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Unifying baryogenesis with dark matter production

Orlando Luongo1,2,3,4 · Nicola Marcantognini1 · Marco Muccino4

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Abstract

We here propose a mechanism that predicts, at early times, both baryon asymmetry and dark matter origin and that recovers the spontaneous baryogenesis during the reheating. Working with *U*(1)-invariant quark *Q* and lepton *L* effective fields, with an interacting term that couples the evolution of Universe's environment field ψ , we require a spontaneous symmetry breaking and get a pseudo Nambu–Goldstone boson θ . The pseudo Nambu–Goldstone boson speeds the Universe up during inflation, playing the role of inflaton, enabling baryogenesis to occur. Thus, in a quasi-static approximation over ψ , we impressively find both baryon and dark matter quasi-particle production rates, unifying *de facto* the two scenarios. Moreover, we outline particle mixing and demonstrate dark matter takes over baryons. Presupposing that θ field energy density dominates as baryogenesis stops and employing recent limits on reheating temperature, we get numerical bounds over dark matter constituent, showing that the most likely dark matter would be consistent with MeV-scale mass candidates. Finally, we briefly underline our predictions are suitable to explain the the low-energy electron recoil event excess between 1 and 7 keV found by the XENON1T collaboration.

Keywords Classical theories of gravity · Baryogenesis · Dark matter · Inflation

B Orlando Luongo orlando.luongo@unicam.it

 \boxtimes Marco Muccino marco.muccino@lnf.infn.it

- ³ Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, via A. Pascoli, 06123 Perugia, Italy
- ⁴ NNLOT, Al-Farabi Kazakh National University, Al-Farabi av. 71, 050040 Almaty, Kazakhstan

Orlando Luongo, Nicola Marcantognini and Marco Muccino have authors contributed equally to this work.

¹ Scuola di Scienze e Tecnologie, Divisione di Fisica, Università di Camerino, Via Madonna delle Carceri 9, 62032 Camerino, Italy

² Dipartimento di Matematica, Università di Pisa, Largo Bruno Pontecorvo 5, 56127 Pisa, Italy

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1 Introduction

Current comprehension of the standard Big Bang paradigm struggled over how to fix considerable issues, above all, the *cosmological constant problem* [\[1,](#page-17-1) [2](#page-17-2)], a *ad hoc* baryon production, named after *baryogenesis* [\[3\]](#page-17-3), *dark matter* (DM) and *dark energy* [\[5](#page-17-4)[–7\]](#page-17-5), *quantum gravity* [\[8](#page-17-6)], and so forth. Similarly, recent experimental tensions suggest the Big Bang model could somehow be theoretically incomplete [\[9\]](#page-17-7). To circumvent the problem of baryogenesis and DM production, we here conjecture a mechanism that unifies both baryon and DM genesis under the same standards.

Within the spontaneous baryogenesis framework [\[14,](#page-17-8) [15\]](#page-17-9), particle production occurs during the reheating due to the coupling of Nambu–Goldstone fields with fermions. The decay of the Nambu–Goldstone field leads to the production of these fermions Further, the interactions of the fermionic fields create a thermal bath thereby reheating the Universe. The Nambu–Goldstone field ψ , described by a complex scalar field with non-vanishing baryon number, and the fermionic quark *Q* and lepton *L* effective fields possess a *U*(1) global invariance, i.e.,

$$
\psi \to e^{i\alpha}\psi \quad , \quad Q \to e^{i\alpha}Q \quad , \quad L \to L, \tag{1}
$$

The field *Q* carries baryon number, whereas *L* does not. Both fields are not endowed with strong interactions, thus cannot describe real quarks and leptons, respectively. In the above approach, the Nambu–Goldstone field plays the role of the inflaton and the corresponding baryon current is generated by the classical rolling down of the inflaton field [\[17,](#page-17-10) [18\]](#page-17-11). Thus, as the field rolls in one direction, it preferentially creates baryons over anti-baryons, while the opposite is true as it rolls in the opposite direction. The decays during reheating are assumed to be baryon number conserving. Finally, no CP violation is required.

We extend the above spontaneous baryogenesis picture by identifying the ψ field with a Universe environment field. This choice is motivated by a recent effective theory involving matter with pressure, which depends upon a scalar field whose time derivative is thermodynamically related to the Universe environment temperature [\[40](#page-17-12)]. Thus, we interpret the pseudo Nambu–Goldstone boson θ , resulting from the baryonic symmetry breaking, as inflaton and show that two stages occur, having a first in which we claim DM to be born, whereas a second providing a dominant baryogenesis over

DM. Particularly, during reheating we recover, in a quasi-static approximation over the ψ field, the abundance of baryons as expected today. We remarkably find the baryon and DM quasi-particle production rates are intertwined between them, unifying *de facto* the two approaches. Further, we describe particle mixing as naive recipe to stop baryogenesis and DM production and qualitatively demonstrate why DM dominates over baryons. Assuming that the θ field energy density dominates when the baryogenesis stops and employing recent limits on the reheating temperature, we extract numerical results on the DM mass constituents, most likely congruent with MeV-scale mass candidates.

The paper is structured as follows. In Sect. [2](#page-2-0) we introduce our effective model and in Sect. [3](#page-7-0) we compute the rate of particle production for our cases and discuss baryogenesis, baryon asymmetry, DM production and mass mixing. The predictions of our model are also critically discussed. We highlight conclusions and perspectives of this work in Sect. [5.](#page-16-0)

2 Baryogenesis

The basic demands of our model is to get leptons formed *before* baryons in order to plausibly describe baryogenesis through the effective fields Q , L and ψ [\[10](#page-17-13)]. The Lagrangian accounts for the evolution of Universe's environment field ψ , associated with the dynamics of the universe. This evolution is provided by a generalized kinetic term of the form $\mathcal{L}_{env} = K(X, \psi)$, where $X = g^{\mu\nu}(\partial_{\mu}\psi)(\partial_{\nu}\psi)/2$ is the kinetic term of the field ψ . For simplicity, in the following we assume that the generalized kinetic term coincides with the canonical one, *i.e.*, $K \equiv X$. Then, we build the Dirac Lagrangian for quarks and leptons, *Q* and *L* with masses m_O and m_L , respectively,

$$
\mathcal{L}_{QL} = \bar{Q}i\gamma^{\mu}\partial_{\mu}Q - m_{Q}\bar{Q}Q + \bar{L}i\gamma^{\mu}\partial_{\mu}L - m_{L}\bar{L}L. \tag{2}
$$

Next, we add the interaction between the fields Q , L and ψ , including the hermitian conjugate terms *h*.*c*.

$$
\mathcal{L}_{int} = [ig\gamma^{\mu}(\partial_{\mu}\psi)\bar{Q} + h\psi\bar{Q}L + h.c.],
$$
\n(3)

where *h* is a coupling constant and *g* is a set of constants.

The set of constants *g* may appear to break the Lorentz invariance since it provides apparent unsaturated Lorentz indices into the action. However, Lorentz invariance is saved whether one assumes the constant to play the role of a set of free constants that act as Stückelberg fields [\[19](#page-17-14)[–21](#page-17-15)]. The idea of considering such a set of constants comes from relativistic hydrodynamics. In particular, it consists of four Lorentz-invariant quantities that contract with the fermionic fields. Indeed, non-dissipative fluids are described by virtue of the pullback formalism [\[22](#page-17-16)[–24\]](#page-17-17) through Carter's covariant formulation [\[25](#page-17-18)]. In order to provide a relativistic effective field theory description of the type of interaction under exam, we consider an observer attached to a particular fluid element by introducing a matter space such that its worldline is identified with a unique point in this space. The coordinates of each matter space serve as labels that

distinguish fluid element worldlines and remain unchanged throughout the evolution. The matter space coordinates can be considered as scalar fields on spacetime, with a unique map relating them to the spacetime coordinates. Thus, the basic assumption of our model is that we assume our free constant *g* to act as a relativistic fluid, being described through additional constant Stückelberg fields, so more conveniently one has to call it by g^{σ} with $\sigma = 0, 1, 2, 3$, instead of simply g. In this way, the action is contracted without violating the Lorentz invariance by contracting the Lorentz indexes for the fermion fields, since the effective quark Q, and the Dirac gamma matrix have two free indexes. However, since we are working in a pure homogeneous and isotropic scenario, neglecting the presence of both perturbations and back-reactions, the set of free constants becomes a fluid with comoving coordinates with an internal time coordinate represented by $g^a \equiv (g, 0, 0, 0)$ only¹ [\[19](#page-17-14), [20](#page-17-19), [26\]](#page-17-20). Clearly, this choice restores the broken diffeomorphisms in four-dimensional spacetimes, permitting the fluid physical properties to be relativistically invariant [\[19](#page-17-14), [27](#page-17-21)[–29](#page-17-22)].

We assumed the minimal choice in Eq. [\(3\)](#page-2-1) by extending the gravitational baryogenesis [\[30](#page-17-23)] through replacing the scalar curvature with fermionic fields. In particular, a gravitational interaction between the derivative of a first field, namely the environment variation $\partial_{\mu}\psi$, and another (external) field Q providing the particle contribution.^{[2](#page-3-1)} In this picture, this interaction causes the reheating and can provide hints toward the dynamically break of the charge–parity–time reversal (CPT) symmetry in an expanding universe. 3

The ψ field vacuum expectation value (VEV) is $\langle \psi \rangle = \psi_0 e^{i\theta}$, where the dimensionless angular field θ is the pseudo Nambu–Goldstone boson. Here, to let θ play the role of the inflaton, we further include in the Lagrangian a potential $V(\theta)$ that agrees with the *Planck* collaboration results [\[11](#page-17-24)]. In particular, such a potential has to be quadratic in θ for small oscillations around $\theta = 0$. We select, among the best candidate, the Starobinsky [\[12\]](#page-17-25) and the T-model [\[13](#page-17-26)] potentials, respectively

$$
V_1(\theta) = \Lambda^4 \left[1 - \exp\left(-\sqrt{\frac{2}{3}} \frac{\psi_0 \theta}{M_{\text{Pl}}} \right) \right]^2 \approx \frac{2}{3} \frac{\Lambda^4 \psi_0^2 \theta^2}{M_{\text{Pl}}^2},\tag{4a}
$$

$$
V_2(\theta) = \Lambda^4 \tanh^2\left(\frac{\psi_0 \theta}{\sqrt{6\alpha} M_{\text{Pl}}}\right) \approx \frac{\Lambda^4 \psi_0^2 \theta^2}{6\alpha M_{\text{Pl}}^2},\tag{4b}
$$

where Λ is the amplitude and M_{Pl} is the Planck mass and $-2 < \log_{10} \alpha < 4$. These choices are licit because, as we will see, the linear term $\partial_{\theta}V(\theta) \equiv V'(\theta) \propto \psi_0^2 \theta$ enters in the equation of motion (EoM) for the θ field. From Eqs. [\(4a\)](#page-3-3)–[\(4b\)](#page-3-4), we define the bare mass of the potentials as $m = \mu \Lambda^2/(\sqrt{3}M_{\rm Pl})$ with $\mu = \{2, 1/\sqrt{\alpha}\}$, respectively.

We now list below our assumptions aimed at simplifying our treatment.

– The condition $h \ll 1$ ensures small enough m_Q and m_L so that the θ field decay produces *Q* and *L*.

¹ Other choices for the set g^{σ} , different from the Stückelberg field, may also be considered, but lie beyond the purposes of this work.

² Here, for simplicity, we do not consider the ψ and *L* coupling to avoid unexpected lepton currents.

³ This represents, however, a far topic from this work that will be investigated in future efforts.

- The SU(1) invariance is for rotations $\alpha = -\theta$.
- To avoid significant additional particle production, we assume $\partial_{\mu}\psi_0 \simeq 0$, which is valid as the reheating approaches its end.

Thus, implementing the above assumptions, the overall Lagrangian is given by Eqs. (2) – (3)

$$
\mathcal{L} = X - V(\theta) + \bar{Q}i\gamma^{\mu}\partial_{\mu}Q - m_{Q}\bar{Q}Q + \bar{L}i\gamma^{\mu}\partial_{\mu}L - m_{L}\bar{L}L + [h\psi_{0}\bar{Q}L + h.c] + \partial_{\mu}\theta J^{\mu},
$$
\n(5)

leading to the Noether baryonic current:

$$
J^{\mu} \equiv \bar{Q}\gamma^{\mu}Q - g\psi_0\gamma^{\mu}(Q + \bar{Q}).
$$
 (6)

Particle production occurs after the inflation, during the reheating. In this epoch, the vacuum energy is converted into radiation energy. To accurately quantify this effect, one has to calculate the production of particles and its back reaction on the inflaton field as it rolls down the potential. It is therefore crucial to study the the equation of motion (EoM) for the inflaton field. To this aim, since inflation already occurred, we assume a spatially flat homogeneous and isotropic background, thus all the fields are functions of the time variable only. Applying the Eulero-Lagrange equation we obtain the EoMs of the fields Q and \overline{Q} , respectively,

$$
4\dot{Q} + i(\gamma_{\mu}m_{Q} - 4\dot{\theta})Q = i\psi_{0}(\gamma_{\mu}hL - 4g\dot{\theta}), \qquad (7a)
$$

$$
4\bar{Q} - i(\gamma_{\mu}m_{Q} - 4\dot{\theta})\bar{Q} = -i\psi_{0}(\gamma_{\mu}h\bar{L} - 4g\dot{\theta}), \qquad (7b)
$$

the EoM of the fields L and \overline{L} , respectively,

$$
4\dot{L} + i\gamma_{\mu}m_{L}L = i\gamma_{\mu}h\psi_{0}Q, \qquad (8a)
$$

$$
4\bar{L} - i\gamma_{\mu}m_{L}\bar{L} = -i\gamma_{\mu}h\psi_{0}\bar{Q}.
$$
 (8b)

By employing Eqs. (7) and taking the VEV in the Heisenberg representation, we write the EOM for the θ field

$$
\psi_0^2(\ddot{\theta} + 3H\dot{\theta}) + V'(\theta) = -ih\psi_0\langle \bar{Q}L - \bar{L}Q\rangle + ihg\psi_0^2\langle L - \bar{L}\rangle - ig\psi_0 m_Q\langle Q - \bar{Q}\rangle.
$$
\n(9)

Solving up Eq. [\(9\)](#page-4-0) requires (a) a semiclassical approach, treating θ and ψ_0 as classical fields and quantizing Q and L, and (b) a perturbative approach $\Xi(t) = \Xi_0(t) + h \Xi_1(t)$ for $h \ll 1$ [\[14](#page-17-8), [15](#page-17-9)], where Ξ_0 generically labels the free *Q* and *L* fields (for $h \simeq 0$) with the condition $\Xi_0 = 0$ and a vacuum expectation value $\langle \Xi_0 \rangle = 0$.

We work up to the order h^2 and assume as solution of Eqs. [\(9\)](#page-4-0) a damped harmonic oscillator $\theta(t) = \theta_i(t) \cos(\Omega t)$ with renormalized mass Ω and amplitude $\theta_i(t)$, varying with time more slowly than the cosine term.

$$
\langle \bar{Q}L - \bar{L}Q \rangle = -\frac{i h}{4\pi} \psi_0 \Omega \dot{\theta} + \frac{i h}{2\pi^2} \psi_0 \Omega^2 \log \left(\frac{2\omega}{\Omega}\right) \theta, \tag{10}
$$

where ω is the particle energy.

The expressions of the other two new terms are detailed in the following. First, by solving Eqs.(7)–(8) for $h = 0$ and $m_O = 0$, we compute the free fields solutions

$$
Q_0(t) = A(t)e^{i\theta(t)} + g\psi_0,
$$
\n(11a)

$$
L_0(t) = B(t)e^{-i\gamma^0 m_L t/4},
$$
\n(11b)

where we imposed the ansatz $A \equiv A(t)$ and $B \equiv B(t)$, leading to $\dot{A} = \dot{B} = 0$ and VEV $\langle A \rangle = \langle B \rangle = 0$. The solutions for $\overline{Q}_0(t)$ and $\overline{L}_0(t)$ are the h.c of Eqs. (11). It is clear that at the zero-th order in *h* we have $\langle Q - \bar{Q} \rangle^{(0)} = \langle L - \bar{L} \rangle^{(0)} = 0$, thus it does not contribute to Eq. [\(9\)](#page-4-0).

Moving to the first order in *h*, i.e., recovering the linear terms in *h* of Eqs. (7)–(8), the perturbative solutions of the fields *Q* and *L* are given by

$$
Q(t) = Q_0(t) + ih\gamma^0 \psi_0 \int d^4 y \mathcal{G}_Q(x, y) L_0(t_y), \qquad (12a)
$$

$$
L(t) = L_0(t) + ih\gamma^0 \psi_0 \int d^4 y \mathcal{G}_L(x, y) Q_0(t_y),
$$
 (12b)

where $\mathcal{G}_O(x, y)$ and $\mathcal{G}_L(x, y)$ are the Green functions for the fields Q and L , respectively, and satisfy the following relations

$$
\[4\partial_t + i\gamma^0 m_Q - 4i\dot{\theta}\] \mathcal{G}_Q(x, y) = \delta(x - y),
$$

$$
\[4\partial_t + i\gamma^0 m_L\] \mathcal{G}_L(x, y) = \delta(x - y),
$$

where the square brackets of the first relation defines the operator \mathcal{O}_O and the square brackets of the second one defines the operator \mathcal{O}_L .

From Eq. [\(12a\)](#page-5-0) and the analogous solution for \overline{Q} , it follows that also at the first order, we have $\langle Q-\overline{Q}\rangle^{(1)} = 0$. Then, by taking the VEV of Eq. [\(12b\)](#page-5-0) and applying the operator O_L to both sides of this equation, we obtain as a solution $\langle L(t) \rangle = b + k_1 e^{-iat}$, where we defined $a \equiv \gamma^0 m_L/4$ and $b \equiv h g \psi_0^2 / m_L$. If we impose the condition $\langle L(t) \rangle_{h=0} = 0$, which follows from Eq. (11), we find $k_1 = 0$ and thus $\langle L - \bar{L} \rangle^{(1)} = 0$.

Finally, we move on to the order h^2 . We replace, into Eq. [\(12a\)](#page-5-0), the term $L_0(t)$ with the solution Eq. $(12b)$. Considering only the highest order term, we find

$$
Q(t) = -h^2 \psi_0^2 \iint d^4 y \, d^4 z \, \mathcal{G}_Q(x, y) \mathcal{G}_L(y, z) \, Q_0(t_z), \tag{13}
$$

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that has a VEV given by

$$
\langle Q(t) \rangle = -h^2 g \psi_0^3 \iint d^4 y \, d^4 z \, \mathcal{G}_Q(x, y) \mathcal{G}_L(y, z). \tag{14}
$$

We apply the product of operators O_LO_Q to both sides of Eq. [\(14\)](#page-6-0). Since the pseudo Nambu-Goldstone boson acquires a mass that largely exceeds the fermionic masses, we can safely assume that m_Q , $m_L \approx 0$ and obtain

$$
16\langle \ddot{Q}(t)\rangle - 16i\dot{\theta}\langle \dot{Q}(t)\rangle - 16i\ddot{\theta}\langle Q(t)\rangle = -h^2 g \psi_0^3.
$$
 (15)

The solution of Eq. [\(15\)](#page-6-1) can be obtained by setting the initial conditions $Y(0) \approx 0$ and $\dot{Y}(0) \approx 0$ and using the damped harmonic oscillator ansatz. Further, considering the case of small oscillations around the bottom of the potential, we expand up to the first order in $\theta(t)$ to get

$$
\langle Q(t) \rangle \simeq -\frac{ih^2 g \psi_0^3}{16} \left[\frac{t^2}{2} + i \frac{\theta_i(t) - \theta(t) + \dot{\theta}(t)t}{\Omega^2} \right].
$$

Because there is no evidence for an ongoing baryogenesis process, we suppose terms proportional to t^2 and $\dot{\theta}t$ to be negligible. Therefore we get

$$
\langle Q - \bar{Q} \rangle^{(2)} = -\frac{i h^2 g \psi_0^3}{8\Omega^2} \left[\theta_i(t) - \theta(t) \right]. \tag{16}
$$

Following an analogous procedure, i.e., replacing into Eq. [\(12b\)](#page-5-0), the term $Q_0(t)$ with the solution Eq. $(12a)$, considering only the highest order term, and applying the VEV, we obtain

$$
\langle L - \bar{L} \rangle^{(2)} = 0. \tag{17}
$$

Finally, plugging Eqs. [\(10\)](#page-5-1), [\(16\)](#page-6-2) and [\(17\)](#page-6-3) into Eq. [\(9\)](#page-4-0), the EoM of the θ field become

$$
\ddot{\theta}(t) + (3H + \Gamma)\dot{\theta}(t) + \Omega^2 \theta(t) + C\theta_i(t) = 0,
$$
\n(18)

where we defined

$$
\Gamma \equiv \frac{h^2}{4\pi} \Omega, \quad C \equiv \frac{h^2 g^2 \psi_0^2 m_Q}{8\Omega^2},\tag{19}
$$

and qualified the renormalized mass Ω by

$$
m^2 \equiv \Omega^2 \left[1 + C + \frac{h^2}{2\pi^2} \log \left(\frac{2\omega}{\Omega} \right) \right].
$$
 (20)

It is important to stress that Γ represents the decay rate of the inflaton field and, thus, the heuristic term $\Gamma \dot{\theta}$ describes the reheating. Since $h \ll 1$, it has to be $\Gamma \ll \Omega$. In addition, being m_O negligible, the constant *C* is negligible too. Therefore, assuming

H $\ll \Gamma$ and applying the initial conditions $\theta(0) = \theta_i$, $\theta(0) \simeq 0$, we get the damped harmonic oscillator solution

$$
\theta(t) = \theta_i e^{-\Gamma t/2} \cos(\Omega t), \qquad (21)
$$

where θ_i is the value of the θ field at the beginning of the reheating epoch.

3 Particle production

We now proceed to calculate the number density of the particles produced during reheating. To the lowest order in perturbation theory, the average number density *n* of particle-antiparticle pairs produced by the decay of the classical scalar field ψ is formalized by [\[14](#page-17-8)]

$$
n = \frac{1}{V} \sum_{s_1, s_2} \int \frac{d^3 p_1}{(2\pi)^3 2p_1^0} \frac{d^3 p_2}{(2\pi)^3 2p_2^0} |A|^2,
$$
 (22)

where *A* is the pair production amplitude and subscripts 1 and 2 refer to the final particles produced. We need to swap it between baryons and DM for reaching baryon and DM amount of particles. By virtue of our Lagrangian couplings, Eq. [\(5\)](#page-4-1) furnishes different kinds of interacting particles, comprising (I) QL and QL pairs, clearly related to the observed baryonic asymmetry [\[16](#page-17-27), [17\]](#page-17-10), (II) $g\psi_0Q$ and $g\psi_0Q$ pairs related to the production of non-baryonic particles. Since inflaton acts as source, we speculate they contribute to DM birth, leading to DM *quasi-particles*^{[4](#page-7-2)}. The reason why it is referred to as a source is that the corresponding Lagrangian couplings potentially generate particle excitation in the field.

3.1 Baryonic matter production

Focusing on baryons, the average number density of QL pairs is computed from Eq. [\(22\)](#page-7-3) by quantizing the fields *Q* and *L*. For the *Q* field we have

$$
Q = \sum_{s} \int \frac{d^3k}{(2\pi)^3 2k^0} [u_k^s b_k^s e^{-ik \cdot x} + v_k^s d_k^{s\dagger} e^{ik \cdot x}], \tag{23}
$$

where b_k^s and $d_k^{s\dagger}$ are annihilation and creation operators obeying the commutation rules $\{b_k^s, b_{k'}^{s'^{\dagger}}\} = \{d_k^s, d_{k'}^{s'^{\dagger}}\} = (2\pi)^3 2k^0 \delta^3(\mathbf{k} - \mathbf{k}') \delta_{ss'}; u_k^s$ and v_k^s are the spinors of particles and antiparticles with momentum *k* and spin *s*, respectively. The field *L* can be written in a fashion similar to Eq. [\(23\)](#page-7-4).

⁴ We here conjecture this mechanism can lead to a non-vanishing DM contribution. Clearly, what we here baptize *DM particles* is better to say as *DM quasi-particles*. Henceforth, we only briefly name them DM particles.

The process pair production amplitude is

$$
A = \langle Q(p_1, s_1), \bar{L}(p_2, s_2) | ih\psi_0 \int d^4x \, \bar{Q}(x) L(x) e^{i\theta(t)} | 0 \rangle,
$$

and, together with Eqs. (22) and (23) , gives

$$
n(Q, \bar{L}) = \frac{1}{V} \sum_{s_1, s_2} \int d\tilde{p}_1 d\tilde{p}_2 \mid \langle Q(p_1, s_1), \bar{L}(p_2, s_2) | ih\psi_0 \int d^4x e^{i\theta(t)} \times \sum_{s'_1} \int d\tilde{k}_1 \left[\bar{u}_{k_1}^{s'_1} b_{k_1}^{s'_1 \dagger} e^{ik_1 \cdot x} + \bar{v}_{k_1}^{s'_1} d_{k_1}^{s'_1} e^{-ik_1 \cdot x} \right] \times \sum_{s'_2} \int d\tilde{k}_2 \left[u_{k_2}^{s'_2} b_{k_2}^{s'_2} e^{-ik_2 \cdot x} + v_{k_2}^{s'_2} d_{k_2}^{s'_2 \dagger} e^{ik_2 \cdot x} \right] |0\rangle|^2, \tag{24}
$$

where $d\tilde{p} \equiv d^3 p/[(2\pi)^3 2p^0]$ and, in general, a state $\langle A(p_1, s_1), \bar{B}(p_2, s_2) \rangle$ corresponds to a final state with an *A* particle of momentum p_1 and spin s_1 and an anti-*B* particle with momentum p_2 and spin s_2 .

Equation (24) can be simplified noting that the only non-zero term is given by

$$
\langle Q(p_1,s_1), \bar{L}(p_2,s_2)|b_{k_1}^{s_1'\dagger}d_{k_2}^{s_2'\dagger}|0\rangle = (2\pi)^6 4p_1^0 p_2^0 \delta^3(\mathbf{p}_1 - \mathbf{k}_1) \delta^3(\mathbf{p}_2 - \mathbf{k}_2) \delta_{s_1s_1'} \delta_{s_2s_2'}.
$$

The δ^3 functions imply that $k_1^0 = p_1^0$ and $k_2^0 = p_2^0$. Next, we exploit the relation $\int d^3x \, e^{-i\mathbf{p}\cdot\mathbf{x}} = (2\pi)^3 \delta^3(\mathbf{p})$ and get a Dirac delta inside the square modulus. The delta squared is naively addressed as $|\delta^3(\mathbf{p}_1 + \mathbf{p}_2)|^2 \equiv \delta^3(\mathbf{p}_1 + \mathbf{p}_2)\delta^3(0)$. Since we are working out a perturbative expansion within a finite volume, we can approximate $\delta^{3}(0) = V/(2\pi)^{3}$. Thus Eq. [\(24\)](#page-8-0) becomes

$$
n(Q, \bar{L}) = \frac{h^2 \psi_0^2}{(2\pi)^3} \sum_{s_1, s_2} \int \frac{d^3 p_1}{2p_1^0} \frac{d^3 p_2}{2p_2^0} \delta^3(\mathbf{p}_1 + \mathbf{p}_2)
$$

$$
\times |\bar{u}_{p_1}^{s_1} v_{p_2}^{s_2}| \int dt e^{i[(p_1^0 + p_2^0)t + \theta(t)]}|^2.
$$
 (25)

The term $\delta^3(\mathbf{p}_1+\mathbf{p}_2)$ kills the integral in d^3p_2 and implies that $\mathbf{p}_1 = -\mathbf{p}_2$. The assumption of negligible fermionic masses, implies that $p^0 = \sqrt{|\mathbf{p}|^2 + m^2} \simeq |\mathbf{p}|$ and leads to the identity $p_1^0 = p_2^0 = \omega$. The sum over the spin states gives $\sum_{s_1, s_2} |\bar{u}_{p_1}^{s_1} v_{p_2}^{s_2}|^2 =$ $4(p_1^0 p_2^0 + |\mathbf{p}_1||\mathbf{p}_2|) = 8\omega^2$. Finally, we write $d^3 p_1 = 4\pi |\mathbf{p}_1|^2 d|\mathbf{p}_1| = 4\pi \omega^2 d\omega$ and obtain

$$
n(Q,\bar{L}) = \frac{h^2 \psi_0^2}{\pi^2} \int d\omega \omega^2 \Big| \int dt e^{i[2\omega t + \theta(t)]} \Big|^2.
$$
 (26)

A similar expression for $n(L, \overline{Q})$ can be obtained by replacing $\theta(t)$ with $-\theta(t)$. Afterwards, we define the baryon number density $n_b \equiv n(Q, \bar{L})$ and the antibaryon number density $n_{\bar{b}} \equiv n(L, Q)$.

Since θ is small, in Eq. [\(26\)](#page-8-1) we can expand $e^{i\theta} \approx 1 + i\theta - \theta^2/2$. The lowest order term gives $\int dt e^{2i\omega t} \propto \delta(2\omega) = 0$. Instead the *i* θ term, when squared, gives the same contribution to particles and antiparticles. So, in order to obtain the lowest order asymmetry, we should consider cross terms. We find

$$
n_{\rm B} = n_b - n_{\bar{b}} = \frac{h^2 \psi_0^2}{\pi^2} \int d\omega \omega^2 \left[\frac{A(\theta) \overline{A(\theta^2)}}{i} + h.c. \right] \tag{27}
$$

where $A(f) = \int_{-\infty}^{+\infty} dt f(t)e^{2i\omega t}$. Finally, using the solution in Eq. [\(21\)](#page-7-5), we find [\[15\]](#page-17-9)

$$
n_b = \frac{1}{2} \Omega \psi_0^2 \theta_i^2 + \frac{h^2}{16\pi} \Omega \psi_0^2 \theta_i^3 \quad , \quad n_{\bar{b}} = \frac{1}{2} \Omega \psi_0^2 \theta_i^2 - \frac{h^2}{16\pi} \Omega \psi_0^2 \theta_i^3,
$$

leading to the final result

$$
n_B = \frac{h^2}{8\pi} \Omega \psi_0^2 \theta_i^3.
$$
 (28)

3.2 Dark matter production

To obtain the DM number density, the amplitude to be accounted for in Eq. [\(22\)](#page-7-3) is now

$$
A_{\rm DM} = \langle Q(p,s)|g^2\psi_0^2 \int d^4x \dot{\theta}(t)\overline{Q}(x)e^{i\theta(t)}|0\rangle.
$$

So, after so quantizing *Q* and solving, analogously to the case of baryons, we get

$$
n(g\psi_0, Q) = \frac{g^2 \psi_0^2}{(2\pi)^3} \int \frac{d^3 p}{2p^0} \delta^3(\mathbf{p}) \sum_s u_p^s \bar{u}_p^s \left| \int dt \, \dot{\theta}(t) e^{i[p_0 t + \theta(t)]} \right|^2.
$$
 (29)

The $\delta^3(\mathbf{p})$ implies that $p^0 = E_p = m_Q$, thus the sum over spin states gives $\sum_s u_p^s \bar{u}_p^s =$ $p_{\mu} \gamma^{\mu} + m_{\Omega} = p_0(\gamma^0 + 1).$

To solve the time integral, we expand $e^{-i\theta} \approx 1 - i\theta$ and consider only the zero-th order term because at the first order we get terms $\alpha \theta \dot{\theta}$, which can be neglected. Finally, considering the solution in Eq. [\(21\)](#page-7-5) with the working assumptions m_Q , $\Gamma \ll \Omega$, the square modulus in Eq. [\(29\)](#page-9-1) gives $\int dt \dot{\theta}(t) e^{im\phi t} \vert^2 \approx \theta_i^2$. Finally, we write Eq. (29) as $n(g\psi_0, Q) \simeq \frac{g^2\psi_0^2}{16\pi^3}\theta_i^2(\gamma^0 + 1)$. For $n(g\psi_0, \bar{Q})$ we have the same result with $(\gamma^0 - 1)$ instead of $(\gamma^0 + 1)$, because $\sum_s v_p^s \bar{v}_p^s = p_\mu \gamma^\mu - m_Q$, ending up with DM asymmetry

$$
n_{\rm DM} = n(g\psi_0, Q) - n(g\psi_0, \bar{Q}) \simeq \frac{g^2 \psi_0^2}{8\pi^3} \theta_i^2.
$$
 (30)

An earlier idea of unifying the creation of DM and baryons was proposed in [\[31,](#page-17-28) [32\]](#page-17-29), albeit with a mechanism profoundly different from our scheme.

3.3 Mass mixing

From the above results, there is evidence for the occurrence of two particle production stages:

- (1) At the beginning, the interaction term $g\psi_0\dot{\theta}Q$ dominates in view of the large value of ψ_0 and $h \ll 1$, producing *de facto* the DM;
- (2) Afterwards, as $\dot{\theta} \rightarrow 0$, DM production becomes negligible, leaving the baryon production dominant.

However, the *Q* and *L* fields are not mass eigenstates, hence, if *Q* and *L* do not decay immediately into stable lighter mass particles with appropriate quark quantum numbers, their mixing may occur. For this reason, Eqs. [\(28\)](#page-9-2) and [\(30\)](#page-9-3) have to account for this phenomenon. The mass mixing in the initial stage can be evaluated from the complete mass matrix

$$
M = \begin{pmatrix} m_Q & -h\psi_0 & \dot{\theta} \\ -h\psi_0 & m_L & 0 \\ \dot{\theta} & 0 & 0 \end{pmatrix},
$$
(31)

which admits as eigenvalues

$$
\lambda_1 = \frac{a}{3} - \frac{\alpha (1 - i\sqrt{3})}{6\sqrt[3]{2}} + \frac{(1 + i\sqrt{3})(3b - a^2)}{3 \times 2^{2/3} \alpha},
$$
(32a)

$$
\lambda_2 = \frac{a}{3} - \frac{\alpha (1 + i\sqrt{3})}{6\sqrt[3]{2}} + \frac{(1 - i\sqrt{3})(3b - a^2)}{3 \times 2^{2/3} \alpha},
$$
(32b)

$$
\lambda_3 = \frac{a}{3} + \frac{\alpha}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}(3b - a^2)}{3\alpha},
$$
\n(32c)

where $a \equiv m_Q + m_L$, $b \equiv m_Q m_L + \dot{\theta}^2 - h^2 \psi_0^2$, and $c \equiv -\dot{\theta}^2 m_L$. Further, we defined

$$
\frac{\alpha^3}{27} \equiv \frac{2a^3}{27} - \frac{ab}{3} - c + \sqrt{\frac{4b^3 - a^2b^2 - 4a^3c}{27} + \frac{2abc}{3} + c^3}.
$$

The eigenvalues λ_i must be real. Taking in mind that the fermionic masses are negligible and the θ field oscillations are small, we can apply the conditions $c \ll a, b \ll 1$ and neglect their high powers. Next, resorting these conditions, we can arrange α in such a way that we can expand it by using the approximation $(1 + x)^\gamma \simeq 1 + \gamma x$, with *x* ≪ 1. We find that $\alpha \simeq (3\sqrt{3}\sqrt{4b^3 - a^2b^2} - 9ab)^{1/3} \simeq \frac{\sqrt{3}}{\sqrt[3]{4}}$ $\frac{3}{\sqrt[3]{4}}$ $\sqrt{4b - a^2}$, and get real eigenvalues

$$
\lambda_1 \simeq \frac{m_Q + m_L}{3} - \frac{1}{2} \sqrt{\Delta m^2 + 4(h^2 \psi_0^2 - \dot{\theta}^2)},
$$
\n(33a)

$$
\lambda_2 \simeq \frac{m_Q + m_L}{3} + \frac{1}{2} \sqrt{\Delta m^2 + 4(h^2 \psi_0^2 - \dot{\theta}^2)},\tag{33b}
$$

$$
\lambda_3 \simeq \frac{m_Q + m_L}{3},\tag{33c}
$$

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where we defined $\Delta m \equiv m_Q - m_L$. With the position $\beta_i = \dot{\theta}/\lambda_i$, the mass eigenstates are given by

$$
\Phi_1 = N_1^{-1} \left[L + \epsilon_1 (Q + \beta_1 g \psi_0) \right], \tag{34a}
$$

$$
\Phi_2 = N_2^{-1} (Q + \epsilon_2 L + \beta_2 g \psi_0), \tag{34b}
$$

$$
\Phi_3 = N_3^{-1} (Q + \epsilon_3 g \psi_0), \tag{34c}
$$

which incorporate the normalizations

$$
N_1 = \sqrt{1 + \epsilon_1^2 (1 + \beta_1^2)},
$$

\n
$$
N_2 = \sqrt{1 + \beta_2^2 + \epsilon_2^2},
$$

\n
$$
N_3 = \sqrt{1 + \epsilon_3^2},
$$

and the definitions

$$
\epsilon_1 = \frac{h\psi_0}{m_Q + \dot{\theta}\beta_1 - \lambda_1},
$$

$$
\epsilon_2 = \frac{h\psi_0}{m_L - \lambda_2},
$$

$$
\epsilon_3 = \beta_3.
$$

Now, the baryon asymmetry is the sum of terms given by the product of a number density of produced particle/antiparticle pairs times the quark content of the pair

$$
n_{\rm B}^{\rm M} = \sum_{i,j\neq i} n(\Phi_i, \bar{\Phi}_j) |\langle Q | \Phi_i \rangle|^2 - n(\Phi_j, \bar{\Phi}_i) |\langle \bar{Q} | \bar{\Phi}_i \rangle|^2
$$

=
$$
\frac{1}{V} \sum_{s_i, s_j} \sum_{\substack{j \neq i}} \int d\widetilde{p}_i d\widetilde{p}_j \xi_{ij} \left[|A_{i\overline{j}}|^2 - |A_{j\overline{i}}|^2 \right],
$$
(35)

where each $n(\Phi_i, \bar{\Phi}_i)$ and $n(\Phi_i, \bar{\Phi}_i)$ have been computed as per Eqs. [\(22\)](#page-7-3) with the positions

$$
\xi_{12} \equiv \frac{1}{N_2^2} - \frac{\epsilon_1^2}{N_1^2},
$$

$$
\xi_{13} \equiv \frac{1}{N_3^2} - \frac{\epsilon_1^2}{N_1^2},
$$

$$
\xi_{23} \equiv \frac{1}{N_3^2} - \frac{1}{N_2^2}.
$$

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The terms $A_{i\bar{j}}$ in Eq. [\(35\)](#page-11-0) can be computed by expressing *Q*, *L* and $g\psi_0$ as linear combinations of Φ_1 , Φ_2 and Φ_3 . Thus, we get

$$
|A_{1\bar{2}}|^2 - |A_{2\bar{1}}|^2 \equiv \zeta_{12} \left[|A_{\bar{Q}L}|^2 - |A_{\bar{L}Q}|^2 \right],\tag{36a}
$$

$$
|A_{1\bar{3}}|^2 - |A_{3\bar{1}}|^2 \equiv \zeta_{13} \left[|A_{\bar{Q}L}|^2 - |A_{\bar{L}Q}|^2 \right],\tag{36b}
$$

$$
|A_{2\bar{3}}|^2 - |A_{3\bar{2}}|^2 \equiv \zeta_{23} \left[|A_{\bar{Q}L}|^2 - |A_{\bar{L}Q}|^2 \right],\tag{36c}
$$

where

$$
\zeta_{12} = \frac{\left[\epsilon_3^2(\beta_2 - \epsilon_3)^2 - \epsilon_1^2 \epsilon_2^2 \epsilon_3^2(\beta_1 - \epsilon_3)^2\right]}{(\beta_2 - \beta_1 \epsilon_1 \epsilon_2 - \epsilon_3 + \epsilon_1 \epsilon_2 \epsilon_3)^4 N_1^{-2} N_2^{-2}},
$$
\n
$$
\zeta_{13} = \frac{\left[(\beta_2 - \epsilon_3)^2(\beta_2 - \beta_1 \epsilon_1 \epsilon_2)^2 - \epsilon_1^2 \epsilon_2^2 \epsilon_3^2(\beta_1 - \beta_2)^2\right]}{(\beta_2 - \beta_1 \epsilon_1 \epsilon_2 - \epsilon_3 + \epsilon_1 \epsilon_2 \epsilon_3)^4 N_1^{-2} N_3^{-2}},
$$
\n
$$
\zeta_{23} = \frac{\left[\epsilon_1^2(\beta_1 - \epsilon_3)^2(\beta_2 - \beta_1 \epsilon_1 \epsilon_2)^2 - \epsilon_1^2 \epsilon_3^2(\beta_1 - \beta_2)^2\right]}{(\beta_2 - \beta_1 \epsilon_1 \epsilon_2 - \epsilon_3 + \epsilon_1 \epsilon_2 \epsilon_3)^4 N_2^{-2} N_3^{-2}}.
$$

Putting together Eqs. (28) and (35) – (37) , we obtain

$$
n_{\rm B}^{\rm M} = \frac{h^2}{8\pi} \Omega \psi_0^2 \theta_i^3 f_{\epsilon}, \quad f_{\epsilon} \equiv \sum_{i=1}^2 \sum_{\substack{j \neq i \\ j > i}}^3 \xi_{ij} \zeta_{ij}.
$$
 (37)

In the case of the DM, the developed machinery provides an asymmetry given by

$$
n_{\rm DM}^{\rm M} = \sum_{i} n(g\psi_0, \bar{\Phi}_i) |\langle g\psi_0 | \Phi_i \rangle|^2 - n(g\psi_0, \bar{\Phi}_i) |\langle g\psi_0 | \bar{\Phi}_i \rangle|^2 = \frac{g^2 \psi_0^2}{8\pi^3} \theta_i^2 \chi_{\epsilon},
$$
\n(38)

where

$$
\chi_{\epsilon} \equiv \frac{\epsilon_1^2 \epsilon_2^2 \epsilon_3^2 + \epsilon_3^2 + (\beta_2 - \beta_1 \epsilon_1 \epsilon_2)^2}{(\beta_2 - \beta_1 \epsilon_1 \epsilon_2 - \epsilon_3 + \epsilon_1 \epsilon_2 \epsilon_3)^2}.
$$

In asymptotic regime $\dot{\theta} \to 0$, we get $\beta_1 = \beta_2 = \beta_3 = 0$, $\epsilon_1 = \epsilon_2 = \epsilon$, and $\epsilon_3 = 0$. In this limit, the spontaneous baryogenesis is recovered [\[14,](#page-17-8) [15\]](#page-17-9) and for $\Delta m = 0$ we have $\epsilon = 1$, taming the asymmetry, *i.e.*, $n_B^M = 0$ and, moreover, the DM production is suppressed.

Confronting Eqs. [\(37\)](#page-12-0) and [\(38\)](#page-12-1), we notice $n_{\rm B}^{\rm M} \propto h^2$ whereas $n_{\rm DM}^{\rm M}$ does not depend upon *h*. Provided the interplay between predominant quantities, e.g. ψ_0 , and negligible terms, say m_Q , qualitatively, the dependence $n_B^M \propto h^2$, with the prescription $h \ll 1$, indicates DM might dominate over baryons. Finally mass mixing ensures that the overall process is not instantaneous and smears baryogenesis out. This is a relief, since DM contribution could, in principle, threaten to blow up.

3.4 Predicting dark matter candidates

Equations [\(37\)](#page-12-0) and [\(38\)](#page-12-1) have been computed in the regime $H \ll \Gamma$, which lasts for about $\Delta t \approx t \approx \Gamma^{-1}$. However, particle production properly begins as $H \approx \Omega \gg \Gamma$. *i.e.*, when θ starts oscillating around the minimum of the potential. In this regime of duration $\Delta t_{\star} \approx t_{\star} \approx \Omega^{-1}$, the Universe's expansion effects turn out to be nonnegligible. During the reheating, the Universe behaves as matter-dominated with a scale factor $a(t) \propto t^{2/3}$. The θ field evolves as $\propto t^{-2/3}$, whereas baryon and DM asymmetries at the beginning are given by

$$
\frac{n_{\rm B\gamma}^{\rm M}}{n_{\rm B}^{\rm M}} = \frac{\Delta t_{\star}}{\Delta t} \left(\frac{t}{t_{\star}}\right)^2 \approx \frac{\Omega}{\Gamma},\tag{39a}
$$

$$
\frac{n_{\text{DM}}^{\text{M}}}{n_{\text{DM}}^{\text{M}}} = \frac{\Delta t_{\star}}{\Delta t} \left(\frac{t}{t_{\star}}\right)^{4/3} \approx \left(\frac{\Omega}{\Gamma}\right)^{1/3},\tag{39b}
$$

implying that the particle production is larger at the beginning. For this reason, hereafter $n_{DM\star}^{\text{M}}$ and $n_{B\star}^{\text{M}}$ are considered as the total asymmetries. Dividing Eq. [\(39a\)](#page-13-1) by Eq. [\(39b\)](#page-13-2), we infer the unknown mass of DM constituent

$$
m_X \simeq \frac{\pi^2 h^2 m_p \Omega \theta_i}{g^2} \left(\frac{f_\epsilon}{\chi_\epsilon}\right) \left(\frac{\Omega_{DM}}{\Omega_B}\right) \left(\frac{\Omega}{\Gamma}\right)^{2/3},\tag{40}
$$

where we compared the produced asymmetries with the cosmic densities, *i.e.*, we assumed $n_{DM}^* \simeq \rho_{cr} \Omega_{DM}/m_X$ and $n_B^* \simeq \rho_{cr} \Omega_B/m_{R}$, with m_p the proton mass and ρ_{cr} the current critical density, namely $\rho_{cr} \equiv 3H_0^2 M_{\rm Pl}^2/(8\pi)$.

Equation [\(40\)](#page-13-3) can be further simplified considering that: (i) recent estimate imply $\Omega_{DM}/\Omega_B \simeq 5$, (ii) $h \ll 1$ leads to $\Omega \simeq m$, (iii) Universe's energy density is dominated by $V(\theta)$, having $H(\theta_i) = \sqrt{4\pi/3}m\psi_0\theta_i/M_{\text{Pl}}$, (iv) $H(\theta_i) = \Gamma$, with $\Gamma(h, m)$ given in Eq. [\(19\)](#page-6-4), yelds to $\theta_i = \sqrt{\frac{3}{4\pi}} \frac{M_{\text{Pl}}}{m \psi_0} \Gamma(h, m)$. Plugging the above into Eq. [\(40\)](#page-13-3), we achieve

$$
m_X \simeq \frac{5\sqrt{3}\pi^{3/2}h^2m_pM_{\rm Pl}m^{2/3}}{2g^2\psi_0} \frac{f_\epsilon}{\chi_\epsilon} \Gamma(h,m)^{1/3},\tag{41}
$$

where f_{ϵ} and χ_{ϵ} are functions of $(h, g, \psi_0, m_Q, m_L, \dot{\theta})$. The function $\dot{\theta}(t)$ can be evaluated as temporal average over $\Delta t_\star \approx \Omega^{-1}$, which is the epoch during which the particle production is more efficient. This average allows us to write $\dot{\theta} \approx -0.5 \, m\theta_i$.

We can now sort out the reheating temperature T_R , requiring all relativistic species energy density, $\rho_{rad} = (\pi^2/30)g^{\star}T_R^4$, is equal to the one estimated for θ field, namely $\rho_{\theta} = 3H^2 M_{\rm Pl}^2/(8\pi)$, at the time $t = \Gamma^{-1}$. We compute

$$
T_R = \left(\frac{45}{4\pi^3 g^*}\right)^{1/4} M_{\text{Pl}}^{1/2} \Gamma^{1/2},\tag{42}
$$

where $g^* \approx 107$ is the effective numbers of relativistic degrees from all particles in thermal equilibrium with photons. Further, from Eqs. [\(37\)](#page-12-0) and [\(42\)](#page-13-4), we can compute baryonic asymmetry parameter $\eta = n_{\rm B\gamma}^{\rm M}/s$, where entropy density is $s = (2\pi^2 g^*/45)T_R^3$ at reheating.

4 Numerical results

Thus our strategy consists in computing the set (h, m_O) with those values that are consistent with current bounds on η and, consequently, solving numerically solving our equations to get m_X and T_R . In so doing, we single out the following bounds $[11, 33]$ $[11, 33]$ $[11, 33]$ $[11, 33]$ *m* ∈ $[10^{10}, 10^{13}]$ GeV, $\psi_0 \in [10^{-6}, 10^{-3}]$ *M*_{Pl}, $\eta = (8.7 \pm 0.10) \times 10^{-11}$, $m_L \approx 0$, having $\Delta m \approx m_Q$ and $m_t < m_Q \ll m$, where $m_t = 173.2$ GeV is the top quark mass. The numerical bound on η defines regions in the space of parameters, as prompted in Fig. [1.](#page-15-0) The contour plots (h, m_O, T_R) (top panels) define the following constraints:

1) for $m = 10^{13}$ GeV and $\psi_0 = 10^{-3} M_{\text{Pl}}$, we have $10^{-5} \ll h \lesssim 10^{-2}$ and

$$
10^{10} \text{ GeV} \lesssim m_Q \ll 10^{13} \text{ GeV},
$$

$$
10^{10} \text{ GeV} \ll T_R \lesssim 10^{13} \text{ GeV}.
$$
 (43)

2) for $m = 10^{10}$ GeV and $\psi_0 = 10^{-6} M_{\text{Pl}}$, we have $10^{-6} \lesssim h \lesssim 10^{-4}$ and

$$
106 GeV \lesssim m_Q \ll 1010 GeV,
$$

$$
107 GeV \lesssim T_R \lesssim 1010 GeV.
$$
 (44)

The above numerical results have been pushed up to $m_O = m$ but, clearly, the ansatz $m_Q \ll m$ has been consistently propagated to the constraints on *h* and T_R . Noteworthy, in the second case the ansatz $m_Q \ll m$ does not introduce absolute lower limits on *h* and T_R . This appears evident by looking at Fig. [1](#page-15-0) (top, right panel).

The estimate of the DM mass constituent depends on the choice of the dimensional constant *g*, as portrayed in Eq. [\(41\)](#page-13-5). On the one hand, large values of this constant bring down the DM mass estimate making it, in principle, consistent with ultralight fields [\[35](#page-17-31)], among which *axions* [\[36](#page-17-32)]. Sterile neutrinos [\[37](#page-17-33)] as fermions, are *a priori* excludable, though their mass range is easily attainable, again, for large values of *g*.

On the other hand, it cannot be $g \ll 1$ due to the large scale energies involved within the epoch in which our computations have been performed. The prize to pay is that *g* turns out to be dimensional, i.e., the theory is not fully-renormalizable. This fact, automatically rules out highly-heavy DM candidates, called variously but mostly as *WIMPZillas* [\[38](#page-17-34)].

To limit our choice of *g*, we decided to rely on the most recent observational signature attributed to DM particles. In this light, we target MeV-scale for DM candidates, as recently proposed to explain the excess of low-energy electron recoil events between 1 and 7 keV measured by the XENON1T collaboration [\[39](#page-17-35)]. Thus, as portrayed in the contour plots (h, m_X, T_R) of Fig. [1](#page-15-0) (bottom panels), we obtain the following bounds:

 $-$ for *m* = 10¹³ GeV and $ψ₀ = 10⁻³M_{Pl}$, *g* ≈ 8 × 10³ GeV^{1/2}, 0.50 MeV $≤ m_X ≤$ 1.99 MeV;

 $-$ for *m* = 10¹⁰ GeV and $\psi_0 = 10^{-6} M_{\text{Pl}}$, *g* ≈ 80 GeV^{1/2}, 0.64 MeV $\lesssim m_X \lesssim$ 2.00 MeV.

5 Conclusions and perspectives

We here introduced a mechanism for unifying baryogenesis and DM production. We preserved spontaneous baryogenesis during reheating, predicting the baryon asymmetry. Further, we turned our attention on how DM could form and mix, proposing DM quasi-particle owing to the couplings between pseudo Goldstone boson and quark fields. In this respect, we set out with a *U*(1) Lagrangian, constructed by means of effective quark *Q* and lepton *L* fields, with a spontaneous symmetry breaking potential, and a further interacting term that couples the evolution of Universe's environment, say ψ_0 , with *Q*. Immediately after the transition, the symmetry breaking potential disappeared and a pseudo Nambu–Goldstone boson, namely the inflaton field, dominated at this stage. Within a quasi-static approximation on the environment field, we highlighted how pairs of baryons and DM particles can be produced, naively described how baryogenesis stops through the mixing process and qualitatively demonstrated why DM dominates over baryonic matter. As byproduct of our manipulations, we are therefore not tied simply to baryogenesis but the overall process yields up two sorts of massive terms, say baryons and DM. Mass mixing ensures how baryogenesis and DM production stop. In particular, examining recent limits on *m* and ψ_0 [\[33](#page-17-30)], and the baryon asymmetry η [\[11\]](#page-17-24), we obtained constraints on h and m_O and found that T_R is consistent with recent estimates [\[33,](#page-17-30) [34](#page-17-36)]. The estimate of the DM mass constituent, in stead, depends upon a dimensional constant *g*, as portrayed in Eq. [\(41\)](#page-13-5). Large values of *g* make m_X consistent with ultralight fields [\[35\]](#page-17-31), among which *axions* [\[36\]](#page-17-32). Sterile neutrinos [\[37](#page-17-33)] as fermions, are *a priori* excludable. The opposite case, i.e. $g \ll 1$, is not possible, due to the large scale energies assumed in our computations, hence automatically ruling out highly-heavy DM candidates, called variously but mostly as *WIMPZillas* [\[38](#page-17-34)]. We decided to target the MeV-scale for DM candidates, as recently proposed to explain the excess of low-energy electron recoil events between 1 and 7 keV measured by the XENON1T collaboration [\[39](#page-17-35)]. Thus, we fixed the value of the constant *g* and extracted numerical bounds on the mass range of the DM constituent, *i.e.*, $0.5 \, \text{MeV} \ll m_X \lesssim 2.00 \, \text{MeV}$. Remarkably, MeV-scale mass particles are suitable DM candidate to successfully explain the currently observed baryonic asymmetry.

Looking ahead, in incoming works we attempt to include quantum chromodynamics and to unify baryogenesis with antecedent inflationary phases [\[40\]](#page-17-12).

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Author Contributions All the authors contributed equally to this work.

Declarations

Conflict of interest All data generated or analysed during this study are included in this published article.

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