for the development of new ideas such as the inverse and behavioral versions of GMCR described in Sect. 7. Moreover, the matrix specification of GMCR is recommended for designing the engine of a DSS for implementing the GMCR methodology in order to have efficient computations, as pointed out in Sect. 8. Another existing challenge is to develop a new generation of DSS for GMCR based on the universal design explained in Sect. 8 and depicted in Fig. 9. In this way, researchers and practitioners will be able to systematically investigate conflicts of direct interest to them, garner valuable strategic advice for supporting and enhancing the decision-making process, and perhaps discover the directions in which the GMCR methodology can be sensibly expanded.

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