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Induced and Linguistic Generalized Aggregation Operators and Their Application in Linguistic Group Decision Making

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Abstract We introduce a wide range of induced and linguistic generalized aggregation operators. First, we present the induced linguistic generalized ordered weighted averaging (ILGOWA) operator. It is a generalization of the OWA operator that uses linguistic variables, order inducing variables and generalized means in order to provide a more general formulation. One of its main results is that it includes a wide range of linguistic aggregation operators such as the induced linguistic OWA (ILOWA), the induced linguistic OWG (ILOWG) and the linguistic generalized OWA (LGOWA) operator. We further generalize the ILGOWA operator by using quasi-arithmetic means obtaining the induced linguistic quasi-arithmetic OWA (Quasi-ILOWA) operator and by using hybrid averages forming the induced linguistic generalized hybrid average (ILGHA) operator. We also present a further extension with Choquet integrals. We call it the induced linguistic generalized Choquet integral aggregation (ILGCIA). We end the paper with an application of the new approach in a linguistic group decision making problem.

Keywords Linguistic aggregation operators · OWA operator · Choquet integral · Linguistic group decision making

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1 Introduction

The ordered weighted averaging (OWA) operator (Yager 1988) is a very well-known aggregation operator for fusing numerical information (Beliakov et al. 2007; Calvo et al. 2002). However, we may find situations where the available information is vague or imprecise and it is not possible to analyze it with numerical values. Thus, it is necessary to use another approach such as a qualitative one that uses linguistic assessments (Zadeh 1975). In the literature, we find different types of OWA operators that use linguistic information (Herrera et al. 2008, 1995; Herrera and Martínez 2000; Xu 2008). In this paper, we follow the ideas of the induced linguistic OWA (ILOWA) and the induced linguistic OWG (ILOWG) operator (Xu 2006a,b). Note that these operators represent a linguistic version of the induced aggregation operators introduced by Yager and Filev (1999). Since its introduction, they have been studied by a lot of authors. For example, Yager (2003, 2004a) developed several properties and an application with the Choquet integral. Merigó and Casanovas (2009) developed an application in decision making with Dempster–Shafer theory of evidence. Wei et al. (2010) studied several types of induced aggregation operators with intuitionistic fuzzy sets.

Another interesting extension of the OWA operator is the generalization that uses generalized means and quasi-arithmetic means. These types of aggregations are known as the generalized OWA (GOWA) operator (Karayiannis 2000; Yager 2004b) and the Quasi-OWA operator (Fodor et al. 1995). They generalize a wide range of aggregation operators such as the average, the OWA and the OWG operator. Recently, Merigó and Gil-Lafuente (2009) have suggested a generalization of the induced OWA (IOWA) operator by using generalized means. This operator is known as the induced generalized OWA (IGOWA) operator and it generalizes a wide range of aggregation operators such as the OWA and the IOWA operator. Note that a further generalization is possible by using quasi-arithmetic means (Quasi-IOWA operator). Further extensions have been introduced recently. For example, Merigó and Casanovas (2010a,b) developed a fuzzy version of the GOWA operator and extended it by using hybrid averages. They also developed a generalization by using heavy aggregations (Merigó and Casanovas 2010c). Zhao et al. (2010) considered the use of intuitionistic fuzzy sets with generalized aggregation operators. Zhou and Chen (2010a,b) developed a generalized logarithmic aggregation operator and a continuous generalized aggregation.

Recently, Merigó and Gil-Lafuente (2008) have suggested the induced linguistic generalized OWA (ILGOWA) operator. Going a step further, one of the objectives of this paper is to analyze the ILGOWA operator in more detail considering a wide range of properties of this aggregation operator. The ILGOWA represents an extension of the IGOWA operator for the cases where the available information is assessed with linguistic variables. It also uses order inducing variables in order to represent complex reordering processes in the aggregation process. Thus, we are able to generalize a wide range of linguistic aggregation operators such as the ILOWA, the linguistic OWA (LOWA), the linguistic weighted average (LWA), the linguistic generalized mean (LGM), the linguistic weighted generalized mean (LWGM) and the linguistic GOWA (LGOWA).

Note that different approaches have been developed for dealing with linguistic information (Bonissone 1982; Zadeh 1975). In this paper, we focus on the ideas of Xu (2004, 2008) where we are able to compute with words directly without losing information in the computation process. Moreover, it is worth noting that Wang and Hao (2006) considered a generalization of the LOWA operator by using quasi-arithmetic means. However, their model is focused on the 2-tuple linguistic approach (Herrera and Martínez 2000) and it does not consider induced aggregation operators. Furthermore, note that our model is different from the model suggested by Xu (2006b). Xu called generalized induced linguistic OWA to a model based on a generalization of the order inducing variables. However, he did not analyze the use of generalized and quasi-arithmetic means.

We also present a further generalization of the ILGOWA operator by using quasi-arithmetic means. We call it the Quasi-ILOWA operator. Note that the Quasi-ILOWA has also been considered by Merigó and Gil-Lafuente (2008). The main advantage of this approach is that it includes the ILGOWA as a special case and a lot of other cases. Thus, we get a more robust formulation of this model.

Moreover, we also extend this approach by using the hybrid average (Xu and Da 2003). By doing so, we are able to use the weighted average and the IOWA in the same formulation and in an uncertain environment that can be assessed with linguistic variables. We call it the induced linguistic generalized hybrid averaging (ILGHA) operator. One of its key features is that it includes a wide range of aggregation operators including the LGOWA and the LWGM. We also generalize this approach by using quasi-arithmetic means obtaining the induced quasi-arithmetic linguistic hybrid average (Quasi-ILHA) operator.

Furthermore, we also present the induced linguistic generalized Choquet integral aggregation (ILGCIA) and the induced linguistic quasi-arithmetic Choquet integral aggregation (Quasi-ILCIA). These aggregation operators represent a generalization of the ILGOWA and the Quasi-ILOWA by using the Choquet integral (Choquet 1953).

Finally, we develop a decision making approach for evaluating university faculty for tenure and promotion based on the ILGHA operator and the ILGOWA operator. That is, we utilize the ILGHA operator to aggregate the individual decision matrix into the overall one, and then we use the ILGOWA operator to obtain the collective preference value of candidates. Thus, we can rank the candidates and select the best one.

This paper is organized as follows. Section 2 presents some basic concepts. In Sect. 3, we present the ILGOWA operator and Sect. 4 introduces the Quasi-ILOWA operator. Section 5 presents the ILGHA and the Quasi-ILHA operators and in Sect. 6 we suggest an extension by using Choquet integrals. In Sect. 7 we develop an application in group decision making. Finally, in Sect. 8 we summarize the main conclusions of the paper.

2 Preliminaries

In this section, we briefly review the linguistic approach, the OWA operator, the IOWA operator, the LOWA operator, the ILOWA operator and the IGOWA operator.

2.1 The Linguistic Approach

Many problems of the real world cannot be assessed in a quantitative form. Instead, it is possible to use a qualitative one, i.e., with vague or imprecise knowledge that uses linguistic assessments instead of numerical values (Zadeh 1975).

We have to select the appropriate linguistic descriptors for the term set and their semantics. For example, a set of seven terms S could be given as follows:

$$S = \{s_1 = N,\ s_2 = VL,\ s_3 = L,\ s_4 = M,\ s_5 = H,\ s_6 = VH,\ s_7 = P\}$$

Note that $N = None$, $VL = Very\ low$, $L = Low$, $M = Medium$, $H = High$, $VH = Very\ high$, $P = Perfect$. Usually, it is required that there exists:

1. A negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g + 1 - i$.
2. The set is ordered: $s_i \leq s_j$ if and only if $i \leq j$.
3. Max operator: $\max(s_i, s_j) = s_i$ if $s_i \geq s_j$.
4. Min operator: $\min(s_i, s_j) = s_i$ if $s_i \leq s_j$.

Different approaches have been developed for dealing with linguistic information (Carlsson and Fuller 2000; Chang and Wen 2010; Xu et al. 2010; Zadeh 1975). In this paper, we follow the ideas of Xu (2004, 2008). Thus, in order to preserve all the given information, we extend the discrete linguistic term set S to a continuous linguistic term set $\hat{S} = \{s_\alpha | s_1 < s_\alpha \leq s_t, \alpha \in [1, t]\}$, where, if $s_\alpha \in S$, we call s_α the original linguistic term, otherwise, we call s_α the virtual linguistic term. Therefore, we are able to compute words directly without losing information in the computation process.

2.2 The OWA Operator

The OWA operator (Yager 1988) is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. It is defined as follows.

Definition 1 An OWA operator of dimension n is a mapping OWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA\,(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the jth largest of the a_i.

Note that different properties can be studied such as the distinction between descending and ascending orders, different measures for characterizing the weighting vector and different families of OWA operators. Note that it is commutative, monotonic, bounded and idempotent. For further reading on recent developments, refer, for example to Emrouznejad and Amin (2010), Merigó (2010), Merigó and Gil-Lafuente (2010), Yager (2010), Yager and Kacprzyk (1997), Zhou et al. (2010).

2.3 The Induced OWA Operator

The IOWA operator (Yager and Filev 1999) is an extension of the OWA operator. Its main difference is that the reordering step is not carried out with the values of the arguments a_i. In this case, the reordering step is developed with order-inducing variables that reflect a more complex reordering process. The IOWA operator also includes as particular cases the maximum, the minimum and the average criteria. It can be defined as follows.

Definition 2 An IOWA operator of dimension n is a mapping $IOWA$: $R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $W = \sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$IOWA\,(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where b_j is the a_i value of the IOWA pair $\langle u_i, a_i \rangle$ having the jth largest u_i, u_i is the order inducing variable and a_i is the argument variable.

Note that it is possible to distinguish between the descending IOWA (DIOWA) operator and the ascending IOWA (AIOWA) operator. The IOWA operator is also monotonic, bounded, idempotent and commutative.

2.4 The Linguistic OWA Operator

In the literature, we find a wide range of linguistic aggregation operators (Merigó and Casanovas 2010d; Merigó et al. 2010; Xu 2009). In this study, we consider the linguistic ordered weighted averaging (LOWA) operator (Xu 2004, 2008) with its particular cases that include among others the linguistic average (LA) and the linguistic maximum and minimum. It can be defined as follows.

Definition 3 A LOWA operator of dimension n is a mapping $LOWA$: $\hat{S}^n \rightarrow \hat{S}$, which has an associated weighting vector W with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$LOWA(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \sum_{j=1}^{n} w_j s_{\beta_j}, \tag{3}$$

where s_{β_j} is the jth largest of the s_{α_i}.

Note that it is possible to distinguish between the descending LOWA (DLOWA) and the ascending LOWA (ALOWA) operator. The weights of these operators are related by $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the DLOWA (or LOWA) operator and w^*_{n+1-j} the jth weight of the ALOWA operator.

The LOWA operator provides a parameterized family of aggregation operators that includes as special cases the LA and the linguistic weighted average (LWA). The LA is obtained when all the weights w_j are equal for all j. The LWA is obtained if the ordered position of the s_{β_j} is the same as the ordered position of the S_{α_i}.

2.5 The Induced Linguistic OWA Operator

The ILOWA operator (Xu 2006b) is an extension of the OWA operator that uses linguistic information and inducing variables in the reordering of the arguments.

Definition 4 An ILOWA operator of dimension n is a mapping $ILOWA: R^n \times S^n \rightarrow S$, which has an associated weighting vector W with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$ILOWA(\langle u_1, s_{\alpha_1}\rangle, \langle u_2, s_{\alpha_2}\rangle, \ldots, \langle u_n, s_{\alpha_n}\rangle) = \sum_{j=1}^{n} w_j s_{\beta_j}, \tag{4}$$

where s_{β_j} is the s_{α_i} value of the ILOWA pair $\langle u_i, s_{\alpha_i}\rangle$ having the jth largest u_i, u_i is the order inducing variable and s_{α_i} is the linguistic variable.

2.6 The Induced Generalized OWA Operator

The IGOWA operator (Merigó and Gil-Lafuente 2009) represents a generalization of the IOWA operator by using generalized means. Thus, it is possible to include in the same formulation, different types of induced operators such as the IOWA operator or the induced OWG (IOWG) operator. It is defined as follows.

Definition 5 An IGOWA operator of dimension n is a mapping $IGOWA: R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$IGOWA\left(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle\right) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda}, \tag{5}$$

where b_j is the a_i value of the IGOWA pair $\langle u_i, a_i\rangle$ having the jth largest u_i, u_i is the order inducing variable, a_i is the argument variable and λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

As we can see, if $\lambda = 1$, we get the IOWA operator. If $\lambda = 0$, the induced ordered weighted geometric (IOWG) operator and if $\lambda = 2$, the induced ordered weighted quadratic averaging (IOWQA) operator. Note that it is possible to further generalize the IGOWA operator by using quasi-arithmetic means. The result is the Quasi-IOWA operator.

Definition 6 A Quasi-IOWA operator of dimension n is a mapping *QIOWA*: $R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$QIOWA\left(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle\right) = g^{-1}\left(\sum_{j=1}^{n} w_j g(b_j)\right), \tag{6}$$

where b_j is the a_i value of the Quasi-IOWA pair $\langle u_i, a_i\rangle$ having the jth largest u_i, u_i is the order inducing variable, a_i is the argument and $g(b)$ is a strictly continuous monotonic function.

As we can see, when $g(b) = b^{\lambda}$, we get the induced generalized OWA (IGOWA) operator. Thus, the Quasi-IOWA operator includes all the particular cases of the IGOWA such as the IOWA and the IOWQA, and a lot of other cases.

2.7 The Hybrid Averaging Operator

The hybrid average (HA) operator (Xu and Da 2003) is an aggregation operator that uses the WA and the OWA operator in the same formulation. Thus, it is possible to consider in the same problem, the attitudinal character of the decision maker and its subjective probability. Since its introduction, it has been used in a lot of applications (Merigó and Casanovas 2010b; Wei 2009; Xu 2006c, 2010a,b). It can be defined as follows.

Definition 7 An HA operator of dimension n is a mapping *HA*: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$HA\left(a_1, a_2, \ldots, a_n\right) = \sum_{j=1}^{n} w_j b_j, \tag{7}$$

where b_j is the jth largest of the $\hat{a}_i$ ($\hat{a}_i = n\omega_i a_i$, $i = 1, 2, \ldots, n$), $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the a_i, with $\omega_i \in [0, 1]$ and the sum of the weights is 1.

From a generalized perspective of the reordering step, we can distinguish between the descending HA (DHA) operator and the ascending HA (AHA) operator. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DHA and w^*_{n-j+1} the jth weight of the AHA operator.

Note that different families of HA operators are found by using a different manifestation in the weighting vector such as the step-HA operator, the window-HA operator, the median-HA operator and the centered-HA operator.

3 The Induced Linguistic Generalized OWA Operator

3.1 Introduction

The ILGOWA operator (Merigó and Gil-Lafuente 2008) is an extension of the IGOWA operator for the cases where the information cannot be assessed with numerical values and it is necessary to use another approach such as a qualitative one that uses linguistic assessments. Note that the ILGOWA operator can also be seen as an aggregation operator that uses the main characteristics of three well-known aggregation operators: the LOWA, the IOWA and the GOWA operator. Thus, it uses linguistic information in a generalized model that uses generalized means. Moreover, it also uses a complex reordering process by using order inducing variables. It can be defined as follows.

Definition 8 An ILGOWA operator of dimension n is a mapping *ILGOWA*: $R^n \times S^n \rightarrow S$, which has an associated weighting vector W with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$ILGOWA(\langle u_1, s_{\alpha_1}\rangle, \langle u_2, s_{\alpha_2}\rangle, \ldots, \langle u_n, s_{\alpha_n}\rangle) = \left(\sum_{j=1}^{n} w_j s_{\beta_j}^{\lambda}\right)^{1/\lambda}, \tag{8}$$

where s_{β_j} is the s_{α_i} value of the ILGOWA pair $\langle u_i, s_{\alpha_i}\rangle$ having the jth largest u_i, u_i is the order inducing variable, s_{α_i} is the linguistic variable and λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

Example 1 Assume the following collection of linguistic values assessed with a set of seven linguistic terms: $S = \{s_1 = N, s_2 = VL, s_3 = L, s_4 = M, s_5 = H, s_6 = VH, s_7 = P\}$, with their corresponding order-inducing variables $\langle u_i, s_i\rangle : \langle 4, s_5\rangle, \langle 9, s_4\rangle, \langle 3, s_6\rangle, \langle 6, s_3\rangle$. Assuming that $W = (0.1, 0.2, 0.3, 0.4)$ and $\lambda = 1$, then, the aggregation formula is the following:

$$0.1 \times s_4 + 0.2 \times s_3 + 0.3 \times s_5 + 0.4 \times s_6 = s_{4.9}.$$

As we can see, the linguistic argument variables are reordered in decreasing order according to the order-inducing variables u_i.

Note that it is possible to distinguish between descending (DILGOWA) and ascending (AILGOWA) orders. Note also that if $\lambda < 0$, we can only use linguistic variables associated with positive numbers S^+, in order to obtain consistent results. Additionally, sometimes the weighting vector may not be normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$. In these cases, the ILGOWA operator can be expressed as:

$$f(\langle u_1, s_{\alpha_1}\rangle, \langle u_2, s_{\alpha_2}\rangle, \ldots, \langle u_n, s_{\alpha_n}\rangle) = \left(\frac{1}{W}\sum_{j=1}^{n} w_j s_{\beta_j}^{\lambda}\right)^{1/\lambda}. \tag{9}$$

The ILGOWA operator is commutative, monotonic, bounded and idempotent. It is commutative because $f(\langle u_1, s_1\rangle, \ldots, \langle u_n, s_n\rangle) = f(\langle u_1, s'_1\rangle, \ldots, \langle u_n, s'_n\rangle)$ where $(\langle u_1, s'_1\rangle, \ldots, \langle u_n, s'_n\rangle)$ is any permutation of the arguments $(\langle u_1, s_1\rangle, \ldots, \langle u_n, s_n\rangle)$. It is monotonic if $f(\langle u_1, s_1\rangle, \ldots, \langle u_n, s_n\rangle) \geq f(\langle u_1, s'_1\rangle, \ldots, \langle u_n, s'_n\rangle)$ with $s_i \geq s'_i$, for all i. It is bounded because $\min\{s_i\} \leq f(\langle u_1, s_1\rangle, \ldots, \langle u_n, s_n\rangle) \leq \max\{s_i\}$. And it is idempotent if $s_i = s_k$, for all i, then $f(\langle u_1, s_1\rangle, \ldots, \langle u_n, s_n\rangle) = s_k$.

Another interesting issue is the problem of ties in the order inducing variables. As it was explained by Yager and Filev (1999), the easiest way to solve this problem consists in replacing each argument of the tied inducing variables by its LGM.

Analysing the applicability of the ILGOWA operator, we can see that it is applicable to similar situations already discussed in other types of induced aggregation operators where it is possible to use linguistic information. For example, we could use it in different decision making problems, etc.

3.2 Families of ILGOWA Operators

The ILGOWA operator provides a parameterized family of aggregation operators that includes the LA, the LWA, the LOWA, the ILOWA, the LGM, the LWGM, the LGOWA and the ILOWG operator, among others.

In order to study these families, we can analyze the weighting vector W or the parameter λ. If we analyze the weighting vector W, then, we find similar results to those found in the OWA operator (Carlsson et al. 2003; Emrouznejad and Amin 2010; Merigó and Gil-Lafuente 2009; Yager 1993). For example:

- If $w_j = 1/n$, we get the LGM.
- The linguistic maximum is obtained if $w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = \text{Max}\{u_i\}$.
- The linguistic minimum is obtained if $w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = \text{Min}\{u_i\}$.
- The LWGM is obtained if $u_i > u_{i+1}$, for all i.
- The LGOWA operator is obtained if the ordered position of u_i is the same as the ordered position of b_j such that b_j is the jth largest of s_i.
- Step-ILGOWA: If $w_k = 1$ and $w_j = 0$, for all $j \neq k$.
- Olympic-ILGOWA: If $w_1 = w_n = 0$, and for all others $w_j = 1/(n-2)$.

If we analyze the parameter λ, we find similar results to those found in the GOWA operator. For example:

- If $\lambda = 1$, then, we get the ILOWA operator.
- If $\lambda \to 0$, we get the ILOWG.
- If $\lambda = 2$, the induced linguistic ordered weighted quadratic averaging (ILOWQA) operator.
- If $\lambda = -1$, the induced linguistic ordered weighted harmonic averaging (ILOWHA) operator.
- If $\lambda = 3$, the induced linguistic ordered weighted cubic averaging (ILOWCA) operator.

- If $\lambda \rightarrow \infty$, we get the linguistic maximum.
- If $\lambda \rightarrow -\infty$, we get the linguistic minimum.

4 Quasi-ILOWA Operators

The induced linguistic ordered weighted quasi-arithmetic averaging (Quasi-ILOWA) operator is a further generalization of the ILGOWA operator by using quasi-arithmetic means (Merigó and Gil-Lafuente 2008). Its main advantage is that it provides a more general formulation because it includes the ILGOWA operator as a particular case. It can be defined as follows.

Definition 9 A Quasi-ILOWA operator of dimension n is a mapping $QILOWA$: $R^n \times S^n \rightarrow S$, which has an associated weighting vector W with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$QILOWA(\langle u_1, s_{\alpha_1}\rangle, \langle u_2, s_{\alpha_2}\rangle, \ldots, \langle u_n, s_{\alpha_n}\rangle) = g^{-1}\left(\sum_{j=1}^{n} w_j g(s_{\beta_j})\right), \tag{10}$$

where s_{β_j} is the s_{α_i} value of the QILOWA pair $\langle u_i, s_{\alpha_i}\rangle$ having the jth largest u_i, u_i is the order inducing variable, s_{α_i} is the linguistic variable and g is a general continuous strictly monotonic function.

As we can see, the ILGOWA operator is a particular case of the Quasi-ILOWA when $g(s) = s^{\lambda}$. Note that all the properties commented in the ILGOWA operator are also applicable in this case such as the distinction between descending and ascending orders and the problem of ties.

As explained in the case of the ILGOWA, if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, then, the Quasi-ILOWA operator can be expressed as:

$$f(\langle u_1, s_{\alpha_1}\rangle, \langle u_2, s_{\alpha_2}\rangle, \ldots, \langle u_n, s_{\alpha_n}\rangle) = g^{-1}\left(\frac{1}{W}\sum_{j=1}^{n} w_j g(s_{\beta_j})\right). \tag{11}$$

Note that all the properties and particular cases commented in the ILGOWA operator are also included in this generalization. For example, we could study different families of Quasi-ILOWA operators such as the Quasi-LA, the Quasi-LWA, the Quasi-ILOWA, the Quasi-olympic-ILOWA and the Quasi-centered-ILOWA.

A further interesting result can be considered by using infinitary aggregation operators (Mesiar and Pap 2008). Thus, we can represent an aggregation process where there are an unlimited number of arguments to be aggregated. Note that $\sum_{j=1}^{\infty} w_j = 1$. By using, the Quasi-ILOWA operator we get the infinitary Quasi-ILOWA (∞-Quasi-ILOWA) operator as follows.

$$\infty - QILOWA(\langle u_1, s_{\alpha_1}\rangle, \langle u_2, s_{\alpha_2}\rangle, \ldots, \langle u_n, s_{\alpha_n}\rangle) = g^{-1}\left(\sum_{j=1}^{\infty} w_j g(s_{\beta_j})\right), \quad (12)$$

where s_{β_j} are the argument of the ∞-QILOWA pair $\left(u_i, s_{\alpha_i}\right)$ having the jth largest u_i and $g(\beta)$ is a general continuous strictly monotonic function such that $g : I \rightarrow R$.

Note that the reordering process is much more complex due to the fact that we never know the largest order-inducing variable because we have an unlimited number of arguments to be considered. For further reading with the usual OWA, see Mesiar and Pap (2008).

5 Using the Hybrid Average in the ILGOWA Operator

A further generalization can be developed by using hybrid averages. Thus, we obtain the induced linguistic generalized hybrid average (ILGHA) operator. The main advantage of this approach is that it is able to deal with the weighted average and the OWA operator in the same formulation. It can be defined as follows.

Definition 10 An ILGHA operator of dimension n is a mapping $ILGHA$: $R^n \times S^n \rightarrow S$, which has an associated weighting vector W with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$ILGHA(\langle u_1, s_{\alpha_1}\rangle, \ldots, \langle u_n, s_{\alpha_n}\rangle) = \left(\sum_{j=1}^{n} w_j s_{\beta_j}^{\lambda}\right)^{1/\lambda}, \quad (13)$$

where s_{β_j} is the $\hat{s}_{\alpha_i}$ value ($\hat{s}_{\alpha_i} = n\omega_i s_{\alpha_i}, i = 1, 2, \ldots, n$), of the ILGHA pair $\left\langle u_n, s_{\alpha_n}\right\rangle$ having the jth largest u_i, u_i is the order inducing variable, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the s_{α_i}, with $\omega_i \in [0, 1]$ and the sum of the weights is 1, and λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

As we can see, if $w_j = 1/n$, for all i, then, the ILGHA operator becomes the LWGM and if $\omega_n = 1/n$, for all i, it becomes the ILGOWA operator. Note that a lot of other families could be studied following the methodology explained in Sect. 3.

Moreover, it is possible to further extend this approach by using quasi-arithmetic means obtaining the induced linguistic quasi-arithmetic HA (Quasi-ILHA) operator. The Quasi-ILHA operator includes the ILGHA as a particular case. It can be defined as follows.

Definition 11 A Quasi-ILHA operator of dimension n is a mapping $QILHA$: $R^n \times S^n \rightarrow S$, which has an associated weighting vector W with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$QILHA\left(\left\langle u_1, s_{\alpha_1}\right\rangle, \ldots, \left\langle u_n, s_{\alpha_n}\right\rangle\right) = g^{-1}\left(\sum_{j=1}^{n} w_j g(s_{\beta_j})\right), \quad (14)$$

where s_{β_j} is the $\hat{s}_{\alpha_i}$ value $\left(\hat{s}_{\alpha_i} = n\omega_i s_{\alpha_i}, i = 1, 2, \ldots, n\right)$, of the QILHA pair $\langle u_n, s_{\alpha_n}\rangle$ having the jth largest u_i, u_i is the order inducing variable, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the s_{α_i}, with $\omega_i \in [0, 1]$ and the sum of the weights is 1, and g is a general continuous strictly monotonic function.

6 Choquet Integrals in the ILGOWA Operator

Following previous papers on Choquet integrals (Choquet 1953; Mesiar 1995; Tan and Chen 2010a,b; Xu 2010a; Yager 2004a,b), it is possible to develop an extension of the ILGOWA and the Quasi-ILOWA operator by using the discrete Choquet integral. Thus, we get the induced linguistic quasi-arithmetic Choquet integral aggregation (Quasi-ILCIA) operator. Before presenting this new result, let us define the concept of fuzzy measure and the Choquet integral. The fuzzy measure (non-additive measure) was introduced by Sugeno (1974) and it is defined as follows.

Definition 12 Let X be a universal set $X = \{x_1, x_2, \ldots, x_n\}$ and $P(X)$ the power set of X. A fuzzy measure on X is a set function on $m : P(X) \to [0, 1]$, that satisfies the following conditions:

(1) $m(\emptyset) = 0, m(X) = 1$ (boundary conditions) and
(2) If $A, B \in P(X)$ and $A \subseteq B$, then $m(A) \leq m(B)$ (monotonicity).

The Choquet integral was introduced by Choquet (1953) and it can be defined as follows in its discrete form.

Definition 13 Let f be a positive real-valued function $f : X \to R^+$ and m be a fuzzy measure on X. The (discrete) Choquet integral of f with respect to m is:

$$C_m(f_1, f_2, \ldots, f_n) = \sum_{i=1}^{n} f_{(i)} \left[m(A_{(i)}) - m(A_{(i-1)})\right], \tag{15}$$

where $(\cdot)$ indicates a permutation on X such that $f_{(1)} \geq f_{(2)} \geq \cdots \geq f_{(n)}$, i.e. $f_{(i)}$ is the ith largest value in the set $\{f_1, f_2, \ldots, f_n\}$, $A_{(i)} = \left\{x_{(1)}, \ldots, x_{(i)}\right\} i \geq 1$ and $A_{(0)} = \emptyset$.

In the following, we present a definition of the Quasi-ILCIA operator based on the use of the Choquet integral with quasi-arithmetic means and linguistic information.

Definition 14 Let S be the set of linguistic values. Let s_i be a positive linguistic value on X, and m be a fuzzy measure on X. An induced linguistic quasi-arithmetic Choquet integral aggregation (Quasi-ILCIA) operator of dimension n is a function *QILCIA*: $\left(R \times S^+\right)^n \to S^+$, which is defined to aggregate the set of second arguments of a list of n tuples $\{(u_1, s_1), \ldots, (u_n, s_n)\}$ according to the following expression:

$$\begin{aligned} &QILCIA\left((u_1, s_1), \ldots, (u_n, s_n)\right) \\ &\quad = \Delta\left(g^{-1}\left(\sum_{j=1}^{n} g(s_{\beta_j})\left[m(A_{(i)}) - m(A_{(i-1)})\right]\right)\right), \end{aligned} \tag{16}$$

where g is a strictly continuous monotonic function such that g: $S \rightarrow R$, s_{β_j} are the linguistic argument values s_i of the Quasi-ILCIA tuples (u_i, s_i) having the jth largest u_i, u_i is the order inducing variable, $A_{(i)} = \{x_{(1)} \ldots, x_{(i)}\} i \geq 1$ and $A_{(0)} = \emptyset$.

A fundamental aspect of this new linguistic aggregation operator is that it includes a wide range of aggregation operators. For example:

- The linguistic quasi-arithmetic Choquet integral aggregation (Quasi-LCIA): When the ordered position of the order inducing variables u_i is the same as the ordered position of j such that j is the jth largest of i.
- The induced linguistic generalized Choquet integral aggregation (ILGCIA): When $g(\beta) = \beta^\lambda$.
- The linguistic generalized Choquet integral aggregation (LGCIA): When $g(\beta) = \beta^\lambda$ and the ordered position of the order inducing variables u_i is the same as the ordered position of j such that j is the jth largest of i.
- The induced quasi-arithmetic Choquet integral aggregation (Quasi-ICIA): When the linguistic values are reduced to the usual exact numbers.
  - The induced generalized Choquet integral aggregation (IGCIA).
  - The quasi-arithmetic Choquet integral aggregation (Quasi-CIA).
  - The generalized Choquet integral aggregation (GCIA).

Moreover, we could also consider a wide range of families of all the previous cases following the methodology explained in Sect. 3. For example, we could analyze the following cases:

- The induced linguistic Choquet integral aggregation.
- The induced linguistic quadratic Choquet integral aggregation.
- The induced linguistic harmonic Choquet integral aggregation.
- The linguistic Choquet integral aggregation.
- The linguistic quadratic Choquet integral aggregation.
- The linguistic harmonic Choquet integral aggregation.

Note that the Quasi-ILCIA operator includes a lot of other particular cases but we believe that those presented here are some of the most relevant.

7 Application in Linguistic Group Decision Making

In this section, we present a new approach based on the ILGHA operator and the ILGOWA operator to group decision making with linguistic preference information. The approach can be used in many decision making problems, such as product management, human resource management, the selection of financial products, and so on.

7.1 An Approach to Group Decision Making with the ILGHA Operator and ILGOWA Operator

In the following, we shall develop an approach based on the ILGHA operator and ILGOWA operator to a multiple attribute group decision making with linguistic preference information.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a discrete set of alternatives, and $C = \{c_1, c_2, \ldots, c_m\}$ be the set of attributes. Let $D = \{d_1, d_2, \ldots, d_m\}$ be the set of decision makers and $W = (w_1, w_2, \ldots, w_l)^T$ be the weight vector of decision makers, where $w_k \geq 0, k = 1, 2, \ldots, l, \sum_{k=1}^{l} w_k = 1$. Suppose that $A^{(k)} = (a_{ij}^{(k)})_{m\times n}$ is the decision matrix, where $a_{ij}^{(k)} \in S$ is a preference value, which takes the form of linguistic variable, given by the decision maker $d_k \in D$, for alternative $x_j \in X$ with respect to attribute $c_i \in C$. The method involves the following steps:

Step 1. Utilize the ILGHA operator:

$$a_{ij} = ILGHA(\langle u_1, a_{ij}^{(1)}\rangle, \langle u_2, a_{ij}^{(2)}\rangle, \ldots, \langle u_l, a_{ij}^{(l)}\rangle) = \left(\sum_{k=1}^{l} w_k (\hat{a}_{ij}^{(k)})^{\lambda}\right)^{1/\lambda},$$
$$i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n$$

to aggregate all the linguistic preference information $A^{(k)} = (a_{ij}^{(k)})_{m\times n}$ into a collective linguistic preference matrix $A = (a_{ij})_{m\times n}$, where $\hat{a}_{ij}^{(k)}$ is the $\overline{a}_{ij}^{(t)}$ value ($\overline{a}_{ij}^{(t)} = l\omega_t a_{ij}^{(t)}, t = 1, 2, \ldots, l$), of the ILGHA pair $\langle u_t, a_{ij}^{(t)}\rangle$ having the kth largest u_t, u_t is the order inducing variable, $\omega = (\omega_1, \omega_2, \ldots, \omega_l)$ is the balance factor of the $a_{ij}^{(t)}$, which satisfying $\omega_t \in [0, 1], t = 1, 2, \ldots, l$, and $\sum_{t=1}^{l} \omega_t = 1$.

Step 2. Utilize the ILGOWA operator:

$$a_j = ILGOWA(\langle u'_1, a_{1j}\rangle, \langle u'_2, a_{2j}\rangle, \ldots, \langle u'_m, a_{mj}\rangle) = \left(\sum_{i=1}^{m} w'_i b_{ij}^{\lambda}\right)^{1/\lambda},$$
$$j = 1, 2, \ldots, n,$$

to aggregate a_{ij} corresponding to the alternative x_i, where $W' = (w'_1, w'_2, \ldots, w'_m)^T$ is the weighting vector of attributes, such that $w'_j \in [0, 1], j = 1, 2, \ldots, m$, and $\sum_{j=1}^{m} w'_j = 1$, b_{ij} is the a_{ij} value of the ILGOWA pair $\langle u_i\prime, a_{ij}\rangle$ having the ith largest u'_i. u'_i is the order inducing variable of a_{ij}.

Step 3. Rank all the alternatives $x_j (j = 1, 2, \ldots, n)$ in descending order and select the best one(s) in accordance with the values of $a_j (j = 1, 2, \ldots, n)$.

Step 4. End.

7.2 Illustrative Example

Let us suppose to evaluate the university faculty for tenure and promotion (adapted from Bryson and Mobolurin 1995). Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ be a finite set of five faculty candidates (alternatives) to be evaluated using the linguistic label set. Suppose that we use three attributes to evaluate the university faculty for tenure and promotion, which include c_1: teaching, c_2: research, and c_3: service. We use the following linguistic label set S to evaluate the five faculty candidates.

Table 1 Decision matrix $A^{(1)}$ provided by d_1

	x_1	x_2	x_3	x_4	x_5
u_1	s_8	s_6	s_5	s_7	s_8
u_2	s_6	s_7	s_8	s_4	s_6
u_3	s_6	s_7	s_7	s_6	s_7

Table 2 Decision matrix $A^{(2)}$ provided by d_2

	x_1	x_2	x_3	x_4	x_5
u_1	s_6	s_5	s_7	s_8	s_8
u_2	s_8	s_6	s_6	s_6	s_7
u_3	s_5	s_7	s_7	s_7	s_6

Table 3 Decision matrix $A^{(3)}$ provided by d_3

	x_1	x_2	x_3	x_4	x_5
u_1	s_7	s_4	s_8	s_7	s_6
u_2	s_8	s_5	s_7	s_5	s_7
u_3	s_6	s_6	s_6	s_8	s_6

Table 4 Decision matrix $A^{(4)}$ provided by d_4

	x_1	x_2	x_3	x_4	x_5
u_1	s_6	s_8	s_7	s_5	s_5
u_2	s_7	s_7	s_6	s_7	s_6
u_3	s_6	s_7	s_8	s_6	s_5

$$S = \{s_1 = extremely\ poor;\ s_2 = very\ poor;\ s_3 = poor;\ s_4 = slightly\ poor; s_5 = fair;\ s_6 = slightly\ good;\ s_7 = good;\ s_8 = very\ good;\ s_9 = extremely\ good\}.$$

Let $D = \{d_1, d_2, d_3, d_4\}$ be a finite set of four experts (or decision makers) who provide the decision matrix $A^{(k)} = (a_{ij}^{(k)})_{3\times 5}$for faculty candidates under these three attributes, $k = 1, 2, 3, 4$, as listed in Tables 1, 2, 3 and 4, respectively.

According to the importance of experts, assume the following order inducing variables: $u_1 = 6, u_2 = 8, u_3 = 5, u_4 = 10$. Consider that the experts weighting vector is: $W = (0.35\ 0.28\ 0.24\ 0.13)^T$ and assume that the balance factor of four experts is: $\omega = (0.24, 0.26, 0.24, 0.26)^T$. Use the parameter $\lambda = 2$ in the *ILGHA* operator.

To get the best candidate, the following steps are involved:

Step 1: Utilize the decision matrix $A^{(k)} = (a_{ij}^{(k)})_{3\times 5}$ and the *ILGHA* operator to derive the overall preference value a_{ij} of alternative x_j according to the following formulas.

$$a_{ij} = ILGHA(\langle u_1, a_{ij}^{(1)}\rangle, \langle u_2, a_{ij}^{(2)}\rangle, \langle u_3, a_{ij}^{(3)}\rangle, \langle u_4, a_{ij}^{(4)}\rangle) = \left(\sum_{k=1}^{4} w_k (\hat{a}_{ij}^{(k)})^{\lambda}\right)^{1/\lambda}, \quad i = 1, 2, 3, \quad j = 1, 2, \ldots, 5,$$

where the weight $W = (0.35\ 0.28\ 0.24\ 0.13)^T$, $\hat{a}_{ij}^{(k)}$ is the $\overline{a}_{ij}^{(l)}$ value ($\overline{a}_{ij}^{(l)} = 4\omega_l a_{ij}^{(l)}$, $l = 1, 2, 3, 4$), of the ILGHA pair $\langle u_l, a_{ij}^{(l)} \rangle$ having the kth largest u_l, u_l is the order inducing variable, $\omega = (0.24, 0.26, 0.24, 0.26)^T$ is the weighting vector of the $a_{ij}^{(l)}$, and $\lambda = 2$.

Thus,

$$\begin{aligned} a_{11} &= ILGHA(\langle u_1, a_{11}^{(1)} \rangle, \langle u_2, a_{11}^{(2)} \rangle, \langle u_3, a_{11}^{(3)} \rangle, \langle u_4, a_{11}^{(4)} \rangle) \\ &= ILGHA(\langle 6, s_8 \rangle, \langle 8, s_6 \rangle, \langle 5, s_7 \rangle, \langle 10, s_6 \rangle) \\ &= [0.35 \times (4 \times 0.26 \times s_6)^2 + 0.28 \times (4 \times 0.26 \times s_6)^2 \\ &\quad + 0.24 \times (4 \times 0.24 \times s_8)^2 + 0.13 \times (4 \times 0.24 \times s_7)^2]^{1/2} \\ &= s_{6.6751} \end{aligned}$$

In similar way, we get

$$\begin{aligned} &a_{12} = s_{6.4559},\ a_{13} = s_{6.8254},\ a_{14} = s_{6.7494},\ a_{15} = s_{6.8786}, \\ &a_{21} = s_{7.3186},\ a_{22} = s_{6.5791},\ a_{23} = s_{6.6751},\ a_{24} = s_{5.9988},\ a_{25} = s_{6.5039} \\ &a_{31} = s_{5.7858},\ a_{32} = s_{6.9671},\ a_{33} = s_{7.3633},\ a_{34} = s_{6.6406},\ a_{35} = s_{5.9597} \end{aligned}$$

Step 2: Utilize the ILGOWA operator to obtain the collective overall preference value a_j of candidate x_j. According to the importance of three attributes, suppose that induced variables of attributes $u'_1 = 5$, $u'_2 = 7$, $u'_3 = 4$, then

$$\begin{aligned} a_1 &= ILGOWA(\langle u'_1, a_{11} \rangle, \langle u'_2, a_{21} \rangle, \langle u'_3, a_{31} \rangle) \\ &= ILGOWA(\langle 5, s_{6.6751} \rangle, \langle 7, s_{7.3186} \rangle, \langle 4, s_{5.7858} \rangle) \\ &= 0.3 \times s_{7.3186} + 0.4 \times s_{6.6751} + 0.3 \times s_{5.7858} = s_{6.6014} \end{aligned}$$

where $W' = (0.3, 0.4, 0.3)^T$ is the weight vector of attributes.

Similarly, we can obtain

$$a_2 = s_{6.6585}, \quad a_3 = s_{6.9417}, \quad a_4 = s_{6.4807}, \quad a_5 = s_{6.4531}$$

Step 3: We can rank the alternatives $(x_1, x_2, x_3, x_4, x_5)$ and select the best one in descending order in accordance with the ranking of a_1, a_2, a_3, a_4, a_5. i.e.,

$$x_3 \succ x_2 \succ x_1 \succ x_4 \succ x_5$$

Therefore, the best candidate is x_3.

8 Conclusions

We have presented a wide range of induced and linguistic generalized aggregation operators. First, we have introduced the ILGOWA operator. It is a generalization of the OWA operator that uses order inducing variables in order to assess complex reordering processes, linguistic information and generalized means. We have analyzed some of its main properties. We have seen that it generalizes a wide range of linguistic aggregation operators such as the LGM, the LGOWA and the LOWA operator.

Moreover, we have developed a further generalization by using quasi-arithmetic means, obtaining the Quasi-ILOWA operator. It includes the ILGOWA as a particular case and a lot of other situations. Thus, we obtain a more robust formulation of the linguistic aggregation operators.

Furthermore, we have presented the ILGHA and the Quasi-ILHA operators. The main advantage of these models is that they are able to deal with the OWA and the weighted average in the same formulation in an uncertain environment that can be assessed with linguistic variables. Additionally, we have also suggested the use of Choquet integrals in the ILGOWA operator obtaining the ILGCIA operator. We have seen a lot of particular cases of these new approaches.

We have applied these new approaches for evaluating university faculty for tenure and promotion in a linguistic group decision making problem. The result shows that the approaches are feasible and effective providing a more robust formulation of the previous models.

In future research, we expect to develop further improvements by adding more characteristics in the model such as the use of other types of aggregation operators and apply it in other decision making problems.

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