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Numerical Methods to Calculate Fuzzy Boundaries for Brownfield Redevelopment Negotiations

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Abstract A numerical method is proposed to represent the likelihood of contamination of a brownfield using fuzzy boundaries, and then to estimate the parameters in a fuzzy real options model for brownfield evaluation from different decision maker perspectives. These different values can be used to facilitate negotiations on redevelopment projects. Linguistic quantifiers and ordered weighted averaging (OWA) techniques are utilized to determine the pollution likelihood at sample locations based on multiple environmental indicators. Risk preferences of decision makers are expressed as different "orness" levels of OWA operators, which affect likelihood estimates. When the fuzzy boundary of a brownfield is generated by interpolation of sample points, the parameters of fuzzy real options, drift rate and volatility, can be calculated as fuzzy numbers. Hence, this proposed method can act as an intermediate between decision makers and the fuzzy real options models, making this model much easier to apply. A potential negotiation support system (NSS) implementing these numerical methods is discussed in the context of negotiating brownfield redevelopment projects. A public–private-partnership will be enhanced through information sharing, scenario generation, and conflict analysis provided by the NSS, encouraging more efficient brownfield redevelopment and leading to greater regional sustainability.

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1 Introduction

A brownfield, the opposite of a greenfield, refers to a developed property that is abandoned or underutilized (Hipel 2010). Brownfields usually occur when an industrialized area evolves into a service-oriented economy (United State Environmental Protection Agency (USEPA) 1997). For instance, Hamilton, one of the major industrial cities in Ontario, Canada, is famous for its steel and chemical plants. However, many factories are relocating to developing countries, and these properties have been left as unproductive brownfields, as suspicion of contamination has prevented redevelopment.

Brownfields represent an unsustainable development pattern because existing infrastructure is wasted and greenfields are irreversibly developed for business or residential purposes. In addition, brownfields usually pose a threat to public health as the hazardous materials left in these properties may eventually leak into groundwater. Hence, leaving brownfields intact reduces the sustainability of cities.

On the other hand, redeveloping brownfields can revive the downtown areas of cities. Historically, many cities were developed around major plants; factories, residential areas, and community facilities constituted the urban core. Hence, redeveloping brownfields reduces not only public health threats, but also unemployment. Therefore, brownfields are challenges to local governments, but also provide opportunities if redevelopment is conducted properly.

Despite willingness to redevelop brownfields by communities and governments, these redevelopment projects are too risky to be undertaken by any single stakeholder. But joint actions from Public–private-Partnerships (PPPs) can accomplish this task (De Sousa 2001).

Negotiations are inevitable in promoting brownfield redevelopment. To facilitate negotiations among stakeholders of brownfield redevelopment, the following difficulties must be addressed:

- *Evaluation techniques* Benefits and costs of brownfield redevelopment projects are highly unpredictable, making deterministic evaluation tools, such as net present value (NPV), inappropriate for pricing brownfields. A better pricing technique called fuzzy real options analysis can be employed in order to evaluate uncertainties involved in brownfield redevelopment (Wang et al. 2009a). The more accurate estimates generated using fuzzy real options provide a solid basis for negotiation.
- *Information sharing* Another obstacle preventing negotiation is the limited information available on brownfield redevelopment, especially for site-specific conditions. Information sharing will be very helpful in building a positive environment for the negotiation.
- *User friendly interface* The utilization of the more complex fuzzy real options model should be automated and concealed so that it will not be an obstacle for decision makers to understand and use this technique. Hence, the proposed method will act as a bridge between the interface, which allows end users to mark their judgments

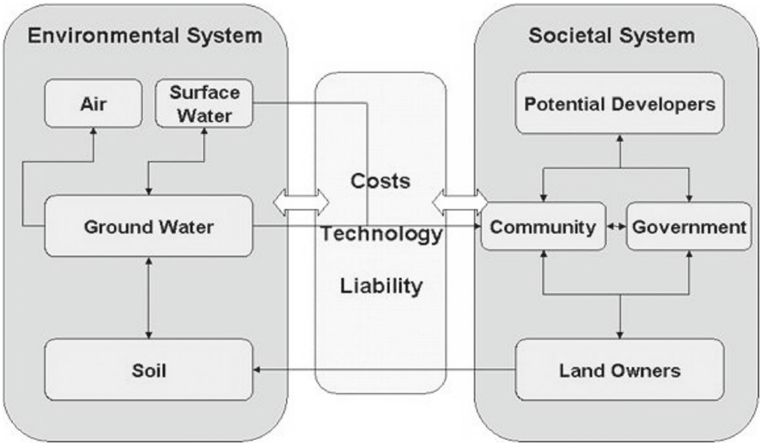


Fig. 1 Brownfield redevelopment as a system of systems (Wang et al. 2008)

on a brownfield map, and the fuzzy real options model for brownfield evaluation, which needs this information for parameter estimation.

This paper will review related works on brownfield redevelopment, risky project evaluation, and negotiation as the basis of this paper. Then proposed numerical methods are explained in the context of negotiation and evaluation of brownfield redevelopment projects. An illustrative example is employed to demonstrate its effectiveness, which is based on a case in the Ralgreen Community, Kitchener, Ontario, Canada. Discussions and future work are presented at the end of this paper.

2 Literature Review

2.1 Brownfield Redevelopment

The brownfield problem is a System of Systems problem (Jamshidi 2008), characterized by its layered subsystems, complex interactions, and non-linear behavior involving high uncertainty (Fig. 1). Various subsystems in brownfields contribute to overall uncertainties in costs, technologies, and liabilities. Malfunctions of the environmental system affect all stakeholders and cooperation is required to redevelop brownfields.

The interaction between groundwater and soil is a key factor in the high uncertainty of brownfield redevelopment. Site-specific hydrogeology plays a critical role in determining the risk of a brownfield (Yu 2009). Pollution levels vary greatly depending on the kinds of pollutants, the phases of contaminants, and the geology of unsaturated zones. In some cases, the whole site has to be cleaned up, which may cost more than the total value of the property. As a result, developers are reluctant to undertake brownfield redevelopment projects because NPV calculations often find these projects unprofitable.

However, there are successful cases in Canada demonstrating that, with joint effort from all stakeholders, private developers have good business opportunities in brownfield redevelopment. For instance, Hamilton, Ontario was one of Canada's major manufacturing centers, famous especially for its steel industry. As the regional economy evolved toward services, heavy industries gradually moved out of the city. The buildings and land they left behind contained hazardous substances.

The City of Hamilton decided to redevelop their brownfields, hoping to boost the city's economy and enhance local sustainability. It launched the Environmental Remediation And Site Enhancement (ERASE) program, which was the main vehicle for brownfield redevelopment. 1,376 hectares of the former industrial sites were listed in ERASE. With full technical support and cost sharing from the City of Hamilton, the ERASE program achieved successful results and earned several awards.

The ERASE program demonstrates that the apparent unprofitability of brownfield redevelopment projects may be due to the failure of the evaluation method used by developers (De Sousa 2001). As a deterministic evaluation method, NPV is usually unable to accurately price risky projects. A system of systems problem requires a more comprehensive systems engineering technique to handle this task.

The success of the ERASE program suggests that a better evaluation technique will be helpful in convincing developers that the value of brownfields can be higher than expected. The fuzzy real options model of brownfield redevelopment, which will be further discussed in Sect. 2.2, adds a fuzzy component to real options analysis in order to include non-market risk and represent expert estimates. Furthermore, a numerical method that can evaluate fuzzy real options regardless of the existence of analytic results is available.

In Ontario, Canada, brownfield redevelopment contains two main stages: redevelopment and long-term monitoring (Ontario Regulation 511/09 2009). The redevelopment process can be further divided into three phases. Phase I is environmental site assessment (ESA) I. An expert investigates the brownfield site and uses his or her judgment to decide whether this site has been contaminated. If so, the scope of Phase II (ESA II) will include surveying, monitoring, and remediation; if not, a record of site condition (RSC) will be submitted to the Ministry of Environment (MOE). Then the site undergoes long-term monitoring. This process is illustrated in Fig. 2.

Human judgment plays a critical role in determining the likelihood that contaminants have affected the property; therefore, subjective uncertainty should be included in pollution estimates. A fuzzy boundary is an appropriate representation for dividing contaminated and clean regions, which can be determined using an OWA operator including judgments on different environmental indicators (Wang and Hall 1996). The proposed numerical method of calculating the fuzzy boundaries will be further discussed later.

2.2 Risky Project Evaluation

NPV is a frequently used technique for project evaluation. Future payoffs are converted to their present values, divided by a riskless discount factor. Subtracting the sum of converted payoffs from the investment provides an estimate of the profitability of a project. However, NPV is a deterministic model, so all inputs must be crisp numbers.

Unfortunately, a risky project has highly unpredictable investment costs and payoffs (incomes). For instance, incomes from a toll highway depend on how many people

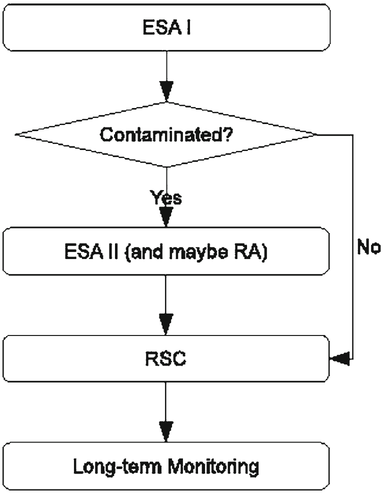


Fig. 2 Brownfield redevelopment process in Ontario, Canada (Ontario 2009)

choose it as their main route. The costs of extracting oil vary greatly, depending on the geology of the oil site. Research and development projects have both highly uncertain incomes and costs. The market for products cannot be accurately predicted, while the costs of development, such as patents and human resources, are also largely unknown ahead of time. As a result, the values of these risky projects are uncertain. Accordingly, the single value calculated using NPV may not be appropriate to help decision makers make the best decisions.

One way to evaluate risky projects, originating from the financial market, is called real options analysis, which explicitly considers uncertainty (Dixit and Pindyck 1994). Since the value of a risky project depends heavily on the market values of its products or consumed resources, real options analysis can be utilized to evaluate risky projects as derivatives in the financial market. A project can be regarded as a cash flow and a portfolio of options reflecting managerial flexibilities, such as closing the project at any time, expanding the scale of the project at some specified times, and so on (Amram and Kulatilaka 1999).

The normal output of real options analysis includes the value of the project, critical values dividing strategy regions, and optimal strategies (Amram and Kulatilaka 1999). Apart from the project value, which can be obtained using NPV, real options analysis also provides critical values, which help decision makers to identify the optimal strategy according to different situations.

Two main applications of real options to brownfield redevelopment are put forward by Erzi-Akcelik (2002) and Lentz and Tse (1995). Erzi-Akcelik (2002) focuses on the impact of uncertainty in brownfield redevelopment on the behavior of developers in the United States and the managerial flexibility of entering or quitting a redevelopment project, concluding that the value derived from real options analysis is often slightly higher than the NPV result. However, the impact of managerial flexibility can be neglected in small projects such as gas stations. Nonetheless, this study overlooks the uncertainty in redevelopment cost, which is often the dominating factor affecting payoffs.

Lentz and Tse (1995) also employed real options modeling to evaluate brownfield sites, modeling both redevelopment investment and payoffs as random variables. Their analytical work reveals that brownfields have higher values than those shown by NPV. The optimal decision of delaying redevelopment conforms to the reluctance of developers in participating in brownfield projects (Lentz and Tse 1995).

Unfortunately, risks considered in both works are assumed to satisfy the requirements given below, which may not be realistic in brownfield redevelopment, especially for the remediation cost.

- *Complete Market* All risks can be hedged by a portfolio of options. In other words, all risks are reflected in the market price and can be replicated as options. In some publications, this approach is also called the Market Asset Disclaimer (MAD);
- *Arbitrage-free Market* Unless a player in the market is willing to take some risk, there is no opportunity for profit (Smith and Nau 1995). In other words, there is no risk-free way of making profit;
- *Frictionless Market* There are no barriers to trading, borrowing, or shorting, and there are no transaction costs for doing so. Furthermore, the underlying assets are infinitely divisible.

Uncertainties in groundwater modeling are better to be modeled as private risk, defined as risk that violates these assumptions because of the difficulty of finding counterparts in the market. The private risk problem can be addressed using fuzzy real options.

Fuzzy real options were first introduced to identify optimal strategies using real options analysis with fuzzy parameters (Carlsson and Fuller 2003). The possibility mean and variance were introduced in combination with real options analysis. This idea can be extended to tackle the private risk problem: random variables are employed to model market uncertainty, while fuzzy representations are used for private uncertainty (Wang et al. 2009a).

Wang et al. (2009a) applied fuzzy real options to brownfield redevelopment. When fuzzy parameters are entered in Lentz and Tse's real options model, brownfield redevelopment showed greater values than under NPV. In addition, since fuzzy variables represent private risk, the value of brownfield sites increases as the fuzzy uncertainty enlarges. This phenomenon is similar to the effect of market risks.

Although fuzzy real options modeling is appropriate for evaluating brownfield redevelopment projects, there are two obstacles to overcome. One is the lack of a numerical method suitable for evaluating fuzzy real options. In many cases, finding the close-form solution of fuzzy real options is impossible, making the application of fuzzy real options limited and inflexible.

The other problem is the numerical determination of parameters of fuzzy real options, which are too complex to be comprehended by non-experts in related fields. Parameter estimation must be simplified. In terms of brownfield redevelopment, estimating the likelihood of contamination on a site normally relies on signs on multiple environmental indicators. Below, a numerical method is proposed to convert multi-criteria judgment into the parameters in a fuzzy real options model for brownfield redevelopment.

2.3 Brownfield Redevelopment Negotiations and Fuzzy Boundaries

A typical brownfield redevelopment negotiation process occurred in the brownfield redevelopment case in the Ralgreen Community, Kitchener, Ontario, Canada (Frontline 2000). At first, the community observed the degradation of some environmental indicators, including odor in basements, sinking of garages, and killing vegetation. They complained to the City of Kitchener, since the landowner was bankrupt. After a year-long negotiation, an air photo showing a landfill site in 1950s under some properties in Ralgreen was found, providing strong evidence of pollution. Since all phenomena indicated that the community has been polluted, corresponding to ESA I, a land survey was conducted by a third-party engineering consulting company (similar to ESA II). A redevelopment plan was proposed and implemented after hazardous materials were detected.

Through this negotiation process, subjective judgments on the likelihood of contamination were critical. This is especially true in ESA I, when surveying and monitoring efforts were minimal. In addition, decision makers may have different judgments based on the same evidence. For instance, residents in a community are more likely to believe their community is contaminated than a landowner. Multi-criteria aggregation with preference characterizes this process.

To facilitate the estimation of contamination likelihood, OWA can be employed to generate fuzzy boundaries around a brownfield, dividing contaminated and clean areas with degrees of fuzzy membership. Unlike crisp boundaries, overlaps between polygons are allowed (Wang and Hall 1996). Fuzzy boundaries reflect the reality that the transition of a contaminated area to a non-contaminated area occurs gradually, rather than abruptly. The transition can be represented using fuzzy membership functions.

Decision makers and experts can mark their judgment at some sample spots on the Conceptual Site Model (CSM), which records the contamination information through the ESA processes, such as site-specific hydrogeology, site layout, and map of surrounding area in Ontario. Their descriptive estimates and preference are represented as fuzzy membership degrees. An OWA operator is applied to compute the likelihood on the spot, where preference is added via linguistic quantifiers (Yager 2007). Interpolating the pollution level on sample spots to the whole site generates the fuzzy boundaries of a brownfield, which is used to determine parameters of its fuzzy real options model (Fig. 3).

OWA operators, originating from fuzzy logic, act as generalized "or" (or "and") operators that can aggregate multiple assessments into one ($I^n \rightarrow I$, where $I = [0, 1]$). For a given vector with the number n assessments, noted as $A = [a_1, a_2, \ldots, a_n]^T$, OWA is defined as a corresponding weight vector $W = [w_1, w_2, \ldots, w_n]'$, which will be applied to a sorted vector B of A. Elements of W satisfy $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0\ (i = 1, 2, \ldots n)$. The final single output can be calculated as WA (Yager 2007). Every OWA operator has an associated "orness" level, which is defined as $Orness = \frac{1}{n-1} \sum_{i=1}^{n} (n - i) w_i$, which is the major indicator of

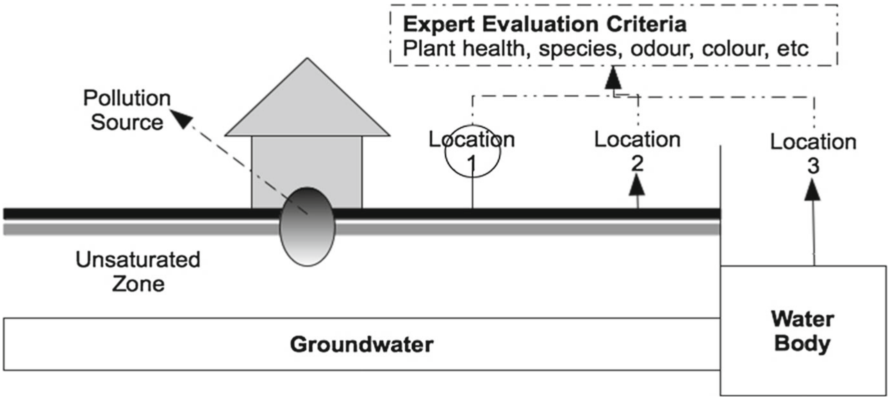


Fig. 3 The conceptual site model of a brownfield

the result. The effect of OWA is to determine a mean value between the maximum value of the assessment ("or" operator) and the minimum value ("and" operator).

As a kind of multi-criteria analysis technique, OWA has the following characteristics: The weight vector does not correspond to judgment on each criterion, but is more related to its orness level as a whole (Yager 2007). Hence, manipulating judgment on a certain criterion provides little strategic advantage for the decision maker in OWA, since this criterion's weight is unknown. In addition, fuzzy variables are easily inputted into OWA, since they fall between 0 and 1 by nature. On the other hand, since OWA can be regarded as a type of logic operator, its output is better treated as a fuzzy number as well. The orness is defined as $\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i$, which ranges from 0 to 1.

There are many ways of determining OWA operators (Xu 2005). In the context of negotiation and expert knowledge representation, the equation of determining weights from the linguistic quantifier can be utilized: $w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n})$, where Q is a function corresponding to a specific linguistic quantifier, such as "all", "most", "some", and "any" (Yager 2007). $Q = x^r$ is normally employed, whose parameter r is set with every linguistic quantifier. Furthermore, an indicator, called Value Of Individual Disapproval (VOID), can be calculated using the formula $VOID(Q) = 1 - \int_0^1 Q(x)dx$, showing a decision maker's "conservativeness" toward all criteria.

When using OWA and a linguistic quantifier to determine the fuzzy boundaries, one issue must be addressed: the common knowledge of linear preference may lead to the behavior of strategic bidding from all decision makers. For instance, when the communities found that the negotiation outcome would be more favorable as the extent (or likelihood) of contamination increased, they would exaggerate the contamination level during their judgment process. Thus, some process should be added in order to encourage decision makers to make accurate estimates.

The numerical method utilizing OWA to facilitate the brownfield redevelopment negotiation will be proposed in Sect. 3.3. Its application in the context of evaluating brownfields with fuzzy real options is proposed in the decision support system.

2.4 Negotiation Facilitation

Although the proposed numerical method focuses on utilizing fuzzy real options to evaluate brownfield redevelopment projects, the ultimate goal is to facilitate brownfield negotiation, and the evaluations will be incorporated into the design of a future Negotiation Support System (NSS). Compared to normal DSS, NSS aims to find optimal multi-party agreements, with innovative models, workflows, and associated communication supports, sometimes with a non-partisan mediator (Kersten and Lai 2007).

NSS can be employed at the stages of preparation, position and interest assessment, and proposal (Wang et al. 2010b). In addition to a DSS component with models based on decision analysis, game theory, or economic theory, NSS often has an electronic communication module based on psychological and behavioral theory (Lim and Benbasat 1993).

In the context of the brownfield negotiation, OWA is appropriate to estimate parameters of fuzzy real options, and hence determine the values of a brownfield to different decision makers, which can be used as the basis of negotiation. The expert judgments of contamination likelihood and risk preferences of decision makers can be expressed graphically, and reflected using fuzzy real options.

A non-partisan professional should be added to facilitate the brownfield negotiation. Qualified persons can be invited to offer professional opinions on the likelihood of contamination, which will be used as the reference point of pollution likelihood. The non-partisan professional can help in trust building and solution identification in the negotiation process.

Maps and fuzzy boundaries will be used to facilitate information sharing and communication based on a GIS module. Decision makers' judgments can be illustrated as fuzzy boundaries for iterative assessment in order to obtain an accurate subjective estimation. The GMCR and report generation functions could be incorporated into the proposed NSS in the future.

In summary, the proposed method will be implemented as a module toward a NSS for brownfield negotiation. An innovative decision model, a negotiation process to prevent strategic bidding, and an intuitive communication method are designed to constitute a better negotiation tool for brownfield redevelopment.

3 Numerical Methods for Brownfield Redevelopment Negotiations

3.1 Fuzzy Real Options

Because it is difficult to find analytic solutions to fuzzy real options problems, numerical methods are often employed in practice. An integrated representation of fuzziness and randomness is required, as fuzzy real options combine fuzzy parameters with stochastic processes.

In this paper, Chance Theory and hybrid processes are utilized. For a systematic study of different uncertainty representations and their axioms (see Li and Liu 2009; Ke and Liu 2007). In Ke and Liu's theory (2007), fuzziness is measured as credibility, whose main difference from probability is that the credibility of the union of multiple events equals the maximum of their individual credibilities.

Because chance space is defined as the product of a credibility space and a probability space, it integrates the two systems. Chance events, chance measurement, and hybrid variables are defined based on the chance space, parallel to probability. The chance measure proves to be subadditive (Li and Liu 2009). People normally weigh facts differently, making subadditivity a more appropriate property in modeling of the assessment of evidence (Klir and Smith 2001).

Among hybrid variables, the normally distributed random variable with a triangle-form fuzzy volatility parameter seems the simplest and most useful. It is employed here. Such a random variable (X) is decided by a density function (φ) depending on a fuzzy variable (y), whose membership function is $m(y)$. This hybrid variable covers all possible combinations of the fuzzy parameter and values of the random variable, denoted B. The measure of this variable on an element $\xi \in B$ (also called an event) is a chance measure, which is denoted as Ch ($\xi \in B$) in (1):

$$Ch(\xi \in B) = \begin{cases} \frac{m(y)}{2} \wedge \int\limits_{B} \phi(x; y)dx, & when\ \frac{m(y)}{2} \wedge \int\limits_{B} \phi(x; y)dx \leq 0.5 \\ \frac{m(y)}{2} \wedge \int\limits_{B^C} \phi(x; y)dx, & when\ \frac{m(y)}{2}) \wedge \int\limits_{B} \phi(x; y)dx > 0.5 \end{cases} \tag{1}$$

Note: (1) uses B^{C} to denote the complementary set of B, and $\alpha \wedge \beta$ the minimum of α and β.

The numerical fuzzy real options method is based on Least Squares Monte-Carlo simulation (LSM), an elegant approach to path-dependent options (Longstaff and Schwartz 2001). But least squares estimation is known to be valid only within the realm of probability theory. Hence, it must be extended to fit the generalized case of hybrid processes.

Extended LSM relies on the three main steps of LSM (Wang et al. 2010a). First, sample paths are generated for the hybrid process in two stages: initially, fuzzy samples are generated for a range of parameter values and associated fuzzy membership degrees; then, the hybrid process is simulated by splitting stochastic processes into groups, each corresponding to certain values of fuzzy parameters.

The second step is backward induction along the sample paths. Least squares estimation is conducted within each group of stochastic processes with the same parameter values (Wang et al. 2010a). Because the separate estimates do not utilize cross-sectional information across different fuzzy parameters, least square estimation remains valid for this process. Each sample path has an associated fuzzy membership degree, derived from the corresponding fuzzy sample of the parameters.

In the third step, expected project price is calculated based on the initial values. After several tests, normalized fuzzy membership was selected over fuzzy expected value definition and plain averaging because of both stability and flexibility (Wang et al. 2010a). Normalized averaging involves two stages: the fuzzy membership function is first normalized; and then weighted averaging is employed to calculate the expected value.

Extended LSM can compute numerical results from fuzzy real options modeling. The numerical and analytic results have been found approximately equal in cases where comparison is possible. The inclusion of fuzzy parameters in real options models usually leads to slightly higher estimates than with crisp parameters, which can be regarded as reflecting the added value of private risks (Wang et al. 2009a). As the skewness of the triangle in the fuzzy membership function changes, the values of the fuzzy real options vary accordingly. Compared to the IVP, extended LSM produces similar results and can thus be regarded as an alternative method of preference representation for private risks.

3.2 OWA and Fuzzy Boundaries

The proposed numerical method contains three main parts. First, OWA is used to aggregate decision makers' judgments on multiple environmental indicators. The three-point estimates are calculated with different preference parameters. While the minimum and maximum corresponds to the two extreme cases, the most likely contamination level will be determined by decision makers' risk preferences expressed in linguistic quantifiers.

Second, once the likelihood of pollution at sample points is determined, an interpolation method must be applied to compute the contamination level in the entire brownfield. Different interpolation techniques are discussed. The method called Inverse Distance Weighting (IDW) is suggested due to its flexibility in dealing with discrete layer boundaries. Then the equations for calibrating parameters in fuzzy real options are derived.

Third, the likelihood of contamination will be visualized and stored as fuzzy objects in a GIS system, enabling iterative modification and negotiation. Ways to store such information in databases are introduced and implemented. All the three parts are further explained in the following subsections.

3.2.1 OWA and Interpolation

Since brownfield redevelopment involves geographic information, spatial analysis and GIS are helpful in decision-making and negotiation. Techniques in GIS software to combine multiple geographic factors include spatial logic operators (i.e. union and intersection) and simple additive weighting (Malczewski and Rinner 2005). OWA can be regarded as a generalization of both methods, so it is entirely appropriate for a spatial decision-making environment.

Because the condition of brownfields varies greatly, there are both generic and site-specific approaches to brownfield redevelopment (Ontario 2009). Criteria in judging pollution level differ case-by-case. Furthermore, the number of criteria may be too great for decision makers to keep them in perspective (Nijkamp et al. 1985). Thus, linguistic quantifiers are helpful in making criteria cognitively manageable (Malczewski and Rinner 2005). Linguistic quantifiers can also be extended to processing descriptive assessments (Zadeh 2004).

The function $Q = x^r$ is employed here to determine VOID and the associated OWA weights. Decision makers can express their risk preferences descriptively, or by changing the parameter r. Using this function, VOID, r, and orness level are linked, where VOID $= \frac{1}{r+1}$ (Yager 2007). This relationship will be shown graphically in the planned NSS, making decision makers aware of the impact of changing their risk preference on the OWA weights generated. The steps involved in determining the likelihood of contamination are as follows:

1. Identify criteria used in judging the contamination level and select appropriate linguistic quantifiers expressing decision makers' risk preferences.
2. Let decision makers express their judgments ranging from 0 to 1 at sample points.
3. Calculate the likelihood of contamination of the entire brownfield site by interpolation, which will be explained below.
4. Map the result and encourage iterative modification if appropriate.

The main interpolation methods are Inverse Distance Weighting and Kriging. In the brownfield redevelopment application, Kriging is inappropriate for the following reasons: Firstly, Kriging is based on stationary spatial stochastic processes with spatial correlations. Since fuzzy real options assume IID processes with exponential growth, Kriging fails to satisfy independence and no correlation (Goovaerts 1997). Secondly, Kriging tends to generate continuous results. However, as geological layers are often discrete, Kriging is inappropriate when some crisp boundaries must be accounted for (Allen et al. 2007). Furthermore, Kriging demands considerable computational power and is difficult to implement. Hence, IDW, with none of these drawbacks, is selected as the interpolation technique.

The idea of IDW is to utilize the distances between each sample point and the estimated positions as the main factor in determining weights (Eq. 1).

$$u(x) = \sum_{k=0}^{N} \frac{w_k(x)}{\sum_{i=0}^{N} w_i(x)} u_k, \quad \text{where } w_k(x) = \frac{1}{d(x, x_k)^p}. \tag{2}$$

The parameter p affecting weights is often greater than 1. Since gravity decreases as the square of distance, $p = 2$ is a frequently-used value. Other possible values can be also tested later.

The combined utilization of OWA, VOID, and IDW enables decision makers to make subjective judgments on the brownfield site. The fuzzy membership degree for each point of the site can be fed into equations to compute the parameters in fuzzy real options and displayed as a map for iterative modification and negotiation, as will be further discussed below.

3.2.2 *Parameter Estimation*

The OWA and interpolation stages produce three maps of the likelihood of contamination. Each of them shows a contamination area with fuzzy membership degrees, corresponding to minimum, most likely, and maximum scenarios. This information can be used to estimate the parameters of fuzzy real options.

Because remediation cost is assumed to satisfy $\frac{dS}{S} = \mu dt + \sigma dz$, there are three parameters to be estimated: the initial (or current) remediation cost S_0, the annual growth rate (for cost) μ, and the volatility σ (Wang et al. 2010a). Initially, the contamination disseminates rapidly in the vodose zone but the rate of spread quickly decreases to nearly zero. Therefore, the contamination volume can be assumed to remain constant after a short initial period (Yu 2009; El-Gamal 2005). Hence, given that remediation cost directly depends on the contamination volume, remediation cost and volatility must be fuzzy numbers, while the growth rate can be assumed to be crisp and independent of remediation volume. The rate of growth of the clean-up cost can be estimated based on market data.

In hydrogeological models, five main factors determine pollutant dissemination: advection, diffusion, dispersion, sorption, and biodegradation (El-Gamal 2005). Parameters for these processes are difficult to calibrate. Uncertainties are represented by fuzzy boundaries around the brownfield site, which reflect the volatility.

$$S_0 = kA^{high}. \tag{3}$$

The initial remediation cost must be proportional to the contaminated volume, which can be regarded as constant as long as pollutants have not entered the saturated zone (Yu 2009). This relationship can be expressed as (2), where k is a coefficient and A^{high} denotes the area value with a high membership degree that is decided by the expert as an empirical parameter, which is similar to A^{low} later. Although the initial remediation might be functionally related to the area in a more complex manner than expressed in (2), this relationship does not have a great impact on the final result and is easy to improve in future work. With three different risk preferences, a three-point estimate of initial cost can be determined.

$$\sigma = \sqrt{\frac{11}{128}(A^{low} - A^{high})}. \tag{4}$$

The volatility will be computed based on the difference between A^{high} and A^{low}, the areas that are certainly polluted and clean, which is shown in (3). This equation is derived from the variance formula for a triangular fuzzy variable,$x = (a, b, c)$, which is $V(x) = \frac{33\alpha^3+21\alpha^2\beta+11\alpha\beta^2-\beta^3}{384\alpha}$, where $\alpha = (b-a)\vee(c-b)$ and $\beta = (b-a)\wedge(c-b)$ (Liu 2008). Because subjective estimates of pollution level are usually linear and fit the triangular form, the transition from contaminated to clean area is assumed to follow a right-angled triangle form. In this case, we can assert that $a = b$, leading to $\alpha = c - b = A^{low} - A^{high}$ and $\beta = 0$.

With (2) and (3), the parameters needed to evaluate fuzzy real options can be calculated. Although the area value used might be replaced by a function of the area, it is believed that (2) is approximately correct due to the layered structure of the hydrogeology (Yu 2009). For now, the equations are simple to use and easy to modify if necessary.

3.2.3 Fuzzy Boundaries

The mapping capacity of GIS can be utilized to facilitate iterative multi-criteria analysis, which is helpful in negotiation. As Jankowski et al. (2001) emphasized, exploratory decision analysis is critical in multi-criteria decision-making. Given that preference and subjective judgment are often expressed with intrinsic vagueness, the mechanism of allowing a decision maker to check the output and modify unsatisfactory inputs will likely be useful. Hence, fuzzy boundaries of a brownfield are proposed in order to employ mapping tools for aid.

As mentioned in the previous subsection, every location in a brownfield site has a fuzzy membership degree measuring likelihood of contamination. Unlike crisp logic used in GIS either within or outside a parcel, fuzzy boundaries of a brownfield are a challenging representation problem for a GIS system. The representation of fuzzy boundaries has been studied in the literature, such as Wang and Hall (1996) and Schneider (2008).

Since representations of geographic features can be classified in vector-based or grid-based storage (Zeiler 2010), efforts may be made in both directions. The representation of fuzzy boundaries in vector form is preferred in normal GIS applications because of its compact format and compatibility to structured query languages like SQL and database.

However, the vector form representation of fuzzy boundaries normally requires a known fuzzy membership function, which may be difficult to obtain. In addition, the performance of spatial operations, such as union, intersection, and buffering, is weaker than that of the grid form. In the brownfield redevelopment application, little information other than the fuzzy membership degree is required, so the advantages of vector form do not compensate for its performance burden. Hence, the grid-form of representation of fuzzy boundaries should be considered.

When more information is to be associated with locations, the vector form representation of fuzzy boundaries can be implemented. The work on fuzzy representations, fuzzy query language, and even a fuzzy relational data model for geographic information can be added to existing work toward a more integrated system that is capable of processing natural language (Wang and Hall 1996). A membership degree will be associated with each tuple (record) in the database, just as for other mandatory geographic attributes (Zeiler 2010).

3.3 Workflows of Negotiation with the Proposed Numerical Methods

The structure of the proposed DSS is illustrated in Fig. 4, which will also be the core module of a NSS. The DSS will be distributed into three places: a server with powerful computational capacity where the core DSS component is installed, a server sharing geographic information publicly, and a mobile device with graphic interface to capture judgments on site:

- *Geographic Information Server* This component provides public information to all decision makers, facilitating negotiation by information sharing. All contaminated

information will be updated here, avoiding information management issues, such as version control, accessibility, and backup.
- *Core Components for Brownfield Evaluation* Because fuzzy real options demand computational power even in a parallel computing environment, the parameter estimation and option evaluation algorithms will be installed and utilized on a powerful server via web services. This mechanism reduces the cost while widening the availability of fuzzy real options models.
- *Mobile Device connected to the Internet* A mobile device is easily carried, enabling decision makers to record and modify their judgments on site or during the negotiation process anywhere. Exploratory decision making and negotiation will be facilitated through this arrangement.

Negotiation using this proposed DSS can follow a process briefly described below:

1. Decision makers bring a mobile device to the brownfield, retrieving maps with appropriate geographic information from local government, and then mark their judgments at sample locations. OWA will be called from another server, to combine multiple assessments and interpolate across the site. The output will be fed back to the mobile device, adding the likelihood of contamination as a layer on the map. Decision makers can modify their estimates if they prefer. Final outputs will be stored on the public server.
2. Once the judgments are fixed, the parameters of fuzzy real options will be computed, and the fuzzy real options model for brownfield evaluation will be called to determine the value of the site, critical values, and optimal decisions for decision makers with different risk preferences.
3. Since the values of the brownfield for decision makers have been determined, any conflicts are now clear. Negotiation can be facilitated through equilibria found using conflict analysis methods. Decision makers can also compromise by adjusting their judgments, changing their attitudes, or adding more options.

Negotiation workflow can be optimized in the future and added as another module on top of the DSS. Better negotiation processes that encourage candid judgment reporting may be added. Another possible improvement would be the additional component dealing with communication.

3.4 Illustrative Example

In this paper, the case of the Ralgreen Community redevelopment in Kitchener, Ontario, Canada is employed to illustrate how to apply proposed numerical methods. This case was selected due to the relatively rich set of available documents and to the long history of controversy concerning the contamination of the site.

Background information on the Ralgreen Community redevelopment is introduced first. Subsequently, steps for determining the likelihood of contamination are shown. Results are discussed in comparison to the case documents.

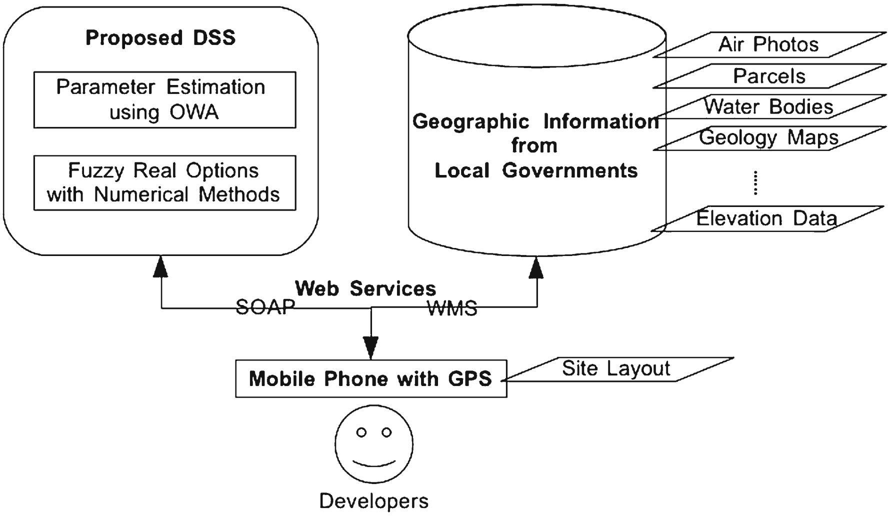


Fig. 4 The structure of the proposed DSS

3.4.1 Ralgreen Community Redevelopment

Until 1948, the Ralgreen property was farmland with a small pond. Then, with the owner's agreement, the City of Kitchener dumped garbage into the pond and surrounding area as landfill. Some twenty years later, the property was developed into a residential community in 1968–1969 (HEATH 1997). On August 22, 1969, 65 and 67 Ralgreen Crescent were devastated by fire, caused by methane gas. During the subsequent investigation, three other semi-detached buildings, 64–66, 68–70, and 94–96 Ralgreen Crescent, and three houses, 1257, 1259, and 1261 Queens Boulevard were found to be in a potentially hazardous situation (HEATH 1997).

In response to this danger and a by-law, the Building By-law (Special Requirements on Filled Lands), was passed by the City of Kitchener in October 1969, requiring venting systems to be installed in all buildings in potential danger. Furthermore, garbage and organic materials were removed and replaced with compacted granular fill (HEATH 1997). All properties passed a methane gas test on June 20, 1978, and were not listed as closed disposal sites by the Ontario Ministry of the Environment.

After 67 Ralgreen Crescent was sold around 1993, the possibility of contamination arose again. At the end of 1995, the homeowner reported a sewage-like water leak in the basement. A high level of combustible gases was detected in April 1996. In the following year, underground monitoring was conducted and contamination was confirmed in the surrounding area, which roughly coincided with the original pond (Frontline 2000).

In 1999, a group of residents undertook legal action against the City of Kitchener. In the following year, an agreement was reached by the parties, under which the City of Kitchener purchased 15 properties in the former pond area and cleaned the land according to the MOE 1997 guidelines (Frontline 2000). In the end, the Ralgreen Community was remediated and redeveloped based on the agreement.

Table 1 Criteria and assessment used in contamination judgment

Criteria	Assessments
Foundation settlement	Confirmed problem (100 %); main structure settlement (80 %); shear cracking (60 %); attached garage settlement (40 %); detached garage settlement (20 %); and no problem (0 %)
Interior methane gas levels	Methane gas level is at least 200 ppm (100 %); 100–200 ppm (50 %); greater than 0 % (25 %); and zero (0 %)
Soil and groundwater quality	At least 5 contaminants (100 %); 4 contaminants (80 %); 3 contaminants (60 %); 2 contaminants (40 %); 1 contaminant (20 %); none (0 %)
Basement water leakage	Confirmed problem (100 %); detected (measured) contaminants (70 %); odor (40 %); none (0 %)
Indoor air quality	Confirmed problem (100 %); detected (measured) contaminants (50 %); none (0 %)

The evidence of pollution found through this process can be classified into five categories: foundation settlement, interior methane gas levels, soil and groundwater quality, basement water leakage, and indoor air quality (HEATH 1997). Each class contains several indicators, around 20 in total, ranging from garage tilting, leaking sewage-like water, and odor, to mould on the wall (Frontline 2000).

3.4.2 Main Steps in Valuing Brownfield Using Proposed Method

The steps involved in determining the value of a brownfield based on subjective judgments are: identify the judgment criteria, assess the likelihood of contamination at sample points, derive the map of the pollution extent of the brownfield as fuzzy boundaries, estimate parameters in a fuzzy real options model of the brownfield, and calculate the value. The case of Ralgreen Community is used to demonstrate this process.

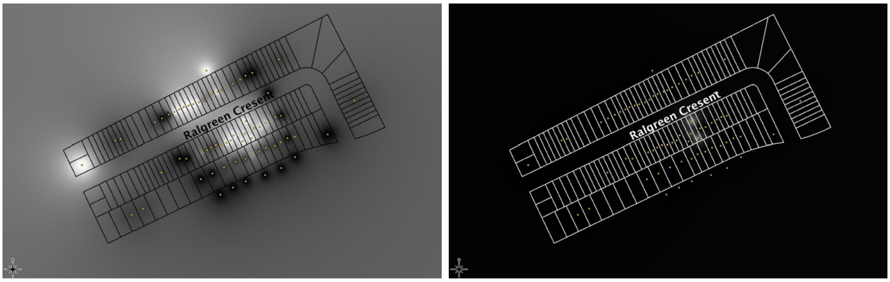
As mentioned in the previous subsection, five criteria were employed to measure the contamination level. The assessment is given in Table 1, summarized from the literature (HEATH 1997).

The linguistic quantifiers used are "most", "average", and "few" for community residents, the non-partisan expert, and the City of Kitchener, respectively, where the parameter r is set as 10, 1, and 0.1. The weights of the OWA operator are listed in Table 2, ordered from largest (applies to maximum assessment) to smallest (applies to minimum assessment).

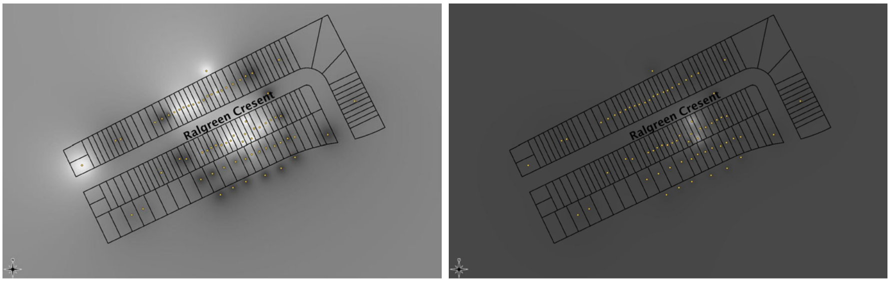
When the IDW system is applied to determine the likelihood of contamination in the Ralgreen community, five maps based on different linguistic quantifiers are generated as shown in Fig. 5. In this figure, the rectangles denote individual properties, the lighter areas are more polluted and the darker shading indicates less pollution. Points spreading in the community are samples where judgments are made. In fact, the lightest patch in this figure is the location of the former pond, which has the highest level of pollution as mentioned in various reports (HEATH 1997). A 50 % possibility of contamination is selected as the high threshold, and 10 % as the low level.

Table 2 OWA weights for different linguistic quantifiers

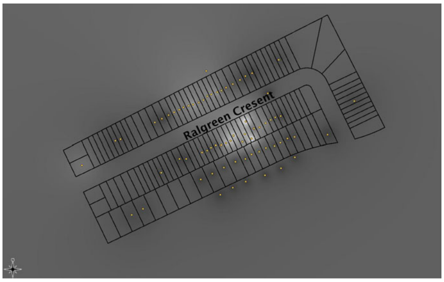
Linguistic quantifiers	Parameter *r*	VOID	Weight 1	Weight 2	Weight 3	Weight 4	Weight 5
Max	Infinity	0	1.0000	0.0000	0.0000	0.0000	0.0000
Most	10	0.0909	0.8926	0.1013	0.0059	0.0001	0.0000
Average	1	0.5	0.2000	0.2000	0.2000	0.2000	0.2000
Few	0.1	0.9091	0.0221	0.0277	0.0378	0.0611	0.8513
Min	0	1	0.0000	0.0000	0.0000	0.0000	1.0000



Max Likelihood Min Likelihood



Likelihood based on “Most” Likelihood based on “Few”



Average Likelihood

Fig. 5 Fuzzy boundaries of pollution levels (*scale* 1 cm = 150 m)

For the areas exceeding the α^{50} and α^{10} thresholds, the parameters of the initial redevelopment cost and volatility are shown in Table 3. The excavation and refill cost is assumed to be \$100 per m^2. The volatility coefficient is set to 10^{-6}. Other parameters are listed in Table 4.

Table 3 Estimated parameters derived from pollution judgment

Scenario	α^{50} Area (m^2)	α^{10} Area (m^2)	S_0 ($)	Volatility
Max	39,845.25	1,137,538.00	3,984,525	0.321790
Most	27,575.00	1,137,311.00	2,757,500	0.325320
Average	1,006.50	1,115, 614.00	110,650	0.326748
Few	37.75	3715.25	3775.00	0.001078
Min	0.00	1511.50	0.00	0.000443

Table 4 Parameters other than fuzzy inputs

Parameter	Value
Riskless rate (r)	5 %
Income drift rate (μ_{in})	2.5 %
Income volatility (σ_{in})	0.2
Initial annual income ($S_{0,in}$)	216,000
Redevelopment drift rate (μ_{cost})	5.5 %

Table 5 Results of fuzzy real options model for brownfields

	Property value ($)	Critical value	Expected redevelopment time (year)
Few	6,812,537.827	0.1811	6.3389
Average	6,607,291.971	0.1976	6.2472
Most	6,439,669.132	0.1353	5.9066

With the inputs shown in the above tables, the fuzzy real options model of brownfields generated the results in Table 5. We see that differing risk preferences among decision makers generates different property values. But the differences are minor compared to the overall value. The less the likelihood of contamination, the less the lower redevelopment cost, and the higher the property value. Since current income/cost ratio exceeds the critical value in each scenario, all decision makers tend to select the option to wait, with an expectend time of 6 years.

4 Discussion and Future Work

4.1 Conclusions and Discussion

As shown in the previous section, the proposed method can help determine the likelihood of contamination and the corresponding value of the brownfield. The results can be understood from the perspectives of fuzzy boundaries, property values, critical values, and expected time.

The fuzzy boundaries of the brownfield reflect the reality of contamination in literature (HEATH 1997; Frontline 2000). Depending on the linguistic quantifiers, the polluted areas of “max”, “most”, and “average” spread around the former pond, although the “average” quantifier is much less probable than the other two. With the linguistic quantifier “few”, the contaminated area is restricted to the two most problematic properties: Ralgreen 65 and 67. The “min” operator even suggests that there was no contamination in the Ralgreen Community. The conflict over pollution extent between residents and developers is clearly shown in these different scenarios.

Comparing the five OWA weight vectors and the scenarios they generated, it is observed that the “most” and “max” cases are similar, as are the “few” and “min” cases. Hence, the derived triangle-form fuzzy estimates are skewed, indicating that decision makers have strong risk preference.

When fuzzy real options analysis is employed, we found minor differences (∼3 %) in values for the brownfield. The major factor affecting values should be the estimates of the initial redevelopment cost. Volatility also has an impact on value, but it is not as influential.

Under all scenarios, critical values are slightly greater than the current income/cost ratio. Hence, the optimal decision should be to wait. However, the values generated using fuzzy real options analysis are also a little higher than those of NPV. A similar result was found in a case study in the United States (Erzi-Akcelik 2002). In particular, only a minor modification or compensation would change the developer’s decision from wait and see to participate immediately.

All expected waiting times are around 6 years, which roughly equals the negotiation process from 1995 (contamination found) to 2001 (redevelopment complete). In contrast to the brownfield value, in which the decision maker became more optimistic about the pollution level, the expected waiting time increases when preference becomes less risk adverse. The reason is the volatility of redevelopment cost, which increases as pollution extent shrinks. Therefore, decision makers tend to wait longer in anticipation of more business opportunities due to higher uncertainty. This result explains why developers are reluctant to redevelop, even though they understand the value of brownfields.

On the other hand, it is unclear whether fuzzy real options can model the behavior of community residents. Since community residents live in the contaminated properties, concerns for public health may be a more important factor than property value.

When the proposed method is applied to the Ralgreen Community case study, outputs reflect the reality of the negotiation process. Conflict among decision makers is shown as different values for the same brownfield, using fuzzy real options and OWA. Even when brownfields have a high value, developers usually select the option to wait, seeking business opportunities that maximize the price. The fuzzy real options model of brownfield pricing thus explains decision makers’ behavior under different policy scenarios.

5 Future Work

In the future, the method proposed here to determine the fuzzy boundaries can likely be improved. Parcel boundaries can be considered to improve the accuracy of subjective contamination estimates. People often regard a building (or property) as a whole. Hence, the combination of crisp parcel boundaries with fuzzy boundaries deserves further study.

It should be possible to input the conflict value of a brownfield derived from fuzzy real options into the graph model for conflict resolution (GMCR) to find equilibria as possible negotiation solutions. Brownfield policies and initiatives could be optimized with the aid of GMCR and fuzzy real options.

Another possible direction is to identify closed-form solutions based on a numerical method that has been proposed elsewhere (Wang et al. 2010a). When more cases and data have been accumulated, experimental formulas can be proposed and tested. The demands for computer performance would be greatly reduced if formulas were found, since simple algebraic computation is much more efficient than numerical simulation.

The NSS proposed in 3.3 can also be improved. Because linguistic quantifiers can be used with the proposed method, text mining techniques can be added in order to facilitate automated information retrieval from brownfield documents (Apache 2009). The proposed NSS will be more intelligent and, therefore, more helpful in facilitating brownfield redevelopment.

In summary, the proposed method builds a foundation for brownfield redevelopment negotiation and policy-making. Various components can be added based on values derived from the proposed method, thus improving development decision making, and enhancing sustainability.

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