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Compatibility Analysis of Intuitionistic Fuzzy Preference Relations in Group Decision Making

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Abstract Compatibility analysis is an efficient and important tool used to measure the consensus of opinions within a given group of individuals. In this paper, we give a compatibility measure between intuitionistic preference values and a compatibility measure between intuitionistic preference relations, respectively, and study their properties. It is shown that each individual intuitionistic preference relation and the collective intuitionistic preference relation is perfectly compatible if and only if all the individual intuitionistic preference relations are perfectly compatible. Based on the compatibility measures, a consensus reaching procedure in group decision making with intuitionistic preference relations is developed, and a method for comparing intuitionistic fuzzy values is pointed out, by which the considered objects are ranked and selected. In addition, we extend the developed measures, procedure and method to accommodate group decision making situations with interval-valued intuitionistic preference relations. Numerical analysis on our results through an illustrative example is also carried out.

Keywords Group decision making · Intuitionistic preference relations · Interval-valued intuitionistic preference relations · Compatibility measure

1 Introduction

Group decision making with preference relations has been being a hot research topic in decision making field over the last decades. Each evaluation value in a preference

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relation is given by a group member (decision maker) for expressing his/her preference degree of one object over another. The provided evaluation values may take various forms, such as non-negative real numbers taken from the closed unit interval (Orlovsky 1978) or from the 1-9 scale (Saaty 1980), interval numbers (Saaty and Vargas 1987; Xu 2004; Xu and Cai 2011), triangular fuzzy numbers (Van Laarhoven and Pedrycz 1983; Xu 2007a) and linguistic labels (Herrera et al. 1996; Xu 2005), etc. However, in the processes of cognition of things, the decision makers may not possess a precise or sufficient level of knowledge of the problem domain, due to the increasing complexity of the socio-economic environment. In such cases, they usually have some uncertainty in providing their preferences over the objects considered, which makes the results of cognitive performance exhibit the characteristics of affirmation, negation and hesitation (Xu and Cai 2010). As the evaluation values in a preference relation described previously cannot be used to completely express all the information in the problems considered, motivated by the idea of Atanassov's intuitionistic fuzzy set (Atanassov 1986), Szmidt and Kacprzyk (2002) used the membership degrees and the hesitation degrees to depict the decision makers' evaluation values in group decision making. All the evaluation values of each decision maker construct an intuitionistic preference relation, which is composed of a preference matrix and a matrix of intuitionistic fuzzy indices. To derive the final decision result, they aggregated all the individual intuitionistic preference relations into a social fuzzy preference relation through the fuzzy majority rule equated with a fuzzy linguistic quantifier (Yager 1988, 1993). In another paper (Szmidt and Kacprzyk 2003), Szmidt and Kacprzyk investigated group decision making problems where individual testimonies are individual intuitionistic preference relations, and pointed out that intuitionistic preference relations that in addition to a membership degree from the closed unit interval include a hesitation margin, can better reflect the very imprecision of individuals' preferences during the consensus-reaching process.

To describe the fuzzy characters of things more detailedly and comprehensively, Xu (2007b) gave a concise concept of intuitionistic preference relation, which is represented by a matrix, each of the elements is composed of a membership degree, a non-membership degree and a hesitation degree, respectively. Xu and Yager (2006) used some intuitionistic fuzzy aggregation operators to fuse all individual intuitionistic preference relations, based on which a group decision making approach was developed. After improving Szmidt and Kacprzyk (2004)'s similarity measure, Xu and Yager (2009) gave an interactive procedure for the evaluation of agreement within a group based on intuitionistic preference relations, and further extended it to the interval-valued intuitionistic fuzzy set theory. For the situations where the membership degrees, the non-membership degrees and the hesitation degrees, provided by the decision makers may not be exact numerical values, but the value ranges can be obtained, Xu and Chen (2007) defined the concept of interval-valued intuitionistic preference relation, and discussed its relationships with the intuitionistic preference relation and the traditional fuzzy preference relation. On the basis of the intuitionistic fuzzy averaging aggregation operator and intuitionistic fuzzy hybrid aggregation operator, they gave a method for group decision making with interval-valued intuitionistic preference relations, and applied it to the partner selection of an enterprise in supply chain management. Recently, Xu et al. (2011) developed two estimation algorithms of intuitionistic preferences. The first algorithm is used to estimate the missing elements using only the known preference values in an acceptable incomplete intuitionistic fuzzy preference relation with the least judgments. The second one is given for the estimation of missing elements of the acceptable incomplete intuitionistic fuzzy preference relations with more known judgments.

Consensus plays an important role in group decision making. Compatibility analysis is an efficient and important tool used to measure the consensus of opinions within a given group of individuals. In this paper, we shall investigate the compatibility of intuitionistic preference relations, propose some novel compatibility measures of intuitionistic fuzzy information, and use them to put forward a consensus reaching procedure in group decision making with intuitionistic preference relations. A method for ranking intuitionistic fuzzy values is also given, and by which the considered objects are ranked and selected. Moreover, we extend the developed measures and procedure to accommodate group decision making situations with interval-valued intuitionistic preference relations, and carry out a numerical analysis of our results through an illustrative example.

2 Compatibility Analysis of Intuitionistic Preference Relations

Let's consider a group decision making problem, in which there is a set of n objects, $X = \{x_1, x_2, \ldots, x_n\}$, and a set of m decision makers, $E = \{e_1, e_2, \ldots, e_m\}$. Each decision maker d_k has own importance weight λ_k, and let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$ be the weight vector of the decision makers $e_k (k = 1, 2, \ldots, m)$. The decision maker e_k is invited to provide his/her evaluation values $r_{ij}^{(k)} = (\mu_{ij}^{(k)}, v_{ij}^{(k)}, \pi_{ij}^{(k)})(i, j = 1, 2, \ldots, n)$ by comparing all pairs of objects, $(x_i, x_j), i, j = 1, 2, \ldots, n$. Each evaluation value $r_{ij}^{(k)} = (\mu_{ij}^{(k)}, v_{ij}^{(k)}, \pi_{ij}^{(k)})$ is an intuitionistic fuzzy value (Xu and Yager 2006; Xu 2007c), where $\mu_{ij}^{(k)}$ denotes the certainty degree to which the object x_i is preferred to the object x_j, and $v_{ij}^{(k)}$ indicates the certainty degree to which the object x_i is not preferred to the object x_j, and $\pi_{ij}^{(k)}$ means the indeterminacy degree or a hesitation degree, with the conditions:

$$\mu_{ij}^{(k)},\ v_{ij}^{(k)} \in [0, 1], \quad \mu_{ij}^{(k)} + v_{ij}^{(k)} \leq 1, \quad \mu_{ij}^{(k)} = v_{ji}^{(k)}, \quad \mu_{ji}^{(k)} = v_{ij}^{(k)},$$
$$\mu_{ii}^{(k)} = v_{ii}^{(k)} = 0.5, \quad \pi_{ij}^{(k)} = 1 - \mu_{ij}^{(k)} - v_{ij}^{(k)}, \text{for all} \quad i, j = 1, 2, \ldots, n \quad (1)$$

All the preference values $r_{ij}^{(k)}(i, j = 1, 2, \ldots, n)$ make up of a decision matrix $R^{(k)} = (r_{ij}^{(k)})_{n\times n}$, which is called an intuitionistic preference relation (Xu 2007b) on the set X.

In what follows, we give a compatibility measure between the evaluation values:

Definition 1 Let $r_{ij}^{(k)} = (\mu_{ij}^{(k)}, v_{ij}^{(k)}, \pi_{ij}^{(k)})$ and $r_{ij}^{(l)} = (\mu_{ij}^{(l)}, v_{ij}^{(l)}, \pi_{ij}^{(l)})$ be two evaluation values, given by two decision makers e_k and e_l by comparing the pair of objects, (x_i, x_j). Then we call

$$c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right) = \frac{\mu_{ij}^{(k)}\mu_{ij}^{(l)} + v_{ij}^{(k)}v_{ij}^{(l)} + \pi_{ij}^{(k)}\pi_{ij}^{(l)}}{\max\left\{\left(\mu_{ij}^{(k)}\right)^2 + \left(v_{ij}^{(k)}\right)^2 + \left(\pi_{ij}^{(k)}\right)^2,\ \left(\mu_{ij}^{(l)}\right)^2 + \left(v_{ij}^{(l)}\right)^2 + \left(\pi_{ij}^{(l)}\right)^2\right\}} \quad (2)$$

the compatibility degree of $r_{ij}^{(k)}$ and $r_{ij}^{(l)}$.

It is clear that the larger the value of $c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right)$, the greater the compatibility degree of $r_{ij}^{(k)}$ and $r_{ij}^{(l)}$. Additionally, we have

Theorem 1 *The compatibility degree $c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right)$ derived from Eq. (2) satisfies the properties:*

(1) $0 \le c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right) \le 1$;
(2) $c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right) = 1$, *if and only if* $r_{ij}^{(k)} = r_{ij}^{(l)}$;
(3) $c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right) = c\left(r_{ij}^{(l)}, r_{ij}^{(k)}\right)$.

Proof It from Eq. (1) that $\mu_{ij}^{(k)},\ v_{ij}^{(k)}, \pi_{ij}^{(k)}, \mu_{ij}^{(l)},\ v_{ij}^{(l)}, \pi_{ij}^{(l)} \in [0, 1]$, then by Eq. (2), we get $c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right) \ge 0$. On the other hand, by using the well-known Cauchy–Schwarz inequality, we have

$$\begin{aligned}
&\mu_{ij}^{(k)}\mu_{ij}^{(l)} + v_{ij}^{(k)}v_{ij}^{(l)} + \pi_{ij}^{(k)}\pi_{ij}^{(l)} \\
&\le \sqrt{\left(\left(\mu_{ij}^{(k)}\right)^2 + \left(v_{ij}^{(k)}\right)^2 + \left(\pi_{ij}^{(k)}\right)^2\right) \cdot \left(\left(\mu_{ij}^{(l)}\right)^2 + \left(v_{ij}^{(l)}\right)^2 + \left(\pi_{ij}^{(l)}\right)^2\right)} \\
&\le \sqrt{\left(\max\left\{\left(\mu_{ij}^{(k)}\right)^2 + \left(v_{ij}^{(k)}\right)^2 + \left(\pi_{ij}^{(k)}\right)^2,\ \left(\mu_{ij}^{(l)}\right)^2 + \left(v_{ij}^{(l)}\right)^2 + \left(\pi_{ij}^{(l)}\right)^2\right\}\right)^2} \\
&= \max\left\{\left(\mu_{ij}^{(k)}\right)^2 + \left(v_{ij}^{(k)}\right)^2 + \left(\pi_{ij}^{(k)}\right)^2,\ \left(\mu_{ij}^{(l)}\right)^2 + \left(v_{ij}^{(l)}\right)^2 + \left(\pi_{ij}^{(l)}\right)^2\right\}
\end{aligned}$$

i.e.,

$$c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right) = \frac{\mu_{ij}^{(k)}\mu_{ij}^{(l)} + v_{ij}^{(k)}v_{ij}^{(l)} + \pi_{ij}^{(k)}\pi_{ij}^{(l)}}{\max\left\{\left(\mu_{ij}^{(k)}\right)^2 + \left(v_{ij}^{(k)}\right)^2 + \left(\pi_{ij}^{(k)}\right)^2,\ \left(\mu_{ij}^{(l)}\right)^2 + \left(v_{ij}^{(l)}\right)^2 + \left(\pi_{ij}^{(l)}\right)^2\right\}} \le 1$$

with equality if and only if $\mu_{ij}^{(k)} = \mu_{ij}^{(l)},\ \ v_{ij}^{(k)} = v_{ij}^{(l)},\ \ \pi_{ij}^{(k)} = \pi_{ij}^{(l)}$, i.e., $r_{ij}^{(k)} = r_{ij}^{(l)}$, which indicates both the properties Eqs.(1) and (2) hold. In addition, the property Eq. (3) can be easily proven directly from

$$c\left(r_{ij}^{(k)}, r_{ij}^{(l)}\right) = \frac{\mu_{ij}^{(k)}\mu_{ij}^{(l)} + v_{ij}^{(k)}v_{ij}^{(l)} + \pi_{ij}^{(k)}\pi_{ij}^{(l)}}{\max\left\{\left(\mu_{ij}^{(k)}\right)^2 + \left(v_{ij}^{(k)}\right)^2 + \left(\pi_{ij}^{(k)}\right)^2,\ \left(\mu_{ij}^{(l)}\right)^2 + \left(v_{ij}^{(l)}\right)^2 + \left(\pi_{ij}^{(l)}\right)^2\right\}}$$
$$= \frac{\mu_{ij}^{(l)}\mu_{ij}^{(k)} + v_{ij}^{(l)}v_{ij}^{(k)} + \pi_{ij}^{(l)}\pi_{ij}^{(k)}}{\max\left\{\left(\mu_{ij}^{(l)}\right)^2 + \left(v_{ij}^{(l)}\right)^2 + \left(\pi_{ij}^{(l)}\right)^2,\ \left(\mu_{ij}^{(k)}\right)^2 + \left(v_{ij}^{(k)}\right)^2 + \left(\pi_{ij}^{(k)}\right)^2\right\}}$$
$$= c\left(r_{ij}^{(l)}, r_{ij}^{(k)}\right)$$

In a similar way, we can define the compatibility measure between the intuitionistic preference relations:

Definition 2 Let $R^{(k)} = (r_{ij}^{(k)})_{n\times n}$ and $R^{(l)} = (r_{ij}^{(l)})_{n\times n}$ be two intuitionistic preference relations, given by two decision makers e_k and e_l by comparing all the pair of objects, $(x_i, x_j), i, j = 1, 2, \ldots, n$. Then we call

$$c\left(R^{(k)}, R^{(l)}\right) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mu_{ij}^{(k)}\mu_{ij}^{(l)} + v_{ij}^{(k)}v_{ij}^{(l)} + \pi_{ij}^{(k)}\pi_{ij}^{(l)}\right)}{\max\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\left(\mu_{ij}^{(k)}\right)^2 + \left(v_{ij}^{(k)}\right)^2 + \left(\pi_{ij}^{(k)}\right)^2\right),\ \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\left(\mu_{ij}^{(l)}\right)^2 + \left(v_{ij}^{(l)}\right)^2 + \left(\pi_{ij}^{(l)}\right)^2\right)\right\}} \quad (3)$$

the compatibility degree of $R^{(k)}$ and $R^{(l)}$. □

Clearly, the larger the value of $c\left(R^{(k)}, R^{(l)}\right)$, the greater the compatibility degree of $R^{(k)}$ and $R^{(l)}$. Similar to Theorem 1, we have the following conclusion:

Theorem 2 *The compatibility degree* $c\left(R^{(k)}, R^{(l)}\right)$ *derived from Eq.* (3) *satisfies the properties:*

(1) $0 \le c\left(R^{(k)}, R^{(l)}\right) \le 1$;
(2) $c\left(R^{(k)}, R^{(l)}\right) = 1$, *if and only if* $R^{(k)} = R^{(l)}$;
(3) $c\left(R^{(k)}, R^{(l)}\right) = c\left(R^{(l)}, R^{(k)}\right)$.

Definition 2 The intuitionistic preference relations $R^{(k)} = (r_{ij}^{(k)})_{n\times n}$ and $R^{(l)} = (r_{ij}^{(l)})_{n\times n}$ are perfectly compatible if $c\left(R^{(k)}, R^{(l)}\right) = 1$, i.e., $R^{(k)} = R^{(l)}$.

Let $R^{(k)}$, $R^{(s)}$ and $R^{(l)}$ be three intuitionistic preference relations, if $c\left(R^{(k)}, R^{(s)}\right) = 1$ and $c\left(R^{(s)}, R^{(l)}\right) = 1$, then it is clear from Theorem 2 that $c\left(R^{(k)}, R^{(l)}\right) = 1$.

Let $R^{(k)}(k = 1, 2, \ldots, m)$ be m individual intuitionistic preference relations, then their aggregation $R = \sum_{k=1}^{m}\lambda_k R^{(k)}$ is also an intuitionistic preference relation (Xu and Yager 2009), which we call the collective intuitionistic preference relation, where $R = (r_{ij})_{n\times n}, r_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$, for all $i, j = 1, 2, \ldots, n$, such that

$$\mu_{ij} = \sum_{k=1}^{m} \lambda_k \mu_{ij}^{(k)},\ v_{ij} = \sum_{k=1}^{m} \lambda_k v_{ij}^{(k)},\ \pi_{ij} = \sum_{k=1}^{m} \lambda_k \pi_{ij}^{(k)}, \quad \text{for all} \quad i, j = 1, 2, \ldots, n \tag{4}$$

By Eq. (3), we can calculate the compatibility degrees $c\left(R^{(k)}, R\right)$ $(k = 1, 2, \ldots, m)$, based on which we have

Theorem 3 *Each individual intuitionistic preference relation and the collective intuitionistic preference relation is perfectly compatible if and only if all the individual intuitionistic preference relations are perfectly compatible, i.e.,* $c\left(R^{(k)}, R\right) = 1(k = 1, 2, \ldots, m)$ *if and only if* $c\left(R^{(k)}, R^{(l)}\right) = 1$, *for all* $k, l = 1, 2, \ldots, m$.

Proof If $c\left(R^{(k)}, R\right) = 1$, for $k = 1, 2, \ldots, m$, then by Theorem 2, we know that $R^{(k)} = R$, for all $k = 1, 2, \ldots, m$, i.e., $R^{(k)} = R^{(l)}$, for all $k = 1, 2, \ldots, m$, which indicates $c\left(R^{(k)}, R^{(l)}\right) = 1$. On the other hand, if $c\left(R^{(k)}, R^{(l)}\right) = 1$, for all $k, l = 1, 2, \ldots, m$, then $R^{(k)} = R^{(l)}$, for all $k, l = 1, 2, \ldots, m$, and thus

$$R = \sum_{k=1}^{m} \lambda_k R^{(k)} = \sum_{k=1}^{m} \lambda_k R^{(l)} = R^{(l)} \sum_{k=1}^{m} \lambda_k = R^{(l)}, \quad \text{for all} \quad l = 1, 2, \ldots, m \tag{5}$$

then by Theorem 2, we have $c\left(R^{(l)}, R\right) = 1(l = 1, 2, \ldots, m)$. This completes the proof of the theorem. □

In practical applications, it is very difficult to construct the perfectly compatible intuitionistic preference relations. As a result, we define the following:

Definition 3 Let $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$ and $R^{(l)} = (r_{ij}^{(l)})_{n \times n}$ be two intuitionistic preference relations. If

$$c\left(R^{(k)}, R^{(l)}\right) \geq \delta_0 \tag{6}$$

then $R^{(k)}$ and $R^{(l)}$ are called of acceptable compatibility, where δ_0 is the threshold value of acceptable compatibility. In general, we take $\delta_0 \in [0.5, 1]$ in practical applications.

Based on the above theoretical analysis, below we develop a consensus reaching procedure in group decision making with intuitionistic preference relations:

Procedure I Step 1. For a group decision making problem, let X, E and λ be defined as before, the decision makers $e_k (k = 1, 2, \ldots, m)$ compare all pairs of alternatives in X, and construct the intuitionistic preference relations $R^{(k)} = (r_{ij}^{(k)})_{n \times n} (k = 1, 2, \ldots, m)$, with $r_{ij}^{(k)} = (\mu_{ij}^{(k)}, v_{ij}^{(k)}, \pi_{ij}^{(k)})$, for all $k = 1, 2, \ldots, m; i, j = 1, 2, \ldots, n$.

Step 2. Utilize Eq. (4) to fuse all individual intuitionistic preference relations $R^{(k)} = (r_{ij}^{(k)})_{n\times n}(k = 1, 2, \ldots, m)$ into the collective intuitionistic preference relation $R = (r_{ij})_{n\times n}$, where $r_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$, for all $i, j = 1, 2, \ldots, n$.

Step 3. Calculate the the compatibility degree $c\left(R^{(k)}, R\right)$ of each individual intuitionistic preference relation $R^{(k)}$ and the collective intuitionistic preference relation R through Eq. (3).

Step 4. Predefine the threshold value δ_0, if all $c\left(R^{(k)}, R\right) \geq \delta_0(k = 1, 2, \ldots, m)$, i.e., each individual intuitionistic preference relation and the collective intuitionistic preference relation is of acceptable compatibility, then the goup reaches an acceptable consensus, go to Step 5; otherwise, there must exist at least one k_0, such that $c\left(R^{(k_0)}, R\right) < \delta_0$. Then utilize Eq. (2) to calculate the compatibility degree $c\left(r_{ij}^{(k_0)}, r_{ij}\right)$ of each pair elements $(r_{ij}^{(k_0)}, r_{ij})$ in $R^{(k_0)}$ and R, respectively. Return the intuitionistic preference relation $R^{(k_0)}$ (together with R and some elements with the smallest compatibility degrees as a reference) to the decision maker e_{k_0} for re-evaluation, and then go to Step 2. Repeat this process until all $c\left(R^{(k)}, R\right) \geq \delta_0(k = 1, 2, \ldots, m)$ or the process will stop as the repetition times reach the maximum number predefined by the group.

Step 5. Aggregate all the elements $r_{ij}(j = 1, 2, \ldots, n)$ in ith line of $R = (r_{ij})_{n\times n}$ by using the intuitionistic fuzzy weighted averaging operator:

$$\mu_i = \frac{1}{n}\sum_{j=1}^{n}\mu_{ij}, \ \in v_i = \frac{1}{n}\sum_{j=1}^{n}v_{ij}, \ \pi_i = \frac{1}{n}\sum_{j=1}^{n}\pi_{ij}, \ i = 1, 2, \ldots, n \quad (7)$$

and get the overall preference values $r_i = (\mu_i, v_i, \pi_i)(i = 1, 2, \ldots, n)$ corresponding to the objects $x_i(i = 1, 2, \ldots, n)$.

Step 6. To rank the overall preference values $r_i(i = 1, 2, \ldots, n)$, we calculate the distance of the overall preference value r_i to both the largest intuitionistic fuzzy value $\alpha^* = (1, 0, 0)$ and the smallest intuitionistic fuzzy value $\alpha_* = (0, 1, 0)$, respectively:

$$d(r_i, \alpha^*) = \frac{1}{2}\left(|1-\mu_i| + |0-v_i| + |0-\pi_i|\right) = \frac{1}{2}\left(1-\mu_i+v_i+\pi_i\right) = 1-\mu_i$$
$$d(r_i, \alpha_*) = \frac{1}{2}\left(|0-\mu_i| + |1-v_i| + |0-\pi_i|\right) = \frac{1}{2}\left(\mu_i+1-v_i+\pi_i\right) = 1-v_i$$

and then calculate the closeness coefficient of each overall preference value:

$$c(r_i) = \frac{d(r_i, \alpha_*)}{d(r_i, \alpha^*) + d(r_i, \alpha_*)} = \frac{1 - v_i}{2 - (\mu_i + v_i)} = \frac{1 - v_i}{1 + \pi_i}, \quad i = 1, 2, \ldots, n \quad (8)$$

Step 7. Rank all the objects $x_i(i = 1, 2, \ldots, n)$ according to the closeness coefficients $c(r_i)(i = 1, 2, \ldots, n)$, the greater the value $c(r_i)$, the better the object x_i.

3 Extended Results in Interval-Valued Intuitionistic Fuzzy Situations

In this section, we shall extend our results to accommodate interval-valued intuitionistic fuzzy situations.

Suppose that the decision maker e_k provides his/her evaluation values over all pairs of alternatives by using interval-valued intuitionistic fuzzy values (Xu and Yager 2009): $\tilde{r}_{ij}^{(k)} = (\tilde{\mu}_{ij}^{(k)}, \tilde{v}_{ij}^{(k)}, \tilde{\pi}_{ij}^{(k)})(i, j = 1, 2, \ldots, n)$, and constructs an interval-valued intuitionistic fuzzy preference relation $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n}$, where $\tilde{\mu}_{ij}^{(k)}$ denotes the certainty degree range to which the object x_i is preferred to the object x_j, and $\tilde{v}_{ij}^{(k)}$ indicates the certainty degree range to which the object x_i is not preferred to the object x_j, and $\tilde{\pi}_{ij}^{(k)}$ means the indeterminacy degree range or a hesitation degree range, with the conditions:

$$\begin{aligned} \tilde{\mu}_{ij}^{(k)} &= [\mu_{ij}^{-(k)}, \mu_{ij}^{+(k)}] \subset [0,1],\ \tilde{v}_{ij}^{(k)} = [v_{ij}^{-(k)}, v_{ij}^{+(k)}] \subset [0,1], \\ &\quad \mu_{ij}^{+(k)} + v_{ij}^{+(k)} \le 1, \quad \tilde{\mu}_{ij}^{(k)} = \tilde{v}_{ji}^{(k)}, \quad \tilde{\mu}_{ji}^{(k)} = \tilde{v}_{ij}^{(k)}, \\ \tilde{\mu}_{ii}^{(k)} &= \tilde{v}_{ii}^{(k)} = [0.5, 0.5],\ \tilde{\pi}_{ij}^{(k)} = [\pi_{ij}^{-(k)}, \pi_{ij}^{+(k)}] \\ &= [1-\mu_{ij}^{+(k)} - v_{ij}^{+(k)}, 1-\mu_{ij}^{-(k)} - v_{ij}^{-(k)}], \quad \text{for all} \quad i, j = 1, 2, \ldots, n \end{aligned} \tag{9}$$

Now we give the compatibility measure between each pair of elements in $\tilde{R}^{(k)}$ and $\tilde{R}^{(l)}$, respectively.

Definition 4 Let $\tilde{r}_{ij}^{(k)} = (\tilde{\mu}_{ij}^{(k)}, \tilde{v}_{ij}^{(k)}, \tilde{\pi}_{ij}^{(k)})$ and $\tilde{r}_{ij}^{(l)} = (\tilde{\mu}_{ij}^{(l)}, \tilde{v}_{ij}^{(l)}, \tilde{\pi}_{ij}^{(l)})$ be any pair of elements in $\tilde{R}^{(k)}$ and $\tilde{R}^{(l)}$, respectively. Then we call

$$c\left(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}\right) = \frac{\left\langle \tilde{r}_{ij}^{(k)},\ \tilde{r}_{ij}^{(l)} \right\rangle}{\max\left\{ \left\| \tilde{r}_{ij}^{(k)} \right\|^2,\ \left\| \tilde{r}_{ij}^{(l)} \right\|^2 \right\}} \tag{10}$$

the compatibility degree of $\tilde{r}_{ij}^{(k)}$ and $\tilde{r}_{ij}^{(l)}$, where

$$\begin{aligned} \left\langle \tilde{r}_{ij}^{(k)},\ \tilde{r}_{ij}^{(l)} \right\rangle &= \mu_{ij}^{-(k)}\mu_{ij}^{-(l)} + \mu_{ij}^{+(k)}\mu_{ij}^{+(l)} + v_{ij}^{-(k)}v_{ij}^{-(l)} + v_{ij}^{+(k)}v_{ij}^{+(l)} + \pi_{ij}^{-(k)}\pi_{ij}^{-(l)} + \pi_{ij}^{+(k)}\pi_{ij}^{+(l)} \\ \left\| \tilde{r}_{ij}^{(k)} \right\|^2 &= \left(\mu_{ij}^{-(k)}\right)^2 + \left(\mu_{ij}^{+(k)}\right)^2 + \left(v_{ij}^{-(k)}\right)^2 + \left(v_{ij}^{+(k)}\right)^2 + \left(\pi_{ij}^{-(k)}\right)^2 + \left(\pi_{ij}^{+(k)}\right)^2 \\ \left\| \tilde{r}_{ij}^{(l)} \right\|^2 &= \left(\mu_{ij}^{-(l)}\right)^2 + \left(\mu_{ij}^{+(l)}\right)^2 + \left(v_{ij}^{-(l)}\right)^2 + \left(v_{ij}^{+(l)}\right)^2 + \left(\pi_{ij}^{-(l)}\right)^2 + \left(\pi_{ij}^{+(l)}\right)^2 \end{aligned}$$

From (10), we can see that the larger the value of $c\left(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}\right)$, the greater the compatibility degree of $\tilde{r}_{ij}^{(k)}$ and $\tilde{r}_{ij}^{(l)}$. The compatibility degree $c\left(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}\right)$ derived from Eq. (10) has also the following properties:

(1) $0 \le c\left(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}\right) \le 1$;

(2) $c\left(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}\right) = 1$, if and only if $\tilde{r}_{ij}^{(k)} = \tilde{r}_{ij}^{(l)}$;

(3) $c\left(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}\right) = c\left(\tilde{r}_{ij}^{(l)}, \tilde{r}_{ij}^{(k)}\right)$.

Similarly, we can define the compatibility measure between the interval-valued intuitionistic preference relations:

Definition 5 Let $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n}$ and $\tilde{R}^{(l)} = (\tilde{r}_{ij}^{(l)})_{n\times n}$ be two interval-valued intuitionistic preference relations. Then

$$c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left\langle \tilde{r}_{ij}^{(k)},\ \tilde{r}_{ij}^{(l)}\right\rangle}{\max\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\left\|\tilde{r}_{ij}^{(k)}\right\|^2,\ \sum_{i=1}^{n}\sum_{j=1}^{n}\left\|\tilde{r}_{ij}^{(l)}\right\|^2\right\}} \tag{11}$$

is called the compatibility degree of $\tilde{R}^{(k)}$ and $\tilde{R}^{(l)}$, where $\left\langle \tilde{r}_{ij}^{(k)},\ \tilde{r}_{ij}^{(l)}\right\rangle$, $\left\|\tilde{r}_{ij}^{(k)}\right\|^2$ and $\left\|\tilde{r}_{ij}^{(l)}\right\|^2$ are given as in Definition 4.

Clearly, the larger the value of $c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right)$, the greater the compatibility degree of $\tilde{R}^{(k)}$ and $\tilde{R}^{(l)}$. The compatibility degree $c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right)$ has also the following properties:

(1) $0 \le c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right) \le 1$;

(2) $c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right) = 1$, if and only if $\tilde{R}^{(k)} = \tilde{R}^{(l)}$;

(3) $c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right) = c\left(\tilde{R}^{(l)}, \tilde{R}^{(k)}\right)$;

(4) If $c\left(\tilde{R}^{(k)}, \tilde{R}^{(s)}\right) = 1$ and $c\left(\tilde{R}^{(s)}, \tilde{R}^{(l)}\right) = 1$, then $c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right) = 1$.

Definition 6 The interval-valued intuitionistic preference relations $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n}$ and $\tilde{R}^{(l)} = (\tilde{r}_{ij}^{(l)})_{n\times n}$ are perfectly compatible if $c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right) = 1$, i.e., $\tilde{R}^{(k)} = \tilde{R}^{(l)}$.

Let $\tilde{R}^{(k)} (k = 1, 2, \ldots, m)$ be m individual interval-valued intuitionistic preference relations, then their aggregation $\tilde{R} = \sum_{k=1}^{m}\lambda_k \tilde{R}^{(k)}$ is also an interval-valued intuitionistic preference relation (Xu and Chen 2007), which we call the collective interval-valued intuitionistic preference relation, where $\tilde{R} = (\tilde{r}_{ij})_{n\times n}, \tilde{r}_{ij} = (\tilde{\mu}_{ij}, \tilde{v}_{ij}, \tilde{\pi}_{ij})$, for all $i, j = 1, 2, \ldots, n$, such that

$$\tilde{\mu}_{ij} = \left[\sum_{k=1}^{m}\lambda_k \mu_{ij}^{-(k)}, \sum_{k=1}^{m}\lambda_k \mu_{ij}^{+(k)}\right], \quad \tilde{v}_{ij} = \left[\sum_{k=1}^{m}\lambda_k v_{ij}^{-(k)}, \sum_{k=1}^{m}\lambda_k v_{ij}^{+(k)}\right],$$
$$\tilde{\pi}_{ij} = \left[\sum_{k=1}^{m}\lambda_k \pi_{ij}^{-(k)}, \sum_{k=1}^{m}\lambda_k \pi_{ij}^{+(k)}\right], \quad \text{for all} \quad i, j = 1, 2, \ldots, n \tag{12}$$

The compatibility degree $c\left(\tilde{R}^{(k)}, \tilde{R}\right)$ $(k = 1, 2, \ldots, m)$ can be calculated by using Eq. (11), and has the following property:

Theorem 4 *Each individual interval-valued intuitionistic preference relation and the collective interval-valued intuitionistic preference relation is perfectly compatible if and only if all the individual interval-valued intuitionistic preference relations are perfectly compatible, i.e.,* $c\left(\tilde{R}^{(k)}, \tilde{R}\right) = 1 (k = 1, 2, \ldots, m)$ *if and only if* $c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right) = 1$*, for all* $k, l = 1, 2, \ldots, m$.

Similar to Definition 3, we define the concept of acceptable compatibility of interval-valued intuitionistic preference relations:

Definition 7 Let $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n}$ and $\tilde{R}^{(l)} = (\tilde{r}_{ij}^{(l)})_{n\times n}$ be two interval-valued intuitionistic preference relations. Then $R^{(k)}$ and $R^{(l)}$are called of acceptable compatibility if

$$c\left(\tilde{R}^{(k)}, \tilde{R}^{(l)}\right) \geq \delta_1 \tag{13}$$

where δ_1 is the threshold value of acceptable compatibility, and in general, $\delta_1 \in [0.5, 1]$.

Based on Eqs.(10)–(13), in what follows, we give a procedure for reaching group consensus based on interval-valued intuitionistic preference relations:

Procedure II Step 1. Let X, E and λ as defined in Sect. 2, the decision makers $e_k (k = 1, 2, \ldots, m)$ compare all pairs of alternatives in X, and construct the interval-valued intuitionistic preference relations $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n} (k = 1, 2, \ldots, m)$, with $\tilde{r}_{ij}^{(k)} = (\tilde{\mu}_{ij}^{(k)}, \tilde{v}_{ij}^{(k)}, \tilde{\pi}_{ij}^{(k)})$, for all $k = 1, 2, \ldots, m; i, j = 1, 2, \ldots, n$.

Step 2. Aggregate all individual interval-valued intuitionistic preference relations $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{n\times n} (k = 1, 2, \ldots, m)$ into the collective interval-valued intuitionistic preference relation $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ through Eq. (12), where $\tilde{r}_{ij} = (\tilde{\mu}_{ij}, \tilde{v}_{ij}, \tilde{\pi}_{ij})$, for all $i, j = 1, 2, \ldots, n$.

Step 3. Calculate the compatibility degree $c\left(\tilde{R}^{(k)}, \tilde{R}\right)$ of each individual interval-valued intuitionistic preference relation $\tilde{R}^{(k)}$ and the collective interval-valued intuitionistic preference relation $\tilde{R}$ through Eq. (11).

Step 4. Predefine the threshold value δ_1, if all $c\left(\tilde{R}^{(k)}, \tilde{R}\right) \geq \delta_1 (k = 1, 2, \ldots, m)$, then the goup reaches an acceptable consensus, go to Step 5; otherwise, there must exist at least one k_0, such that $c\left(\tilde{R}^{(k_0)}, \tilde{R}\right) < \delta_1$. Then utilize Eq. (10) to calculate the compatibility degree $c\left(\tilde{r}_{ij}^{(k_0)}, \tilde{r}_{ij}\right)$ of each pair elements $(\tilde{r}_{ij}^{(k_0)}, \tilde{r}_{ij})$ in $\tilde{R}^{(k_0)}$ and $\tilde{R}$, respectively. Return the interval-valued intuitionistic preference relation $R^{(k_0)}$ (together with $\tilde{R}$ and some elements with the smallest compatibility degrees as a reference) to the decision maker

e_{k_0} for re-evaluation, and then go to Step 2. Repeat this process until all $c\left(\tilde{R}^{(k)}, \tilde{R}\right) \geq \delta_1 (k = 1, 2, \ldots, m)$ or the process will stop as the repetition times reach the maximum number predefined by the group.

Step 5. Aggregate all the elements $\tilde{r}_{ij} (j = 1, 2, \ldots, n)$ in ith line of $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ by using the interval-valued intuitionistic fuzzy weighted averaging operator:

$$\tilde{\mu}_i = \left[\frac{1}{n}\sum_{j=1}^{n}\mu_{ij}^-, \frac{1}{n}\sum_{j=1}^{n}\mu_{ij}^+\right], \quad \tilde{v}_i = \left[\frac{1}{n}\sum_{j=1}^{n}v_{ij}^-, \frac{1}{n}\sum_{j=1}^{n}v_{ij}^+\right],$$

$$\tilde{\pi}_i = \left[\frac{1}{n}\sum_{j=1}^{n}\pi_{ij}^-, \frac{1}{n}\sum_{j=1}^{n}\pi_{ij}^+\right], \quad i = 1, 2, \ldots, n \tag{14}$$

and get the interval-valued overall preference values $\tilde{r}_i = (\tilde{\mu}_i, \tilde{v}_i, \tilde{\pi}_i) (i = 1, 2, \ldots, n)$ corresponding to the objects $x_i (i = 1, 2, \ldots, n)$.

Step 6. To rank the interval-valued overall preference values $\tilde{r}_i (i = 1, 2, \ldots, n)$, we calculate the distance of the interval-valued overall preference value $\tilde{r}_i$ to both the largest interval-valued intuitionistic fuzzy value $\tilde{\alpha}^* = ([1, 1], [0, 0], [0, 0])$ and the smallest interval-valued intuitionistic fuzzy value $\tilde{\alpha}_* = ([0, 0], [1, 1], [0, 0])$:

$$\begin{aligned} d(\tilde{r}_i, \tilde{\alpha}^*) &= \frac{1}{4}\left(\left|1-\mu_i^-\right| + \left|1-\mu_i^+\right| + \left|0-v_i^-\right| + \left|0-v_i^+\right| + \left|0-\pi_i^-\right| + \left|0-\pi_i^+\right|\right) \\ &= \frac{1}{4}\left(1-\mu_i^- + 1-\mu_i^+ + v_i^- + v_i^+ + \pi_i^- + \pi_i^+\right) = 1 - \frac{1}{2}\left(\mu_i^- + \mu_i^+\right) \\ d(\tilde{r}_i, \tilde{\alpha}_*) &= \frac{1}{4}\left(\left|0-\mu_i^-\right| + \left|0-\mu_i^+\right| + \left|1-v_i^-\right| + \left|1-v_i^+\right| + \left|0-\pi_i^-\right| + \left|0-\pi_i^+\right|\right) \\ &= \frac{1}{4}\left(\mu_i^- + \mu_i^+ + 1 - v_i^- + 1 - v_i^+ + \pi_i^- + \pi_i^+\right) = 1 - \frac{1}{2}\left(v_i^- + v_i^+\right) \end{aligned}$$

and then calculate the closeness coefficient of each interval-valued overall preference value:

$$\begin{aligned} c(\tilde{r}_i) &= \frac{d(\tilde{r}_i, \tilde{\alpha}_*)}{d(\tilde{r}_i, \tilde{\alpha}^*) + d(\tilde{r}_i, \tilde{\alpha}_*)} = \frac{1 - \frac{1}{2}\left(v_i^- + v_i^+\right)}{1 - \frac{1}{2}\left(v_i^- + v_i^+\right) + 1 - \frac{1}{2}\left(\mu_i^- + \mu_i^+\right)} \\ &= \frac{2 - (v_i^- + v_i^+)}{2 + (\pi_i^- + \pi_i^+)}, \quad i = 1, 2, \ldots, n \end{aligned} \tag{15}$$

Step 7. Rank all the objects $x_i (i = 1, 2, \ldots, n)$ according to the closeness coefficients $c(\tilde{r}_i) (i = 1, 2, \ldots, n)$.

4 Numerical Analysis

In the following, we use a practical problem of the strategic alliance partner selection of a software company (adapted from Fan and Liu 2010) to illustrate our procedures.

Eastsoft is one of the top five software companies in China. It offers a rich portfolio of businesses, mainly including industry solutions, product engineering solutions, and related software products and platform and services. It is dedicated to becoming a globally leading IT solutions and services provider through continuous improvement of organization and process, competence development of leadership and employees, and alliance and open innovation. To improve the operation and competitiveness capability in the global market, Eastsoft plans to establish a strategic alliance with a transnational corporation. After numerous consultations, four transnational corporations would like to establish a strategic alliance with Eastsoft; they are HP (x_1), PHILIPS (x_2), EMC (x_3), and SAP (x_4). To select the desirable strategic alliance partner, five decision makers $d_k (k = 1, 2, 3, 4, 5)$ (whose weight vector is $\lambda = (0.2, 0.2, 0.2, 0.2, 0.2)^T$) are invited to participate in the decision analysis, who come from the operation management department, the engineering management department, the finance department, the human resources department, and the business process outsourcing department of Eastsoft, respectively. The preference information on the potential alliance partners provided by the five decision makers takes the form of the intuitionistic preference relations $R^{(k)} = (r_{ij}^{(k)})_{4\times 4} (k = 1, 2, 3, 4, 5)$, listed as follows:

$$R^{(1)} = \begin{pmatrix} (0.5, 0.5, 0.0) & (0.2, 0.6, 0.2) & (0.3, 0.4, 0.3) & (0.6, 0.2, 0.2) \\ (0.6, 0.2, 0.2) & (0.5, 0.5, 0.0) & (0.5, 0.4, 0.1) & (0.6, 0.4, 0.0) \\ (0.4, 0.3, 0.3) & (0.4, 0.5, 0.1) & (0.5, 0.5, 0.0) & (0.3, 0.2, 0.5) \\ (0.2, 0.6, 0.2) & (0.4, 0.6, 0.0) & (0.2, 0.3, 0.5) & (0.5, 0.5, 0.0) \end{pmatrix}_{4\times 4}$$

$$R^{(2)} = \begin{pmatrix} (0.5, 0.5, 0.0) & (0.3, 0.5, 0.2) & (0.4, 0.4, 0.2) & (0.5, 0.2, 0.3) \\ (0.5, 0.3, 0.2) & (0.5, 0.5, 0.0) & (0.5, 0.3, 0.2) & (0.7, 0.2, 0.1) \\ (0.4, 0.4, 0.2) & (0.3, 0.5, 0.2) & (0.5, 0.5, 0.0) & (0.3, 0.3, 0.4) \\ (0.2, 0.5, 0.3) & (0.2, 0.7, 0.1) & (0.3, 0.3, 0.4) & (0.5, 0.5, 0.0) \end{pmatrix}_{4\times 4}$$

$$R^{(3)} = \begin{pmatrix} (0.5, 0.5, 0.0) & (0.3, 0.4, 0.3) & (0.2, 0.3, 0.5) & (0.7, 0.3, 0.0) \\ (0.4, 0.3, 0.3) & (0.5, 0.5, 0.0) & (0.6, 0.4, 0.0) & (0.6, 0.3, 0.1) \\ (0.3, 0.2, 0.5) & (0.4, 0.6, 0.0) & (0.5, 0.5, 0.0) & (0.4, 0.2, 0.4) \\ (0.3, 0.7, 0.0) & (0.3, 0.6, 0.1) & (0.2, 0.4, 0.4) & (0.5, 0.5, 0.0) \end{pmatrix}_{4\times 4}$$

$$R^{(4)} = \begin{pmatrix} (0.5, 0.5, 0.0) & (0.5, 0.4, 0.1) & (0.3, 0.3, 0.4) & (0.5, 0.4, 0.1) \\ (0.4, 0.5, 0.1) & (0.5, 0.5, 0.0) & (0.4, 0.4, 0.2) & (0.7, 0.3, 0.0) \\ (0.3, 0.3, 0.4) & (0.4, 0.4, 0.2) & (0.5, 0.5, 0.0) & (0.4, 0.2, 0.4) \\ (0.4, 0.5, 0.1) & (0.3, 0.7, 0.0) & (0.2, 0.4, 0.4) & (0.5, 0.5, 0.0) \end{pmatrix}_{4\times 4}$$

$$R^{(5)} = \begin{pmatrix} (0.5, 0.5, 0.0) & (0.3, 0.4, 0.3) & (0.3, 0.6, 0.1) & (0.5, 0.3, 0.2) \\ (0.4, 0.3, 0.3) & (0.5, 0.5, 0.0) & (0.4, 0.5, 0.1) & (0.5, 0.4, 0.1) \\ (0.6, 0.3, 0.1) & (0.5, 0.4, 0.1) & (0.5, 0.5, 0.0) & (0.4, 0.3, 0.3) \\ (0.3, 0.5, 0.2) & (0.4, 0.5, 0.1) & (0.3, 0.4, 0.3) & (0.5, 0.5, 0.0) \end{pmatrix}_{4\times 4}$$

In the following, we use Procedure I to check and reach the group consensus for this problem:

We first utilize Eq. (4) to fuse all individual intuitionistic preference relations $R^{(k)} = (r_{ij}^{(k)})_{4\times 4} (k = 1, 2, 3, 4, 5)$ into the collective intuitionistic preference relation $R = (r_{ij})_{4\times 4}$:

$$R=\begin{pmatrix} (0.5,0.5,0.0) & (0.32,0.46,0.22) & (0.30,0.40,0.30) & (0.56,0.28,0.16) \\ (0.46,0.32,0.22) & (0.5,0.5,0.0) & (0.48,0.40,0.12) & (0.62,0.32,0.04) \\ (0.40,0.30,0.30) & (0.40,0.48,0.12) & (0.5,0.5,0.0) & (0.36,0.24,0.40) \\ (0.28,0.56,0.16) & (0.32,0.62,0.04) & (0.24,0.36,0.40) & (0.5,0.5,0.0) \end{pmatrix}_{4\times 4}$$

Then by Eq. (3), we calculate the the compatibility degree $c\left(R^{(k)},R\right)$ of each individual intuitionistic preference relation $R^{(k)}$ and the collective intuitionistic preference relation R:

$$c\left(R^{(1)},R\right)=0.965,\quad c\left(R^{(2)},R\right)=0.982,\quad c\left(R^{(3)},R\right)=0.941$$
$$c\left(R^{(4)},R\right)=0.966,\quad c\left(R^{(5)},R\right)=0.962$$

Suppose that the threshold value $\delta_0=0.95$, then $c\left(R^{(k)},R\right)\geq\delta_0(k=1,2,4,5)$, but $c\left(R^{(3)},R\right)=0.941<0.95$. Then by Eq. (2), we calculate all the compatibility degrees $c\left(r_{ij}^{(3)},r_{ij}\right)$ $(i,j=1,2,3,4)$:

$$c\left(r_{11}^{(3)},r_{11}\right)=1,\ c\left(r_{12}^{(3)},r_{12}\right)=0.955,\ c\left(r_{13}^{(3)},r_{13}\right)=0.868,\ c\left(r_{14}^{(3)},r_{14}\right)=0.821$$
$$c\left(r_{21}^{(3)},r_{21}\right)=0.955,\ c\left(r_{22}^{(3)},r_{22}\right)=1,\ c\left(r_{23}^{(3)},r_{23}\right)=0.862,\ c\left(r_{24}^{(3)},r_{24}\right)=0.966$$
$$c\left(r_{31}^{(3)},r_{31}\right)=0.868,\ c\left(r_{32}^{(3)},r_{32}\right)=0.862,\ c\left(r_{33}^{(3)},r_{33}\right)=1,\ c\left(r_{34}^{(3)},r_{34}\right)=0.978$$
$$c\left(r_{41}^{(3)},r_{41}\right)=0.821,\ c\left(r_{42}^{(3)},r_{42}\right)=0.966,\ c\left(r_{43}^{(3)},r_{43}\right)=0.978,\ c\left(r_{34}^{(3)},r_{34}\right)=1$$

Thus, we need to return the intuitionistic preference relation $R^{(3)}$ (together with R and some elements with the smallest compatibility degrees, such as $r_{14}^{(3)}, r_{23}^{(3)}$ and $r_{13}^{(3)}$) to the decision maker d_3 for re-evaluation. Assume that the decision maker d_3 improves $r_{14}^{(3)}=(0.7,0.3,0.0)$ as $\dot{r}_{14}^{(3)}=(0.6,0.3,0.1)$, $r_{23}^{(3)}=(0.6,0.4,0.0)$ as $\dot{r}_{23}^{(3)}=(0.5,0.4,0.1)$, and $r_{13}^{(3)}=(0.2,0.3,0.5)$ as $\dot{r}_{13}^{(3)}=(0.3,0.3,0.4)$, then the re-evaluated intuitionistic preference relation is:

$$\dot{R}^{(3)}=\begin{pmatrix} (0.5,0.5,0.0) & (0.3,0.4,0.3) & (0.3,0.3,0.4) & (0.6,0.3,0.1) \\ (0.4,0.3,0.4) & (0.5,0.5,0.0) & (0.5,0.4,0.1) & (0.6,0.3,0.1) \\ (0.3,0.3,0.4) & (0.4,0.5,0.1) & (0.5,0.5,0.0) & (0.4,0.2,0.4) \\ (0.3,0.6,0.1) & (0.3,0.6,0.1) & (0.2,0.4,0.4) & (0.5,0.5,0.0) \end{pmatrix}_{4\times 4}$$

and then, we utilize Eq. (4) to fuse all individual intuitionistic preference relations $R^{(k)}(k=1,2,4,5)$ and $\dot{R}^{(3)}$ into the collective intuitionistic preference relation:

$$\dot{R}=\begin{pmatrix} (0.5,0.5,0.0) & (0.32,0.46,0.22) & (0.32,0.40,0.28) & (0.54,0.28,0.18) \\ (0.46,0.32,0.22) & (0.5,0.5,0.0) & (0.46,0.40,0.14) & (0.62,0.32,0.04) \\ (0.40,0.32,0.28) & (0.40,0.46,0.14) & (0.5,0.5,0.0) & (0.36,0.24,0.40) \\ (0.28,0.54,0.18) & (0.32,0.62,0.04) & (0.24,0.36,0.40) & (0.5,0.5,0.0) \end{pmatrix}_{4\times 4}$$

Using Eq. (3), we calculate the compatibility degrees:

$$c\left(R^{(1)}, \dot{R}\right) = 0.960, \quad c\left(R^{(2)}, \dot{R}\right) = 0.980, \quad c\left(\dot{R}^{(3)}, \dot{R}\right) = 0.985$$
$$c\left(R^{(4)}, \dot{R}\right) = 0.962, \quad c\left(R^{(5)}, \dot{R}\right) = 0.975$$

Then all $c\left(R^{(k)}, \dot{R}\right) \geq 0.95(k = 1, 2, 4, 5)$ and $c\left(\dot{R}^{(3)}, \dot{R}\right) \geq 0.95$, and thus, the goup reaches an acceptable consensus.

After that, we use Eq. (7) to aggregate all the elements $\dot{r}_{ij}(j = 1, 2, 3, 4)$ in ith line of $\dot{R}$, and get the overall preference values:

$$\dot{r}_1 = (0.420, 0.410, 0.170), \quad \dot{r}_2 = (0.510, 0.385, 0.105)$$
$$\dot{r}_3 = (0.415, 0.380, 0.205), \quad \dot{r}_4 = (0.335, 0.505, 0.160)$$

To rank the overall preference values $\dot{r}_i(i = 1, 2, 3, 4)$, we calculate the closeness coefficient of each object by using Eq. (8):

$$c(x_1) = 0.504, \quad c(x_2) = 0.557, \quad c(x_3) = 0.515, \quad c(x_4) = 0.427$$

by which we rank all the objects as $x_2 \succ x_3 \succ x_1 \succ x_4$, and thus the best object is x_2.

If we use Xu and Yager (2009)'s method to solve the problem, then in a similar way, we utilize Eq. (4) to fuse all $R^{(k)} = (r_{ij}^{(k)})_{4\times4}(k = 1, 2, 3, 4, 5)$ into $R = (r_{ij})_{4\times4}$, and use the following formula [a revised version of Eq. (18) in Xu and Yager (2009)]:

$$s\left(r_{ij}^{(k)}, r_{ij}\right) = \begin{cases} 1, & \text{if } \mu_{ij}^{(k)} = v_{ij}^{(k)} = \mu_{ij} = v_{ij} \\ \frac{|\mu_{ij}^{(k)}-v_{ij}|+|v_{ij}^{(k)}-\mu_{ij}|+|\pi_{ij}^{(k)}-\pi_{ij}|}{|\mu_{ij}^{(k)}-\mu_{ij}|+|v_{ij}^{(k)}-v_{ij}|+|\pi_{ij}^{(k)}-\pi_{ij}|+|\mu_{ij}^{(k)}-v_{ij}|+|v_{ij}^{(k)}-\mu_{ij}|+|\pi_{ij}^{(k)}-\pi_{ij}|}, & \text{otherwise} \end{cases} \tag{16}$$

to calculate the similarity degree between $r_{ij}^{(k)}$ and r_{ij}:

$$s\left(r_{11}^{(1)}, r_{11}\right) = s\left(r_{22}^{(1)}, r_{22}\right) = s\left(r_{33}^{(1)}, r_{33}\right) = s\left(r_{44}^{(1)}, r_{44}\right) = 1$$
$$s\left(r_{12}^{(1)}, r_{12}\right) = s\left(r_{21}^{(1)}, r_{21}\right) = 0.667, \ s\left(r_{13}^{(1)}, r_{13}\right) = s\left(r_{31}^{(1)}, r_{31}\right) = 1$$
$$s\left(r_{14}^{(1)}, r_{14}\right) = s\left(r_{41}^{(1)}, r_{41}\right) = 0.818, \ s\left(r_{23}^{(1)}, r_{23}\right) = s\left(r_{32}^{(1)}, r_{32}\right) = 0.833$$
$$s\left(r_{24}^{(1)}, r_{24}\right) = s\left(r_{42}^{(1)}, r_{42}\right) = 0.794, \ s\left(r_{34}^{(1)}, r_{34}\right) = s\left(r_{43}^{(1)}, r_{43}\right) = 0.615$$
$$s\left(r_{11}^{(2)}, r_{11}\right) = s\left(r_{22}^{(2)}, r_{22}\right) = s\left(r_{33}^{(2)}, r_{33}\right) = s\left(r_{44}^{(2)}, r_{44}\right) = 1$$
$$s\left(r_{12}^{(2)}, r_{12}\right) = s\left(r_{21}^{(2)}, r_{21}\right) = 0.818, \ s\left(r_{13}^{(2)}, r_{13}\right) = s\left(r_{31}^{(2)}, r_{31}\right) = 0.500$$

$$s\left(r_{14}^{(2)}, r_{14}\right) = s\left(r_{41}^{(2)}, r_{41}\right) = 0.725,\ \ s\left(r_{23}^{(2)}, r_{23}\right) = s\left(r_{32}^{(2)}, r_{32}\right) = 0.605$$

$$s\left(r_{24}^{(2)}, r_{24}\right) = s\left(r_{42}^{(2)}, r_{42}\right) = 0.768,\ \ s\left(r_{34}^{(2)}, r_{34}\right) = s\left(r_{43}^{(2)}, r_{43}\right) = 0.500$$

$$s\left(r_{11}^{(3)}, r_{11}\right) = s\left(r_{22}^{(3)}, r_{22}\right) = s\left(r_{33}^{(3)}, r_{33}\right) = s\left(r_{44}^{(3)}, r_{44}\right) = 1$$

$$s\left(r_{12}^{(3)}, r_{12}\right) = s\left(r_{21}^{(3)}, r_{21}\right) = 0.667,\ \ s\left(r_{13}^{(3)}, r_{13}\right) = s\left(r_{31}^{(3)}, r_{31}\right) = 0.500$$

$$s\left(r_{14}^{(3)}, r_{14}\right) = s\left(r_{41}^{(3)}, r_{41}\right) = 0.724,\ \ s\left(r_{23}^{(3)}, r_{23}\right) = s\left(r_{32}^{(3)}, r_{32}\right) = 0.625$$

$$s\left(r_{24}^{(3)}, r_{24}\right) = s\left(r_{42}^{(3)}, r_{42}\right) = 0.868,\ \ s\left(r_{34}^{(3)}, r_{34}\right) = s\left(r_{43}^{(3)}, r_{43}\right) = 0.800$$

$$s\left(r_{11}^{(4)}, r_{11}\right) = s\left(r_{22}^{(4)}, r_{22}\right) = s\left(r_{33}^{(4)}, r_{33}\right) = s\left(r_{44}^{(4)}, r_{44}\right) = 1$$

$$s\left(r_{12}^{(4)}, r_{12}\right) = s\left(r_{21}^{(4)}, r_{21}\right) = 0.400,\ \ s\left(r_{13}^{(4)}, r_{13}\right) = s\left(r_{31}^{(4)}, r_{31}\right) = 0.500$$

$$s\left(r_{14}^{(4)}, r_{14}\right) = s\left(r_{41}^{(4)}, r_{41}\right) = 0.647,\ \ s\left(r_{23}^{(4)}, r_{23}\right) = s\left(r_{32}^{(4)}, r_{32}\right) = 0.500$$

$$s\left(r_{24}^{(4)}, r_{24}\right) = s\left(r_{42}^{(4)}, r_{42}\right) = 0.841,\ \ s\left(r_{34}^{(4)}, r_{34}\right) = s\left(r_{43}^{(4)}, r_{43}\right) = 0.800$$

$$s\left(r_{11}^{(5)}, r_{11}\right) = s\left(r_{22}^{(5)}, r_{22}\right) = s\left(r_{33}^{(5)}, r_{33}\right) = s\left(r_{44}^{(5)}, r_{44}\right) = 1$$

$$s\left(r_{12}^{(5)}, r_{12}\right) = s\left(r_{21}^{(5)}, r_{21}\right) = 0.667,\ \ s\left(r_{13}^{(5)}, r_{13}\right) = s\left(r_{31}^{(5)}, r_{31}\right) = 0.600$$

$$s\left(r_{14}^{(5)}, r_{14}\right) = s\left(r_{41}^{(5)}, r_{41}\right) = 0.838,\ \ s\left(r_{23}^{(5)}, r_{23}\right) = s\left(r_{32}^{(5)}, r_{32}\right) = 0.167$$

$$s\left(r_{24}^{(5)}, r_{24}\right) = s\left(r_{42}^{(5)}, r_{42}\right) = 0.639,\ \ s\left(r_{34}^{(5)}, r_{34}\right) = s\left(r_{43}^{(5)}, r_{43}\right) = 0.615$$

Then using the following formula:

$$s\left(R^{(k)}, R\right) = \frac{1}{4^2}\sum_{i=1}^{4}\sum_{j=1}^{4} s\left(r_{ij}^{(k)}, r_{ij}\right) \tag{17}$$

we can calculate the similarity degree between $R^{(k)}$ and R:

$$s\left(R^{(1)}, R\right) = 0.841,\quad s\left(R^{(2)}, R\right) = 0.739,\quad s\left(R^{(3)}, R\right) = 0.773$$
$$s\left(R^{(4)}, R\right) = 0.711,\ \ s\left(R^{(5)}, R\right) = 0.691$$

Suppose that the threshold value $\alpha_0 = 0.70$, then $s\left(R^{(k)}, R\right) \geq \alpha_0 (k = 1, 2, 3, 4)$, but $s\left(R^{(5)}, R\right) = 0.691 < 0.70$. Then, we need to return the intuitionistic preference relation $R^{(5)}$ (together with R and some elements with the smallest similarity degrees, such as $r_{23}^{(5)}$) to the decision maker d_5 for re-evaluation. Assume that the decision maker d_5 improves $r_{23}^{(5)} = (0.4, 0.5, 0.1)$ as $\dot{r}_{23}^{(5)} = (0.5, 0.4, 0.1)$, then the re-evaluated intuitionistic preference relation is:

$$\dot{R}^{(5)} = \begin{pmatrix} (0.5, 0.5, 0.0) & (0.3, 0.4, 0.3) & (0.3, 0.6, 0.1) & (0.5, 0.3, 0.2) \\ (0.4, 0.3, 0.3) & (0.5, 0.5, 0.0) & (0.5, 0.4, 0.1) & (0.5, 0.4, 0.1) \\ (0.6, 0.3, 0.1) & (0.4, 0.5, 0.1) & (0.5, 0.5, 0.0) & (0.4, 0.3, 0.3) \\ (0.3, 0.5, 0.2) & (0.4, 0.5, 0.1) & (0.3, 0.4, 0.3) & (0.5, 0.5, 0.0) \end{pmatrix}_{4\times 4}$$

and then, we utilize Eq. (4) to fuse all individual intuitionistic preference relations $R^{(k)}(k = 1, 2, 3, 4)$ and $\dot{R}^{(5)}$ into the collective intuitionistic preference relation:

$$\dot{R} = \begin{pmatrix} (0.5, 0.5, 0.0) & (0.32, 0.46, 0.22) & (0.32, 0.40, 0.28) & (0.54, 0.28, 0.18) \\ (0.46, 0.32, 0.22) & (0.5, 0.5, 0.0) & (0.50, 0.38, 0.12) & (0.62, 0.32, 0.04) \\ (0.40, 0.32, 0.28) & (038, 0.50, 0.12) & (0.5, 0.5, 0.0) & (0.36, 0.24, 0.40) \\ (0.28, 0.54, 0.18) & (0.32, 0.62, 0.04) & (0.24, 0.36, 0.40) & (0.5, 0.5, 0.0) \end{pmatrix}_{4\times 4}$$

By Eqs.(16) and (17), we calculate the similarity degrees:

$$s\left(R^{(1)}, \dot{R}\right) = 0.844, \quad s\left(R^{(2)}, \dot{R}\right) = 0.753, \quad s\left(R^{(3)}, \dot{R}\right) = 0.776$$
$$s\left(R^{(4)}, \dot{R}\right) = 0.711, \quad s\left(\dot{R}^{(5)}, \dot{R}\right) = 0.777$$

Then all $s\left(R^{(k)}, \dot{R}\right) \geq 0.70(k = 1, 2, 3, 4)$ and $s\left(\dot{R}^{(5)}, \dot{R}\right) \geq 0.70$, and thus, each individual intuitionistic preference relation and the collective intuitionistic preference relation are of acceptable similarity.

From the numerical results above, we can see that the individuals with the greatest differences (i.e., the smallest similarity degree and the smallest compatibility degree) from the collective preference relation are not same. That is because that Xu and Yager (2009)'s method mainly examines if the compared values are more similar or more dissimilar to each other so as to avoid drawing conclusions about strong similarity between two intuitionistic fuzzy values on the basis of the small distances between these values, which can be seen clearly from the similarity degre $s\left(r_{23}^{(5)}, r_{23}\right)$; while Procedure I developed in this paper focuses on the compatibility degree between each pair of intuitionistic fuzzy values themselves. In addition, compared to Xu and Yager (2009)'s method, Procedure I gives a simple approach for comparing intuitionistic fuzzy values, by which the considered objects are ranked and selected.

In the case where the the preference information provided by the five decision makers in the above example takes the form of the interval-valued intuitionistic preference relations $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{4\times 4}(k = 1, 2, 3, 4, 5)$:

$$\tilde{R}^{(1)} = \begin{pmatrix} ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.2, 0.3], [0.5, 0.6], [0.1, 0.3]) \\ ([0.5, 0.6], [0.2, 0.3], [0.1, 0.3]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \\ ([0.4, 0.5], [0.2, 0.4], [0.1, 0.4]) & ([0.3, 0.4], [0.4, 0.6], [0.0, 0.3]) \\ ([0.2, 0.3], [0.5, 0.6], [0.1, 0.3]) & ([0.3, 0.4], [0.5, 0.6], [0.0, 0.2]) \end{matrix}$$

$$\left.\begin{matrix} ([0.2, 0.4], [0.4, 0.5], [0.1, 0.4]) & ([0.5, 0.6], [0.2, 0.3], [0.1, 0.3]) \\ ([0.4, 0.6], [0.3, 0.4], [0.0, 0.3]) & ([0.5, 0.6], [0.3, 0.4], [0.0, 0.2]) \\ ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.3, 0.4], [0.2, 0.3], [0.3, 0.5]) \\ ([0.2, 0.3], [0.3, 0.4], [0.3, 0.5]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \end{matrix}\right)_{4\times 4}$$

$$\tilde{R}^{(2)} = \left(\begin{matrix} ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.3, 0.4], [0.5, 0.6], [0.0, 0.2]) \\ ([0.5, 0.6], [0.3, 0.4], [0.0, 0.2]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \\ ([0.4, 0.5], [0.3, 0.4], [0.1, 0.3]) & ([0.3, 0.4], [0.4, 0.5], [0.1, 0.3]) \\ ([0.2, 0.3], [0.4, 0.5,], [0.2, 0.4]) & ([0.2, 0.3], [0.6, 0.7], [0.0, 0.2]) \end{matrix}\right.$$

$$\left.\begin{matrix} ([0.3, 0.4], [0.4, 0.5], [0.1, 0.3]) & ([0.4, 0.5], [0.2, 0.3], [0.2, 0.4]) \\ ([0.4, 0.5], [0.3, 0.4], [0.1, 0.3]) & ([0.6, 0.7], [0.2, 0.3], [0.0, 0.2]) \\ ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.2, 0.3], [0.3, 0.4], [0.3, 0.5]) \\ ([0.3, 0.4], [0.2, 0.3], [0.3, 0.5]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \end{matrix}\right)_{4\times 4}$$

$$\tilde{R}^{(3)} = \left(\begin{matrix} ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.3, 0.4], [0.4, 0.5], [0.1, 0.3]) \\ ([0.4, 0.5], [0.3, 0.4], [0.1, 0.3]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \\ ([0.3, 0.4], [0.3, 0.4], [0.2, 0.4]) & ([0.3, 0.4], [0.4, 0.5], [0.1, 0.3]) \\ ([0.2, 0.3], [0.5, 0.6], [0.1, 0.3]) & ([0.3, 0.4], [0.5, 0.6], [0.0, 0.2]) \end{matrix}\right.$$

$$\left.\begin{matrix} ([0.3, 0.4], [0.3, 0.4], [0.2, 0.4]) & ([0.5, 0.6], [0.2, 0.3], [0.1, 0.3]) \\ ([0.4, 0.5], [0.3, 0.4], [0.1, 0.3]) & ([0.5, 0.6], [0.3, 0.4], [0.0, 0.2]) \\ ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.3, 0.4], [0.2, 0.4], [0.2, 0.5]) \\ ([0.2, 0.4], [0.3, 0.4], [0.2, 0.5]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \end{matrix}\right)_{4\times 4}$$

$$\tilde{R}^{(4)} = \left(\begin{matrix} ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.4, 0.5], [0.4, 0.5], [0.0, 0.2]) \\ ([0.4, 0.5], [0.4, 0.5], [0.0, 0.2]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \\ ([0.3, 0.3], [0.3, 0.4], [0.3, 0.4]) & ([0.3, 0.4], [0.4, 0.4], [0.2, 0.3]) \\ ([0.3, 0.4], [0.5, 0.6], [0.0, 0.2]) & ([0.2, 0.3], [0.6, 0.7], [0.0, 0.2]) \end{matrix}\right.$$

$$\left.\begin{matrix} ([0.3, 0.4], [0.3, 0.3], [0.3, 0.4]) & ([0.5, 0.6], [0.3, 0.4], [0.0, 0.2]) \\ ([0.4, 0.4], [0.3, 0.4], [0.2, 0.3]) & ([0.6, 0.7], [0.2, 0.3], [0.0, 0.2]) \\ ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.3, 0.4], [0.2, 0.3], [0.3, 0.5]) \\ ([0.2, 0.3], [0.3, 0.4], [0.3, 0.5]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \end{matrix}\right)_{4\times 4}$$

$$\tilde{R}^{(5)} = \left(\begin{matrix} ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.3, 0.4], [0.4, 0.4], [0.2, 0.3]) \\ ([0.4, 0.4], [0.3, 0.4], [0.2, 0.3]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \\ (0.4, 0.5], [0.3, 0.4], [[0.1, 0.3]) & ([0.3, 0.5], [0.4, 0.5], [0.0, 0.3]) \\ ([0.2, 0.3], [0.5, 0.6], [0.1, 0.3]) & ([0.3, 0.3], [0.5, 0.7], [0.0, 0.2]) \end{matrix}\right.$$

$$\left.\begin{matrix} ([0.3, 0.4], [0.4, 0.5], [0.1, 0.3]) & ([0.5, 0.6], [0.2, 0.3], [0.1, 0.3]) \\ ([0.4, 0.5], [0.3, 0.5], [0.0, 0.3]) & ([0.5, 0.7], [0.3, 0.3], [0.0, 0.2]) \\ ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.3, 0.5], [0.2, 0.3], [0.2, 0.5]) \\ ([0.2, 0.3], [0.3, 0.5], [0.2, 0.5]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \end{matrix}\right)_{4\times 4}$$

then we can use Procedure II to check and reach the group consensus. The following is the solution process:

By Eq. (12), we aggregate all individual interval-valued intuitionistic preference relations $\tilde{R}^{(k)} (k = 1, 2, 3, 4, 5)$ into the collective interval-valued intuitionistic preference relation:

$$\tilde{R} = \begin{pmatrix} ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.30, 0.40], [0.44, 0.52], [0.08, 0.26]) \\ ([0.44, 0.52], [0.30, 0.40], [0.08, 0.26]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \\ (0.36, 0.44], [0.28, 0.40], [[0.16, 0.36]) & ([0.30, 0.42], [0.40, 0.50], [0.08, 0.30]) \\ ([0.22, 0.32], [0.48, 0.58], [0.10, 0.30]) & ([0.26, 0.34], [0.54, 0.66], [0.00, 0.20]) \end{pmatrix.$$
$$\begin{pmatrix} ([0.28, 0.40], [0.36, 0.44], [0.16, 0.36]) & ([0.48, 0.58], [0.22, 0.32], [0.10, 0.30]) \\ ([0.40, 0.50], [0.30, 0.42], [0.08, 0.30]) & ([0.54, 0.66], [0.26, 0.34], [0.00, 0.20]) \\ ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) & ([0.28, 0.40], [0.22, 0.34], [0.26, 0.50]) \\ ([0.22, 0.34], [0.28, 0.40], [0.26, 0.50]) & ([0.5, 0.5], [0.5, 0.5], [0.0, 0.0]) \end{pmatrix}_{4\times 4}$$

Then we use Eq. (11) to calculate the the compatibility degree $c\left(\tilde{R}^{(k)}, \tilde{R}\right)$ of each individual interval-valued intuitionistic preference relation $\tilde{R}^{(k)}$ and the collective interval-valued intuitionistic preference relation $\tilde{R}$:

$$c\left(\tilde{R}^{(1)}, \tilde{R}\right) = 0.989, \quad c\left(\tilde{R}^{(2)}, \tilde{R}\right) = 0.983, \quad c\left(\tilde{R}^{(3)}, \tilde{R}\right) = 0.993$$
$$c\left(\tilde{R}^{(4)}, \tilde{R}\right) = 0.981, \quad c\left(\tilde{R}^{(5)}, \tilde{R}\right) = 0.987$$

Suppose that the threshold value $\delta_1 = 0.98$, then all $c\left(\tilde{R}^{(k)}, \tilde{R}\right) \geq \delta_1 (k = 1, 2, 3, 4, 5)$, and thus, the goup reaches an acceptable consensus.

Then we aggregate all the elements $\tilde{r}_{ij} (j = 1, 2, 3, 4)$ in ith line of $\tilde{R}$ by using Eq. (14), and get the interval-valued overall preference values:

$$\tilde{r}_1 = ([0.390, 0.470], [0.380, 0.445], [0.085, 0.230])$$
$$\tilde{r}_2 = ([0.470, 0.545], [0.340, 0.415], [0.040, 0.190])$$
$$\tilde{r}_3 = ([0.360, 0.440], [0.350, 0.435], [0.125, 0.290])$$
$$\tilde{r}_4 = ([0.300, 0.375], [0.450, 0.535], [0.090, 0.250])$$

To rank the interval-valued overall preference values $\tilde{r}_i (i = 1, 2, 3, 4)$, we calculate their closeness coefficients:

$$c(\tilde{r}_1) = 0.508, \quad c(\tilde{r}_2) = 0.558, \quad c(\tilde{r}_3) = 0.503, \quad c(\tilde{r}_4) = 0.434$$

by which we rank all the objects as $x_2 \succ x_1 \succ x_3 \succ x_4$, and thus x_2 is the best object.

The ranking of the objects derived by Procedure II is slighly different from that derived by Procedure I because of the change of preference information provided by the decision makers.

5 Conclusions

We have focused on group deicision making situations where the preference information given by the decision makers is expressed as intuitionistic preference relations or interval-valued intuitionistic preference relations. We have defined some compatibility measures for intuitionistic preference values, interval-valued intuitionistic preference values, intuitionistic preference relations and interval-valued intuitionistic preference relations, respectively, and studied their desirable properties. Based on these compatibility measures, we have developed a group consensus reaching procedure with intuitionistic preference relations, and a group consensus reaching procedure with interval-valued intuitionistic preference relations, respectively. Numerical analysis on the developed procedures has been conducted through a practical problem of the strategic alliance partner selection of a software company in China. Our measures and procedures have developed the theory of group decision making with preference relations, and laid a theoretic basis for the applications of intuitionistic preference relations and interval-valued intuitionistic preference relations in actual group decision making problems under uncertainty.

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