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A Representative Committee by Approval Balloting

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Abstract A new voting rule for electing committees is described. Specifically, we use approval balloting and propose a new voting procedure that guarantees that if there is a committee that represents (with a given proportion of representatives) all of the existing voters, then the selected committee has to represent all of voters in at least the same proportion. This property is a way of selecting a committee that represents completely all of voters when such a committee exists. The usual voting rules in this context do not satisfy this condition.

Keywords Approval balloting · Committee election · Unanimity · Justified representation · Representativeness

JEL Classification D71 · D72

1 Introduction

The problem of selecting a representative committee has been widely studied. In fact, there are several approaches to this problem in the literature. Much of the known multi-winner rules are based on the assumption that voters have totally or partially ordered preferences regarding the candidates (see, for instance, Chamberlin and Courant 1983; Monroe 1995; Potthoff and Brams 1998; Ratliff 2003, 2006; Elkind et al. 2014).

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In contrast, this paper focuses on approval-based rules by which voters have dichotomous preferences (each voter sorts the candidates as either *good* or *bad*). Approval Voting (Brams and Fishburn 1978; Fishburn and Brams 1981) is an appealing procedure for single-winner elections whenever voters have dichotomous preferences. Voters approve as many candidates as they wish (approval balloting), and the candidate with the most approvals wins. Approval Voting is known to fulfill many good properties when voters have dichotomous preferences. Recently, in Gehrlein and Lepelley (2015), different scenarios using both types of preferences (dichotomous and complete orders) have been analyzed in order to compare Approval Voting with other scoring rules for single-winner elections.

Approval balloting is also useful in committee elections where a subset of candidates should be selected. There are different ways of aggregating approval ballots in order to choose a committee of size s. Kilgour (2010) and Kilgour and Marshall (2012) surveyed methods using approval ballots in multi-winner elections. Some of them are generalizations of the Approval Voting procedure; others are related to threshold or centralization procedures, where representativeness plays a more significant role. Besides *Approval Voting* (AV), other interesting procedures are *Representativeness* (Monroe 1995; Potthoff and Brams 1998), *Proportional Approval* (PAV) (Simmons 2001; Thiele 1890), *Threshold-Majority* (Fishburn and Pekeč 2004), *Minimax Approval Voting* (MAV) (Kilgour et al. 2006), *Satisfaction Approval* (SAV) (Brams and Kilgour 2014), etc.

Some of the proposed rules present a lack of representativeness, suffering from what is known as the *tyranny of the majority* (50.01% of the voters could determine the selected committee, leaving almost half of the voters without representation). When the goal is to choose a committee that represents the greatest number of voters this lack of representativeness is an important handicap.

Monroe (1995) and Elkind et al. (2014) propose examples where representation is a focal objective. Although they use a non dichotomous approach, their examples (the selection of a set of newspapers for a graduate common room (Monroe 1995), or the selection of movies for a long-distance flight (Elkind et al. 2014)) are clear situations in which having a representative for the greatest number of voters would be the main goal and the information about the preferences might be dichotomous in a natural way. Committees that are constituted not to make final decisions (where the committee should be a reflection of the electorate) but rather to elaborate pre-proposals where each voter's opinion should be considered, are quite common. This paper focuses on such types of contexts, where the goal is to select committees that represent the largest number of voters.

Sometimes, in order to ensure some representation using dichotomous preferences, restrictions are established on the admissible ballots to obtain more representative committees.[1] Nevertheless, in line with approval balloting philosophy, a modification in the process of aggregating ballots would be more adequate than a limitation on voters' opinions.

[1] For the elections to the Senate of Spain (The Upper House of the Spanish Parliament) each province elects four senators regardless of their population. In order to obtain a more representative committee, under current legislation, each voter is allowed to mark three or fewer candidates' names.

In recent works, Aziz et al. (2015), Aziz and Walsh (2014) and Sánchez-Fernández et al. (2016) study new properties related to representativeness: Justified Representation (JR), Extended Justified Representation (EJR), Proportional Justified Representation (PJR), among others. Justified Representation is the weakest of these conditions and it should be a necessary requirement when representation of the different opinions of the voters is important. Justified Representation ensures that a large group of voters with shared preferences should have at least one representative allocated to them. Aziz et al. (2015) show that most of the standard approval-based multi-winner rules do not satisfy JR and, for some of them, they propose natural modifications to overcome this lack of representation.

In this work we investigate a new property related to representation, which we call α-unanimity. The idea is that when there is a committee that represents all of the voters (in a given proportion α), then the selected committee has to represent all of the voters in at least the same proportion. We think that in the afore-mentioned context, where the main objective is to select a committee that represents the greatest number of voters, it is indeed an appealing property. As far as we know, none of the standard approval-based multi-winner rules hold α-unanimity for positive values of α. We define a new voting rule, the *Lexiunanimous Approval Voting rule* (LUAV), which fulfills α-unanimity. This rule allows us to achieve the broadest degree of consensus and to choose a committee in which, if possible, all the voters are represented. We prove that the LUAV rule also satisfies JR.

2 Preliminaries

We assume throughout this paper that $n > 1$ and $k > 1$ are the numbers of voters and candidates, respectively, and we denote the set of voters by N and the set of candidates by K.

An approval ballot is a binary k-vector $v = (v_1, v_2, \ldots, v_k) \in \{0, 1\}^k$ where the value 1 or 0 for v_i indicates the approval or disapproval, respectively, of candidate i. For each $v = (v_1, v_2, \ldots, v_k) \in \{0, 1\}^k$, $|v| = \sum_{i=1}^{k} v_i$ indicates the number of approved candidates in the ballot v.

An approval ballot profile is a vector $V = \left(v^1, v^2, \ldots, v^n\right) \in \{0, 1\}^{nk}$ where $v^j \in \{0, 1\}^k$, $j \in N$, is the voter j's approval ballot.

Although we sometimes use the candidates' numbers to denote a committee, it can also be represented by a binary vector $C = (c_1, c_2, \ldots, c_k) \in \{0, 1\}^k$ where the value 1 or 0 for c_i, $i \in K$, indicates whether candidate i is or not a member of the committee C. We also use $i \in C$ to denote that candidate i belongs to the committee C. As before $|C| = \sum_{i=1}^{k} c_i$ denotes the number of members in this committee. For any approval ballot v and any committee C, $vC = \sum_{i=1}^{k} v_i c_i$ denotes how many candidates in committee C are approved in the ballot v. For any two ballots v and w, $v \odot w = (v_1 w_1, v_2 w_2, \ldots, v_k w_k)$ represents the agreement between ballots v and w and then $| \odot_{j \in N^*} v^j |$ (where N^* is a subset of N) indicates the number of candidates approved by all voters in N^*.

We denote by ξ the set of feasible committees, and by ϑ the set of admissible ballots. In general, $\xi \subseteq \{0, 1\}^k \setminus \left\{ \overrightarrow{0} \right\}$ and $\vartheta \subseteq \{0, 1\}^k \setminus \left\{ \overrightarrow{0} \right\}$. It seems reasonable that the selected committee must contain at least one candidate. On the other hand, blank ballots will not be considered for the aggregation procedure. The case of ballots with all the possible candidates marked could be treated as inadmissible with no differences in the results. Nevertheless, we will consider the possibility that a voter could support all the candidates, because in some real situations such ballots are not discarded.

In real-life elections there are other a priori restrictions on ξ and ϑ and they can not be the whole set $\{0,1\}^k \setminus \left\{\overrightarrow{0}\right\}$. A typical restriction on ξ is that only committees of a given size are possible. In this case, we will denote by ξ_s the set of committees with exactly s members.

Ballot restrictions are often imposed in fixed-size committee elections. Usual examples are when ballots that name more than a prefixed number of candidates are discarded. We are not going to consider ballot restrictions because they mask the fact that the election is conducted under approval balloting. Nevertheless, in contexts where this kind of restriction exists our proposal could be applied without modifications.

Representational requirements are also very usual (balanced representation of men and women, representation from different geographical regions, representation from different faculties or departments, etc.). These representational requirements are usually guaranteed by restricting the set of admissible committees (Kilgour 2010; Fishburn and Pekeč 2004).

In general, an approval-based committee election problem can be represented as $\left(N, K, \vartheta \subseteq \{0,1\}^k \setminus \left\{\overrightarrow{0}\right\}, \xi \subseteq \{0,1\}^k \setminus \left\{\overrightarrow{0}\right\}\right)$. A voting rule in this context is a function, possibly multi-valued, $\psi : \vartheta^n \longrightarrow \xi$, where $C^V \in \psi(V)$ represents an elected committee when $V \in \vartheta^n$ is the approval ballot profile provided by the voters. Whenever the voting rule does not uniquely select a committee, we suppose that ties are broken according to a pre-fixed priority order over the admissible committees.

As afore-mentioned, the size of the elected committee is a common restriction usually established in advance. We are going to focus on this particular context, in which the family of feasible committees is ξ_s, $1 \leq s < k$. Moreover we do not restrict the voters' opinions, so $\vartheta = \{0,1\}^k \setminus \left\{\overrightarrow{0}\right\}$. In this case we call the voting rule the s-committee voting rule.

Given a ballot profile $V = \left(v^1, v^2, \ldots, v^n\right)$, for any candidate $i \in K$ its approval score is the number of voters approving candidate i, that is, $App(V,i) = \sum_{j=1}^{n} v_i^j$. A natural way to use approval ballots to select a committee is to choose the committee $C \in \xi$ maximizing $\sum_{i \in C} App(V,i)$. This rule, which has been adopted by several academic and professional societies, is known as *Simple Approval Voting* (AV) and it is an obvious generalization of Approval Voting for single-winner elections. Since AV for single-winner elections has been widely studied and it has appealing properties, a desirable requirement for committee selection rules is that, when applied to the particular case $s = 1$, it selects a candidate who wins under Approval Voting.

Definition 1 (Kilgour and Marshall 2012) An s-committee voting rule is *approval-based* if and only if when $s = 1$ it selects the candidate who win in single-winner Approval Voting.

Not all the committee selection rules defined in the literature are based on Approval Voting (see Kilgour and Marshall 2012).

When a committee of size $s > 1$ has to be selected, as commented, properties on representativeness may be crucial and it is known that the voting rules in the literature may generate outcomes that are highly unrepresentative of the electorate. Within this approach Aziz et al. (2015) study an interesting property.

Definition 2 (Aziz et al. 2015) A committee C of size s provides *justified representation* for the approval-based committee election problem (N, K, ϑ, ξ_s) if there does not exist a set of voters $N^* \subseteq N$ with $|N^*| \geq \frac{n}{s}$ such that $| \odot_{j \in N^*} v^j| \geq 1$ and $v^j C = 0$ for all $j \in N^*$. An s-committee voting rule based on approval voting satisfies justified representation (JR) if it selects committees that provide justified representation.

Justified representation is a weak condition establishing that if a subset of voters of a considerable size agree unanimously on at least one candidate, a candidate approved by at least one voter in this group should be elected. AV, SAV and PAV voting rules do not fulfill JR (see Aziz et al. 2015). Committees fulfilling JR always exists. Aziz et al. (2015) use an algorithm (*Greedy Approval Voting* (GAV)) selecting committees providing justified representation. Aziz et al. (2015) and Sánchez-Fernández et al. (2016) analyze other, more restrictive, types of representation properties but, as Sánchez-Fernández et al. (2016) claim, justified representation is an interesting axiom, and it must be a necessary requirement (maybe not sufficient) for considering the use of an approval-based multi-winner voting rule when it is desired that the winning committee represents the different opinions or preferences of the agents involved in the election.

We are interested in voting rules that provide committees that represent the largest number of voters as possible. In fact, we look for a rule that selects a committee that represents all voters when such a committee exists. The concept of α-unanimity tries to capture this idea. We first define, for each proportion $\alpha \in [0, 1]$, each ballot profile V, and each committee C, the α-agreement level as

$$d_\alpha(V, C) = \left| \left\{ j \in N : \frac{v^j C}{\min\left(|v^j|, |C|\right)} \geq \alpha \right\} \right|.$$

The parameter α can be interpreted as the proportion of the committee that should coincide with any voter's ballot in order to consider that the committee represents the voter. Therefore, α can be seen as the degree of representation required. The extreme cases appear when $\alpha = 0$ (no representation at all) and $\alpha = 1$ (total agreement on the committee). In order to illustrate the use of $d_\alpha(V, C)$ consider the following example.

Example 1 Four voters have to choose a committee of two from four candidates, $K = \{1, 2, 3, 4\}$. Table 1 shows the approved candidates for each voter.

Note that although a two-member committee is elected, the first voter supports three candidates; then he fully agrees with any committee that does not include the candidate 4. On the other hand, the third voter only supports one candidate, so as he knows that the committee will contain two members, he fully agrees with any committee that includes the candidate 2. Obviously $d_0(V, C) = 4$ for any two-committee C. In the case of $\alpha = 0.5$, the voters that are represented by at least half of the committee,

Table 1 Example 1 ballots profile

# Voters	Candidates			
	1	2	3	4
1	1	1	1	0
1	1	0	0	1
1	0	1	0	0
1	0	0	1	1
4	2	2	2	2

$d_{0.5}(V, C) = 3$ for any two-committee except for $C = \{2, 4\}, d_{0.5}(V, \{2, 4\}) = 4$. Finally, for $\alpha = 1$ (full agreement),

$$d_1(V, \{1, 2\}) = d_1(V, \{2, 3\}) = 2$$
$$d_1(V, \{1, 3\}) = d_1(V, \{1, 4\}) = d_1(V, \{2, 4\}) = d_1(V, \{3, 4\}) = 1.$$

Definition 3 For a fixed $\alpha \in [0, 1]$, we say that committee C is α-*unanimous* for the voting profile V if $d_\alpha(V, C) = n$.

Then, α-unanimity means that committee C represents all voters at least to the proportion α. In the previous example, all two-committees are 0-unanimous, only $C = \{2, 1\}$ is 0.5-unanimous and there are not 1-unanimous committees.

Definition 4 An s-committee voting rule satisfies α-*unanimity* if, when an α-unanimous committee exists, the procedure selects α-unanimous committees. That is, if for any ballot profile V there is a committee $S \in \xi_s$ such that $d_\alpha(V, S) = n$, then for any C selected by the voting rule, $d_\alpha(V, C) = n$.

The property of α-unanimity implies that when there is a committee so that all voters agree at least to a proportion α, the selected committee should be also acceptable for all voters to the same degree. Of course, all procedures satisfy 0-unanimity because for every C and V, $d_0(V, C) = n$. On the other hand, for $\alpha = 1$, $d_1(V, S) = n$ means that all voters unanimously agree with committee S, hence it seems reasonable to select a committee with this maximum degree of consensus. Note that when using approval balloting, several committees supported by all the voters may exist. The 1-unanimity is related to the strong unanimity property in Elkind et al. (2014), which conveys that if a committee supported by all the voters exists, the committee selected must therefore be supported by all the voters as well.

Justified representation and α-unanimity are independent properties. Committees fulfilling JR may not be α-unanimous for positive values of α, as Example 2 shows. On the other hand, Example 3 shows that α-unanimity does not imply JR.

Example 2 Six voters have to choose a committee of two from three candidates, $K = \{1, 2, 3\}$. Table 2 shows the approved candidates for each voter.

Voting rules that select $C_1 = \{1, 2\}$ or $C_2 = \{1, 3\}$ fulfill JR. But α-unanimity is not fulfilled for $\alpha = 0.5$, because $d_{0.5}(V, C) = 6$ when $C = \{2, 3\}$ and $d_{0.5}(V, C_1) =$

Table 2 Example 2 ballots profile

# Voters	Candidates		
	1	2	3
2	1	1	0
2	1	0	1
1	0	1	0
1	0	0	1
6	4	3	3

Table 3 Example 3 ballots profile

# Voters	Candidates		
	1	2	3
6	1	0	0
5	0	1	0
1	0	0	1
12	6	5	1

$d_{0.5}(V, C_2) = 5$. Then an α-unanimous voting rule should select the committee $C = \{2, 3\}$, which also fulfills JR.

Example 3 Twelve voters have to choose a committee of two from three candidates, $K = \{1, 2, 3\}$. Table 3 shows the approved candidates for each voter.

In this case, as no candidate is supported by all the voters, an α-unanimous voting rule could select any two-committee, for instance $C = \{2, 3\}$. Note that $C = \{2, 3\}$ does not fulfill JR because half of the voters only support candidate 1 and they are not represented.

In the context described in this work, in order for all voters to get some representation, solutions fulfilling α-unanimity for some $\alpha > 0$ would be desirable. Unfortunately, none of the usual voting rules satisfy this property, even for small positive values. For instance Example 2 shows that *Simple Approval Voting* (AV) does not fulfill α-unanimity. AV selects $C_1 = \{1, 2\}$ or $C_2 = \{1, 3\}$ while, as we have seen, any α-unanimous rule should select $C = \{2, 3\}$. In a similar way it is easy to figure up ballot profiles showing that *Proportional Approval* (PAV) (Simmons 2001; Thiele 1890), *Minimax Approval Voting* (MAV) (Kilgour et al. 2006) and *Satisfaction Approval* (SAV) (Brams and Kilgour 2014) do not fulfill α-unanimity.

3 Lexiunanimous Approval Voting Rule

To define a voting rule satisfying α-unanimity for positive values of α, we need some previous notation. Although $d_\alpha(V, C)$ can be computed for any $\alpha \in [0, 1]$, the quotient

$$\frac{v^j C}{\min\left(|v^j|, |C|\right)} \tag{1}$$

Table 4 Example 2: $U^{\alpha_s}(V, C)$ vectors

$\overrightarrow{\alpha_s}$	0	1/2	1
$C \in \xi_2$	$U_1^{\alpha_s}(V, C)$	$U_2^{\alpha_s}(V, C)$	$U_3^{\alpha_s}(V, C)$
(1, 1, 0)	6	5	3
(1, 0, 1)	6	5	3
(0, 1, 1)	6	6	2

can reach only a finite number of values. The reached values depend on s, C and the number of candidates supported by each voter. Given an s-election problem (N, K, ϑ, ξ_s), let

$$\Lambda_s = \left\{\alpha \in [0, 1] : \alpha = \frac{r}{t}, t = 1, 2, \ldots, s;\ r = 0, 1, 2, \ldots, t\right\}.$$

Λ_s is a finite set and contains all the possible values for the quotient (1), and therefore the relevant values of the parameter α. We denote by $\overrightarrow{\alpha_s} = (\alpha_1, \alpha_2, \ldots, \alpha_m)$ the ordered vector of the different values in Λ_s ($p < q$ if and only if $\alpha_p < \alpha_q$). Note that, for each $\overrightarrow{\alpha_s}$, the first element is $\alpha_1 = 0$ and the last element is $\alpha_m = 1$.

For a given voters' profile V and each committee C in the s-election problem (N, K, ϑ, ξ_s), we compute the vector $U^{\alpha_s}(V, C)$, with the same number of elements as $\overrightarrow{\alpha_s}$, defined by $U_p^{\alpha_s}(V, C) = d_{\alpha_p}(V, C)$ for all $p = 1, 2, \ldots, m$. It is easy to observe that for each voters' profile V and each committee C, the elements of $U^{\alpha_s}(V, C)$ are non-increasing ($p < q$ implies $U_p^{\alpha_s}(V, C) \geq U_q^{\alpha_s}(V, C)$) and, moreover, $U_1^{\alpha_s}(V, C) = n$ (since $\alpha_1 = 0$). These vectors will allow us to define our procedure for s-election problems. We first provide an example.

When selecting a committee of two candidates the possible values of quotient (1) are 0, $\frac{1}{2}$ and 1, so $\overrightarrow{\alpha_s} = \left(0, \frac{1}{2}, 1\right)$. In the particular case of Example 2 the vectors $U^{\alpha_s}(V, C)$ for the ballot profile V and the committees in ξ_2 are displayed in Table 4.

Definition 5 Given an s-election problem (N, K, ϑ, ξ_s), the *Lexiunanimous Approval Voting rule* (LUAV) assigns to a given ballot profile V any committee C such that $U^{\alpha_s}(V, C)$ is maximum with respect to the lexicographical order.[2]

In Example 2 (see Table 4) the maximum with respect to the lexicographical order is the vector (6, 6, 2) and then the LUAV rule selects the committee $C = \{2, 3\}$.

The following result summarizes important properties of the Lexiunanimous Approval Voting rule.

[2] The lexicographical order is defined as $(x_1, x_2, \ldots, x_m) \succ_L (y_1, y_2, \ldots, y_m)$ if and only if there is $p \in \{1, 2, \ldots, m\}$ such that $x_q = y_q$ for all $q < p$ and $x_p > y_p$. The weak relation $(x_1, x_2, \ldots, x_m) \succcurlyeq_L (y_1, y_2, \ldots, y_m)$ means $(x_1, x_2, \ldots, x_m) \succ_L (y_1, y_2, \ldots, y_m)$ or $(x_1, x_2, \ldots, x_m) = (y_1, y_2, \ldots, y_m)$. Note that being the set of vectors $\left\{U^{\alpha_s}(V, C) : C \in \xi_s\right\}$ finite, the lexicographical order always reaches a unique maximum (although several committees may provide this maximum).

Theorem 1 *The Lexiunanimous Approval Voting rule*

(i) *is α-unanimous, for any $\alpha \in [0, 1]$;*
(ii) *is an approval-based voting rule;*
(iii) *satisfies JR.*

Proof (i) Let $\alpha^* \in [0, 1]$ and suppose there are $S \in \xi_s$ and $V \in \vartheta^n$ such that $d_{\alpha^*}(V, S) = n$. By construction, for each $\alpha' < \alpha^*$, $d_{\alpha'}(V, S) = n$. Let us consider a committee C^L selected with the LUAV rule. Then $U^{\alpha_s}\left(V, C^L\right) \succsim_L U^{\alpha_s}(V, S)$. So, $d_{\alpha^*}\left(V, C^L\right) = n$ and C^L is an α^*-unanimous committee.

(ii) The possible values for α in a single-winner election problem are $\alpha = 0$ and $\alpha = 1$. As $d_0(V, \{i\}) = n$ for each candidate i, the Lexiunanimous Approval Voting rule selects the candidate with the greatest value $d_1(V, \{i\})$; that is, the candidate receiving the most approval votes.

(iii) Let $C^L \in \xi_s$ be a committee selected with the LUAV rule. Then

$$U^{\alpha_s}\left(V, C^L\right) \succsim_L U^{\alpha_s}(V, C)$$

for all $C \in \xi_s$. By way of contradiction let us suppose that there is a coalition $N^* \subseteq N$ with $|N^*| = n^* \geq \frac{n}{s}$ such that $\left|\odot_{j \in N^*} v^j\right| \geq 1$ and $v^j C^L = 0$ for all $j \in N^*$. That is, no candidate supported by any voter in N^* is selected.
It is clear that for all $C \in \xi_s$,

$$U_1^{\alpha_s}(V, C) = n$$

and

$$U_2^{\alpha_s}(V, C) = \left|\left\{j \in N : v^j C \neq 0\right\}\right| = n - \left|\left\{j \in N : v^j C = 0\right\}\right|.$$

Then, $U_2^{\alpha_s}\left(V, C^L\right) \leqslant n - n^*$.
Consider the set of candidates rearranged in the following way: first the s candidates selected in C^L ordered in such a way that if $i < i'$ then $App(V, i) \geqslant App(V, i')$; next the candidate $s + 1$ is a candidate supported by all the voters in N^*; and then the rest of candidates in any order.
We denote by

$$b_1(V, C) = \left|\left\{j \in N : v_1^j = 1\right\}\right|$$

and for each $i \in K$, $i \neq 1$

$$b_i(V, C) = \left|\left\{j \in N : v_i^j = 1, \text{and } i' < i \Rightarrow v_{i'}^j = 0\right\}\right|$$

that is, the number of voters supporting candidate i but none of the previous candidates.
We can compute $U_2^{\alpha_s}\left(V, C^L\right)$ as

$$U_2^{\alpha_s}\left(V, C^L\right) = \sum_{i=1}^{s} b_i(V, C^L)$$

If for some $i^* \in C^L$, $b_{i^*}(V, C^L) < n^*$ we can consider the committee $C \in \xi_s$ formed by the candidates in C^L but changing candidate i^* for candidate $s+1$. Observe that for all $j \in N^*$, $v^j_{s+1} = 1$ and $v^j_{i'} = 0$ for all $i' < s+1$, since $v^j C^L = 0$ for all $j \in N^*$. Thus, $b_{s+1}(V, C) \geq n^* > b_{i^*}(V, C^L)$ and $U_2^{\alpha_s}(V, C) > U_2^{\alpha_s}\left(V, C^L\right)$ that contradicts the maximality of $U^{\alpha_s}(V, C)$.
Then for all $i \in C^L$, $b_i(V, C^L) \geqslant n^*$ and consequently

$$U_2^{\alpha_s}\left(V, C^L\right) = \sum_{i=1}^{s} b_i(V, C^L) \geqslant \sum_{i=1}^{s} n^* = sn^* \geqslant s\frac{n}{s} = n$$

A contradiction because as we have seen $U_2^{\alpha_s}\left(V, C^L\right) \leqslant n - n^* < n$.
Therefore, the LUAV rule satisfies JR. □

Aziz et al. (2015) define a stronger version of the JR axiom that they refer to as *Extended Justified Representation* (EJR). EJR conveys the idea that a very large group of voters with similar preferences may deserve several representatives. Sánchez-Fernández et al. (2016) propose a modification of EJR, *Proportional Justified Representation* (PJR) with the same objective. Both properties are related to proportionality and not to the fact that the largest number of voters obtain representation. The LUAV rule does not fulfill even EJR or PJR. Example 2 in Aziz et al. (2015) shows this fact.

Example 4 (Example 2 in Aziz et al. 2015) One hundred voters have to choose a three-member committee from four candidates, $K = \{1, 2, 3, 4\}$. Ninety-eight voters approve candidates 1 and 2, one voter approves only candidate 3 and the last voter approves only candidate 4.

The LUAV rule proposes committees $\{1, 3, 4\}$ or $\{2, 3, 4\}$ which violate EJR and PJR.

4 Final Comments

In contexts looking for the widest representativeness of a committee, α-unanimity for positive values of α is an appealing property. It is, in some way, related to threshold rules (Fishburn and Pekeč 2004) in the sense that the main goal is to select a committee that represents the largest number of voters while the relative number of agents supporting each candidate is ignored. Threshold rules also take into account interdependencies among candidates and therefore they provide, in general, different committees from that provided by the LUAV rule, as the following example shows (Kilgour 2010).

Example 5 (Example 2 in Kilgour 2010) Nine voters have to choose a three-member committee from 8 candidates, $K = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Table 5 shows the voters' ballot profile. The LUAV rule selects the committee $\{2, 3, 4\}$. But, for instance, the threshold solution in Kilgour (2010) selects the committee $\{1, 2, 3\}$ at a constant threshold $t = 2$.

As α-unanimity focus on the objective of selecting committees which represent the largest number of voters (all of them, when it is possible), consequently the opinion of

Table 5 Example 5 ballots profile

# Voters	Candidates							
	1	2	3	4	5	6	7	8
2	1	1	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	0	0	1	0	0	0	1	0
1	0	0	0	1	1	0	0	0
1	0	0	0	1	0	1	0	0
1	0	0	0	1	0	0	1	0
1	0	0	0	1	0	0	0	1
9	3	3	2	4	1	1	1	1

single voters may have important influences in the final result (see for instance Example 4). It would be possible to ask for weaker conditions than unanimity. Of course, these proposals will be less sensible to minorities but we will lose representativeness. A possible weakening of unanimity could be $\delta-$majority representation, replacing $d_\alpha(V, C) = n$ by $d_\alpha(V, C) = \delta n$, with $\delta \in (1/2, 1]$. The parameter δ will determine the sensibility of the voting rule to minorities.

The rules we have mentioned throughout the paper differ from a computational perspective. As indicated in Aziz et al. (2015), for some of these rules, namely, AV and SAV, a winning committee can be computed in polynomial time. In contrast, PAV and MAV are computationally hard. This is also the case for the LUAV rule,[3] that is computationally hard. Some of these computationally hard rules admit efficient approximation algorithms (see Aziz et al. 2015). This analysis is out of the scope of our research.

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[3] We are grateful to H. Aziz for pointing out this fact in a private communication.

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