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A Decision Rule Aggregation Approach to Multiple Criteria-Multiple Participant Sorting

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Abstract A system that aggregates case-based linguistic decision rules using a hybrid of the dominance-based rough set approach (DRSA) and the Dempster–Shafer (DS) theory of evidence is proposed for multiple criterion-multiple participant sorting. First, DRSA is employed to infer linguistic decision rules that estimate the preferences of a few participants by means of their evaluations of representative case sets. Next, DS theory is applied to aggregate the decision rules triggered by all participants' evaluations of an alternative, thereby generating an overall decision recommendation for the alternative. The method is demonstrated on a numerical example.

Keywords Group decision making · Multiple criteria-multiple participant sorting · Dominance-based rough set approach · Dempster–Shafer theory of evidence · Decision rule aggregation

1 Introduction

Increasingly complex economic, societal and environmental issues force humans to tackle problems crossing many disciplines. Raiffa et al. (2002) categorized decision

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analysis methodologies as descriptive, normative, or prescriptive. Descriptive decision analysis addresses the actual decisions that are made in practice, normative decision analysis addresses how decisions should made, and prescriptive decision analysis provides practical methods for improving decisions. As a research area devoted mainly to perspective and normative analyses, group decision and negotiation (GDN) focuses on a common but essential activity for individuals, corporations, governments, and other organizations around the world. At its best, GDN can be an informative process that results in judgements that are particularly effective for strategic planning, for example by integrating expert feedback. Because organizational performance is usually improved by consideration of multiple perspectives, such processes can be a key to success for any enterprise.

GDN processes typically fall into one of two classes:

- Group decision making (GDM): A multi-party decision problem in which two or more independent, concerned parties must make a joint decision.
- Negotiation: A multi-party decision problem in which two or more independent, concerned parties may make a joint decision, or may choose to make no decision at all. In other words, negotiators have the option of “walking away”.

Support for GDM consists mainly of procedures to aggregate opinions, since the final result typically balances the points of view of the participants. Support for negotiation, on the other hand, is usually based on interactive analysis of the choices of different participants, and on the search for equilibria (stable outcomes) consistent with some set of assumptions about human behavior. Obviously, a GDM process is more controllable, and it is often assumed, explicitly or implicitly, that a “leader” is in charge. This leader usually does not express preferences over the alternatives under discussion, but rather controls the selection of analysis tools and aggregation methods, and may even adjust the weight given to different stakeholder inputs. In contrast, a negotiation is less controllable: There can be no leader, and the final result may be that there is no agreement, and therefore no decision.

The applications of GDM and negotiation are different because of these distinctions. For example, GDM may be carried out democratically, according to one of the many different voting procedures that can be used to determine group choices on social issues. In contrast, negotiation follows a procedure only if all participants accept it, and conflicts among individuals, corporations, or governments often remain unresolved, and may even evolve, over long periods of time.

The main theoretical tool for the analysis of negotiation is game theory, a set of models and methods that attempts to capture mathematically the behavioral issues in an interaction in which each side has preferences over all sides’ choices (Myerson 1997). Variants of game theory designed for the analysis of negotiation include metagame theory (Howard 1971), conflict analysis (Fraser and Hipel 1984), and the graph model for conflict resolution (Fang et al. 1993).

Research on GDM has a long history. Over two hundred years ago, Condorcet pioneered the mathematical analysis of social choice, asking how individual preferences could be aggregated into a collective preference or group decision (Arrow

1963). Condorcet's voting paradox pinpoints a fundamental problem: Used to aggregate preference, majority rule can fail to preserve transitivity if there are three or more alternatives—from a set of three or more individuals, a majority can prefer A over B, another majority can prefer B over C, and yet another majority can prefer C over A.

In a complex society, GDM processes must inevitably take many criteria (factors) into account. Thus, research on GDM that explicitly incorporates multiple criteria has been a major focus, and has made significant progress with the rapid development of operations research, management science, systems engineering, and other disciplines. Hwang and Lin (1987) seems to have been the first study to explore systematically how multiple criteria could be used in GDM. Very influential ideas on the use of multiple criteria decision analysis (MCDA) for GDM appeared in a 1998 issue of the journal *Group Decision and Negotiation*, Vol. 7(1).

Recent advances in information technology have made possible decision support systems (DSSs) designed explicitly for multiple criteria GDM. An early approach was Bui (1987), in which MCDA methods for individual preference elicitation and preference aggregation were proposed for the analysis, design and implementation of group DSSs to support GDM. More recently, Wallenius et al. (2008) provided a comprehensive summary of advances in the application of multiple-criteria methods to both negotiation and GDM.

This paper addresses multiple-criteria sorting, a fundamental problem of MCDA, in a GDM context. The objective is to integrate the opinions of several participants into a sorting. The hybrid approach proposed here combines dominance-based rough sets with the Dempster–Shafer theory of evidence. Specifically, the major contributions of this article include:

- *Preference elicitation by means of holistic judgements on representative case sets*: With representative case sets, participants need only specify preference directions over criteria. No measure of value (utility) is required. Then a DRSA generates linguistic decision rules, which effectively represent preferences. This process greatly eases preference elicitation.
- *Flexible information aggregation*: A procedure implementing three basic construction strategies is designed to capture decision information from decision rules. The Dempster–Shafer (DS) theory of evidence then provides a flexible mechanism for efficient information aggregation.

Thus, the proposed methodology sidesteps arguments about which specific models of utility are appropriate, and at the same time simplifies the measurement of individual preference.

The remainder of this paper is organized as follows: aggregation-based group DSSs are discussed in Sect. 2; applicable principles from MCDA are described in Sect. 3; the DRSA is explained in Sect. 4; and the Dempster–Shafer (DS) theory of evidence is applied to aggregate the decision rules of different participants in Sect. 5. Section 6 contains a demonstration of the method using a numerical example, and Sect. 7 presents some conclusions.

2 Rule Aggregation-based Group Decision Support

2.1 Multiple Criteria-Multiple Participant Sorting

The paper proposes a decision rule that aggregates viewpoints in order to solve a multiple criteria-multiple participant sorting (MCMPS) problem. An MCMPS problem is defined by the sets listed below (note that $|\mathbf{X}|$ represents the cardinality of the set $\mathbf{X}$):

- The set of alternatives, $\mathbf{A} = \{a^1, \ldots, a^i, \ldots, a^{|\mathbf{A}|}\}$, under assessment;
- The set of criteria, $\mathbf{C} = \{c_1, \ldots, c_j, \ldots, c_{|\mathbf{C}|}\}$, representing the measures by which the alternatives are evaluated;
- The set of participants, $\mathbf{E} = \{e_1, \ldots, e_k, \ldots, e_{|\mathbf{E}|}\}$, who are considered to have expert opinions;
- A set of classes, $\mathbf{Cl} = \{cl_1, \ldots, cl_l, \ldots, cl_{|\mathbf{Cl}|}\}$, that constitute the pre-defined groupings into which the alternatives of **A** are to be assigned. Note that the classes of **Cl** are often associated with preference; for example, it may be assumed that if $1 \leq g < h \leq |\mathbf{Cl}|$, then the participants see every alternative in cl_g as preferable to any alternative in cl_h.

To solve the MPMCS problem is to design a procedure that aggregates the views of the participants, in **E** as to the class in **Cl** to which each alterative in **A** should be assigned, consistent with the criteria in **C**. The procedure should be both theoretically sound and practical. It is assumed that the decision makers (DMs) (or analysts), who are in charge of the process for solving the MPMCS problem, do not have, or do not express, any explicit preferences over **A**. Their role is to choose the aggregation procedure and adjust the relative weightings of the participants.

MCMPS problems have a variety of practical applications (Zopounidis and Doumpos 2002). For example, for efficient management of inventories, which may include hundreds of items, it is usual to classify items into a few pre-defined categories based on expert assessments of criteria such as criticality, cost, and replenishment time. Then efficient inventory management can usually be achieved by following different policies for items in each class (Chen et al. 2008a). For example, in ABC analysis, a well-known inventory classification procedure, items are sorted into three groups, **A** (greatest management effort and attention), **B** (less effort), and **C** (least effort) (Silver et al. 1998).

Majority voting may seem to be a good procedure to carry out sorting by aggregating the views of different participants. In practical problems, however, it has two major shortcomings:

- *Quality of participants' recommendations*: Natural variability in participants' backgrounds, such as education and experience, often reduces the number who are experts in a particular sorting problem, though it may be useful to include others' judgements. Even if every participant has a realistic assessment of his or her own knowledge, voting procedures in which voters signal level of uncertainty along with assessment are difficult to design and implement. In MCDA it is common practice for the DM to assign a relative weight to each participant to reflect competence and aggregate opinions taking that weight into account. However, it would be preferable if such input information came from the participants and not the DM.

- *Process Efficiency*: Ties are common when participants have diverse opinions. For example, if there are three participants and three classes, each participant may assign an alternative to a different class, so that majority voting fails to determine a class for the alternative. The alternative may then be assigned to the middle class by default, or by some averaging procedure. Because the number of alternatives is often much larger than the number of categories, ties are both likely and problematic. Another issue associated with large sets of alternatives is that participants may be unable to assess them all consistently.

To make each input opinion meaningful while avoiding these problems, this paper's strategy is to identify the participants' preference patterns over alternatives and then aggregate them as they are applied to the entire alternative set to generate the final sorting. This idea is related to the development of an expert system (or knowledge-based system) that integrates knowledge from several experts (Ignizio 1991). Specifically, a preference elicitation and aggregation system is designed to identify participants' decision patterns in terms of linguistic decision rules; then these rules are aggregated to produce final results. The proposed system works within a MCDA framework, as presented next.

2.2 The Overall Framework

Figure 1 gives an overview of the group support system based on aggregation of participants' input. It includes two main components, the DRSA for decision rule *elicitation* and the Dempster–Shafer (DS) theory of evidence for decision rule *aggregation*. The steps, beginning with the basic MCDA structure (the set of alternatives, **A**, the set of criteria, **C**)) and the MCMPS participants $\left(\mathbf{E} = \left\{e_1, \ldots, e_k, \ldots, e_{|\mathbf{E}|}\right\}\right)$ are all under the control of the analysts. They are now summarized:

1. Elicitation of Individual Participants' Preferences
   - *Case Set Identification*: For each participant, e_k, a representative case set, $\mathbf{T}_k = \left\{t_k^1, \ldots, t_k^i, \ldots, t_k^{|\mathbf{T}_k|}\right\}$ must be identified. Each participant may receive a different case set; alternatively, a representative case set, $\mathbf{T} = \mathbf{T}_1 = \mathbf{T}_2 = \cdots = \mathbf{T}_{|\mathbf{E}|}$, may be assigned to all participants. Each participant's sorting of the case set is noted.
   - *DRSA Construction and Calculation*: The DRSA is then applied to each participant's response to analyze the sorting.
   - *Linguistic Decision Rules*: Linguistic decision rules are generated for each participant. For $i = 1, \ldots, |\mathbf{E}|$, participant e_k's linguisitic rules are denoted $\mathbf{R}_k = \left\{r_k^1, \ldots, r_k^i, \ldots, r_k^{|\mathbf{R}_k|}\right\}$.
2. Preference Aggregation based on Decision Rules
   - *Triggered Decision Rules from Participants*: To evaluate alternative a^i, where $i = 1, \ldots, |\mathbf{A}|$, apply all applicable linguistic decision rules and note which of the participants' decision rules are triggered.

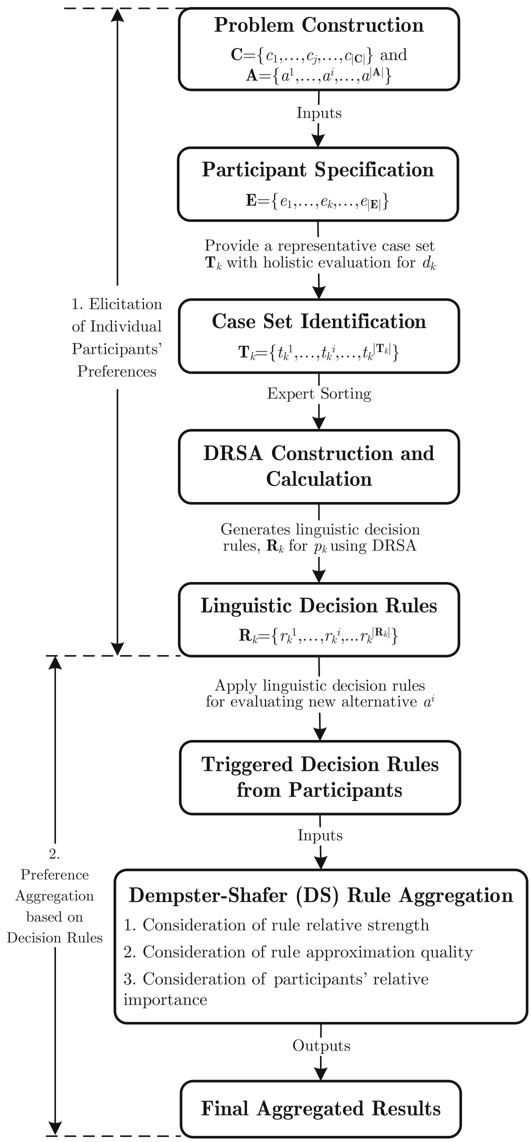

Fig. 1 The overall procedure of rule aggregation-based group decision support

- *DS Rule Aggregation*: Use DS theory to aggregate the triggered rules, considering such additional information as rule relative strength, rule approximation quality, and participants' relative importance.
- *Final Aggregated Results*: Select the most plausible result based on the outcome of the aggregated rules.

A detailed explanation of the above procedure is given below, following a brief introduction to MCDA. Descriptions of DRSA and DS theory, and their application to MCMPS appear in Sects. 4 and 5, respectively.

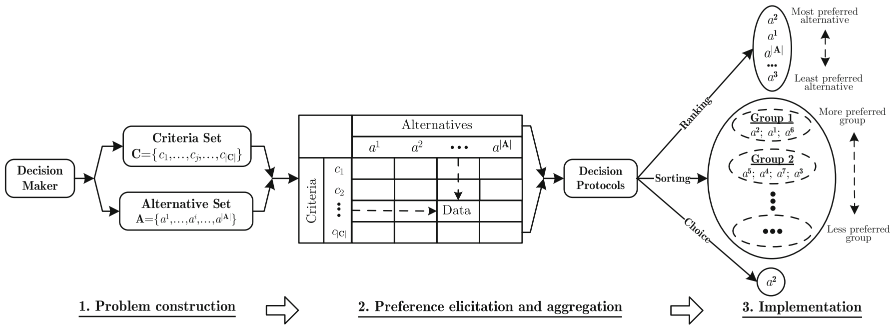

Fig. 2 Steps in multiple criteria decision analysis (Chen et al. 2008b)

3 Multiple Criteria Decision Analysis

3.1 General Description

The field of MCDA includes not only theory and methodological development but also practical techniques to help a DM identify, compare and evaluate alternatives. There are always two or more criteria, usually conflicting; for example, it is common for social, economic and environmental criteria to be included in a decision problem. According to Roy (1996), MCDA has three fundamental decisions involving alternatives from a set **A**:

- *Choice*. Choose the best alternative from **A**.
- *Sorting*. Sort the alternatives of **A** into relatively homogeneous groups, arranged in preference order.
- *Ranking*. Rank the alternatives of **A** from best to worst.

A typical MCDA analysis procedure is illustrated in Fig. 2. There are three key steps:

1. *Problem construction*: Define the DM's objectives, translate them into criteria by which alternatives can be measured, identify all available alternatives, and measure the *consequence* of each alternative on each criterion. Criteria may be qualitative or quantitative, but all consequences are expressed numerically.
2. *Preference elicitation and aggregation*: Model the DM's preferences for performance on each criterion and the DM's relative weights for the different criteria, thereby obtaining an overall evaluation of each alternative.
3. *Implementation*: Use the evaluations to assess all alternatives in order to choose from, sort, or rank the set **A**.

3.2 Case-Based Preference Elicitation

The DM's preferences are a fundamental input to any MCDA problem, so their elicitation and representation are obviously important. Generally speaking, there are two methodologies for preference elicitation: direct input and holistic elicitation. In direct input methods such as the multiattribute utility theory (MAUT) (Keeney and Raiffa 1976) and analytic hierarchy process (AHP) (Saaty 1980), the DM supplies explicit preference information that is used to calibrate a model of preference, so that all preference parameters are known explicitly.

Holistic elicitation methods, especially case-based methods, have been investigated extensively in recent years. They include the DRSA (Greco et al. 2001), the UTA method (Jacquet-Lagrèze and Siskos 1982), ELECTRE TRI Assistant (Mousseau and Slowinski 1998), UTADIS (UTilités Additives DIScriminantes) and MHDIS (Multi-group Hierarchical DIScrimination) (Doumpos and Zopounidis 2002), and the case-based distance approach (Chen et al. 2007). Most case-based elicitation methods are well-suited to sorting problems.

In a case-based holistic elicitation method, the DM furnishes a holistic (global) judgement on representative cases, which are then input to an optimization program that calibrates a preference model to approximate the DM's decisions as closely as possible. Generally, case-based approaches to preference elicitation in MCDA include three steps:

1. *Representation*: Identify representative cases from the full set of alternatives, or elsewhere, and present them to the DM for preference assessment.
2. *Inference*: Find preference parameters that reproduce the DM's judgement on the case set as accurately as possible.
3. *Application*: If the best-fit preference parameters reproduce the DM's judgement with sufficient clarity and accuracy, apply them to obtain preferences on the full set of alternatives. If there is too much ambiguity, add more cases to the case set and repeat. If the reproduction of the DM's judgement is too inaccurate, query the DM about possible contradictions in the judgements on the case set.

4 The Dominance-Based Rough Set Approach (DRSA)

4.1 Rough Set Theory

Pawlak (1982) introduced rough sets as a tool to describe dependencies among attributes and to evaluate the significance of individual attributes. Because of its ability to handle the inherent uncertainty or vagueness of data, rough set theory complements probability theory, evidence theory, fuzzy set theory, and other approaches. Recent advances in rough set theory have made it a powerful tool for data mining, pattern recognition, and information representation. Pawlak and Skowron (2007) is a comprehensive literature review of rough set theory, including summaries of research issues and applications.

As pointed out by Greco et al. (2001), the original rough set approach cannot efficiently extract information (DMs' preferences) from the analysis of a DM's judgements on a case set. The DRSA, an extension of rough set theory, works well in MCDA by replacing the *indiscernibility* relation with *dominance*, thereby allowing for inconsistent preference comparisons of alternatives over criteria and of orderings over sorting classes (groups). The main ideas of DRSA are summarized next.

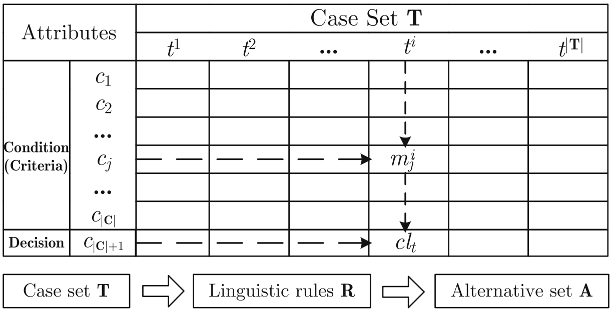

Fig. 3 The DRSA procedure

4.2 The Structure of DRSA

In the standard MCDA sorting problem, a DM assigns each alternative in the set $\mathbf{A} = \{a^1, \dots, a^i, \dots, a^{|\mathbf{A}|}\}$ to a member of $\mathbf{Cl} = \{cl_1, \dots, cl_l, \dots, cl_{|\mathbf{Cl}|}\}$ in accordance with the set of criteria $\mathbf{C} = \{c_1, \dots, c_j, \dots, c_{|\mathbf{C}|}\}$. The subset of $\mathbf{A}$ assigned to each of the classes in $\mathbf{Cl}$ must be non-empty, and for $1 \leq g < h \leq |\mathbf{Cl}|$, the DM must prefer every alternative in cl_g to any alternative in cl_h. Hence, $\mathbf{Cl}$ is often written as $cl_1 \succ cl_2 \succ \dots \succ cl_{|\mathbf{Cl}|}$, where $\succ$ is pronounced "is preferred to."

To implement case-based preference elicitation, the DM is asked to assign each alternative in the case set $\mathbf{T} = \{t^1, \dots, t^i, \dots, t^{|\mathbf{T}|}\}$ to a class in $\mathbf{Cl}$. Again, for all $g = 1, \dots, |\mathbf{Cl}|$, $cl_g \neq \emptyset$ and $\bigcup_{g=1}^{|\mathbf{Cl}|} cl_g = \mathbf{T}$. (Typically, $|\mathbf{A}| >> |\mathbf{T}| >> |\mathbf{Cl}|$.) Using DRSA, a set of decision rules, $\mathbf{R}$, can be obtained from the sorting of $\mathbf{T}$ and then applied to sort $\mathbf{A}$.

The DRSA-based sorting procedure is illustrated in Fig. 3. Here, m_j^i is the performance measurement (consequence) of alternative t^i according to criterion (or condition) c_j. It is assumed that the DM's preferences over each criterion are monotonic, i.e., for each $j = 1, \dots, |\mathbf{C}|$, c_j is either a positive criterion (the greater the performance, the more preferred the alternative, *ceteris paribus*) or a negative criterion (the greater the performance, the less preferred the alternative). Once the DM specifies the preference direction for each criterion, DRSA extracts a set of linguistic rules, $\mathbf{R}$, that captures the preference information in the DM's sorting of the case set. Then $\mathbf{R}$ is applied to sort $\mathbf{A}$ as required.

It is useful to define the *upward union* (signalled by the superscript "$\succeq$") and the *downward union* (signalled by superscript "$\preceq$") of a sorting $cl_1 \succ cl_2 \succ \dots \succ cl_{|\mathbf{Cl}|}$. For $h = 1, \dots, |\mathbf{Cl}|$, let $cl_h^{\succeq} = \bigcup_{g \leq h} cl_g$, and $cl_h^{\preceq} = \bigcup_{g \geq h} cl_g$. It is easy to show that $cl_{|\mathbf{Cl}|}^{\succeq} = cl_1^{\preceq} = \mathbf{Cl}$, and $cl_1^{\succeq} = cl_1$ and $cl_1^{\preceq} = cl_{|\mathbf{Cl}|}$.

4.3 Rough Approximation

Now assume that the DM's assignment of the cases in $\mathbf{T}$ to the classes in $\mathbf{Cl}$ is given. Let $\mathbf{P} \subseteq \mathbf{C}$ be a non-empty subset of criteria and define $D_{\mathbf{P}}$, a binary relation on $\mathbf{T}$, by $t^i D_{\mathbf{P}} t^l$ iff t^i is at least as good as t^l with respect to all the criteria in $\mathbf{P}$, where

$t^i, t^l \in \mathbf{T}$. It is assumed that $D_{\mathbf{P}}$ is a complete preorder, i.e. a reflexive, complete and transitive binary relation. For fixed $\mathbf{P} \subseteq \mathbf{C}$ and $t^i \in \mathbf{T}$, define the $\mathbf{P}$-*dominating set* and $\mathbf{P}$-*dominated set* of t^i to be $D_{\mathbf{P}}^{+}\left(t^i\right) = \left\{t^l \in \mathbf{T} : t^l D_{\mathbf{P}} t^i\right\}$ and $D_{\mathbf{P}}^{-}\left(t^i\right) = \left\{t^l \in \mathbf{T} : t^i D_{\mathbf{P}} t^l\right\}$, respectively.

Next, define the $\mathbf{P}$-*lower approximation* and the $\mathbf{P}$-*upper approximation* to $cl_h^{\succeq}$ by $\underline{\mathbf{P}}\left(cl_h^{\succeq}\right) = \left\{t^i \in \mathbf{T} : D_{\mathbf{P}}^{+}\left(t^i\right) \subseteq cl_h^{\succeq}\right\}$ and $\overline{\mathbf{P}}\left(cl_h^{\succeq}\right) = \left\{t^i \in \mathbf{T} : D_{\mathbf{P}}^{-}\left(t^i\right) \cap cl_h^{\succeq} \neq \emptyset\right\}$, respectively. Similarly, the $\mathbf{P}$-lower and $\mathbf{P}$-upper approximation to $cl_h^{\preceq}$ are $\underline{\mathbf{P}}\left(cl_h^{\preceq}\right) = \left\{t^i \in \mathbf{T} : D_{\mathbf{P}}^{-}\left(t^i\right) \subseteq cl_h^{\preceq}\right\}$ and $\overline{\mathbf{P}}\left(cl_h^{\preceq}\right) = \left\{t^i \in \mathbf{T} : D_{\mathbf{P}}^{+}\left(t^i\right) \cap cl_h^{\preceq} \neq \emptyset\right\}$. It is easy to verify that $\underline{\mathbf{P}}\left(cl_h^{\succeq}\right) \subseteq cl_h^{\succeq} \subseteq \overline{\mathbf{P}}\left(cl_h^{\succeq}\right)$ and $\underline{\mathbf{P}}\left(cl_h^{\preceq}\right) \subseteq cl_h^{\preceq} \subseteq \overline{\mathbf{P}}\left(cl_h^{\preceq}\right)$.

Define the $\mathbf{P}$-boundaries of $cl_h^{\succeq}$ and $cl_h^{\preceq}$ by $Bn_{\mathbf{P}}\left(cl_h^{\succeq}\right) = \overline{\mathbf{P}}\left(cl_h^{\succeq}\right) - \underline{\mathbf{P}}\left(cl_h^{\succeq}\right)$ and $Bn_{\mathbf{P}}\left(cl_h^{\preceq}\right) = \overline{\mathbf{P}}\left(cl_h^{\preceq}\right) - \underline{\mathbf{P}}\left(cl_h^{\preceq}\right)$, respectively. Then a natural measure of the quality of classification of $\mathbf{T}$ into $\mathbf{Cl}$ using only the criteria of $\mathbf{P}$ is

$$\gamma_{\mathbf{P}}(\mathbf{Cl}) = \frac{1}{|\mathbf{T}|}\left|\mathbf{T} - \bigcup_{cl_h \in \mathbf{Cl}} \left(Bn_{\mathbf{P}}\left(cl_h^{\succeq}\right) \cup Bn_{\mathbf{P}}\left(cl_h^{\preceq}\right)\right)\right|.$$

It is clear that $0 < \gamma_{\mathbf{P}}(\mathbf{Cl}) \leq 1$.

4.4 Decision Rules

The approximations obtained through dominance analysis can be used to construct decision rules capturing the preference information contained in the sorting of a case set. For $c_j \in \mathbf{C}$, let $D_{c_j} = D_{\{c_j\}}$. Criterion c_j is positive iff $m_j\left(t^i\right) \geq m_j\left(t^l\right)$ implies $t^i D_{c_j} t^l$ for all $t^i, t^l \in \mathbf{T}$. First, assume that all criteria are positive. The decision rules that can be generated from a non-empty set of criteria $\mathbf{P} \subseteq \mathbf{C}$ to sort $\mathbf{A}$ into $\mathbf{Cl}$ are of one of three types, as follows:

- $D_{\succeq}$-decision rule: If $m_j\left(t^i\right) \geq r_j$, for all $c_j \in \mathbf{P}$, then $t^i \in cl_h^{\succeq}$.
- $D_{\preceq}$-decision rule: If $m_j\left(t^i\right) \leq r_j$, for all $c_j \in \mathbf{P}$, then $t^i \in cl_h^{\preceq}$.
- $D_{\succeq\preceq}$-decision rule: If $m_j\left(t^i\right) \geq r_j$, for all $c_j \in \mathbf{O} \subset \mathbf{P}$ and $m_j\left(t^i\right) \leq r_j$ for all $c_j \in \mathbf{P} - \mathbf{O}$, then $t^i \in cl_h \cup cl_{h+1} \cup \cdots \cup cl_g = cl_h^{\succeq} \cap cl_g^{\preceq}$.

For each $c_j \in \mathbf{C}$, $r_j \in \mathbb{R}$ is called the generated performance threshold for criterion c_j. If criterion c_j is a negative criterion, the condition $m_j\left(t^i\right) \geq r_j$ is replaced by $m_j\left(t^i\right) \leq r_j$, and $m_j\left(t^i\right) \leq r_j$ by $m_j\left(t^i\right) \geq r_j$.

A set of decision rules is *complete* with respect to $\mathbf{T}$ if it classifies every alternative of $\mathbf{T}$ into one or more groups. A set of decision rules is *minimal* if it is complete and non-redundant, i.e. dropping any rule makes the set incomplete (Greco et al. 2001).

Let $\mathbf{R}$ be a set of decision rules generated from $\mathbf{T}$. The relative strength of decision rule $r \in \mathbf{R}$ is $\alpha(r)$, the ratio of the number of cases supporting r to the total number of upper or lower approximation classes with which r is associated. Note that $0 < \alpha(r) \leq 1$. The most popular rule induction system for DRSA is DOMLEM (Greco et al. 2002), which has been implemented in the 4eMka2 software (ICS 2008).

4.5 Rule Application

Given a new alternative, $a^i \in \mathbf{A}$, a procedure for application of a set of generated decision rules $\mathbf{R}$ was proposed by (Blaszczynski et al. 2007).

- If a^i triggers no rules, assign a^i to all classes.
- If a^i triggers one or more rules, begin by taking the intersection of the unions from all triggered "at least" ($D_{\succeq}$) rules and identify the highest class of the intersection, cl_h, as the lower bound for the assignment of a^i. Then take the intersection of the unions from all triggered "at most" ($D_{\preceq}$) rules and identify the lowest class of the intersection, cl_g, as the upper bound for the assignment of a^i. Then a^i is assigned according to the following rules:
  - If $h = g$, then assign a^i to $cl_h = cl_g$;
  - if $h < g$, then assign a^i to $cl_h \cup cl_{h+1} \cup \cdots \cup cl_g = cl_h^{\succeq} \cap cl_g^{\preceq}$ with no possibility of refinement because of imprecise information;
  - if $h > g$, then assign a^i to $cl_g \cup cl_{g+1} \cup \cdots \cup cl_h = cl_g^{\succeq} \cap cl_h^{\preceq}$ with no possibility of discernment because of contradictory information.

Note that this procedure does not consider available information gauging the precision of decision rules, such as the overall approximation quality, $\gamma_{\mathbf{P}}(\mathbf{Cl})$, and the relative strength of decision rules, $\alpha(r)$, for $r \in \mathbf{R}$. By appropriate integration of this information, the overall classification quality can be improved.

Of course, this DRSA sorting procedure is designed for a single DM. Clearly, it can be applied to data provided by several different DMs to produce decision rules based on the same, or different, case sets. To solve the MCMPS problem, we next propose an approach to aggregating decision rules. Information about classification quality will be used in this procedure to help address inconsistencies between the decision rules. We turn now to a description of the procedure, which is based on the Dempster–Shafer theory of evidence (DS).

5 Dempster–Shafer (DS) Rule Aggregation

5.1 The Dempster–Shafer Theory of Evidence

Dempster (1968) and Shafer (1976) developed a mathematical theory of evidence, based on belief functions and plausibility reasoning, which can be used to combine separate pieces of information (evidence) to assess the probability of an event. The Dempster–Shafer (DS) theory of evidence is summarized next.

Let $\mathbf{\Theta} = \left\{h_1, \ldots, h_t, \ldots, h_{|\mathbf{\Theta}|}\right\}$ be the universal set, consisting of all states under consideration. The power set of $\mathbf{\Theta}$, called $\mathbb{P}(\mathbf{\Theta})$, is the set of all subsets of $\mathbf{\Theta}$, including the empty set. Any $\mathbf{H} \in \mathbb{P}(\mathbf{\Theta})$, i.e. $\mathbf{H} \subseteq \Theta$, is called a *proposition*; the proposition $\mathbf{H}$ is a *singleton* iff it contains exactly one state of $\mathbf{\Theta}$.

The DS theory begins with a belief mass function defined on each element of the power set. Formally, $m : \mathbb{P}(\mathbf{\Theta}) \to [0, 1]$ is a *basic belief assignment (BBA)* if it satisfies $m(\emptyset) = 0$ and $\sum_{h_t \in \mathbb{P}(\mathbf{\Theta})} m(h_t) = 1$. Using a BBA, two measures of confidence for a proposition $\mathbf{E} \subseteq \mathbf{\Theta}$ can be defined.

- The *belief* in $\mathbf{E}$, $Bel(\mathbf{E})$, is the sum of the masses associated with all subsets of $\mathbf{E}$. Formally, $Bel(\mathbf{E}) = \sum_{\mathbf{F} \subseteq \mathbf{E}} m(\mathbf{F})$.
- The *plausibility* of $\mathbf{E}$, $Pl(\mathbf{E})$ is the sum of the masses associated with all sets that intersect $\mathbf{E}$. Formally, $Pl(\mathbf{E}) = \sum_{\mathbf{F} \cap \mathbf{E} \neq \emptyset} m(\mathbf{F})$.

Dempster's rule of combination provides a way to integrate two BBAs, m_1 and m_2, considered as two independent pieces of evidence, to produce the BBA $m_1 \oplus m_2$, defined as follows:

$$(m_1 \oplus m_2)(\mathbf{G}) = \begin{cases} 0, & \text{if } \mathbf{G} = \emptyset, \\ \frac{1}{1-K} \sum_{\mathbf{E} \cap \mathbf{F} = \mathbf{G}} m_1(\mathbf{E}) m_2(\mathbf{F}), & \text{if } \mathbf{G} \neq \emptyset, \end{cases}$$

where $K = \sum_{\mathbf{E} \cap \mathbf{F} = \emptyset} m_1(\mathbf{E}) m_2(\mathbf{F})$. Note that K is a measure of the amount by which mass functions m_1 and m_2 conflict.

Recently, many refined belief elicitation and aggregation approaches have been developed, suggesting a range of new methods to acquire, test and integrate pieces of evidence. For example, Hurley and Shogren (2005) used an experiment to test whether an induced probability can be recovered using an elicitation mechanism based on predictions about a random event; Chambers and Melkonyan (2008) developed an algorithm to approximate a DM's beliefs for a very general class of decision-theoretic models that includes many common preference structures; and Yager et al. (1994) collected a useful set of articles on DS, fuzzy reasoning, and neural computing.

5.2 Application of DS for Rule Aggregation

The DS theory is an alternative to the Bayesian approach that models changing beliefs by manipulating subjective probabilities. In most cases, a BBA can be associated with any hypotheses—a singleton, a larger subset of $\mathbf{\Theta}$, or even the entire universe. In some applications of DS theory to MCDA, including DeKorvin and Shipley (1993), Yang and Singh (1994) and Butler et al. (1995), the BBA is based on uncertainty in preference over alternatives, reflecting conflicting criteria. A recent series of papers, including Beynon et al. (2000) and Beynon (2002, 2005, 2006), systematically utilizes DS theory to develop a DS-AHP procedure for GDM that improves the "standard" AHP analysis (Saaty 1980) by reducing the number of pairwise comparisons and eliminating the need to check for inconsistent pairwise judgements.

As explained above, DS and DRSA are two different methods of processing and integrating information. DS assesses the probability of events defined by particular preferences, while DRSA evaluates preferences on the basis of a data sample and uses them to generate linguistic rules that represent preference in the presence of uncertainty or vagueness. The two approaches can be complementary, as shown in Fig. 1, where DRSA is used for preference elicitation and then DS provides information aggregation.

To connect DRSA with DS, three types of "BBA" are designed to transfer the analysis results from the DRSA, as explained below. In particular, DS theory can be applied to classify an alternative, $a^i \in \mathbf{A}$, into one or more classes of $\mathbf{Cl} = \{cl_1, \ldots, cl_{|\mathbf{Cl}|}\}$ by aggregating decision rules across participants, thereby solving the MPMCS problem. Begin by identifying possibilities and classes, i.e. by setting the universal set as $\mathbf{\Theta} = \mathbf{Cl}$. Note that a proposition is then the assignment of a^i to a class, or set of classes. Carry out the following steps:

1. *Identify each participant's propositions.* Obtain different propositions for a^i using triggered decision rules from each participant. As explained in Sect. 4.4, the result $a^i \in cl_h^{\succeq}$, obtained from a $D_{\succeq}$-decision rule (when all criteria are positive), can be interpreted as "a^i is at least in class cl_h", and hence coded as $\mathbf{H} = \{cl_1, \ldots, cl_{h-1}, cl_h\}$. Similarly, $a^i \in cl_h^{\preceq}$ can be interpreted as "a^i is at best in class cl_h", and coded as $\mathbf{H} = \left\{cl_h, cl_{h+1}, \ldots, cl_{|\mathbf{Cl}|}\right\}$. Finally, $a^i \in cl_h^{\succeq} \cap cl_g^{\preceq}$, obtained from a $D_{\succeq\preceq}$-decision rule, can be coded as $\mathbf{H} = \left\{cl_h, cl_{h+1}, \ldots, cl_{g-1}, cl_g\right\}$.
2. *Select a Basic Belief Assignment for each participant.* To construct a BBA, $m : \mathbb{P}(\mathbf{\Theta}) \rightarrow [0, 1]$ for a participant, three types of information about the participants and the precision of the rules they generated can be considered.
   - I_1. Each triggered rule, r, has a relative strength, $\alpha(r)$, as calculated above. Note that $0 < \alpha(r) \leq 1$.
   - I_2. The overall approximation quality of the set of decision rules generated by a participant, e_k, is $\gamma(e_k)$, as defined above. Note that $0 < \gamma(e_k) \leq 1$.
   - I_3: The relative importance of participants in the MPMCS problem is determined by a weight vector $\mathbf{w} = (w_1, \ldots, w_k, \ldots, w_{|\mathbf{E}|})$. Formally, w_k is the weight to be assigned to participant e_k's input to the sorting. Note that the weight vector must satisfy $\sum_{k=1}^{|\mathbf{E}|} w_k = 1$ and $w_k > 0$ for $k = 1, \ldots, |\mathbf{E}|$. Participant weights are similar to criterion weights in MCDA, and give the DM or analyst the opportunity to emphasize or de-emphasize certain experts according to their perceived or measured ability. Of course, all weights may be equal. Here it is simply assumed that an unambiguous weight vector is available.

   Three methods of constructing a BBA, $m_k(\cdot) = m(\cdot)$, representing the beliefs of participant $e_k \in \mathbf{E}$ are now suggested.
   - Consideration of I_1 only. Determine the relative strength, $\beta(\mathbf{H})$, that e_k associates with each proposition, $\mathbf{H}$. Let $\mathbf{R_H}$ denote the set of all rules of e_k that produce $\mathbf{H}$ and set $\beta(\mathbf{H}) = \max_{r \in \mathbf{R_H}} \alpha(r)$. Now assign $m(\mathbf{H}) = \beta'(\mathbf{H})$, the normalized relative strength of e_k's belief in $\mathbf{H}$, calculated as follows:

     $$\beta'(\mathbf{H}) = \frac{\beta(\mathbf{H})}{\sum_{\mathbf{H} \subseteq \mathbf{Cl}} \beta(\mathbf{H})}.$$

     Note that if there is only one rule in $\mathbf{R_H}$, then $m(\mathbf{H}) = \beta(\mathbf{H})$ and $m(\mathbf{\Theta}) = 1 - \beta(\mathbf{H})$.
   - Consideration of I_1 and I_2. First, determine the relative strength $\beta(\mathbf{H})$ of e_k's belief in each proposition $\mathbf{H}$, as above. Continue by calculating $\beta'(\mathbf{H})$, the normalized relative strength, as above. Then, for each of e_k's propositions, set $\beta''(\mathbf{H}) = \beta'(\mathbf{H}) \cdot \gamma(e_k)$, where "$\cdot$" indicates multiplication. Finally, set $m(\mathbf{H}) = \beta''(\mathbf{H})$. Note that $m(\mathbf{\Theta}) = 1 - \gamma(e_k)$. If there is only one proposition, then $m(\mathbf{H}) = \beta(\mathbf{H}) \cdot \gamma(e_k)$ and $m(\mathbf{\Theta}) = 1 - \beta(\mathbf{H}) \cdot \gamma(e_k)$.

- Consideration of I_1, I_2, and I_3. Proceed as above, calculating the normalized relative strength, $\beta'(\mathbf{H})$, of e_k's belief in proposition $\mathbf{H}$, for each proposition $\mathbf{H}$. But now set $\beta'''(\mathbf{H}) = \beta'(\mathbf{H}) \cdot \gamma(e_k) \cdot w_k$ for all propositions $\mathbf{H}$ of e_k. Finally, set $m(\mathbf{H}) = \beta'''(\mathbf{H})$ and $m(\mathbf{\Theta}) = 1 - \gamma(e_k) \cdot w_k$. If e_k has only one proposition, then $m(\mathbf{H}) = \beta(\mathbf{H}) \cdot \gamma(e_k) \cdot w_k$ and $m(\mathbf{\Theta}) = 1 - \beta(\mathbf{H}) \cdot \gamma(e_k) \cdot w_k$.

3. Finally, treat propositions from different participants as evidence from different sources and apply Dempster's rule of combination to aggregate them. Then classify a^i into cl_h, where cl_h is a singleton proposition with the maximum value of $Bel(cl_h)$.

6 Numerical Example

6.1 Basic Information

A numerical example borrowed from Greco et al. (2002) is used to demonstrate the proposed procedure. A set of ten firms was chosen and subjected to a risk analysis according to twelve criteria and then classified into three pre-defined classes. Hence, $\mathbf{T} = \{t^1, \ldots, t^{10}\}$, $\mathbf{C} = \{c_1, \ldots, c_{12}\}$ and $\mathbf{Cl} = \{cl_1, cl_2, cl_3\}$. The detailed measurements (consequences) of the ten firms over the twelve criteria are listed in Table 1.

The classes in $\mathbf{Cl}$ are cl = "acceptable", cl_2 = "uncertain", and cl_3 = "unacceptable". The criteria of $\mathbf{C}$ are c_1: earnings before interest and taxes as a fraction of total assets, c_2: net income as a fraction of net worth, c_3: total liabilities divided by total assets, c_4: total liabilities divided by annual cash flow, c_5: interest expenses divided by sales, c_6: general and administrative expenses as a percentage of sales, c_7: managers' work experience, c_8: subjective measure of market niche or position, c_9: subjective measure of technical structure and facilities, c_{10}: organization and personnel, c_{11}: assessment of special competitive advantage, and c_{12}: market flexibility (Greco et al. 2002).

Table 1 Measurements of ten firms according to twelve criteria

T	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
t^1	1	1	2	1	1	1	3	3	4	4	2	3
t^2	3	5	2	1	1	1	3	2	3	4	1	3
t^3	2	2	1	1	1	1	3	3	3	4	3	4
t^4	2	1	1	1	1	3	2	2	4	4	2	3
t^5	1	1	3	1	2	1	3	4	4	4	3	4
t^6	2	1	2	1	1	2	4	3	3	2	1	2
t^7	2	2	2	2	1	3	5	3	5	4	2	4
t^8	4	5	2	3	3	3	5	4	5	5	4	5
t^9	3	5	1	1	2	2	5	3	5	5	3	5
t^{10}	2	3	2	1	2	4	5	2	5	4	3	4

Table 2 Expert assignments of the ten firms

T	e_1	e_2	e_3
t^1	3	3	2
t^2	3	2	3
t^3	3	3	2
t^4	2	3	3
t^5	2	1	1
t^6	2	2	2
t^7	1	2	3
t^8	1	2	1
t^9	1	1	2
t^{10}	1	1	1

Table 3 Decision rules for e_1

	Decision rules	Relative strength
1.	If $c_6 \leq 1$ & $c_8 \leq 3$, then at most cl_3	1
2.	If $c_7 \leq 4$, then at most cl_2	1
3.	If $c_9 \geq 5$, then at least cl_1	1
4.	If $c_6 \geq 2$, then at least cl_2	0.857
5.	If $c_3 \geq 3$, then at least cl_2	0.143
	Overall approximation quality	$\gamma(e_1) = 1$

The first six criteria are quantitative (financial ratios) and the last six are qualitative. The six qualitative criteria take values in an ordinal scale from 4 or 5 (best) to 1 (worst). The six quantitative criteria were transformed to ordinal by splitting the original scales into intervals coded by integers from 4 or 5 (best) to 1 (worst) (Greco et al. 2002).

6.2 Decision Rule Generation

Three experts ($\mathbf{E} = \{e_1, e_2, e_3\}$) have used their judgments to assign the ten alternatives of ($\mathbf{T}$) into the pre-defined three classes of ($\mathbf{Cl}$). (Note that $\mathbf{T} = \mathbf{T}_1 = \mathbf{T}_2 = \mathbf{T}_3$). The assessments are shown in Table 2.

Next, the software 4eMka2 (ICS 2008) is employed to generate decision rules using the minimal decision algorithm for each participant. All criteria are set as positive. The decision rules for e_1, e_2 and e_3 are summarized in Tables 3, 4 and 5, respectively.

6.3 Aggregation of Decision Rules using DS

To demonstrate the proposed DS-based aggregation procedure, the case set $\mathbf{T}$ is reclassified using the decision rules obtained from each participant. Then a sorting is obtained by assigning each alternative in $\mathbf{T}$ to a class by using DS Theory to combine the three

Table 4 Decision Rules for e_2

	Decision rules	Relative strength
1.	If $c_7 \leq 2$, then at most cl_3	0.333
2.	If $c_6 \leq 1$ & $c_3 \leq 3$, then at most cl_3	0.333
3.	If $c_1 \leq 1$ & $c_{12} \leq 3$, then at most cl_3	0.333
4.	If $c_5 \leq 1$, then at most cl_2	1
5.	If $c_{12} \geq 5$, then at most cl_2	1
6.	If $c_6 \geq 4$, then at least cl_1	0.5
7.	If $c_3 \geq 3$, then at least cl_1	0.5
8.	If $c_7 \geq 4$, then at least cl_2	0.714
9.	If $c_2 \geq 5$, then at least cl_2	0.429
	Overall approximation quality	$\gamma(e_2) = 0.8$

Table 5 Decision Rules for e_3

	Decision rules	Relative strength
1.	If $c_7 \leq 2$, then at most cl_3	0.5
2.	If $c_{11} \leq 1$ & $c_8 \leq 2$, then at most cl_3	0.5
3.	If $c_5 \leq 1$, then at most cl_2	0.857
4.	If $c_3 \leq 1$, then at most cl_2	0.429
5.	If $c_8 \geq 4$, then at least cl_1	0.667
6.	If $c_6 \geq 4$, then at least cl_1	0.333
7.	If $c_{11} \geq 3$, then at least cl_2	1
8.	If $c_{12} \leq 2$, then at least cl_2	0.333
9.	If $c_4 \geq 2$ & $c_{12} \leq 4$, then at least cl_2	0.333
10.	If $c_1 \leq 2$ & $c_{12} \leq 3$, then at least cl_2	0.333
	Overall approximation quality	$\gamma(e_3) = 0.7$

experts' triggered decision rules. In the illustration, the aggregation procedures are used only to classify alternatives in the case set. Of course, the same techniques can be applied to new alternatives, not in any case set, once the DRSA decision rules are available.

6.3.1 Aggregation Based on I_1 Only

First, rule aggregation is demonstrated using only I_1, the relative strength of rules, obtained as described in Sect. 5.2. The final result is summarized in the middle columns of Table 6 (under the heading I_1). In this example, DS-based aggregation is consistent with majority voting, yet it provides more information, including the degrees of belief and plausibility of the final result (fourth column of Table 6). As well, it works even when majority voting is inconclusive; for instance, each DM assesses $a^7 = t^7$ differently, but the system unambiguously assigns it to cl_2, as elaborated below.

The details of the calculation for $a^7 = t^7$ illustrate how DS-based aggregation works:

- For e_1, the triggered decision rules concerning a^7 are Rule No. 3 in Table 3, if $c_9 \geq 5$, then a^7 is at least in cl_1, with $\alpha = 1$, and Rule No. 4 in Table 3, if $c_6 \geq 2$, then a^7 is at least cl_2, with $\alpha = 0.857$. Hence, $\mathbf{H}_1 = \{cl_1\}$ and $\mathbf{H}_2 = \{cl_1, cl_2\}$ with $m(\mathbf{H}_1) = 0.538$ and $m(\mathbf{H}_2) = 0.462$ (after normalization of the rule strengths, α).
- For e_2 and e_3, similar calculations are carried out; the details are omitted here. The propositions generated for e_2 are $\mathbf{H}_3 = \{cl_1, cl_2\}$ and $\mathbf{H}_4 = \{cl_2, cl_3\}$ with $m(\mathbf{H}_3) = 0.417$ and $m(\mathbf{H}_4) = 0.583$. The propositions generated for e_3 are $\mathbf{H}_5 = \{cl_2, cl_3\}$ and $\mathbf{H}_6 = \{cl_1, cl_2, cl_3\}$ with $m(\mathbf{H}_5) = 0.857$ and $m(\mathbf{H}_6) = 0.143$.
- Dempster's rule of combination is then used to integrate the three experts' propositions. The detailed calculations are suppressed, but the results are $Bel(cl_2) = 0.879$, $Bel(cl_1) = 0.065$ and $Bel(cl_1, cl_2) = 1$. Since its singleton proposition value, 0.879, is maximal, $a^7 = t^7$ is assigned to cl_2.

6.3.2 Aggregation Based on I_1 and I_2

Using both I_1 and I_2 means considering the relative strength of rules and the overall approximation quality in the aggregation, as described in Sect. 5.2. The final result based on the combination of I_1 and I_2 is shown in the right-hand columns of Table 6, which facilitates comparison with the aggregation based solely on I_1 (middle columns of the same table). Some observations about the comparison follow:

- The two sortings are relatively close. Eight of the ten alternatives are assigned to the same class, and the remaining two are assigned to adjacent classes.
- For the two divergent classifications (for t^4 and t^5), the $I_1\&I_2$-based aggregation is not consistent with majority voting, and obviously favors the recommendation

Table 6 Final aggregated sortings based on I_1 and $I_1\&I_2$

T	Participant			I_1		I_1 & I_2	
	e_1	e_2	e_3	Final result	$[Bel, Pl]$	Final result	$[Bel, Pl]$
t^1	3	3	2	3	[0.625, 1]	3	[0.6, 1]
t^2	3	2	3	3	[0.603, 0.881]	3	[0.563, 0.895]
t^3	3	3	2	3	[0.435, 0.696]	3	[0.483, 0.805]
t^4	2	3	3	3	[0.374, 0.711]	2	[0.303, 0.731]
t^5	2	1	1	1	[0.4, 1]	2	[0.438, 0.859]
t^6	2	2	2	2	[0.658, 1]	2	[0.543, 1]
t^7	1	2	3	2	[0.879, 0.935]	2	[0.63, 0.801]
t^8	1	2	1	1	[0.723, 1]	1	[0.668, 1]
t^9	1	1	2	1	[0.45, 0.835]	1	[0.48, 0.891]
t^{10}	1	1	1	1	[0.796, 1]	1	[0.745, 1]

of e_1. This emphasis reflects the relative strengths of the approximation quality from the three participants, as $\gamma(e_1) = 1 > \gamma(e_2) = 0.8 > \gamma(e_3) = 0.7$.
- For most alternatives, the belief intervals, $[Bel, Pl]$, using $I_1\&I_2$-based aggregation are narrower than those generated by I_1-based aggregation. Since more belief is assigned to the universal set, i.e. $m(\mathbf{\Theta}) = m(\{cl_1, cl_2, cl_3\})$, when information I_2 is incorporated, it is easy to understand why these beliefs are more compressed. The introduction of I_2 into the aggregation adjusts (or "corrects for") the inconsistency of the experts' holistic assessments of the case set provided according to the pre-defined preference directions (relationships), as explained in Sect. 4. Opinions of more consistent participants have more bearing on the final result, and overall belief intervals are decreased because judgement inconsistencies have been detected in the assessments of participants e_2 and e_3.

Similar calculations can be carried out to aggregate decision rules and produce a sorting based on information $I_1\&I_2\&I_3$, which incorporates different weights for the participants (I_3). Both I_2 and I_3 are measures of the amount of emphasis to be given to different participants; the difference between them is that I_2 reflects objective information and approximation, while I_3 is subjective, provided by the DM who is in charge of the decision process, based on experience.

7 Conclusions

A system to carry out multiple criterion-multiple participant sorting is proposed. The method aggregates case-based linguistic decision rules using a hybrid of the DRSA and the Dempster–Shafer (DS) theory of evidence. It requires relatively little input information—preference directions for criteria, plus one or more holistic sortings of representative case sets for training. The feasibility of the method is demonstrated using a numerical example. In future work, it would also be good to compare the DS assessment with some methods based on possibility theory to find out if similar results could be obtained.

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