



# Black holes in 4D AdS Einstein Gauss Bonnet gravity with power: Yang Mills field

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Received: 1 October 2022 / Accepted: 2 December 2022 / Published online: 15 December 2022

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## Abstract

In this paper we construct an exact spherically symmetric black hole solution with a power Yang–Mills (YM) source in the context of 4D Einstein Gauss–Bonnet gravity (4D EGB). We choose our source as  $(F_{\mu\nu}^{(a)} F^{\mu\nu(a)})^q$ , where  $q$  is an arbitrary positive real number. Thereafter we study the horizon structure, thermodynamic issues like thermal stability and black hole phase transition. Our purpose here is to analyse the black hole space-time under the net effect coming from the Gauss–Bonnet coupling parameter  $\alpha$  and the nonlinear parameter  $q$ . We then evaluate all important thermodynamic quantities to establish the Smarr formula and the first law of thermodynamics in extended phase space. The behaviour of heat capacity as a function of horizon radius is thoroughly studied to understand the thermal stability of the black hole solution. An interesting phenomena of existence/absence of thermal phase transition occur due to the nonlinearity of YM source. For some values of the parameters, we find that the solution exhibits a first-order phase transition, like a van der Waals fluid. In addition, we also verify Maxwell’s equal area law numerically by crucial analysis of Gibbs free energy as a function of temperature. Moreover, the critical exponents are derived and showed the universality class of the scaling behaviour of thermodynamic quantities near criticality.

**Keywords** Black hole Thermodynamics · Einstein Gauss-Bonnet Gravity · Yang Mills Fields

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## 1 Introduction

Understanding towards the formulation of quantum gravity has been initiated successfully by the work of Hawking and Bekenstein [1–3] in the branch of black hole physics. It was argued then the space-time geometry of black holes are related through the thermodynamic parameters like temperature and entropy following the discovery of black hole radiation [2]. Later on an important development has happened due to Hawking and Page in the study of thermodynamic phase transition of Anti-de Sitter (AdS) black hole [4]. However, this particular phase transition has a wonderful connection with the confinement/deconfinement transition of the Yang–Mills theory in the context of AdS/CFT correspondence [5–7]. A close resemblance was observed in [8, 9], between a first order phase transition of charged AdS black hole with the liquid–gas phase transition of Van der Waals (VdW) fluid. This analogy of first order phase transition has taken considerable interest when the cosmological constant  $\Lambda$  ( $\Lambda \sim -\frac{1}{l^2}$  in 4 dimensions, where  $l$  is the AdS length scale) is interpreted as the thermodynamic pressure and at the same time the mass of the black hole is identified with the enthalpy [10–12] of the black hole system. Therefore treating the cosmological constant as pressure  $P$  and its conjugate thermodynamic volume  $V$  completes the similarity between AdS black holes and Van der Waals fluid in extended black hole thermodynamics.<sup>1</sup>

In order to explain some astrophysical observations in the regime of strong gravity around black holes (see [14, 15] for recent reviews) Einstein’s General Relativity which is successful in the weak field limits needs to be modified. A natural candidate of the modified theory of gravity can be obtained by adding the higher order derivative terms with the existing Einstein Hilbert (EH) action. In the low energy limit, string theories provide an effective model of gravity in higher dimensions with the action augmented by the higher order curvature terms [16–20]. In this regards Lovelock theories are considered as the natural generalization of the Einstein General Relativity in higher derivative gravities [21, 22]. Gauss Bonnet (GB) gravity is the special case of Lovelock gravity of second order curvature correction terms added to the first order EH Lagrangian, contribute to the equation of motion for space-time dimensions  $D > 4$ . Recently the focus has been shifted from  $D > 4$  to the  $D = 4$  space time dimensions in the domain of GB gravity. It is known that in the four dimensional description, the integral of the GB term is topological invariant and does not contribute to the field equations. Nevertheless in Ref. [23] Glavan and Lin suggested a novel 4D Einstein Gauss Bonnet (EGB) gravity by rescaling the GB coupling constant  $\alpha \rightarrow \frac{\alpha}{D-4}$  and taking the limit  $D \rightarrow 4$  at the level of field equations. Now the GB terms contribute to the gravitational dynamics by avoiding the conditions imparted by Lovelock’s theorem [21, 22]. Though this effective GB gravity admits solutions of spherically symmet-

<sup>1</sup> See recent review [13] and references therein for related phenomena of phase transition of black hole systems.

ric black hole in four dimensions, but the dimensional regularization scheme due to Glavan and Lin [23] was contradicted following many works in Refs. [24–30]. The inconsistency arises from the fact that there is no manifestly covariant novel 4D EGB gravity model with only two degrees of freedom to satisfy the Lovelock's theorem. Recently other approaches have been made in Refs. [31–36] to find well-defined theories of 4D EGB gravity. Though, the authors in [31–36] have found consistent theory of 4D EGB gravity but obtained gravity with  $2 + 1$  degrees of freedom in general. Very recent Aoki–Gorji–Mukohyama in [37, 38] have proposed a well defined theory of 4D EGB gravity based on Arnowitt–Deser–Misner (ADM) decomposition and by breaking, a part of the 4D diffeomorphism invariance, that is to keep the 3-dimensional spatial diffeomorphism invariance and break the temporal diffeomorphism. Finally it was argued in [39] that the  $D = 4$  limit of any  $D$ -dimensional solution of EGB gravity produced in [23] is a solution of the consistent Aoki–Gorji–Mukohyama theory even though the matter fields are minimally coupled. Several authors have explored a wide variety of black hole solutions with electrically charged AdS black hole [40], magnetically charged nonsingular black holes [41, 42] and Born–Infeld (BI) AdS black hole solution [43] in this new gravity theory. Other black hole solutions where 4D EGB gravity coupled with nonlinear electrodynamics (NED) are obtained in [44, 45]. The study of 4D EGB black hole in AdS space within the formulation of extended thermodynamics have received much attention while considering Maxwell charge [46–51] in this gravity model. Further investigation of extended thermodynamics in charged Gauss–Bonnet Black Hole in de Sitter space-time was done in [52]. In particular, due to GB coupling constant  $\alpha$  this class of black hole shows VdW like phase transition at its charge neutrality. The study of phase transitions in this gravity theory have been extended for AdS black hole with BI charged [53], NED charged [54, 55], magnetically charged Bardeen black hole [56] and Hayward AdS black hole [57].

Now, it is fascinating to study nonlinear electrodynamics (NED) which is minimally coupled with gravity, has expectedly opened up a new window to understand nonsingular nature of the black hole space-time [58–61]. In that context black hole solution charged under BI electric field has been derived in [62] in 4D EGB gravity. A large number of other black hole solutions in 4D EGB gravity with nonlinear electromagnetic field invariants are presented in [45, 63–68]. Having discussed the various form of Abelian electromagnetic fields invariant coupled with 4D EGB gravity one can immediately extend this problem by incorporating non-Abelian Yang–Mills (YM) fields invariant which are inherently nonlinear. Earlier, authors of Refs. [69–72] have made significant progress in that direction after exploring black hole solutions in higher dimensions with GB and Lovelock terms. Therefore, as depicted in [73], one can go further to consider spherically symmetric black holes in arbitrary  $D$  dimensions which are sourced by the power of YM's invariant  $(F_{\mu\nu}^{(a)} F^{\mu\nu(a)})^q$ , ( $q$  is a real number)<sup>2</sup> coupled with GB gravity. We are presenting here the standard Yang–Mills (YM) invariant raised to the power  $q$ , as the source of our 4D EGB gravity and obtain the possible black hole solutions in AdS space-time. In this paper the emphasis has been put on the analysis of the horizon structure of the black hole solution, thermal stability of the black hole, thermodynamic phase transition in extended phase space, Maxwell's

<sup>2</sup> Where  $F_{\mu\nu}^{(a)}$  is the YM field with its internal index  $1 \leq a \leq \frac{1}{2}(D-2)(D-1)$ .

equal area law and critical exponents of some specific thermodynamic quantities. Our aim here is to capture the net effect of GB coupling parameter  $\alpha$  and the non-linearity parameter  $q$  of black holes in this process of studying various features of extended thermodynamics.

The paper is organized as follows: In Sect. 2, we obtain a black hole solution and discuss the horizon structure of 4D Einstein Power Yang–Mills Gauss–Bonnet Gravity (EPYMGB) in AdS space. In Sect. 3 we probe the extended thermodynamic behaviours of the black hole and study the thermal stability. We also analyze in Sect. 4  $P-v$  criticality (where  $v$  is the specific volume) and study the Gibbs free energy to comment on the Maxwell’s equal area law. In Sect. 5 we elaborate the discussion on critical exponents of various thermodynamic parameters for this class of black holes. We complete this paper with a conclusion in Sect. 6.

## 2 Black hole solutions in 4D Gauss Bonnet gravity

We consider the  $D$  dimensional action given in [73] for EPYMGB gravity in  $D$  dimensions with a cosmological constant  $\Lambda$ , is given by ( $8\pi G = 1$ )

$$I = \frac{1}{2} \int d^D x \sqrt{-g} \left( \mathcal{R} - \frac{(D-2)(D-1)}{3} \Lambda + \alpha \mathcal{L}_{GB} - \mathcal{F}^q \right), \tag{1}$$

Here  $\alpha$  is the Gauss–Bonnet coupling constant and the Gauss–Bonnet Lagrangian is given by

$$\mathcal{L}_{GB} = R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2. \tag{2}$$

The YM invariant  $\mathcal{F}$  is

$$\mathcal{F} = \sum_{a=1}^{\frac{(D-1)(D-2)}{2}} Tr(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}), \tag{3}$$

$R$  is the Ricci Scalar and  $q$  is a positive real parameter. The YM field is defined as

$$F_{\lambda\sigma}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + \frac{1}{2\sigma} C_{(b)(c)}^{(a)} A_\mu^{(b)} A_\nu^{(c)}. \tag{4}$$

Here  $C_{(b)(c)}^{(a)}$  are the structure constants of  $\frac{(D-1)(D-2)}{2}$  parameter Lie group  $G$  and  $\sigma$  is a coupling constant,  $A_\mu^{(a)}$  are the  $SO(D-1)$  gauge group  $YM$  potentials. The equation of motion for space-time metric  $g_{\mu\nu}$  and the gauge potential  $A_\mu^{(a)}$  are given by

$$G_{\mu\nu} + \frac{(D-2)(D-1)}{3} \Lambda g_{\mu\nu} + \alpha H_{\mu\nu} = T_{\mu\nu}, \tag{5}$$

and

$$d(*\mathbf{F}^{(a)}\mathcal{F}^{q-1}) + \frac{1}{\sigma}C_{(b)(c)}^{(a)}\mathcal{F}^{q-1}\mathbf{A}^{(b)} \wedge *\mathbf{F}^{(c)} = 0, \tag{6}$$

where hodge star  $*$  denotes duality.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

$$H_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\sigma}R_{\nu}^{\sigma} - 2R^{\sigma\delta}R_{\mu\sigma\nu\delta} + R_{\mu}^{\sigma\delta\lambda}R_{\nu\sigma\delta\lambda}) - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}. \tag{7}$$

The energy momentum tensor is

$$T_{\nu}^{\mu} = 2q\mathcal{F}^{q-1}Tr(F_{\nu\delta}^{(a)}F^{(a)\mu\delta}) - \frac{1}{2}\delta_{\nu}^{\mu}\mathcal{F}^q. \tag{8}$$

The metric ansatz for  $D$  dimensions is chosen as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\Omega_{(D-2)}^2, \tag{9}$$

where  $d\Omega_{D-2}^2$  is the line element of the unit  $(D - 2)$ -dimensional sphere. For the YM field we introduce the magnetic Wu–Yang ansatz [71, 72, 74, 75] where the Yang–Mills magnetic gauge potential one-forms are expressed as

$$\mathbf{A}^{(a)} = \frac{Q}{r^2}(x_i dx_j - x_j dx_i), \quad r^2 = x_1^2 + x_2^2 + \dots + x_{D-1}^2, \tag{10}$$

where the indices  $a, i$  and  $j$  run in the following ranges  $2 \leq j + 1 \leq i \leq D - 1$ , and  $1 \leq a \leq \frac{(D-1)(D-2)}{2}$ . The above ansatz Eq. (10) satisfy the Yang–Mills equations Eq. (6) and yields the following form of the energy momentum tensor after using Eq. (8)

$$T_i^t = T_r^r = -\frac{1}{2}\mathcal{F}^q, \tag{11}$$

$$T_{\theta_i}^{\theta_i} = -\frac{1}{2}\left(1 - \frac{4q}{D-2}\right)\mathcal{F}^q. \tag{12}$$

Whereas the form of the Yang–Mills invariant  $\mathcal{F}$  takes the form as

$$\mathcal{F} = \frac{(D-2)(D-3)Q^2}{r^4}, \tag{13}$$

$$Tr(F_{\theta_i\sigma}^{(a)}F^{(a)\theta_i\sigma}) = \frac{1}{D-2}\mathcal{F}, \tag{14}$$

with  $i$  run form 1 to  $D - 2$ .

For  $D = 4$  dimensions the integral over the Gauss–Bonnet term is a topological invariant, thus not contributing to the dynamics. However, as found recently in [23], by rescaling the coupling constant as  $\alpha \rightarrow \frac{\alpha}{D-4}$ , and then considering the limit  $D \rightarrow 4$  the  $(r, r)$  component of Eq. (5) reduces to

$$2(r^3 - 2\alpha r g(r))g'(r) + 2\alpha g^2(r) + 2r^2 g(r) + 2\Lambda r^4 + \frac{(2Q^2)^q}{r^{4(q-1)}} = 0, \tag{15}$$

in which  $f(r) = 1 + g(r)$  and prime denotes the derivative with respect to  $r$ . Integration of the above equation gives the following solutions of the 4D black holes in  $AdS$  space of Einstein Power Yang–Mills Gauss Bonnet gravity,

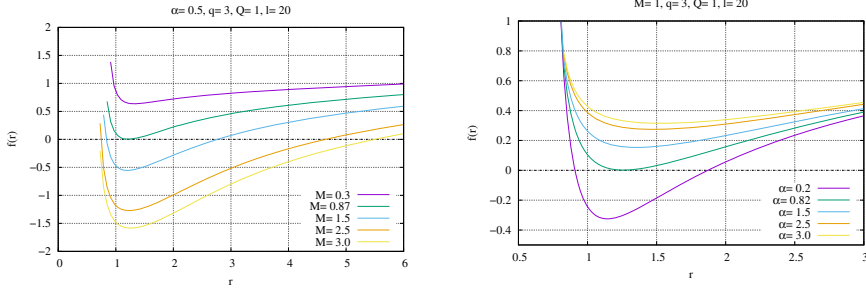
$$f(r) = 1 + \frac{r^2}{2\alpha} \left( 1 \pm \sqrt{1 + \frac{8\alpha M}{r^3} - \frac{4\alpha}{l^2} - \frac{2\alpha(2Q^2)^q}{(4q-3)r^{4q}}} \right). \tag{16}$$

This solution describe an exact four dimensional black hole solution in the 4D EGB gravity coupled to power Yang–Mills field with negative cosmological constant  $\Lambda = -\frac{3}{l^2}$ . The dimensional regularization scheme [23] adopted here in order to obtain static spherically symmetric black hole solution Eq. (16), which is also a solution of consistent Aoki Gorji–Mukohyama theory of 4D EGB gravity as follows from [39]. These class of solutions again characterized by the mass  $M$ , cosmological constant  $\Lambda = -\frac{3}{l^2}$ , Yang–Mills magnetic charge  $Q$ , nonlinear parameter  $q$  and Gauss–Bonnet coupling constant  $\alpha$ . For  $Q = 0$  solution (16) reduces to the 4D EGB solution presented in [23]. Setting the nonlinear parameter  $q = 1$  gives us the black hole solution derived in [76]. Further by taking the limit of vanishing  $GB$  coupling constant  $\alpha$  the solution of plus sign branch reduces to the solution of nonlinearly charged black hole in Einstein Yang–Mills gravity with a negative mass and unphysical charge whereas the negative branch solution correctly reproduces the physical black hole solution as presented in [77–79]. So the present study will be confined in the negative branch of the solution (16).

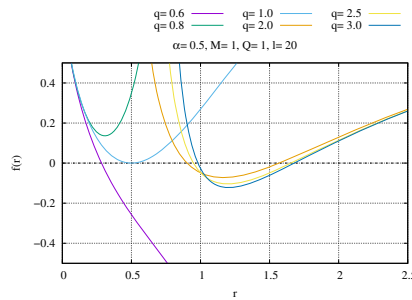
The location of horizon radii are given by all the real positive roots of  $f(r_+) = 0$  which follows from the equation

$$\frac{r_+^4}{l^2} + r_+^2 - 2Mr_+ + \frac{(2Q^2)^q r_+^{4-4q}}{2(4q-3)} + \alpha = 0. \tag{17}$$

We solve the above equation numerically for the set of parameters and study all those positive real roots of the Eq. (17) and have been illustrated in Fig. 1. There are various subplots in Fig. 1, according to those subplots there can be two horizons namely inner Cauchy and outer Event horizons, an extremal black hole with degenerate horizon and finally no horizon i.e. naked singularity appear for the particular set of parameters. For the given set of values of Gauss–Bonnet coupling  $\alpha$ , nonlinear charge parameter  $q$ , magnetic charge  $Q$  and  $AdS$  length scale  $l$  various plots are presented for different mass parameter  $M$  in Fig. 1a. It is quite obvious from the above mentioned plot that the number of zeros are changing with the values of  $M$ . This analysis shows that the line



(a) green line indicates degenerate horizon at  $M = M_{ext}$  (b) green line indicates degenerate horizon at  $\alpha = \alpha_{ext}$



(c) blue line indicates degenerate horizon at  $q = q_{ext}$

**Fig. 1** Plot of metric function  $f(r)$  versus  $r$  for various values of parameters

element Eq. (16) describes a naked singularity for  $M < M_{ext}$  and a black hole with an outer event horizon and an inner Cauchy horizon appear for  $M > M_{ext}$ . Finally, for  $M = M_{ext}$ , the horizon is degenerate and Eq. (16) represents an extremal black hole with horizon radius  $r_+ = r_{ext}$ . For extremely large values of  $M$  for the domain of  $M > M_{ext}$ , black hole solutions obtained with a single horizon like uncharged Schwarzschild black hole. Other two subplots Fig. 1b and c show the horizon radius of black hole solutions with the variation of  $\alpha$  and  $q$  values respectively. Again we can obtain  $\alpha = \alpha_{ext}$  and  $q = q_{ext}$ , where the extremal black hole with single degenerate horizon exist. Nonextremal black holes with two distinct horizons are appeared for  $\alpha < \alpha_{ext}$  whereas the nohorizon condition appear for  $\alpha > \alpha_{ext}$ . Finally we focus on the analysis of the black hole solutions (16) from the perspective of non-linearity parameter  $q$ . At  $q = q_{ext} = 1$  for given values of the other parameters as shown in Fig. 1c produces an extremal black hole. But there are set of curves for values  $q < q_{ext}$  where  $q < 0.75$  the single horizon formed and it will disappear to produce naked singularity when  $q > 0.75$ . However for  $q > q_{ext}$  we always obtain nonextremal black hole solutions in the 4D power Yang–Mills Gauss–Bonnet gravity theory. In the following analysis we consider such values of parameter for which the black hole event horizon should always exist.

### 3 Thermodynamics and thermal stability of 4D AdS EPYMGB black hole

In this section we investigate the various thermodynamic quantities which are important to show first law of thermodynamics of black holes in 4D Einstein Gauss–Bonnet gravity coupled with power Yang–Mills fields in AdS space-time. Later we are interested to study the combined effect of  $GB$  coupling parameter  $\alpha$  and non-linear parameter  $q$  on the thermodynamic stability of this class of black hole by calculating the specific heat capacity in canonical ensemble. The mass of the black hole which is identified as the enthalpy of the black hole in extended thermodynamics can be expressed in terms of the horizon radius  $r_+$  from Eq. (17) as

$$M = \frac{1}{2} \left( r_+ + \frac{r_+^3}{l^2} + \frac{\alpha}{r_+} + \frac{(2Q^2)^q}{2(4q-3)r_+^{4q-3}} \right). \quad (18)$$

The Hawking temperature associated with this black hole can be obtained from the relation  $T_H = \frac{f(r)}{4\pi} |_{r_+}$  as follows

$$T = \frac{1}{4\pi r_+(r_+^2 + 2\alpha)} \left( r_+^2 - \alpha - 2^{q-1} Q^{2q} r_+^{4-4q} + \frac{3r_+^4}{l^2} \right). \quad (19)$$

For  $Q = 0$  Eq. (19) reduces to the temperature of 4D *EGB* black holes in *AdS* space-time [35]

$$T = \frac{3r_+^4 + l^2(r_+^2 - \alpha)}{4\pi r_+ l^2 (r_+^2 + 2\alpha)}. \quad (20)$$

On the other hand if non-linearity parameter  $q = 1$  then the above temperature (19) takes the form as given in [76].

However we can derive the entropy of the black hole following the approach of [80, 81] and using the expression

$$S = \int \frac{dM}{T} = \int \frac{1}{T} \left( \frac{\partial M}{\partial r_+} \right) dr_+ + S_0, \quad (21)$$

where  $S_0$  is an integration constant. Substituting (18) and (19) into (21), we obtain the entropy of the Gauss–Bonnet black holes (16) as

$$S = \pi r_+^2 + 4\pi\alpha \ln |r_+| + S_0. \quad (22)$$

Following the arguments discussed in [82] we fix the constant  $S_0 = -2\pi\alpha \ln |\alpha|$ . So the Eq. (22) turns into the following simple form

$$S = \pi r_+^2 + 4\pi\alpha \ln \left( \frac{r_+}{\sqrt{\alpha}} \right). \quad (23)$$



The first term of the above entropy formula exactly coming from the Bekenstein–Hawking entropy area law. The second term appears here solely due to the effect of higher derivative curvature terms in Gauss–Bonnet theory. So all together this entropy (22) of the black hole does not satisfy the area formula of Bekenstein and Hawking. Another point to be noted here is that Yang–Mills charge has no effect on the entropy (22) of this 4D Einstein Gauss–Bonnet Power Yang–Mills black hole. As it is known that the thermodynamic pressure  $P$  is connected with the cosmological constant  $\Lambda$  through the relation  $P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi l^2}$  in the extended phase-space thermodynamics. The corresponding thermodynamic volume is given by

$$V = \left( \frac{\partial M}{\partial P} \right)_{S, Q, \alpha} = \frac{4}{3} \pi r_+^3. \tag{24}$$

The Yang–Mills potential due to non-linearly charged Yang–Mills black holes can be measured at infinity with respect to the horizon, as expressed in [83]

$$\Phi_q = \frac{2^{q-1} q Q^{2q-1}}{(4q - 3) r_+^{4q-3}}. \tag{25}$$

In the extended phase space the GB coupling is considered as thermodynamic variable to satisfy the first law of thermodynamics. So its conjugate potential can be calculated from Eqs. (18), (23) and (19) which comes out as

$$\Phi_\alpha = \frac{1}{2r_+} \left( 1 - 2T_H \ln \left( \frac{r_+}{\sqrt{\alpha}} \right) \right). \tag{26}$$

The Smarr relation for 4D EPYMGB black hole in the extended phase space is obtained by using all the above quantities and considering the mass  $M$  as the enthalpy of the black hole [10],

$$M = 2T_H S + \left( \frac{2q - 1}{q} \right) \Phi_q Q - 2VP + 2\Phi_\alpha \alpha. \tag{27}$$

However Eqs. (19), (18), (23), (26) for all those thermodynamic quantities must satisfy the first law of black hole thermodynamics in extended phase space

$$dM = TdS + \Phi_q dQ + VdP + \Phi_\alpha d\alpha. \tag{28}$$

The heat capacity of the black hole (16) is a very essential thermodynamic quantity which study gives the information regarding stability under thermal fluctuation of this class of black holes. The heat capacity can be calculated from the following relation for fixed charge ensemble

$$C_Q = \left( \frac{\partial M}{\partial T} \right)_Q = \left( \frac{\partial M}{\partial r_+} \right)_Q \left( \frac{\partial r_+}{\partial T} \right)_Q. \tag{29}$$

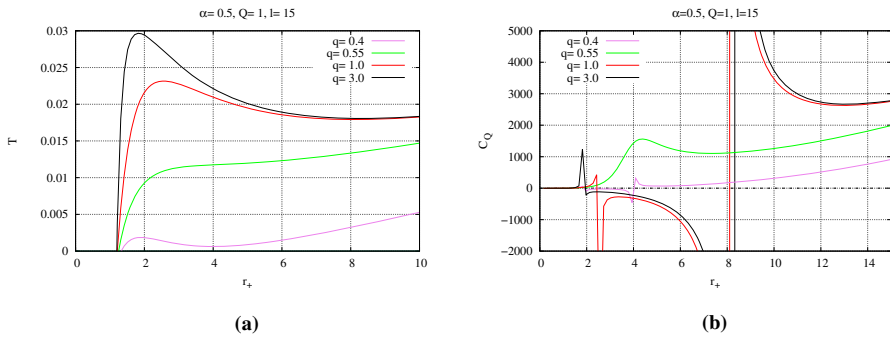


Fig. 2  $T$  (left panel),  $C_Q$  (right panel) versus  $r_+$  plot for various parameter values

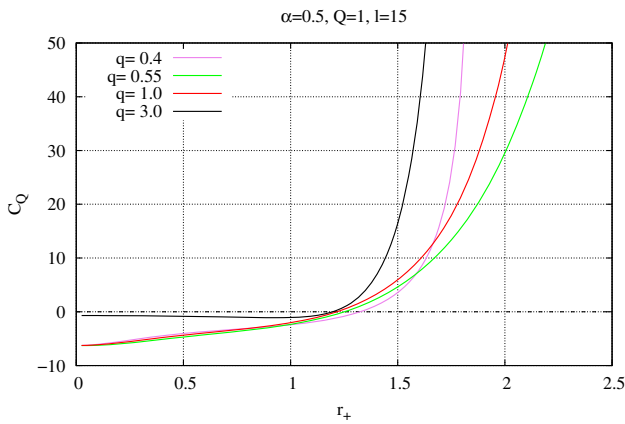


Fig. 3  $C_Q$  versus  $r_+$  plot for various parameter values

Using Eqs. (18) and (19) for  $M$  and  $T$  we can derive the expression for heat capacity as a function of horizon radius  $r_+$  in the following form

$$C_Q = \frac{A}{B}. \tag{30}$$

where  $A$  and  $B$  take the following form

$$A = 2\pi(r_+^2 + 2\alpha)^2 \{-6r_+^{4q+4} + l^2(2^q(Q^2)^q r_+^4 + 2r_+^{4q}(\alpha - r_+^2))\}, \tag{31}$$

$$B = 2^q l^2 Q^{2q} r_+^4 \{\alpha(8q - 6) + (4q - 1)r_+^2\} + 2r_+^{4q} \{2\alpha^2 l^2 + 5\alpha^2 r_+^2 + (18\alpha - l^2)r_+^4 + 3r_+^6\}. \tag{32}$$

In order to analyze the thermal stability and the thermodynamic phase transitions for EPYMGB black holes we simply plot  $C_Q$  (30) along side with the temperature (19) as a function of  $r_+$  to learn several features of extended thermodynamics from both the curves. The analytical solutions of Eq. (30) for  $r_+$  where the numerator  $A = 0$

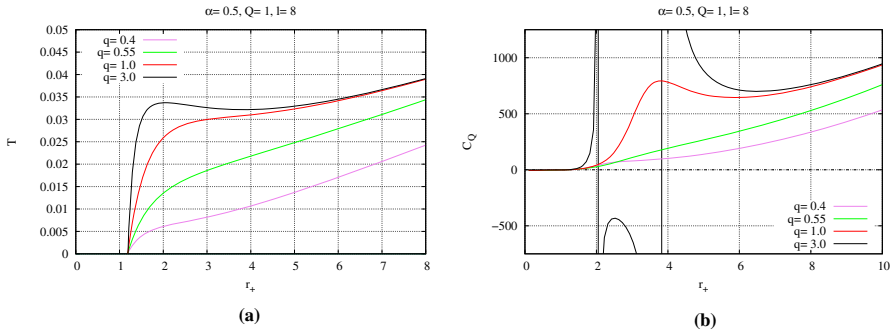


Fig. 4  $T$  (left panel),  $C_Q$  (right panel) versus  $r_+$  plot for various parameter values

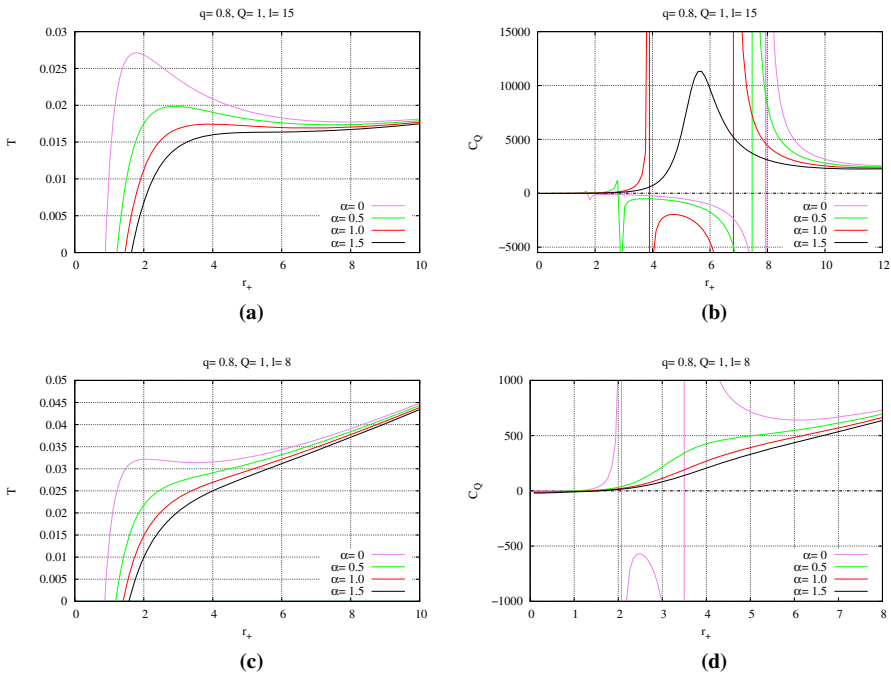


Fig. 5  $T$  (left panel),  $C_Q$  (right panel) versus  $r_+$  plot for various parameter values

in one hand and on the other hand the denominator  $B = 0$  are not obvious. Here we can only go for numerical analysis after presenting all the plots in Figs. 2, 3, 4 and 5 for various values of the parameters like GB coupling and non linear YM charge parameter. As shown in Fig. 2a and b there is a critical horizon radius  $r_{+c}$  where both the temperature and the specific heat is zero and this is the size of the extremal black hole we are considering here. Hence the black hole with larger radius than the critical radius for particular set of parameters are stable have positive specific heat and temperature. This particular situation have been shown in Fig. 3. These curves show that the zeros of the Eq. (30) depend on the the value of the parameter  $q$  we fixed here, that means

the value of  $r_{+c}$  increases as the parameter  $q$  decreases. It is evident in  $C_Q - r_+$  plots that the divergences occur at larger horizon radius larger than  $r_{+c}$  and these divergences signify the phase transition between small and large black holes through some intermediate thermally unstable black hole phases, where intermediate phases have negative  $C_Q$ . These divergences occur at those  $r_+$  values where temperature plots show its extremum. Here it is observed in Fig. 2a and b for very small values of  $q$  phase transition occur but as the value of  $q$  increases the divergences in the function of  $C_Q$  disappear and get a smooth behaviour with positive  $C_Q$ , that signifies the solutions are thermally stable. On the other hand if  $q$  changes to even larger values then again those divergences appear, which means that phase transition happens between small and large size of black holes. This is due to the non-linearity effect of power-invariant YM fields appearing here in the thermodynamic phase transition of the black hole in EGB gravity theory which has already been studied for non-linear electromagnetic fields in [84]. Again this type of appearing and disappearing of divergences in the specific heat are not seen if we tune the *AdS* length scale  $l$  to some lower values (as shown in Fig. 4). Another point in this plot to be noted that as  $q$  changes the critical horizon radius  $r_{+c}$  remain fixed in this case. The effects of GB coupling parameter  $\alpha$  on the thermal structure of this class of black holes in  $4D$  have been depicted in the Fig. 5. Here the effect is quite different from the effect of the  $q$  parameter on the behaviour of the temperature and the specific heat of EPYMGB black holes. As we can see that the increment in the values of  $\alpha$ , the divergences of  $C_Q$  and the extrema of  $T$  ceased to exist, hence the system does not show any thermodynamic phase transition. Therefore, we always see thermodynamically stable large black hole phase if the GB coupling constant is tuned to some larger values for the given set of parameters.

#### 4 $P-v$ criticality and Gibbs free energy

In this subsequent study follows from the previous discussion we are intend to consider black hole phase transition of Van der Waals (VdW) type by using  $P-v$  isotherms. In the extended phase space the cosmological constant is related to the thermodynamical pressure  $P$  of the black hole. According to Eq. (19) the equation of state for the class of black holes we are considering as follows

$$P = \frac{T}{v} + \frac{8\alpha T}{v^3} - \frac{1}{2\pi v^2} + \frac{2\alpha}{\pi v^4} + \frac{2^{5q-4} Q^{2q}}{\pi v^{4q}}, \quad (33)$$

where  $v$  is denoted as the specific volume and identified with the horizon radius  $r_+$  of the black hole through the relation  $v = 2r_+$ .

Following [12] we present the critical temperature for the critical isotherm curve from the condition

$$\frac{\partial P}{\partial v} = \frac{\partial^2 P}{\partial v^2} = 0. \quad (34)$$

**Table 1**  $q=3, Q=1$

$\alpha$	$v_c$	$P_c$	$T_c$	$\rho = \frac{P_c v_c}{T_c}$
0	3.49293	0.0108709	0.0828457	0.458336
0.3	4.293	0.00398985	0.0457575	0.37433
0.5	5.15293	0.00251083	0.0360567	0.358827
1.0	7.19453	0.00126366	0.0255539	0.355776
1.2	7.87901	0.00105319	0.0233285	0.355708
1.5	8.80788	0.000842609	0.0208661	0.355678

Solving those above equations one will get

$$v^4 - 48\alpha v^2 - 192\alpha^2 - 3 \times 2^{5q+1} \alpha q (4q - 3) Q^{2q} v^{4-4q} - 2^{5q-2} q (4q - 1) Q^{2q} v^{6-4q} = 0. \tag{35}$$

We calculate all the thermodynamic quantities at critical phase transition point using the Eq. (35) for arbitrary positive values of  $\alpha$  and  $q$ . Furthermore it has been checked numerically that for  $\alpha > 0$  and  $q \neq \frac{3}{4}$  always obtained single positive real root for the critical specific volume  $v_c$ , which leads to the system will have one critical phase transition point. This observation finally concludes that this type of AdS EPYMGB black hole system shows Van der Waals like phase transition for specific values of those parameters. Though the analytical solutions for critical values of pressure  $P_c$ , temperature  $T_c$  and specific volume  $v_c$  are not obvious so we use numerical method to show the universal dimensionless ratio  $\rho = \frac{P_c v_c}{T_c}$  in the tabular form in Tables 1 and 2. However for  $\alpha = 0$  the exact value of  $\rho$  can be given in the following form

$$\rho = \frac{4q - 1}{8q}. \tag{36}$$

The above  $\rho$  does not depend on YM charge  $Q$  but has an explicit dependence on the nonlinear parameter  $q$ . For  $q = 1$  the Eq. (36) reduces to the value of  $\rho$  for Van der Waals fluid which is 0.375 that means we recover the results for the linear charged black hole in the YM theory. One can see from Eq. (36) the effects of nonlinearity sets into the theory does modify the universal ratio  $\rho$ . In Fig. 6a and b we have presented the  $P-v$  diagrams for different parameter values of the EPYMGB black holes. We get the critical isotherm at  $T = T_c$  whereas temperature below criticality the black hole system undergoes VdW-like phase transition. Above criticality one should get thermally stable black hole as discussed previously.

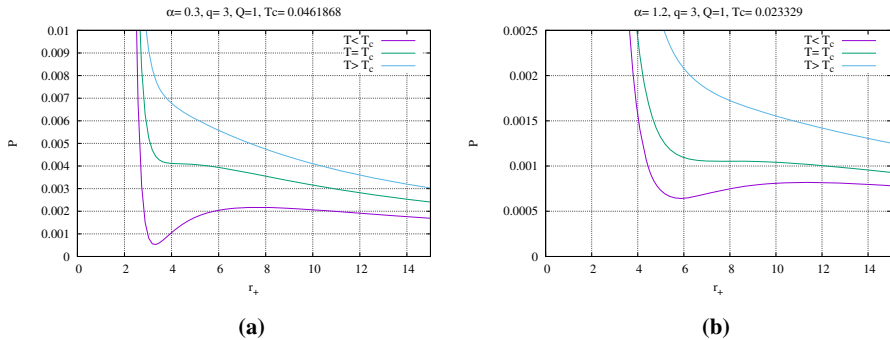
In order to understand first order phase transition one can also derive the Gibbs free energy for canonical ensemble (fixed  $Q$ ) from the following definition

$$G = M - TS \tag{37}$$

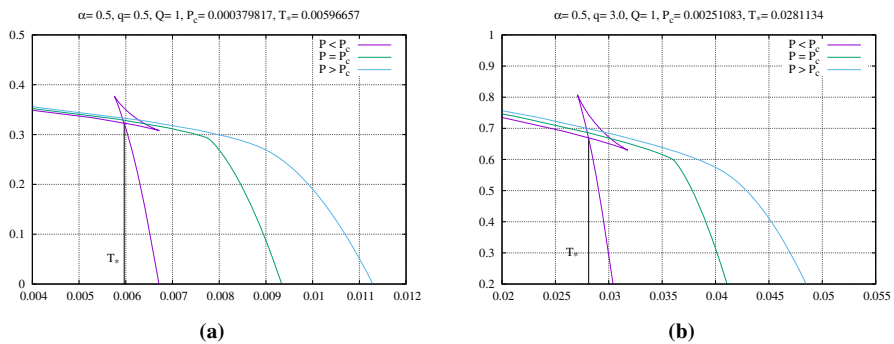
Since the corresponding expression for the thermodynamic potential  $G$  for this EPYMGB black hole calculated using Eqs. (18), (19), (23) is very cumbersome so

**Table 2**  $\alpha=0.5, Q=1$

$q$	$v_c$	$P_c$	$T_c$	$\rho = \frac{P_c v_c}{T_c}$
0.5	7.47677	0.000379817	0.000775783	0.366056
1.0	7.12971	0.00141321	0.0275925	0.365164
1.5	6.03845	0.00206579	0.0332854	0.374764
2.0	5.52327	0.00235547	0.0351919	0.369685
3.0	5.15293	0.00251083	0.0360567	0.358827



**Fig. 6**  $P$  versus  $r_+$  plot for different temperature



**Fig. 7** Gibbs free energy versus temperature plot for 4D EPYGB black hole in AdS space-time

we do not present here instead the behaviour of  $G$  is depicted in Fig. 7 in terms of Hawking temperature. The subplots (a) and (b) of Fig. 7 are displayed for GB coupling parameter  $\alpha = 0.5$  for two different values of nonlinear parameter  $q$ , however the value of YM charge  $Q = 1$  for both the cases. In these  $G-T$  plots characteristic swallow tail behaviour observed for pressure  $P < P_c$  and the system is undergoing first order phase transition between small and large black holes. There is an intersection point at temperature  $T = T_*$  shown in both the subplots are the coexistence temperature at which small and large size black holes have the same free energy. It has also been shown that at critical pressure  $P_c$  the cross over behaviour of free energy disappear. Beyond the critical point free energy become a smooth decreasing function

of the temperature, hence no phase transition occur. By the Fig. 7, one can obtain the coexistence temperature  $T_*$  numerically from the intersection point. Substituting the value of  $T_*$  into Eq. (33) we get the values of  $v_1, v_2$  and  $v_3$  for  $P-v$  diagram, here,  $v_1, v_2$  and  $v_3$  denote the three values of  $v$  from small to large size black hole corresponding to isobar  $P = P_*$  in  $P-v$  diagram. We use these values of  $v$  to calculate area  $\mathcal{A}_1$  and  $\mathcal{A}_2$  of Maxwell’s equal area law in  $P-v$  isotherm following the equation

$$\mathcal{A}_1 = P_*(v_3 - v_1) = \int_{v_1}^{v_3} P dv = \mathcal{A}_2. \tag{38}$$

Here we are also intended to verify Maxwell’s area law [85] for isotherm considered in  $P-V$  plane just by calculating  $V_1, V_2$  and  $V_3$  which denote the three values of  $V$  from small to large thermodynamic volume corresponding to  $P = P_*$  with the help of Eqs.(33) and (38) by taking the volume  $V$  as the variable at the place  $v$ . The results of this numerical study of Maxwell’s area law for the class of black hole in EPYMGB gravity have been presented in Tables 3 and 4. For different set of parameters we have shown in Tables 3 and 4 that the relative errors calculated for isotherm in  $(P, v)$  plane are very large while relative errors in the  $(P, V)$  plane are extremely small.

So it is concluded from numerical study that Maxwell’s equal area law valid for  $P-V$  diagram and fails for  $P-v$  diagram as similarly found in [86].

### 5 Critical exponents

In this section we would like to analyse the critical behaviour of some physical quantities in extended phase space by computing the critical exponents. Here we find critical exponents  $\alpha', \beta, \gamma, \delta$  which determine the following quantities near critical point

$$C_v \propto |t|^{-\alpha'}, \tag{39}$$

$$\eta \propto |t|^\beta, \tag{40}$$

$$\kappa_T \propto |t|^{-\gamma}, \tag{41}$$

$$|P - P_c| \propto |v - v_c|^\delta. \tag{42}$$

In order to compute the critical exponent  $\alpha'$ , we consider entropy  $S$  from Eq. (23) which is independent of temperature  $T$ . So that the specific heat at constant volume  $C_v = T \frac{\partial S}{\partial T} |_v$  vanishes. We conclude that the critical exponent  $\alpha' = 0$  in this case. To calculate other exponents, let us define  $t = \frac{T}{T_c} - 1, \epsilon = \frac{v}{v_c} - 1$  and  $p = \frac{P}{P_c}$ . Using the above definition we expand the equation of state (33) around the critical point as following:

$$p = 1 + p_{10}t + p_{01}\epsilon + p_{11}t\epsilon + p_{02}\epsilon^2 + p_{03}\epsilon^3. \tag{43}$$

**Table 3** For isotherms in  $P-v$  plane

$(\alpha, q)$	$T_*$	$P_*$	$v_1$	$v_2$	$v_3$	$\mathcal{A}_1$	$\mathcal{A}_2$	Relative error
(0.5, 0.5)	0.00596657	0.0001899085	4.01054	10.2917	19.2279	0.00288991	0.00249289	0.13738
(0.5, 3.0)	0.0281134	0.001255415	2.96014	7.20953	13.7382	0.0135309	0.0115275	0.148063



**Table 4** For isotherms in  $P - V$  plane

$(\alpha, q)$	$T_*$	$P_*$	$V_1$	$V_2$	$V_3$	$\mathcal{A}_1$	$\mathcal{A}_2$	Relative error
(0.5, 0.5)	0.00596657	0.0001899085	33.776	570.767	3722.12	0.700448	0.700304	$2 \times 10^{-4}$
(0.5, 3.0)	0.0281134	0.001255415	13.5811	196.209	1357.27	1.68689	1.68908	$1.3 \times 10^{-3}$

The non zero expansion coefficient for this EGBP YM black holes are given by

$$\begin{aligned}
 p_{10} &= \frac{T_c}{P_c v_c^3} (8\alpha + v_c^2), \\
 p_{11} &= -\frac{T_c}{P_c v_c^3} (24\alpha + v_c^2), \\
 p_{03} &= \frac{1}{P_c} \left( -\frac{40\alpha}{\pi v_c^4} - \frac{80\alpha T_c}{v_c^3} + \frac{2}{\pi v_c^2} - \frac{T_c}{v_c} - \frac{2^{5q-2} q Q^{2q} (4q + 1)^2}{3\pi v_c^{4q}} \right), \tag{44}
 \end{aligned}$$

where  $p_{01} = p_{02} = 0$ . During the phase transition the pressure of large black hole with volume  $\epsilon_l$  is equal to the pressure of the small black hole with volume  $\epsilon_s$ . So the equation of state (43) can be written in the following manner

$$p = 1 + p_{10}t + p_{11}t\epsilon_l + p_{03}\epsilon_l^3 = 1 + p_{10}t + p_{11}t\epsilon_s + p_{03}\epsilon_s^3. \tag{45}$$

Equation (45) will be simplified to the form below

$$p_{11}t(\epsilon_l - \epsilon_s) + p_{03}(\epsilon_l^3 - \epsilon_s^3) = 0. \tag{46}$$

However using Maxwell’s area law we also obtain

$$\int_{\epsilon_s}^{\epsilon_l} \epsilon (p_{11}t + 3\epsilon^2 p_{03}) d\epsilon = 0. \tag{47}$$

The above integration has been performed to get the following expression

$$p_{11}t + \frac{3}{2} p_{03}(\epsilon_s^2 + \epsilon_l^2) = 0 \tag{48}$$

With Eqs. (46) and (48) one will get the nontrivial solutions for  $\epsilon_s$  and  $\epsilon_l$  as

$$\epsilon_l = -\epsilon_s = \sqrt{\frac{-p_{11}t}{p_{03}}} \tag{49}$$

$p_{11}$  and  $p_{03}$  can easily be evaluated from Eq. (44) for certain parameters value of the EGBP YM black holes. The order parameter  $\eta$  can be calculated as

$$\eta = v_l - v_s = v_c(\epsilon_l - \epsilon_s) = 2v_c\epsilon_s = 2v_c \sqrt{\frac{-p_{11}t}{p_{03}}} \propto \sqrt{-t}. \tag{50}$$

Hence we have  $\beta = \frac{1}{2}$ . Now to estimate the value of  $\gamma$  as given in Eq. (41) we use the definition of isothermal compressibility  $\kappa_T = -\frac{1}{v} \frac{\partial v}{\partial p} |_T$ . So differentiating Eq. (45) to

get

$$\kappa_T \propto -\frac{1}{\frac{\partial P}{\partial \epsilon}} = -\frac{1}{P_{11}t}. \tag{51}$$

The Eq. (51) indicates that the critical exponent  $\gamma = 1$ . For critical isotherm at  $T = T_c$ ,  $t = 0$  and one should obtain from Eq. (45)

$$|P - P_c| = \frac{P_c}{v_c} p_{03} |v - v_c|^3 \propto |v - v_c|^3. \tag{52}$$

Which leads to the corresponding value of the exponent  $\delta = 3$ . This study again confirm that the scaling law behaviour of certain physical quantities remain unchanged near critical point of the phase transition for this class of 4D EPYMGB black hole.

### 6 Conclusion

In this work we have found an exact solution of charged AdS black hole sourced by a power of YM's invariant in the context of 4D EGB gravity. The power of the invariant form of nonabelian YM fields have chosen as  $(F_{\mu\nu}^{(a)} F^{\mu\nu(a)})^q$ , where  $q$  is a positive real number. A dimensional regularization technique [23] has been used to get this solution. However according to [39] this spherically symmetric nonlinear charged solution (16) happens to be a solution of the consistent theory proposed by Aoki–Gorji–Mukohyama [37] using temporal diffeomorphism breaking regularization scheme. By making the nonlinear parameter  $q = 1$  our black hole solution reduces to the solution of [76]. This black hole can have two horizons, one degenerate horizon, no horizon and some time single horizon of Schwarzschild type depending on various black hole parameters  $M$ ,  $Q$ ,  $l$ ,  $\alpha$  and  $q$ .

We also have studied extended thermodynamics and the thermal stability of EPYMGB black holes by calculating the Hawking temperature, entropy, other potentials due to YM charge, heat capacity etc. We have obtained first law of thermodynamics of this novel 4D EPYMGB black hole in AdS space and verified the Smarr relation. In the analysis of thermal stability we have determined specific heat at constant charge and plotted with respect to the horizon radius. There are zeros and divergences in the function of  $C_Q$  which signify the extremality and the thermodynamic phase transition of the black hole. The divergences of  $C_Q$  are identified with the extrema of temperature as shown using the plots presented in Figs. 2, 4 and 5. An interesting phenomena happened for larger values of  $l$ , where existence/absence of phase transition occur for various values of nonlinearity parameter  $q$ . After studying specific heat as a function of  $r_+$  we conclude that small and large size black hole phases are thermodynamically stable due to positive specific heat ( $C_Q > 0$ ) and there are unstable phases for which  $C_Q < 0$  as shown in plots (Figs. 2, 4, and 5).

Next we examined the equation of state  $P = P(T, v)$  and presented  $P-v$  diagrams to investigate the phase transition and critical behaviour of the black hole system at fixed temperature  $T$ . The universal ratio  $\rho$  also has been determined for various

parameter values. The first order phase transition between small and large size black hole were studied from the isotherm in the  $P-v$  plane, has similar phase structure like liquid gas phase transition of VdW fluids. On the other hand this structure of phase transition we studied from the curve in  $G-T$  plane and showed a characteristic swallow tail behaviour below the critical pressure. From the  $G-T$  plot we obtained coexistence temperature at which the free energy is equal for small and large black hole solutions. Further this coexistence temperature was used to show that the Maxwell's equal area law holds for  $P-V$  isotherms and fails for  $P-v$  isotherms. We have further studied the behaviour of certain physical quantities near the critical point and calculated those critical exponents  $\alpha' = 0$ ,  $\beta = \frac{1}{2}$ ,  $\gamma = 1$  and  $\delta = 3$ .

**Acknowledgements** We are also grateful to the anonymous referee(s) for valuable comments and suggestions to improve this paper.

**Data availability** All data generated or analysed during this study are included in this article.

## Declarations

**Conflict of interest** The author did not receive support from any organization for the submitted work and has no relevant financial or non-financial interests to disclose.

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