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Visual and Interactive Comparative Analysis of Individual Opinions: A Group Decision Support Tool

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Abstract This study proposes the use of the visual, interactive and comparative analysis (VICA) methodology to encourage consensus-building in decision making processes involving multiple criteria and multiple participants working in cooperative groups. The tool was applied to the Electre TRI (VICA-Electre TRI) method which utilises comparative analyses plus visuals and the interactive exchange of individuals' opinions within the group. It aims at reducing complexity, presenting updates about each member's progress in the decision making process and fostering the search for consensus. The methodology was implemented in a spreadsheet format (Microsoft Excel) to make it as accessible as possible while also facilitating its acceptance and efficient use within organizations.

1 Introduction

Cooperative groups, according to Dias and Clímaco (2005), distinguish themselves from negotiation groups. Some characteristics distinguishing them include: convergence of targets and goals; relationships of power and interdependence; the ability to share information; and the behaviour considered acceptable by participants (the possibility of abandoning negotiations, for example). In this paper, the cooperative group consists of individuals that either desire or need to arrive at a consensus and are, therefore, likely to contribute to the mutual understanding of a relevant question.

Combining multiple criteria among multiple decision-makers in order to arrive at a decision is a difficult task. Considering alternatives in accordance with varying criteria which, in turn, are often based on a combination of objective measures and qualitative judgements of each of the decision-makers can prove highly demanding and complex.

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This is especially true when the goal is to achieve not only quality decisions but to do so in a collective and transparent manner. In addition to these difficulties, it must be taken into account that the information exchanged in the process is in constant modification as a result of group interaction and that there are often lack of precision and uncertainties regarding information in the decision-making process. This challenge requires a strict control of the process, procedures, methods and tools to support the group's decision.

This work presents the principles of the visual, interactive and comparative analysis (VICA) methodology as applied to the Electre TRI method (VICA-Electre TRI). VICA aims at contributing to the search for justifiable, transparent and collectively constructed solutions through the application of the procedures of multicriteria decision aiding (MCDA). It proposes a way to reduce complexity and tackle two important challenges in the context of multiple criteria and multiple person classification (Zopounidis and Doumpos 2002): presenting information about the state of each member of the decision-making process and supporting the group in its quest for consensus.

Consensus has traditionally meant unanimity among group members to the extent that any lack of agreement of any single member of the group, in any aspect, could block consensus and therefore a possible course of action. There is, however, a more broadly defined approach to consensus which can incorporate compromise and even passive disagreement. In this case, Ness and Hoffman (1998) argue that consensus can be reached when a certain percentage of group members (to be pre-determined or not) agrees with the decision and the discordant members feel that they have had reasonable opportunity to influence the result and they, therefore, agree to support the proposed solution. This study adopts this latter definition of consensus.

The VICA methodology (Bezerra et al. 2008) is a feedback-based approach adapted from a proposal by Vetschera (1991), incorporating both visual and interactive tools as suggested in Hodgkin et al. (2005). VICA has been tested with the Electre TRI method supporting a group decision-making process, while taking into consideration both shared parameters and those determined by personal preferences. Starting from the individual contributions of each participant, the methodology offers numerous tools for the visualisation and comparison of results. Mechanisms for incorporating "*What if*" analyses (in the form of direct inputs, tables, scenarios, etc.) as well as "*How to*" (optimisation) analyses allow members to interact, modify and revise individual opinions in a wide range of manners. Thus, the group is better supported in searching the bases for a consensual decision.

Microsoft Excel spreadsheets were chosen to develop the tools so that the work would be as accessible as possible. According to Moreno-Jiménez et al. (2005), Microsoft Excel is available on many computers and provides Visual Basic for user interaction improvements. Moreover, it contains the optimisation tool Solver. Hyde and Maier (2006) argue that this software allows the analysis and manipulation of data as well as their immediate visualisation. Belton and Stewart (2003, p. 282) point out that "*If customised software is not available most multicriteria analyses can be carried out with the help of a spreadsheet*". Ragsdale (2004) also states that spreadsheets have become the standard vehicle for introducing business and engineering students to the concepts and tools in courses of *Management Science/ Operations Research*. Moreover, most business people consider spreadsheets as their most important analytical tool, with the exception of their own brains.

Section 2 of this paper discusses the conceptual bases of the VICA methodology, while Sect. 3 briefly reviews Electre TRI. In Sect. 4, the VICA-Electre TRI model, as well as its structure and function, are presented. A quantitative (numerical) example illustrates the model in Sect. 5. Finally, contributions and future developments are discussed.

2 Feedback Approach, Consensus and Visual Concepts

Consensus may be defined as a function of the degree of agreement among group members and also as a characteristic of the process as perceived by its participants. Thus, Martz and Shepherd (2004) propose the concept of perceived consensus, which would result from the relationship between an acceptable consensus and the estimate of consensus perceived by the group members.

Consensus is intimately related with feedback: participants often reconsider or alter their own opinions after coming to understand the opinions of other group members. According to Pruitt (1971), and Vetschera (1991), the phenomenon is empirically observable. Ben-Arieh and Easton (2007, p. 714) stated that "In the second type of consensus, the experts are expected to modify their opinion in order to reach a closer agreement in opinions". Thus, revisions or changes of opinion of individuals during the decision-making process, the so-called "feedback effect", have a central role in moving the group toward consensus. An implicit assumption is that the changes that occur, are due, at least in part, to the desire of members to contribute toward the search for a collective decision This motivation does not necessarily exclude other reasons for changes of opinion, such as knowledge acquired during the process or some other type of influence exerted by group members. This approach views consensus as a collective construction based on interactions and opinion modifications.

Achieving consensus in a group project is an important goal and is often considered a performance measure both for the process and the group. Nunamaker et al. (1997) pointed out that, in certain circumstances, consensus is simply not possible. In that case, the information that is gained about the process is that insurmountable differences exist and that their causes require investigation.

Ben-Arieh and Easton (2007) argue that there are four possible approaches to measuring consensus:

i. **Counting** the number of members who share the group opinion ;
ii. **Measuring distances** between the participants;
iii. **Comparing similarities** and differences between group members;
vi. **Ordering** the alternatives according to the group and its members.

Melo (2005) reviews numerical measures of consensus in the absence of unanimity, including statistical measures of correlation. These are particularly aimed at measuring consensus based on individual ranking preferences. Cook (2006) offered an extensive review of solutions and measures based on distance and ad hoc models, also for measuring consensus on ordering.

The VICA methodology is intended to help a group in its search for consensus based on a feedback approach adapted from a proposal by Vetschera (1991) and incorporating both visual and interactive tools as suggested in Hodgkin et al. (2005).

The next paragraphs summarize the proposal of Vetschera (1991) and some other feedback approaches. After that, visual and interactive tools are reviewed.

Of the feedback approaches, we call attention to that of Vetschera (1991), who points out that ***G**roup **D**ecision **S**upport **S**ystem* (GDSS) are usually composed of two levels, the first representing individual evaluations and the second represented by the group. Some GDSSs are designed as extensions of ***D**ecision **S**upport **S**ystem* (DSS) and focus on data bases and on the presentation and analysis of data. Others concentrate on methods for evaluating and aggregating preferences. The author then goes on to propose an alternative feedback approach incorporating both levels in which preference changes by decision makers (DMs) can take place in two ways: through a change in the evaluation structure (increased or decreased attention paid to an attribute, for example) or by the introduction of the group opinion as an attribute or additional criterion, giving to it some weight.

This feedback approach was expanded on TriGdist (Melo 2005) and the work of Han and Ahn (2005) and Herrera-Viedma et al. (2002). TriGdist is a GDSS developed to assist in structuring the problem as well as to encourage consensus regarding problems of classification. It provides mechanisms for measuring the difference between individual positions. Such distances are presented in the form of the alterations that a DM would have to make to obtain the same result as another DM. Han and Ahn (2005) present an interactive procedure partly based on a measure of intensity of group preference for each of the alternatives and points out the direction that alterations in individual preferences must be made in order for consensus to be achieved. Herrera-Viedma et al. (2002) present a system for modelling and substituting the actions of a mediator in an automatic process designed to guide groups toward consensus. It is based on the use of a measure of consensus to identify the group position and in a measure of proximity that locates the distance of each DM from the collective opinions.

The work presented in this paper has similarities with the approach presented in Vetschera (1991), in that both use feedback and change of individual preferences to achieve acceptable consensus for group decision (as exemplified in Sect. 5). The main difference is that whereas the original approach was applied to MAUT-like methods, VICA is applied, in this paper, to an outrank-like method, incorporating shared veto. An additional difference is that an explicit group evaluation is not attempted in the original work (which focus on a single pair of DMs), whereas VICA consensus building iterates through “group-centered” solutions. Finally, rather than considering a C-consensus for a pair of individuals (proposed in Vetschera, 1991, as consensus on the top C results), VICA uses a different definition called G-consensus (with G being defined iteratively by the group process) describing full agreement for G group member evaluations.

Other approaches to search for consensus do not explicitly represent the differences (or proximity) among the DMs. Damart et al. (2007) utilise the aggregation/disaggregation approach for the Electre TRI method. They proposed that the DMs assign sample actions in pre-defined categories. The group defines how to classify the sample actions consensually with the help of the IRIS software (Dias and Mousseau 2003). The sequential interactions allow the construction of a collective model, in which the parameters are defined by mathematical programming. The VIP methodology (Dias and Clímaco 2005) aggregates multicriteria performances through functions of additive value with imprecise information. The version for groups, VIP-G helps achieve majority or consensus-based decisions. The software intends to reflect the consequences of different inputs from group members, furnishing feedback for the DMs to compare with their own individual models with one acceptable to the group (either by all or by a pre-established majority). Costa et al. (2003) establish that to achieve consensus it is necessary to support three layers of interaction. At the **individual level**—the participants process information focused on their points of view. At the **interpersonal level**—group members learn about the opinion of others and incorporate these opinions into their own individual structures. The group works at the **collective level** while processing information in order to arrive at a common solution.

Information provided by DMs is often represented through simple visuals and/or synthesised into models. Well-designed visual and interactive interfaces are powerful tools for exploring the implications of alterations in the values of the models. VICA recognises and applies visual resources, as many GDSS projects, as we exemplify bellow.

Visual interactive sensitivity analysis (VISA) allows members to compare strategies aiding in the modelling and visual and interactive analysis of problems. According to Belton et al. (1997), it is based on a multi-attribute value function in which the criteria are usually structured into value trees. The extension *groupware* VISA enables all group members to visualise an array of group results within a local network in order that the points of agreement and disagreement might be made obvious.

Some multivariate statistical techniques have been used as a tool to demonstrate and compare alternatives evaluated by multiple criteria (Stewart 1981; Brans and Mareschal 1994). These include factor analysis and principal component analysis (PCA-Biplot). In Losa et al. (2001), the techniques are applied to the conflict diagram as well as to the effect of alterations in the model's parameters.

Hodgkin et al. (2005, p. 175) define the application of MCDA in groups as a collective learning process about a certain subject. They believe it is important to perceive and understand divergent points of view and individual preferences. They state that "for the majority of people, visual interactive displays are the most powerful means of communication". They further argue that the tools available for visuals in the generic MCDA packages are not yet sufficiently adequate to help analyse sensitivity and communicate results. Therefore, they suggest that future developments should provide new mechanisms capable of combining quantitative measures with visual tools in order to emphasise similarities and differences between individuals. This, they believe, will serve to discover more systematic and powerful multi-criteria, group decision-support tools.

To sum up, the VICA methodology is based on the principle that to help the search for consensus, in a group decision process, it is necessary to provide, explicitly or implicitly, some sort of information about the comparison of individual preferences and associated outputs, allowing for several revision interactions. Moreover the provided information must be presented in some visual and interactive form in order to decrease the cognitive burden upon the DMs. In the next sections we will present the VICA applied to the Electre TRI method (VICA-Electre TRI).

3 Sorting Problems and Electre TRI

Electre TRI (Yu 1992) is one of the outranking MCDA methods and was specifically designed to deal with sorting problems. These problems consider a situation in which each alternative of a set $A = \{a_1, a_2, \ldots, a_i, \ldots, a_n\}$ should be allocated to a class or category. Such assignment is the result of successive comparisons between their performance and the performance of reference alternatives or profiles $b_h \in B = \{b_1, b_2, \ldots, b_h, \ldots, b_{k-1}\}$, in each of the criteria $j = 1, 2, \ldots, t$. The profiles function as lower and/or upper limits located between k ordered categories. The profiles may be real or fictitious alternatives.

The method requires the calculation of the partial concordance indices $c_j(a_i, b_h)$ followed by the general concordance index $C(a_i, b_h)$, discordance indices $D_j(a_i, b_h)$ and credibility index$\sigma(a_i, b_h)$ for the outranking relation $a_i S b_h$ ("a_i is at least as good as b_h"), taking the performance $g_j(a_i)$ and $g_j(b_h)$ in each criterion into account. These indices employ the following parameters: indifference threshold q_j, preference threshold p_j, and veto threshold $v_{j,}$, defined for all criteria or established for each of the comparisons, by profile, when taking the forms: $q_j(b_h)$, $p_j(b_h)$, and $v_j(b_h)$, respectively. The profiles must be coherent so that the reference actions of the highest categories should have better performance than the lower ones in all criteria.

Once these parameters are defined, the partial concordance indices $c_j(a_i, b_h)$ can be obtained for the statement $a_i S b_h$ based on the formula:

$$\Delta_j = \begin{cases} g_j(a_i) - g_j(b_h) & \text{if the criterion is to be maximised} \\ g_j(b_h) - g_j(a_i) & \text{if the criterion is to be minimised} \end{cases}$$

$$c_j(a_i) \begin{cases} 1 & \textit{if } \Delta_j \geq -q_j \\ \frac{p_j+\Delta_j}{p_j-q_j} & \textit{if } -p_j \leq \Delta_j < -q_j \\ 0 & \textit{if } \Delta_j < -p_j \end{cases}$$

The general concordance index $C(a_i, b_h)$ is calculated by aggregating the partial concordance indices $c_j(a_i, b_h)$ for all criteria. In order to perform this aggregate, the relative importance of each of the criteria must be defined and weighted w_j. The following formula calculates $C(a_i, b_h)$:

$$C(a_i, b_h) = \frac{\sum_{j=1}^{t} w_j c_j(a_i, b_h)}{\sum_{j=1}^{t} w_j}$$

The veto thresholds can delineate discordance indices which may prohibit the statement "a_i outranks b_h". The discordance indices are calculated for each criterion through the following formula:

$$D_j(a_i, b_h) = \begin{cases} 1 & \textit{if } -\Delta_j > v_j \\ 0 & \textit{if } -\Delta_j \leq p_j \\ \frac{p_j+\Delta_j}{p_j-v_j} & \textit{otherwise} \end{cases}$$

In the case of significant discordance indices generated by veto effects, the method requires the adjustment or reduction of the general concordance. When the general concordance index is combined with the discordance indices, the credibility index σ (a_i, b_h) can be obtained as follows:

Considering $\bar{F} = \left\{ j | D_j(a_i, b_h) > C(a_i, b_h) \right\}$

$$\sigma\,(a_i, b_h) = \begin{cases} C\,(a_i, b_h) & if\ \bar{F} = \emptyset \\ C\,(a_i, b_h) \prod\limits_{j \in \bar{F}} \frac{1-D_j(a_i,b_h)}{1-C(a_i,b_h)} & otherwise \end{cases}$$

The credibility index is a fuzzy measure defined in [0,1]. Therefore, in order to establish an outranking relation $a_i S b_h$ (or—$(a_i S b_h)$—the rejection of the statement "a_i is at least as good as b_h") a cut threshold λ must be defined. This is the lowest value belonging to [0.5;1], which is compatible with the statement $a_i S b_h$. Only when $\sigma(a_i, b_h) \geq \lambda$, can a_i outranks b_h.

Finally, two possible procedures exist for assigning alternatives to categories. The **pessimistic** procedure compares an alternative with $b_{k-1}, b_{k-2,} \ldots, b_{1,}$ from the best to the worst profile and, then, assigns it to the best category so that $a_i S b_h$. In other words, after a series of comparisons, a_i is assigned to the first category, from the best to the worse, for which $\sigma(a_i, b_h) \geq \lambda$.

The **optimistic** procedure compares an alternative first with b_1, then with b_2, …, b_{k-1}, from the worst to the best profile, then assigns it to the worst so that $b_h S a_i$ and - $(a_i S b_h)$, where $\sigma(a_i, b_h) < \lambda$.

4 Application of VICA to Electre TRI

We shall assume that the group has established common criteria in order to evaluate the alternatives. We must call attention to the importance of this step. The set of goals or basic points of view must not only reflect the group values but also be subject to operationalisation. In other words, the information must be fully available as the measures used to maximise or minimise. Roy and Bouyssou (1993) have identified three further necessary requirements for a coherent family of criteria: exhaustivity, non-redundancy and cohesion.

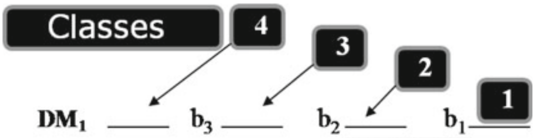
We shall assume that the group shares categories, alternatives and profile performance ($g_j(a_i)$ and $g_j(b_h)$). Parameters q_j, p_j, v_j, and the cutting level λ are also shared, although these may be altered during the process.

Parameters reflecting the relative importance of the criteria (w_j, $j = 1, 2, \ldots, t$) are defined as being a function of individual preferences and can be elicited directly, using techniques as proposed by Figueira and Roy (2002), or inferred from examples, as in the approach suggested by Mousseau and Slowinski (1998).

Figure 1 presents an example of the Electre TRI method using structured matrices as presented on a spreadsheet. This structure facilitates the calculation of Δ_j; $c_j(a_i, b_h)$; $C(a_i, b_h)$; $D_j(a_i, b_h)$ and $\sigma(a_i, b_h)$ as well as aiding in the use of visuals, the comparison of results and the search for a group decision.

Fig. 1 Matrix comparing (a_i, b_h) by criterion

	g_j			
	b_{K-1}	b_{K-2}	...	b_1
a_1				
a_2				
⋮				
a_n				
q				
p				
v				
w				



DM_1	b_3	b_2	b_1
a_1	0.00	0.20	1.00
a_2	0.36	0.70	1.00
a_3	0.60	0.80	0.90
a_4	0.40	0.64	1.00
a_5	0.40	1.00	1.00

Fig. 2 Example of the interpretation of a result

4.1 Presentation of Results

VICA-Electre TRI can represent the evaluation of each individual (d) by his/her credibility indices which, in this paper, are referred to as the Matrix of Individual Credibilities, containing $\sigma^d(a_i, b_h)$, or by the simple result DM_d, $d = 1, 2, \ldots, M$.

The elements of the credibility matrix, with values equal to or greater than the threshold cut and which imply the outranking relation $a_i S b_h$, can be identified by numbers and colours/patterns associated with the person who created them. Thus, each member of the group has an evaluation, a result (DM_d) identified with a configuration which characterises them and can take on differing forms/contours as they revise or alter their preferences.

As an example, Fig. 2 shows the display for the DM_1 set of preferences. Their importance coefficients (w_j, $j = 1, 2, \ldots, t$) coincide with the performance evaluations (a_i versus b_h) for each criterion j, indicating concordance and discordance indices and the credibility for $a_i S b_h$ according to the individual "DM_1". These credibility indices and the cut threshold established by the group determine the allocations of the alternatives into the pre-determined classes, according to the preference of this DM. In the model, the pessimistic Electre-TRI procedure was used. The results with $\lambda = 0.6$ can easily be displayed in a visual similar to that of Fig. 2 which illustrates that a_1 is allocated to class 2, a_2, a_4 and a_5 to class 3 while a_3 is allocated to class 4.

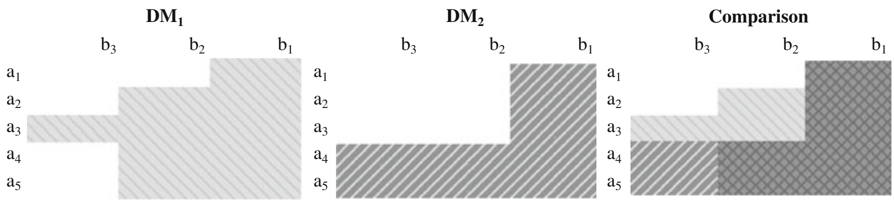


Fig. 3 Comparison of the results for two individuals

The results of the allocations of a_i in class K_h according to $\mathrm{DM}_d(a_i \xrightarrow{d} K_h)$ also appear in the proposed model in the form of a matrix column$X^d = \left[x_i^d\right]$. A $R^d = \left[r_{ih}^d\right]$ $(n.k)$ matrix must be constructed as the first step towards obtainingX^d when following the pessimistic Electre TRI procedure, as follows.

$$r_{i1}^d = 1; r_{i(h+1)}^d = \begin{cases} 1 \; if \; a_i S b_h \Leftrightarrow \sigma^d(a_i, b_h) \geq \lambda \\ 0 \; otherwise \end{cases} ; \quad h = 1, \ldots, k-1$$

$$x_i^d = \sum\nolimits_{h=1}^{k} r_{ih}^d$$

Presenting results in the form of a matrix column permits ordering and filtering functions on spreadsheets, easily leading to numerical results by category or by alternative, according to each DM, in a clear and organised fashion.

Diagrams (visuals) allow the team to pairwise compare results and immediately see any alterations due to changes in DM's opinions. This occurs by comparing the matrix elements $\sigma^d(a_i, b_h)$, which compose the results (DM_d) of each DM in combination with tools for conditional formatting that spreadsheets provide.

Comparisons of results identify two types of agreement and two types of disagreement. In agreement **type 1**, both results show credibility indices equal to or greater than the threshold, thus $a_i S b_{h,.}$. Agreement **type 2** implies that both $\sigma^d(a_i, b_h) < \lambda$, thus $\neg\ (a_i S b_h)$. The disagreement is **type 1** when $a_i S b_h$ for individual 1 and $\neg\ (a_i S b_h)$ for individual 2, while disagreement **type 2** exists when $\neg\ (a_i S b_h)$ for person 1 and $a_i S b_h$ for person 2.

The display of VICA-Electre TRI utilises the types of agreement and disagreement to illustrate the similarities and differences between results of two DMs. The display uses diagrams (comparative matrices) in which the characteristics such as colours/patterns and numbers are associated with the agreement/disagreement between the individual elements.

Disagreement **type 1** is indicated with the colours/pattern and number associated with person 1 and **type 2** is associated with the individual 2. Type 1 agreement is displayed by some combination of colours/patterns which seems to reflect the sum of the characteristics of both persons. A **type 2** agreement would be indicated by the absence of filling. Figure 3 illustrates a comparison of results for two decision makers, DM_1 and DM_2. A simple look at the **comparison** matrix clearly demonstrates that $a_1$is allocated to class 2. Although the DMs agree that $a_2$is at least in class 2, DM_1 places $a_2$in class 3 while DM_2 places it firmly in class 2. The disagreement is obviously greater for alternative a_3: both agree that a$_3$is at least in class 2 but DM$_1$places it two classes above, in class 4, while DM$_2$places it in class 2. Both allocate $a_4$and a_5 at least to class 3 while DM$_2$ places them in class 4 but DM$_1$in class 3. Faced with this disagreement, the DMs can either classify the alternatives at the lowest common category, that is, a_1, a_2 and a_3 are allocated to class 2, and $a_4$and a_5 are allocated to class 3, or revise their parameters, as explained in the next sub-sections, trying to achieve a less disagreeing situation. Naturally, the classification of other group members has to be considered as well.

This comparison can also be used to immediately and visually analyse sensitivity for each DM. To that end one must compare the results of DM$_d$ and DM$_{d'}$, where DM$_{d'}$ illustrates the results of DM$_d$ after modification of the parameters under study. The results of an individual can also be compared to the aggregate results of a group.

The visuals (diagrams, pictures and tables) delineated in this Section (and also in the next sub-sections) are helpful on presenting the results (that can change over time) do the DMs during the building consensus process, however some analytical tools must be used in order to inform the DMs about the need for changing opinions (parameters) and the impact of those changes on the results. The next sub sections will focus on the development of those analytical tools. The visual and the analytical tools can be used in an unstructured manner by the participants of the decision group, but in Sect. 4.2.2 a flowchart of a possible group process towards consensus is presented. This is an admissible structure for the group process. Other structured group processes could be built with the same tools (visual and analytical).

4.2 In Search of one Solution for the Group: the G-consensus

We shall designate by G-consensus the result obtained by a significant subgroup of members composed of a representative majority of the whole, capable of assigning each alternative into one of two classes. M will represent the number of group members and $G \in \mathrm{N} : M/2 < G \leq M$ the necessary majority as defined by the group as indicating G-consensus. If we designate:

$$V(a_i, b_h) = \sum_{d=1}^{M} v^d(a_i, b_h)$$

Then:

$V(a_i, b_h) \geq G \Rightarrow$ G-**consensus** for $a_i S b_h$
$V(a_i, b_h) \leq M - G \Rightarrow$ G-**consensus** for $\neg(a_i S b_h)$
$M - G < V(a_i, b_h) < G \Rightarrow$ **Divergence (lack of consensus)** for $a_i S b_h$ and for $\neg(a_i S b_h)$

G-consensus exists for the assignment of an alternative in the absence of divergence (G elements of the group agree that an alternative outranks the profile or, the opposite, G elements of the group agree that an alternative does not outrank the profile). That is, if: $V(a_i, b_h) \geq G$ or$V(a_i, b_h) \leq M - G, b_h \in B = \{b_1, b_2, \ldots, b_{k-1}\}$. G-consensus for the set A of alternatives exists if it exists for $\forall a_i \in A$.

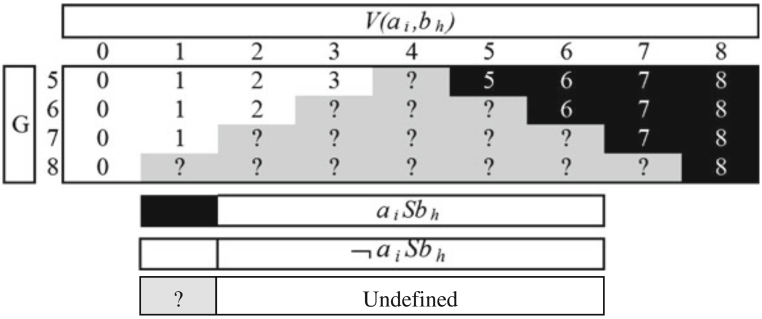


Fig. 4 Possible $V(a_i, b_h)$ results for a_iSb_h

The results of assigning a_i to category K_h according to group ($a_i \xrightarrow{G} K_h$) are demonstrated through a $V(a_i, b_h)$ matrix in which distinctive formats highlight the comparisons leading to a_iSb_h, and those that lead to $\neg(a_iSb_h)$.

Just as happens with individual results, group results can be presented in a table column where each cell has the number of the category associated with each alternative a_i, when G-consensus exists for a_i. Without G-consensus, however, there is divergence and a final assignment is impossible ($a_i \xrightarrow{G} ?$). If M is an odd number, $G = 0.5\ (M+1)$ immediately results in consensus. However, as G approaches M, the chance of divergence increases.

Spreadsheets are an extremely flexible instrument for analysing sensitivity as a result of changes in multiple parameters while they further offer immediate visualisation of results. Figure 4 exemplifies a case in which $M = 8$ and $G \in \{5, 6, 7, 8\}$. For each possible result of $V(a_i, b_h)$, the matrix identifies the existence of a G-consensus for a_iSb_h or for $\neg(a_iSb_h)$.

As our interest is in identifying the factors capable of contributing to consensus or divergence for a group solution, their causes must be analysed. Any of the comparisons between the credibility indices $\sigma^d(a_i, b_h)$, $d = 1, 2, \ldots, M$ and $h = 1, 2, \ldots, k-1$ may lead to divergence. Therefore, an analysis of the factors contributing to consensus is recommended.

In a case in which there are just two individuals, DM_1 and DM_2, whose credibility indices are $\sigma^1(a_i, b_h) = x$ and $\sigma^2(a_i, b_h) = y$, respectively, (in order to simplify the notation) and the binary variable χ suggests the existence of consensus ($\chi = 1$) or not ($\chi = 0$) as a function of the values of x, y and λ, consensus will exist between the credibility indices of DM_1 and DM_2, $\chi^{1,2}(a_i, b_h) = 1$, if $V(a_i, b_h) = 2$ or $V(a_i, b_h) = 0$. Indeed, we have:

If $(x \geq \lambda, y \geq \lambda) \Longleftrightarrow (x - \lambda \geq 0, y - \lambda \geq 0) \Rightarrow V(a_i, b_h) = 2,\ \chi = 1$
Or if $(x < \lambda, y < \lambda) \Longleftrightarrow (x - \lambda < 0, y - \lambda < 0) \Rightarrow V(a_i, b_h) = 0,\ \chi = 1$
Otherwise, $V(a_i, b_h) = 1$ and $\chi = 0$.
That is, $((x - \lambda) \cdot (y - \lambda) \geq 0) \Longleftrightarrow \chi = 1$.
Concluding, $((\sigma^1(a_i, b_h) - \lambda) \cdot (\sigma^2(a_i, b_h) - \lambda) \geq 0) \Longleftrightarrow \chi = 1$.

Consensus depends not only on the proximity of the credibility indices but also on the threshold cut. If, for illustrative purposes, we define λ as 0, there will be no divergence and any alternative will be allocated in the best class, since x and $y \in [0, 1]$.

4.2.1 Revising Relative Importance of Criteria

Let us now analyze how χ relates with the difference between the credibility indices of the DMs. The indices are equal to $C(a_i, b_h)$ if we consider that q_j and p_j are shared and the v_j are inactive (non-existent or sufficiently large relative to the performance of the alternative). Thus, for the analysis we can consider the difference between the general concordance of DM$_1$and DM$_2$:

$$C^{1\to 2}(a_i, b_h) = C^1(a_i, b_h) - C^2(a_i, b_h).$$

Assuming $\sum_{j=1}^{t} w_j = 1$:

$$C^{1\to 2}(a_i, b_h) = \sum_{j=1}^{t} w_j^1 c_j(a_i, b_h) - \sum_{j=1}^{t} w_j^2 c_j(a_i, b_h) = \sum_{j=1}^{t} \left(w_j^1 - w_j^2\right) c_j(a_i, b_h)$$

and similarly

$$C^{2\to 1}(a_i, b_h) = \sum_{j=1}^{t} \left(w_j^2 - w_j^1\right) c_j(a_i, b_h).$$

It can be observed that, if the veto effect is ignored, the shared evaluation of the alternatives and the profiles lead to a situation in which the difference between indices $\sigma(a_i, b_h)$ is a direct function of the differences between the importance levels of the criteria.

Taking into consideration the v_j (active and defined by the group) as well as the other thresholds, we can analyse the difference between indices $\sigma(a_i, b_h)$ of DM$_1$and DM$_2$, now under the influence of the veto effect (E):

$$\sigma^{1\to 2}(a_i, b_h) = \left[\sum_{j=1}^{t} w_j^1 c_j(a_i, b_h)\right] E^1 - \left[\sum_{j=1}^{t} w_j^2 c_j(a_i, b_h)\right] E^2$$

$$\begin{aligned}\sigma^{1\to 2}(a_i, b_h) &= \sum_{j=1}^{t} E^1 w_j^1 c_j(a_i, b_h) - \sum_{j=1}^{t} E^2 w_j^2 c_j(a_i, b_h)\\ &= \sum_{j=1}^{t} \left(E^1 w_j^1 - E^2 w_j^2\right) c_j(a_i, b_h)\end{aligned}$$

where $E^d = \prod_j e_j^d$, and

$$e_j^d = \begin{cases} 1 & if \;\; D_j(a_i, b_h) \leq C^d(a_i, b_h) \\ \frac{1-D_j(a_i,b_h)}{1-C^d(a_i,b_h)} & otherwise \end{cases}$$

With both $c_j(a_i, b_h)$ and v_j shared, the $D_j(a_i, b_h)$ indices will be the same for both DMs and the difference between the $\sigma\ (a_i, b_h)$ indices for DM_1 and DM_2will be a function of the differences between their $C(a_i, b_h)$ indices, which, in turn, is a function of the difference between the relative importance of the criteria. Without loss of generality, let us consider the first term of the sum in $\sigma^{1\rightarrow 2}\ (a_i, b_h)$:

$$\begin{aligned}\left(E^1 w_1^1 - E^2 w_1^2\right) &= w_1^1 \prod_{j=1}^{t} e_j^1 - w_1^2 \prod_{j=1}^{t} e_j^2 \\ &= w_1^1 \prod_{j \in \bar{F}^1} \frac{1 - D_j(a_i, b_h)}{1 - C^1(a_i, b_h)} - w_1^2 \prod_{j \in \bar{F}^2} \frac{1 - D_j(a_i, b_h)}{1 - C^2(a_i, b_h)}\end{aligned}$$

Where:

$$\bar{F}^d = \left\{ j : D_j\ (a_i, b_h) > \sum_{r=1}^{t} w_r^d c_r\ (a_i, b_h) \right\}$$

The intersection of the sets $\bar{F}^1$ and $\bar{F}^2$ is equal to:

$$\bar{F}^1 \cap \bar{F}^2 = \left\{ j : D_j\ (a_i, b_h) > \sum_{r=1}^{t} w_r^1 c_r\ (a_i, b_h) \bigwedge D_j\ (a_i, b_h) > \sum_{r=1}^{t} w_r^2 c_r\ (a_i, b_h) \right\}$$

The differences between the sets $\bar{F}^1$ and $\bar{F}^2$ and the sets $\bar{F}^2$ and $\bar{F}^1$ depend directly on the differences between the relative importance of the criteria of both DMs:

$$\bar{F}^1 - \bar{F}^2 = \left\{ j : \sum_{r=1}^{t} w_r^2 c_r\ (a_i, b_h) \geq D_j\ (a_i, b_h) > \sum_{r=1}^{t} w_r^1 c_r\ (a_i, b_h) \right\}$$

$$\bar{F}^2 - \bar{F}^1 = \left\{ j : \sum_{r=1}^{t} w_r^1 c_r\ (a_i, b_h) \geq D_j\ (a_i, b_h) > \sum_{r=1}^{t} w_r^2 c_r\ (a_i, b_h) \right\}$$

Rewriting the previous equation in terms of these sets, and considering $\sum_{r=1}^{t} w_r^2 c_r\ (a_i, b_h) \geq \sum_{r=1}^{t} w_r^1 c_r\ (a_i, b_h)$, which implies $\bar{F}^2 - \bar{F}^1 = \emptyset$ and $\bar{F}^1 \cap \bar{F}^2 = \bar{F}^2$ (but it is possible that $\bar{F}^1 - \bar{F}^2 \neq \emptyset$), we obtain:

$$\left(E^1 w_1^1 - E^2 w_1^2\right) = \prod_{j \in \bar{F}^2} \left[1 - D_j(a_i, b_h)\right] \left[w_1^1 \prod_{j \in \bar{F}^2} \left(\frac{1}{1 - \sum_{r=1}^{t} w_r^1 c_r\left(a_i, b_h\right)} \right) \right.$$
$$\left. \prod_{j \in \bar{F}^1 - \bar{F}^2} \left(\frac{1 - D_j(a_i, b_h)}{1 - \sum_{r=1}^{t} w_r^1 c_r\left(a_i, b_h\right)} \right) - w_1^2 \prod_{j \in \bar{F}^2} \frac{1}{1 - \sum_{r=1}^{t} w_r^2 c_r\left(a_i, b_h\right)} \right]$$

Let us now assume that through the concession process, w^1 is changed to be closer to w^2. In this case, one can see from the definitions that some criterion indexes that belonged to the set $\bar{F}^1$ and do not belong to the set $\bar{F}^2$ will leave the set $\bar{F}^1$.

If $\sum_{r=1}^{t} w_r^1 c_r\left(a_i, b_h\right) \geq \sum_{r=1}^{t} w_r^2 c_r\left(a_i, b_h\right)$ then $\bar{F}^1 - \bar{F}^2 = \emptyset$, $\bar{F}^1 \cap \bar{F}^2 = \bar{F}^1$ and it is possible that $\bar{F}^2 - \bar{F}^1 \neq \emptyset$. That is:

$$\left(E^1 w_1^1 - E^2 w_1^2\right) = \prod_{j \in \bar{F}^1} \left[1 - D_j(a_i, b_h)\right] \left[w_1^1 \prod_{j \in \bar{F}^1} \left(\frac{1}{1 - \sum_{r=1}^{t} w_r^1 c_r\left(a_i, b_h\right)} \right) \right.$$
$$\left. - w_1^2 \prod_{j \in \bar{F}^1} \frac{1}{1 - \sum_{r=1}^{t} w_r^2 c_r\left(a_i, b_h\right)} \prod_{j \in \bar{F}^2 - \bar{F}^1} \left(\frac{1 - D_j(a_i, b_h)}{1 - \sum_{r=1}^{t} w_r^1 c_r\left(a_i, b_h\right)} \right) \right]$$

If through the concession process, w^1 is changed to be closer to w^2, some criterion indexes that belong to the set set $\bar{F}^2$ and did not belong to the set $\bar{F}^1$ will join the set $\bar{F}^1$ as the vector w^1 gets closer to w^2(maintaining unchanged all the other parameters).

In the concessions process, as shown, $\bar{F}^1$ will tend to converge to $\bar{F}^2$. If in the concessions process the relative importance of the criteria of both DMs have been already changed and are close enough for $\bar{F}^1 = \bar{F}^2$, we obtain:

$$\left(E^1 w_1^1 - E^2 w_1^2\right) = \prod_{j \in \bar{F}^2} \left[1 - D_j(a_i, b_h)\right]$$
$$\left[w_1^1 \prod_{j \in \bar{F}^2} \frac{1}{1 - \sum_{r=1}^{t} w_r^1 c_r\left(a_i, b_h\right)} - w_1^2 \prod_{j \in \bar{F}^2} \frac{1}{1 - \sum_{r=1}^{t} w_r^2 c_r\left(a_i, b_h\right)} \right]$$

This difference is a monotonous growing function with the differences between the parameters of relative importance, thus it is possible to achieve consensus by approximating the DMs' relative importance parameters. It's important to notice that, for agreement between DMs, the modifications required don't need to make the credibility indexes equal. They just need to be enough for:

$$\left[\sigma^1\left(a_i, b_h\right) - \lambda\right] \cdot \left[\sigma^2\left(a_i, b_h\right) - \lambda\right] \geq 0.$$

However, if the veto thresholds of the DMs are different, the difference between the $\sigma(a_i, b_h)$ indices will not merely be a function of the difference between their $C(a_i, b_h)$ agreements (and, as a result, the difference between the w_j). It will also be a function of the difference between the discordance indices $D_j(a_i, b_h)$ which, in turn, are a function of the differences between the v_j (with the other shared thresholds held constant). Consequently, achieving consensus by just approximating the DMs' relative importance parameters is not assured.

Therefore, without the veto effect, or with shared v_j, reducing the difference between the importance parameters of the criteria may very well approximate the credibility indices and achieve consensus. Within the model this can be obtained on a pair by pair basis, in which person o seeks to approach person p and vice versa, via concessions. Note that the threshold cut, and other parameters, is shared and kept fixed in the process. For different threshold cuts, the needed change on the relative importance parameters may be different in order to achieve consensus.

In the event that $W^d = \left[w_j^d\right]$ is the importance vector of criteria j according to person d, preference changes can be undertaken through α concessions between people:

$$W^{o\to p} = W^o - \alpha_{o\to p}(W^o - W^p),$$

and

$$W^{p\to o} = W^p - \alpha_{p\to o}(W^p - W^o)$$

4.2.2 Consensus Building

The Excel "What if" capacity can help groups in their search for collective solutions by means of its infinite number of possible scenarios for analysing the variation of individual and shared parameters.

The "Data table" tool of Excel can explore the results for a function: the number of alternatives in G-consensus for combinations of values assigned to G and λ, for example. It is also possible to create scenarios with different parameter values and verify the effect on some results, for example: individual allocations, agreements and divergences in the group, the number of alternatives per category, etc.

The "What if" analysis not only allows the identification of the combinations of G and λ which lead to consensus but it also permits the identification, for each of these combinations, of the assignment of alternatives in G-consensus according to the group ($a_i \overset{G}{\to} K_h$) as well as the divergences ($a_i \overset{G}{\to} ?$). In the same manner, other parameters (v_j, p_j and q_j) can be analysed, either separately or in combination.

Using the "Data table", combinations of concessions α between two people that might lead to G-consensus can be obtained and visualised, either for a single alternative or for the entire set A. In this case, minimal concessions required for $\chi = 1$ can be identified with a high level of precision, considering: bilateral concessions($\alpha_{o\to p}, \alpha_{p\to o}$); unilateral concessions based on $\mathrm{DM}_o(0, \alpha_{p\to o})$; or unilateral ones based on $\mathrm{DM}_\mathrm{p}(\alpha_{o\to p}, 0)$.

Information from these types of analyses can aid the group in several types of interaction. One possibility is that the DMs, in addition to the initial weights, establish limits in the form of $\left[\underline{W}_j^d, \overline{W}_j^d\right]$ intervals, within which they may vary. The "Solver" of Excel obtains the maximum α concessions of each DM for each of the other DMs, taking into consideration the constraints imposed by the intervals, as follows:

$$\begin{aligned}
\text{Maximise: } & \alpha_{o\to p} \\
\text{Subject to: } & \left(w_j^p - w_j^o\right)\alpha_{o\to p} \le \overline{W_j^o} - w_j^o \\
& \left(w_j^p - w_j^o\right)\alpha_{o\to p} \ge \underline{W_j^o} - w_j^o \\
& \alpha_{o\to p} \in [0, 1]
\end{aligned}$$

The information on the necessary minimum two by two concessions and on the maximum number permitted when combined enables the exploration of diverse mechanisms for viable modifications which lead to consensus. It is also possible to verify the necessary individual concessions for consensus, around what we call the closest or most central DM. In this case, the $\mathrm{DM_d}$ with the vector W^d whose greatest distance between the vectors W^d is the smallest should be identified, and then the minimum necessary concessions are calculated from each $\mathrm{DM_o}$ to that central $\mathrm{DM_p}$. These calculations can use the Euclidian distance in $\mathbb{R}^t$:

$$W^o - W^p = \sqrt{\left(w_1^o - w_1^p\right)^2 + \left(w_2^o - w_2^p\right)^2 + \cdots + \left(w_t^o - w_t^p\right)^2}$$

The VICA-Electre TRI model can be used for various types of interactions, taking into consideration the decision-making context and the group preferences. The flowchart of a possible group process towards consensus appears in Fig. 5 below.

After setting up the framework and arriving at the first individual results with one common arbitrary threshold cut, still in **Phase a**, it is defined $G = M$ and a set L of possible values for λ. In **Phase b**, using the data table tool, it is determined, for each of the values $\lambda \in L$, whether or not a G-consensus solution exists for an entire set of alternatives. If none is found, the value of G is reduced and the search repeated.

Once G-consensus solutions are found, **Phase c** begins. Herein, the choice of one of the solutions may potentially complete the process. In the absence of a solution, or if those available prove to be unsatisfactory, or in the case that the group wishes to continue the search, the process outlined in **Phase b** may be repeated.

Phase c includes the presentation of the minimum concessions necessary on the part of each DM to reach a unanimous consensus, focused on each of the solutions encountered. In order to do this, the model compares each solution with one of the individual results. Thus, the existence of DMs who **do not** need to make concessions in order to arrive at consensus are identified. The results of these DMs then come to fulfil the role of guiding the modifications of the others. The data base tool can then identify the minimal $\alpha_{o\to p}$ concessions required from $\mathrm{DM}_{o,}$ to those DM_p whose results might act as guides in the search for unanimous consensus.

If no guiding results are found, the reduction of G, combined with the variation of λ, might still identify a guiding concession in **Phase d**, in which, as long as the group concurs, the search for the **closest DM is carried out**.

After successive group interactions, especially those indicated by grey in Fig. 5, the process finishes in **Phase e** with consensus or without it.

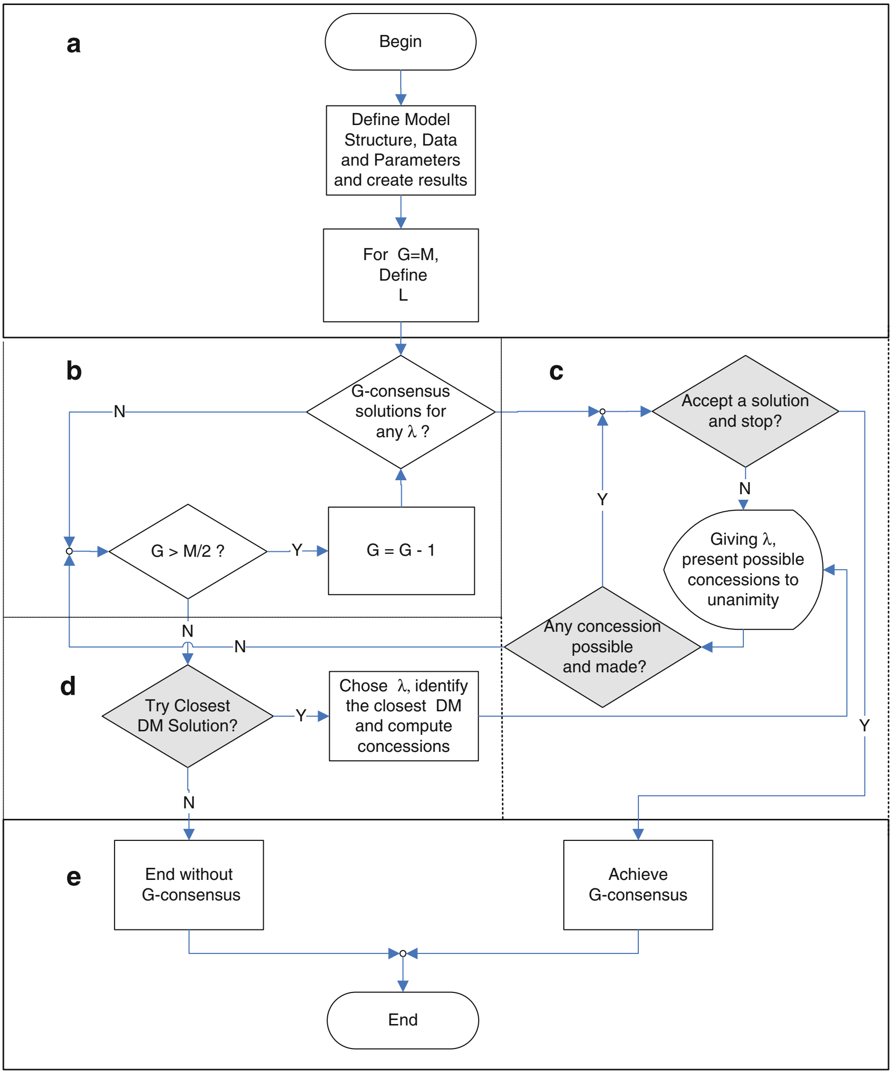


Fig. 5 Process flowchart

It is important to point out that the model can be used to organise decision-making processes in completely different manners. One such way would be to begin the process starting from two-by-two interactions and concessions until a majority grouping or a unanimous solution were achieved.

5 A Numerical Example

In order to demonstrate the model, we shall use a well-known case originally presented in Dimitras et al. (1995), with 39 companies assigned to 3 categories of bankruptcy risk. The original case was modified in Dias et al. (2000), who added one company (alternative $a_{0,}$ the better than all the others), which brought the number of companies to 40, and also increased the number of categories from 3 to 5. In this work, we present the modified version of the problem, in which we introduce 4 DMs (with their respective preference sets) and, in some of the analyses, v_j values (which had not been previously defined). The data of the case appear in "Appendix".

The analysis of the problem had as a starting point the shared evaluation of the alternatives and the identification of the beginning individual results with an arbitrary threshold λ = 0.6 (Fig. 6).

With the results of **Phase a**, a common solution was sought in **Phase b** (search for the group's starting G-consensus). The Excel "What if" and "Data table" tools, searched for the results for the function: number of alternatives in G-consensus considering $G = 4$, combined with each element of the set $L = \{0.5, 0.51, 0.52, \ldots, 1\}$.

It was confirmed that, in **Phase b**, there was no instance of any element of set L in combination with $G = 4$ with the result: the number of alternatives in G-consensus were 40 (unanimous consensus for A). Not even for $\lambda = 1$ was it possible to achieve a consensual result, since there was none for a_{28}. The divergence in a_{28} is due to the fact that DM_3 assigned a weight of 0 to criterion 5, where the performance of a_{28} is significantly less. Thus, for $\mathrm{DM}_3 \sigma(a_{28}, b_1) \geq 1$, while for all the others $\sigma(a_{28}, b_1) < 1$. In this case, direct[1] modifications in W^3or concessions $\alpha_{3\to d}$ could have aided the search for a unanimous solution.

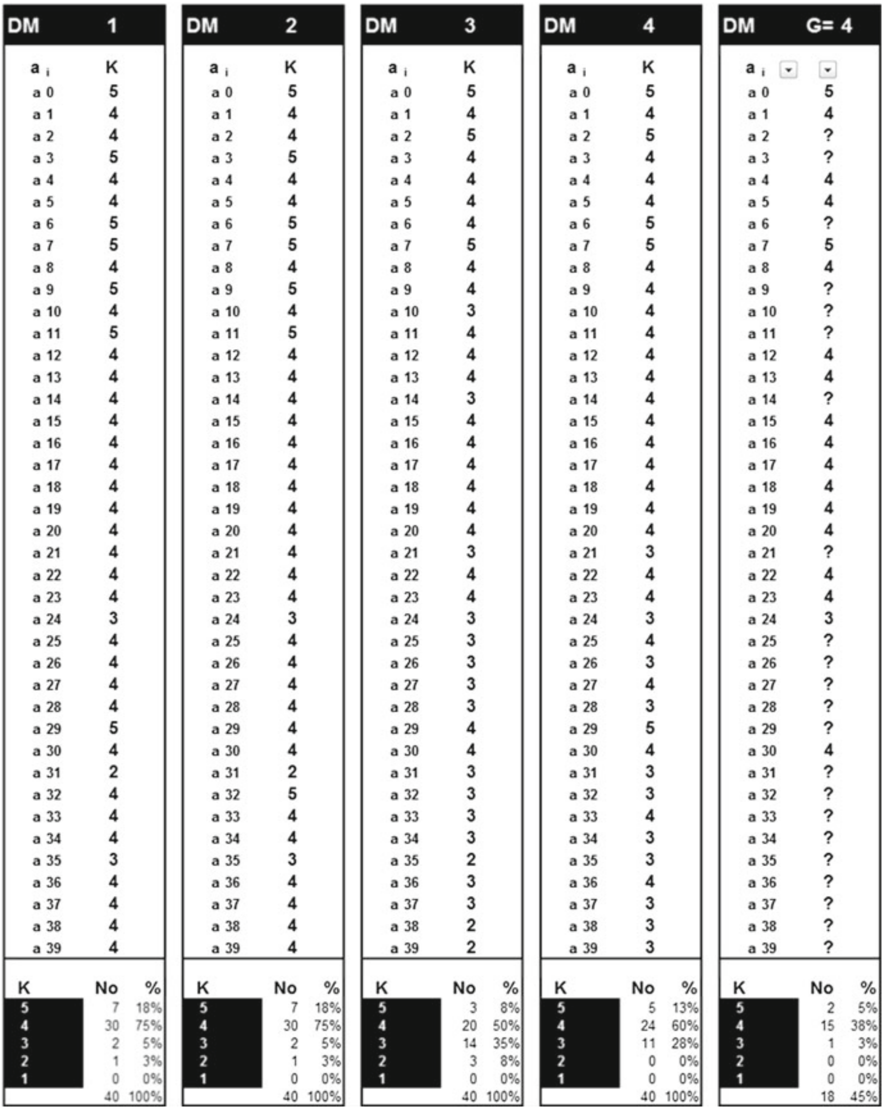
As no consensus solutions for $G = 4$ were found in Phase b, it was noted through the data table that for $G = 3$ there was a subset of values $L^3 = \{0.84, 0.85, 0.92, 0.94, 0.95, \ldots, 1\}$ for which a G-consensus existed. With the help of Excel tools, it was demonstrated for G and λ combinations, the assignment of the consensus alternatives and the divergences.

In **Phase c**, based on the comparison between individual and group results, the changes necessary from each DM in order to achieve unanimous consensus for A were calculated, taking into account the different possible values of λ.

Unanimous consensus with λ = 0.84 can be achieved through the changes of DM_1 and $\mathrm{DM}_{3.}$ Note that, in this case, the assignments resulting from DM_2 and $\mathrm{DM}_4$are equal to those of the group. Therefore, the concessions of DM_1 and DM_3 can occur either in the direction of $\mathrm{DM}_2(\alpha_{1\to 2} = 0.91$ and $\alpha_{3\to 2} = 0.72)$, as well as in the direction of $\mathrm{DM}_4(\alpha_{1\to 4} = 0.33$ and $\alpha_{3\to 4} = 0.89)$. Tables 1 and 2 present the required changes.

Let us now consider the unanimous consensus with λ = 0.92. In this case, only the assignments resulting from DM_4 are equal to those of the group consensual solution, therefore, the concessions of $\mathrm{DM}_{1,}$ DM_2 and DM_3carried out in the direction of $\mathrm{DM}_4(\alpha_{1\to 4} = 0.33; \alpha_{2\to 4} = 0.53;$ and $\alpha_{3\to 4} = 0.57)$. Tables 3, 4 and 5 demonstrate the changes.

[1] If the DM_3 assigned weight 0.001 to criterion 5 instead of zero, a unanimous solution could have been found for λ = 1. The veto in criterion 5, however, was not sufficient to eliminate the divergence, since $D_5(a_{28}, b_1) = C(a_{28}, b_1) = 1$, in this case.



a_i	DM 1 K	DM 2 K	DM 3 K	DM 4 K	DM G= 4
a 0	5	5	5	5	5
a 1	4	4	4	4	4
a 2	4	4	5	5	?
a 3	5	5	4	4	?
a 4	4	4	4	4	4
a 5	4	4	4	4	4
a 6	5	5	4	5	?
a 7	5	5	5	5	5
a 8	4	4	4	4	4
a 9	5	5	4	4	?
a 10	4	4	3	4	?
a 11	5	5	4	4	?
a 12	4	4	4	4	4
a 13	4	4	4	4	4
a 14	4	4	3	4	?
a 15	4	4	4	4	4
a 16	4	4	4	4	4
a 17	4	4	4	4	4
a 18	4	4	4	4	4
a 19	4	4	4	4	4
a 20	4	4	4	4	4
a 21	4	4	3	3	?
a 22	4	4	4	4	4
a 23	4	4	4	4	4
a 24	3	3	3	3	3
a 25	4	4	3	4	?
a 26	4	4	3	3	?
a 27	4	4	3	4	?
a 28	4	4	3	3	?
a 29	5	4	4	5	?
a 30	4	4	4	4	4
a 31	2	2	3	3	?
a 32	4	5	3	3	?
a 33	4	4	3	4	?
a 34	4	4	3	3	?
a 35	3	3	2	3	?
a 36	4	4	3	4	?
a 37	4	4	3	3	?
a 38	4	4	2	3	?
a 39	4	4	2	3	?

K	DM 1 No	DM 1 %	DM 2 No	DM 2 %	DM 3 No	DM 3 %	DM 4 No	DM 4 %	G= 4 No	G= 4 %
5	7	18%	7	18%	3	8%	5	13%	2	5%
4	30	75%	30	75%	20	50%	24	60%	15	38%
3	2	5%	2	5%	14	35%	11	28%	1	3%
2	1	3%	1	3%	3	8%	0	0%	0	0%
1	0	0%	0	0%	0	0%	0	0%	0	0%
	40	100%	40	100%	40	100%	40	100%	18	45%

Fig. 6 Individual and group allocations for $G = M$ with $\lambda = 0.6$

Table 1 DM_1 changes with $\lambda = 0.84$

Changes DM_1	g_1 (%)	g_2 (%)	g_3 (%)	g_4 (%)	g_5 (%)	g_6 (%)	g_7 (%)
W_1	15.0	30.0	15.0	15.0	15.0	5.0	5.0
$W^{1\to 2}$	8.9	48.4	8.9	8.9	8.9	8.0	8.0
$W^{1\to 4}$	14.8	24.8	14.8	14.8	14.8	8.1	8.1

Table 2 DM_3 changes with $\lambda = 0.84$

Changes DM_3	g_1 (%)	g_2 (%)	g_3 (%)	g_4 (%)	g_5 (%)	g_6 (%)	g_7 (%)
W_3	14.3	14.3	14.3	0.0	0.0	28.6	28.5
$W^{3\to 2}$	10.0	40.1	10.0	6.0	6.0	14.0	14.0
$W^{3\to 4}$	14.3	14.3	14.3	12.7	12.7	15.9	15.8

Table 3 DM_1 changes with $\lambda = 0.92$

Changes DM_1	g_1 (%)	g_2 (%)	g_3 (%)	g_4 (%)	g_5 (%)	g_6 (%)	g_7 (%)
W_1	15.0	30.0	15.0	15.0	15.0	5.0	5.0
$W^{1\to 4}$	14.8	24.8	14.8	14.8	14.8	8.1	8.1

Table 4 DM_2 changes with $\lambda = 0.92$

Changes DM_2	g_1 (%)	g_2 (%)	g_3 (%)	g_4 (%)	g_5 (%)	g_6 (%)	g_7 (%)
W_2	8.3	50.2	8.3	8.3	8.3	8.3	8.3
$W^{2\to 4}$	11.5	31.2	11.5	11.5	11.5	11.5	11.5

Table 5 DM_3changes with $\lambda = 0.92$

Changes DM_3	g_1 (%)	g_2 (%)	g_3 (%)	g_4 (%)	g_5 (%)	g_6 (%)	g_7 (%)
W_3	14.3	14.3	14.3	0.0	0.0	28.6	28.5
$W^{3\to 4}$	14.3	14.3	14.3	8.1	8.1	20.4	20.4

The changes in DM_3 are sufficient to achieve unanimous consensus with $\lambda = 1$: the assignments resulting from DM_1, DM_2 and DM_4 are equal to those of the group, thus the concessions of DM_3 may be made by any of the other DMs.

The choice of the starting point for efficiently guiding the concessions can be made based on the distance between the W^d vectors. Table 6 presents the Euclidian distances in $\mathbb{R}^7$ calculated pair by pair between W^d vectors. DM_4 is seen to present the weighted vector with the *Minimax* distance between all the DMs' vectors and this is the DM with the weighted vector specifically closest to DM_3.

The modifications of DM_3, with $\alpha_{3\to 4} = 0.0001$ and $\lambda = 1$ appear in Table 7.

Thresholds 0.96, 0.97, 0.98 and 0.99 also lead to unanimous consensus when the above-mentioned change is taken into account.

Depending on the context, the identified changes may be made either directly by the DMs or automatically, when viable, in other words, in the event that they will lead to a weight vector whose components fall within pre-established $\left[\underline{W}_j^d, \overline{W}_j^d\right]$ limits.

Note that in examples presented in this section the required changes in weights are 'drastic' in some cases and the threshold cut is higher than usual. This can lead to the DMs' rejection of the required changes. The method does not guarantee that a

Table 6 Distance between weight vectors

	DM_1	DM_2	DM_3	DM_4	Max
DM_1	–	0.247	0.425	0.205	0.425
DM_2	0.247	–	0.482	0.388	0.482
DM_3	0.425	0.482	–	0.286	0.482
DM_4	0.205	0.388	0.286	–	0.388
Minimax distance between vectors					0.388

Table 7 DM_3changes with $\lambda = 1$

Changes DM_3	g_1 (%)	g_2 (%)	g_3 (%)	g_4 (%)	g_5 (%)	g_6 (%)	g_7 (%)
W_3	14.3	14.3	14.3	0.0	0.0	28.6	28.5
$W^{3\to4}$	14.300	14.300	14.300	0.001	0.001	28.599	28.499

G-consensus will be achieved on every situation, however due to the characteristics of the Electre TRI, the consequence of not achieving G-Consensus is that, on output, the alternatives are put in lower classes.

6 Future Developments and Anticipated Contributions

This paper has presented the principles of the VICA methodology intended to aid cooperating groups in the search for consensus. Tools for the interactive comparison of opinions of individual group members through visuals help reduce the inherit complexity of decision making among groups using multiple criteria. VICA-Electre TRI was introduced and tested by applying it to a well-known quantitative case. The process proved useful and mostly pointed out its flexibility and potential for use in diverse problems. In the future, the adaption and use of the tool might be tested in an experiment in which DMs and analysts might ascertain the efficiency of the tool in different, comparable processes. We are preparing a field test of the VICA-Electre TRI on the evaluation of the seller team of a Brazilian company which supplies independent retailers with beer and soft drinks. The sellers are quarterly evaluated by a group of managers according to their performance on several aspects. A bonus salary is associated with this evaluation. The principles established here could be applied to different problems and in varying contexts while using other MCDA methods.

This study supports previous ones to the extent that a solid MCDA method, in conjunction with a familiar and accessible tool such as Excel, can contribute to the search for transparent, justifiable and collectively constructed consensus. It is our hope that this project will contribute to this fertile area of study about the knowledge and implementation of MCDA methods to aid real life cooperating groups involved in multi-criteria decision making.

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Appendix

	g_1	g_2	g_3	g_4	g_5	g_6	g_7
a_0	35.80	67.00	19.70	0.00	0.00	5.00	4.00
a_1	16.40	14.50	59.80	7.50	5.20	5.00	3.00
a_2	35.80	24.00	64.90	2.10	4.50	5.00	4.00
a_3	20.60	61.70	75.70	3.60	8.00	5.00	3.00
a_4	11.50	17.10	57.10	4.20	3.70	5.00	2.00
a_5	22.40	25.10	49.80	5.00	7.90	5.00	3.00
a_6	23.90	34.50	48.90	2.50	8.00	5.00	3.00
a_7	29.90	44.00	57.80	1.70	2.50	5.00	4.00
a_8	8.70	5.40	27.40	4.50	4.50	5.00	2.00
a_9	25.70	29.70	46.80	4.60	3.70	4.00	2.00
a_{10}	21.20	24.60	64.80	3.60	8.00	4.00	2.00
a_{11}	18.30	31.60	69.30	2.80	3.00	4.00	3.00
a_{12}	20.70	19.30	19.70	2.20	4.00	4.00	2.00
a_{13}	9.90	3.50	53.10	8.50	5.30	4.00	2.00
a_{14}	10.40	9.30	80.90	1.40	4.10	4.00	2.00
a_{15}	17.70	19.80	52.80	7.90	6.10	4.00	4.00
a_{16}	14.80	15.90	27.90	5.40	1.80	4.00	2.00
a_{17}	16.00	14.70	53.50	6.80	3.80	4.00	4.00
a_{18}	11.70	10.00	42.10	12.20	4.30	5.00	2.00
a_{19}	11.00	4.20	60.80	6.20	4.80	4.00	2.00
a_{20}	15.50	8.50	56.20	5.50	1.80	4.00	2.00
a_{21}	13.20	9.10	74.10	6.40	5.00	2.00	2.00
a_{22}	9.10	4.10	44.80	3.30	10.40	3.00	4.00
a_{23}	12.90	1.90	65.00	14.00	7.50	4.00	3.00
a_{24}	5.90	-27.70	77.40	16.60	12.70	3.00	2.00
a_{25}	16.90	12.40	60.10	5.60	5.60	3.00	2.00
a_{26}	16.70	13.10	73.50	11.90	4.10	2.00	2.00
a_{27}	14.60	9.70	59.50	6.70	5.60	2.00	2.00
a_{28}	5.10	4.90	28.90	2.50	46.00	2.00	2.00
a_{29}	24.40	22.30	32.80	3.30	5.00	3.00	4.00
a_{30}	29.50	8.60	41.80	5.20	6.40	2.00	3.00
a_{31}	7.30	-64.50	67.50	30.10	8.70	3.00	3.00
a_{32}	23.70	31.90	63.60	12.10	10.20	3.00	2.00
a_{33}	18.90	13.50	74.50	12.00	8.40	3.00	3.00
a_{34}	13.90	3.30	78.70	14.70	10.10	2.00	2.00
a_{35}	-13.30	-31.10	63.00	21.20	23.10	2.00	1.00
a_{36}	6.20	-3.20	46.10	4.80	10.50	2.00	1.00
a_{37}	4.80	-3.30	71.10	8.60	11.60	2.00	2.00
a_{38}	0.10	-9.60	42.50	12.90	12.40	1.00	1.00
a_{39}	13.60	9.10	76.00	17.10	10.30	1.00	1.00

	g_1	g_2	g_3	g_4	g_5	g_6	g_7
q_j	1	4	1	1	0	0	0
p_j	2	6	3	2	3	0	0
v_j	NA	NA	NA	NA	NA	NA	NA
w_j^1	0.150	0.300	0.150	0.150	0.150	0.050	0.050
w_j^2	0.083	0.502	0.083	0.083	0.083	0.083	0.083
w_j^3	0.143	0.143	0.143	0.000	0.000	0.286	0.285
w_j^4	0.143	0.143	0.143	0.143	0.143	0.143	0.143

	C_5	C_4	C_3	C_2	C_1
g_1	25	8	0	−10	NA
g_2	30	−20	−40	−60	NA
g_3	35	60	75	90	NA
g_4	10	18	23	28	NA
g_5	14	22	32	40	NA
g_6	5	4	2	0	NA
g_7	4	3	2	0	NA

Class	Description
C_5	Very low risk
C_4	Low risk
C_3	Medium risk
C_2	High risk
C_1	Very high risk

Criterion	Description	Goal
g_1	Earnings before interest and taxes/total assets	Max
g_2	Net income/net worth	Max
g_3	Total liabilities/total assets	Min
g_4	Interest expenses/sales	Min
g_5	General and administrative expenses/sales	Min
g_6	Managers work experience	Max
g_7	Market niche/position	Max

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