Gen Relativ Gravit (2016) 48:100
DOI 10.1007/s10714-016-2095-5

RESEARCH ARTICLE

A statistical representation of the cosmological constant from finite size effects at the apparent horizon

Stefano Viaggiu[1]

Received: 18 February 2016 / Accepted: 11 June 2016 / Published online: 21 June 2016

Abstract In this paper we present a statistical description of the cosmological constant in terms of massless bosons (gravitons). To this purpose, we use our recent results implying a non vanishing temperature T_Λ for the cosmological constant. In particular, we found that a non vanishing T_Λ allows us to depict the cosmological constant Λ as composed of elementary oscillations of massless bosons of energy $\hbar\omega$ by means of the Bose–Einstein distribution. In this context, as happens for photons in a medium, the effective phase velocity v_g of these massless excitations is not given by the speed of light c but it is suppressed by a factor depending on the number of quanta present in the universe at the apparent horizon. We found interesting formulas relating the cosmological constant, the number of quanta N and the mean value $\overline{\lambda}$ of the wavelength of the gravitons. In this context, we study the possibility to look to the gravitons system so obtained as being very near to be a Bose–Einstein condensate. Finally, an attempt is done to write down the Friedmann flat equations in terms of N and $\overline{\lambda}$.

Keywords Cosmological constant · Gravitons · Apparent horizon · Bose–Einstein condensation

1 Introduction

The standard concordant cosmological model is obtained by a spatially flat Friedmann metric endowed with a cosmological constant Λ representing about 68 % of the present universe matter-energy content. Despite the enormous success of this model

✉ Stefano Viaggiu
viaggiu@axp.mat.uniroma2.it

1 Dipartimento di Matematica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica, 1, 00133 Rome, Italy

in explaining the main cosmological data, the real nature of this dark energy remains obscure and a fundamental issue in modern cosmology. Many attempts have been done in the literature: (see for example [1–19] and references therein) quintessence, k-essence, phantom models, clock effects, holographic dark energy, Bose–Einstein condensate (BEC), extended theories of gravity to cite someone of the most relevant. In particular, the recent detection of gravitational waves [20] (GW150914 event) represents the born of the gravitational wave astronomy. Gravitational wave astronomy, as noticed in [19], opens the door to test general relativity against extended theories of gravity (see [19] and references therein). In these extended theories a possible scalar component of the gravitational radiation arises than can be verified by the study of the signal of gravitational waves. Another important issue concerning the cosmological constant Λ is due to its very small value $\Lambda \simeq 10^{-52}/\mathrm{m}^2$ (where 'm' stands for meters), i.e. 10^{122} orders smaller than the value expected for vacuum energy in quantum field theory. Also a thermodynamic description of a Friedmann universe endowed with a cosmological constant is a complicated and debated task [21–27]. First of all, the universe is a dynamical expanding system out from equilibrium. Moreover, it is not yet clear how to describe [27] the dynamical degrees of freedom related to the expansion of the universe. As a consequence, a physically sound statistical mechanics description of the cosmological constant in terms of a fluid or gas is still lacking. The knowledge of a thermodynamic description of the actual universe (at its 'thermodynamic radius', i.e. the apparent horizon [28,29] of our universe) is an important step for a better physical understanding of the cosmological constant.

Recently [30–33], we have generalized the Bekenstein–Hawking entropy formula suitable for black holes embedded in Friedmann universes. In particular, our technology can be applied to the apparent horizon [31–33] of Friedmann universes. As a first important consequence [31,32], we have obtained $U_h = const. = 0$. This can be interpreted with the fact that, according to an old conjecture [34], the gravitational degrees of freedom encoded with an expanding universe are included in our tractation in such a way that the Misner–Sharp energy $M_s c^2$ at the apparent horizon is exactly balanced by the negative gravitational expanding energy for a universe whose spatial sections are flat. Another important consequence of our setups is that [33] the de Sitter universe, filled only with a non-vanishing positive cosmological constant Λ, is the only Friedmann solution that is in thermal equilibrium with its surrounding. As a result, to a cosmological constant Λ can be associated a non-zero temperature given by the one of the apparent horizon. After introducing [32] the Planck constant $\hbar$, a non vanishing internal energy U_h with $U_h \sim T_\Lambda$ is allowed. All these facts permit us to explore a statistical description of Λ.

In Sect. 2 we write down the first law at the apparent horizon for Friedmann spacetimes. In Sect. 3 we analyze the cosmological constant as composed of massless bosons (gravitons), while in Sect. 4 the thermodynamic limit is discussed. Finally, Sect. 5 is devoted to some conclusions and final remarks.

2 First law at the apparent horizon for Friedmann spacetimes

A Friedmann spacetime is given in comoving coordinates by

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \tag{1}$$

where $k = -1, 0, +1$. The spacetimes (1) filled with a matter content satisfying the weak energy condition are equipped with an apparent horizon at the proper areal radius L_h given by

$$L_h = \frac{c}{\sqrt{H^2 + \frac{kc^2}{a(t)^2}}}. \tag{2}$$

To the apparent horizon (2) can be associated an holographic temperature (see for example [11–13,17,18,21,24,25,27–29]) T_h, namely the Hawking temperature $T_h = c\hbar/(2\pi k_B L_h)$. In this paper we adopt the normalization used in [32,33] and the holographic temperature at the apparent horizon (2) is

$$T_h = \frac{c\hbar}{4\pi k_B L_h}. \tag{3}$$

As well known [25], at the apparent horizon the Friedmann equations can be written in a form similar to the first law of thermodynamic

$$T_h dS = dU_h + W_h dV_h, \quad W_h = \frac{p-\rho c^2}{2}, \tag{4}$$

where W_h is the work term and $U_h = c^4 L_h/(2G)$ is the Misner–Sharp energy term, ρ, p respectively the energy density and the pressure, provided that, as happens for static asymptotically flat black holes, the apparent horizon is equipped with the entropy $S_h = \frac{k_B A_h}{4L_P^2}$. However, the Friedmann spacetimes are dynamical and non asymptotically flat and we expect, on general physical grounds, a modification for the usual entropy law $S \sim A/4$. Moreover, as stated in [26], we have not a consistent way to calculate the dynamical degrees of freedom for a non-static gravitational field.

In [30–33], by using suitable theorems for the formation of trapped surfaces in Friedmann spacetimes, we proposed a new formula for the entropy of black holes embedded in Friedmann universes given by

$$S_h = \frac{k_B A_h}{4L_P^2} + \frac{3k_B}{2cL_P^2} V_h H - \frac{3kk_B}{4L_P^2}\frac{L_h V_h}{a(t)^2}, \tag{5}$$

where $V_h = 4\pi L_h^3/3$ and H the Hubble constant. In our proposal, thanks to the holographic principle, the (5) is supposed to be the entropy of the whole universe at its apparent horizon where the entropy bound [30–33] is saturated as happens for the event horizon of a static black hole. The new first law at L_h becomes [31–33] the (4) but with

$$dU_h = \frac{c^4}{2G}dL_h + \frac{c^3}{2G}L_h^2 dH + \frac{kc^4}{2G}\frac{L_h^3}{a^3}da, \tag{6}$$

$$W_h = -\frac{kc^4}{4\pi G a^2} + \frac{3c^3 H}{8\pi G L_h}. \tag{7}$$

A first consequence of (6) is that, according to an old conjecture [34], the internal energy only for Friedmann flat solutions is vanishing [31–33].[1]

Moreover [33], our new formulas (5)–(7) imply that the only Friedmann solution that is in thermal equilibrium with its surrounding is the de Sitter one, i.e. a Friedmann flat solution with a non-vanishing cosmological constant. This means that the temperature of the apparent horizon (3) is nothing else but the temperature of a de Sitter cosmological universe T_Λ, i.e. $T_h = T_\Lambda$ for a de Sitter universe.[2] This is in contradiction with the usual setups (see [26] and references therein) where a zero temperature is usually attributed to Λ. A non vanishing T_Λ can be interpreted as a finite size effect. In fact, the Friedmann universe is thermodynamically a spherical object of areal radius L_h. The usual results can be seen as the limit for $L_h \to \infty$ where $T_h \to 0$. Obviously, this finite volume effect is negligible for our actual universe, but also a small but non-vanishing T_Λ is conceptually important. In particular, as stated in [33], the cosmological constant can be depicted as a fluid satisfying the equality

$$T_h^3 V_h = \frac{1}{48\pi^2}\left(\frac{c\hbar}{k_B}\right)^3, \tag{8}$$

that looks like the equation of a reversible adiabatic transformation. Hence, according to this hypothesis [33], a fluid such that $T^3 V$ is less than the second member of (8) is a phantom fluid.

The first law for the Friedmann flat case can be written as [32]

$$T_h dS_h = dU_h + c^2 \rho \, dV_h. \tag{9}$$

In the following two sections our reasonings do apply to a de Sitter universe equipped only with a positive non-vanishing cosmological constant Λ with $a(t) = e^{ct\sqrt{\frac{\Lambda}{3}}}$, $k = 0$ and without dark matter and electromagnetic radiation. At the conclusions, Sect. 5, we discuss the possible modifications of our results in presence of dark matter and electromagnetic radiation.

Since a de Sitter expanding universe is in thermal equilibrium with its surrounding, we must have, for a cosmological constant with density ρ_Λ, $c^2\rho_\Lambda = |p_\Lambda|$: the apparent horizon is stationary ($L_h = const$) and the work term W_h must reduce to the usual expression $W = p\, dV$. The usual relation $p_\Lambda = -c^2\rho_\Lambda$, as noticed in [27], can be regained by the fact that the first law at the apparent horizon has a different heat-sign convention with respect to the usual prescription.

[1] As shown in [32], when the Planck constant $\hbar$ is introduced, a non vanishing U_h is allowed.

[2] This permit us to trace back [33] the thermal history of the universe in terms of $T_u - T_h$, being T_u the temperature of the matter energy inside L_h where a de Sitter phase emerges when $T_u = T_h$.

We are now in the position to explore the possible statistical consequences of the relation (8).

3 Bose–Einstein distribution for the dark energy

In this section we describe the cosmological constant of a de Sitter universe in terms of the usual Bose–Einstein distribution. To start with, we must evaluate the internal energy of a de Sitter universe. As stated in [31,33], for all Friedmann flat spacetimes the internal energy U_h is a constant of motion that for dimensional arguments (with only the constants Λ, G, c at our disposal) can be set to zero. In ordinary thermodynamics, to a vanishing internal energy can be associated a vanishing temperature. However, as stated above, thanks to finite size effects at L_h and introducing the Planck constant $\hbar$, a non vanishing temperature T_Λ [31,32] satisfying the (8) can be attributed to Λ. In [32] we have also discussed the possible ultraviolet modification to the Friedmann flat equations caused by the introduction of the Planck constant.

In this paper we consider a de Sitter universe filled with the cosmological constant Λ composed of massless gravitons with a quanta of energy ϵ given by the usual relation $\epsilon = \hbar\omega$. Concerning the chemical potential μ_Λ, we may suppose that gravitons are interacting. We have a certain number of interacting gravitons in thermal equilibrium within L_h. The situation is similar to the one of photons emitting a black body radiation in thermal equilibrium with a surrounding body. Hence, also in such a situation the number of gravitons is determined by the thermal equilibrium condition for the free energy F given by $dF = 0$ and consequently $\mu_\Lambda = 0$.

In a similar manner to the photons case, we can introduce the wave number $\mathbf{k}$ and the number of oscillations in $d^3k = 4\pi k^2 dk$ are given by $V_h/(2\pi)^3 4\pi k^2 dk$. After introducing the usual relation $\omega = ck$ and since for gravitons we have only two independent polarizations corresponding to physical gravitons, as usual we have for the number of gravitons N_h within $L_h = c/H_\Lambda$

$$N_h = \frac{V_h}{\pi^2 c^3}\frac{k_B^3 T_h^3}{\hbar^3}\Gamma(3)\xi(3), \tag{10}$$

where $\Gamma(3) = 2$ and $\xi(3) \simeq 1.21$. It is easy to see that, thanks to (8), $N = \Gamma(3)\xi(3)/(48\pi^4) \ll 1$. This result is reasonable only if we suppose a BEC of gravitons filling the ground state with $\epsilon = 0$ at $T < T_h$. Considering gravitons as massless bosons, condensation cannot happen in ordinary situations [35].

Nevertheless, note that, thanks to the interactions among photons and the electrons of a medium, photons propagating in a medium with an effective phase velocity different from c. In fact, in a medium with index of refraction η, the effective phase velocity of a photon is c/η. In a similar way, we suppose that the interactions among gravitons physically justifies a slowing down of the gravitons phase velocity. As a consequence, we may suppose that the following relation holds: $\omega = \gamma|\mathbf{k}|c$, where the effective phase velocity v_g is $v_g = c\gamma$. The parameter γ must be fixed by the overall Bose–Einstein statistics satisfied by gravitons. With the introduction of γ, the formula (10) changes to

$$N_h = \frac{V_h}{\pi^2 c^3} \frac{k_B^3 T_h^3}{\gamma^3 \hbar^3} \Gamma(3)\xi(3). \tag{11}$$

After using the equality (8) characterizing the dark energy, we obtain for γ:

$$\gamma = \left(\frac{\Gamma(3)\xi(3)}{48\pi^4}\right)^{\frac{1}{3}} \frac{1}{N_h^{\frac{1}{3}}}. \tag{12}$$

Formula (12) shows that the effective phase velocity of the gravitons depends only from the number of gravitons present at the horizon L_h and is always smaller than c, a reasonable result. This result is a direct consequence of the constraint (8).

To specify the system, we must evaluate the internal energy of the system at the apparent horizon. As stated in [32], quantum fluctuations do imply a non-vanishing internal energy U_h. In this paper we consider only positive quantum fluctuations.[3] In that case we can write [32] $U_h = |c_0| c\hbar\sqrt{\Lambda}$, with c_0 a dimensionless constant. The total internal energy can be obtained, as usual, multiplying dN_h for $\hbar\omega$ and integrating with respect to ω: we obtain

$$U_h = \frac{V_h}{c^3} \frac{k_B^4 T_h^4}{\gamma^3 \hbar^3} \frac{\pi^2}{15}. \tag{13}$$

After using $U_h = |c_0| c\hbar\sqrt{\Lambda}$ and Eqs. (8) and (13) we have

$$U_h = k N_h c\hbar\sqrt{\Lambda}, \quad k = \frac{\pi^3}{60\sqrt{3}\Gamma(3)\xi(3)}, \tag{14}$$

i.e. $|c_0| \sim N$. Another interesting quantity is the mean value $\overline{\omega}$ of the frequency ω. We get

$$\overline{\omega} = \frac{\pi^4 k_B T_h}{15\Gamma(3)\xi(3)\hbar}. \tag{15}$$

Hence, the following formula holds for U_h:

$$U_h = N_h \hbar\, \overline{\omega}, \tag{16}$$

which is the energy of N oscillators with frequency $\overline{\omega}$. Note that the mean frequency results independent on the number of gravitons N_h. However, this does not happens for the mean proper wavelength $\overline{\lambda}$ (denoting with $\overline{\lambda}_c$ the wavelength in comoving coordinates, we have $\overline{\lambda} = e^{ct\sqrt{\frac{\Lambda}{3}}}\, \overline{\lambda}_c$). In fact we have $\overline{\lambda} = 2\pi\gamma c/\overline{\omega}$ and as a consequence

$$\overline{\lambda} = \frac{120}{\pi^2}\Gamma(3)\xi(3)\left[\frac{\Gamma(3)\xi(3)}{48\pi^4 N_h}\right]^{\frac{1}{3}} \sqrt{\frac{3}{\Lambda}}. \tag{17}$$

[3] For the case of negative quantum fluctuations, a quantum field theory formalism is necessary [36].

The (17) is an interesting formula relating the mean value for the wavelength of the gravitons, the cosmological constant and the number of excitations inside L_h. The (17) can also be written in the form $\overline{\lambda} N_h^{1/3}/L_h \simeq 2.37$. Since $\Lambda \simeq 10^{-52}/\text{m}^2$, we have $\overline{\lambda} \simeq 2.37 \times 10^{26}/(N_h^{1/3})$ m. This formula permits us some numerical investigations. As an example, for gravitons with $\overline{\lambda} \sim 10^{15}$ m, we obtain $N_h \sim 10^{33}$, while for $\overline{\lambda} \sim 10^6$ m (binaries source) we obtain $N_h \sim 10^{60}$.

Concerning the mean frequency $\overline{\varpi}$ given by (15), for $\Lambda \sim 10^{-52}/\text{m}^2$ we obtain, thanks to the formula $k_B T_h/\hbar = c/(4\pi)\sqrt{\Lambda/3}$, $\overline{\varpi} \sim 10^{-18}$ Hz, i.e. a very low frequency that we expect, for example, in a system near condensation.

In fact, thanks to the large number of gravitons $N_h \gg 1$ expected at the apparent horizon L_h of a de Sitter universe,[4] we have that $v_g \ll c$ and for $N_h \to \infty$ we have $v_g \to 0$. In the next sections we explore this possibility by presenting a suitable thermodynamic limit.

As a final consideration for this section, we study the thermalization of gravitons in an expanding universe. Gravitons are expected to weakly interact among themselves. Moreover, the expansion of the universe can take more difficult thermalization. Hence we expect that thermalization can certainly happen for a sufficiently slow expansion and for a huge gravitons number N_h. The very low value for Λ works in such a direction. To be more quantitative, we introduce the interaction rate Γ representing the mean interactions frequency. As usual, we can assume that thermalization follows provided that $\Gamma > H(t) = c\sqrt{\frac{\Lambda}{3}}$. For Γ we could adopt the usual expression, i.e. $\Gamma = \frac{N_h}{V_h}\sigma_g v_g$, where σ_g is the graviton–graviton cross section and $v_g = c\gamma$. With the help of (11) and (12) we have:

$$N_h^{\frac{2}{3}} > \left[\frac{\Gamma(3)\xi(3)}{48\pi^4}\right]^{-\frac{1}{3}} \frac{4\pi}{\Lambda\sigma_g}. \tag{18}$$

Since we have not a consolidated quantum gravity theory on a curved spacetime, the cross section σ_g in (18) is left unspecified. Nevertheless, the (18) clearly shows that, also for a very small σ_g, a sufficiently large N_h makes the job.

4 Dark energy as an 'almost' Bose–Einstein condensate

In the section above we have depicted the cosmological constant as composed of N_h interacting gravitons with phase velocity v_g given by $v_g = c\gamma$, as happens for photons interacting with a given medium. The dimensional factor γ given by (12) is calculated by the properties of the system at its thermodynamic radius, i.e. the apparent horizon $L_h = \sqrt{3/\Lambda}$. Since $\Lambda \sim 10^{-52}/\text{m}^2$ for our actual universe, we expect $N_h \gg 1$ and consequently $v_g \simeq 0$, i.e. the gravitons phase velocity slow down up to a value closed to zero, as expected for a boson system near the critical temperature. In particular, thanks to the (8), we have for the temperature T_Λ of Λ, $T_\Lambda = T_h \sim 10^{-29}$ K, i.e. a

[4] Practically of the same order of the one predicted by the concordance ΛCDM model at present cosmological time.

very cold temperature that is typical of BEC matter. The mean frequency $\overline{\omega}$ is very small $\overline{\omega} \sim 10^{-18}$ Hz and the mean energy $\overline{\epsilon}$ of gravitons is of the order of $\overline{\epsilon} \sim 10^{-52}$ J: this implies that the gravitons are near the ground state with $\overline{\omega} \simeq 0$. In this regard, the temperature T_h, fixed thanks to the (8), can be seen as a critical temperature. Hence, the very low value for Λ can well be a consequence of the fact that the cosmological constant is composed of gravitons at very low temperatures, namely the 'critical' temperature T_h. The value of T_h and the fact that $\gamma \ll 1$ imply that effectively gravitons are very near to be in a BEC state. In this regard, thanks to (8) and (12), gravitons at $T_\Lambda \simeq 0$ are practically frozen ($\overline{\omega} \simeq 0$ and $v_g \simeq 0$). As an example, for $N_h \sim 10^{60}$ (a relatively 'small' value), we have $v_g \sim c/10^{20}$, i.e gravitons employ 10^{15} seconds to travel one kilometer. We stress that this almost BEC state of gravitons at $T = T_h$ is due to the fact that $v_g \simeq 0$. Also note that, since for the critical temperature T_c we have $T_c = T_\Lambda \simeq 0$, fluctuations allow to a non-zero fraction of gravitons to fill the state with $T < T_h$, therefore with exactly $\omega = 0$. As a consequence of these reasonings, one may expect that gravitons at $T = T_h$ can acquire a kind of effective mass m_g. We can estimate this effective mass in the following way. From (15) we see that the mean energy for graviton $\overline{\epsilon}$ is given by $\overline{\epsilon} = \hbar\overline{\omega} \simeq k_B T_h$. For a particle the relativistic dispersion relation is given by $\overline{\epsilon} = \sqrt{c^2 p^2 + m^2 c^4}$. Since gravitons are frozen in a quasi BEC state we can assume $p = 0$. Hence we obtain $m = \frac{\overline{\epsilon}}{c^2} \simeq 10^{-65}$ g for the effective mass acquired by gravitons. The same result can be obtained by considering the effective mass as an effect due to the finiteness of the apparent horizon $L_h \sim 10^{26}$ m. We obtain $m_g = \hbar/(cL_h) \sim 10^{-65}$ g, that is of the same order of the estimation above. Note that also in [35], thanks to the 'finite' dye-filled optical microcavity where experiment is performed, photons acquire an effective mass near the BEC ground state.

This phenomenon does not happen for huge values of the cosmological constant, for example at the primordial inflation $t_I \sim 10^{-37} - 10^{-35}$ s, where $\overline{\omega} \gg 0$ and N_h, thanks to the very small dimensions of the universe at the begin of the primordial inflation, is expected to be certainly much more less than the actual expected value and as a consequence v_g at $t = t_I$ is not negligible. Obviously, the tractation of section above is still valid for a finite value of Λ expected at the primordial inflation.

In the following we perform the thermodynamic limit. This a subtle issue since a Friedmann flat solution is spatially infinite. However, we have identified the apparent horizon L_h with the thermodynamic radius of the system. Hence, according with this identification, the thermodynamic limit must be performed by sending $N_h \to \infty$ and $L_h \to \infty$ (i.e. $\Lambda \to 0$) in such a way that $\frac{N_h}{V_h} = \rho_g = const$. In this way the cosmological constant Λ 'disappears' in the thermodynamic limit, but the gravitons density ρ_g remains finite. This does not means that in the thermodynamic limit we have a Minkowski spacetime, but merely that a finite Λ at a finite L_h is spread on an infinite region. This is a mathematical procedure to eliminate fluctuations depending on N from the BEC phenomenology.

As a first consequence, in the thermodynamic limit above defined, we have $T_h = T_\Lambda \to 0$. From (15) we deduce that $\overline{\omega} \to 0$ an also the mean energy for gravitons $U_h/N_h \to$ with obviously $U_h \to \infty$ (spatially infinite system). These features are typical of the BEC phenomenology. In particular, the mean frequency $\overline{\omega}$ must be vanishing in the ground state. Moreover, thanks to (12) we have $\gamma \to 0$ and as a result $v_g \to 0$. Finally, for $\overline{\lambda}$ we have

$$\overline{\lambda} \to \frac{1,47}{\rho_g^{\frac{1}{3}}}. \tag{19}$$

The fact that the proper mean wavelength $\overline{\lambda}$ is not vanishing in the thermodynamic limit means that effectively the gravitons are not vanishing in this limit.

Summarizing, in the thermodynamic limit we have frozen ($v_g = 0$) gravitons filling the ground state with zero mean energy and finite mean wavelength: this implies that effectively exactly at $T_\Lambda = 0$ gravitons are in a BEC state.

The actual very small value of the cosmological constant, thanks to our setups, is an indication that the identification of the actual Λ as composed of gravitons in a state very near to the ground state of a BEC condensate is a reasonable and viable possibility.

5 Conclusions and final remarks

In this paper, following our recent results [30–33] concerning the thermodynamic of Friedmann universes, we presented a statistical description of the cosmological constant. In particular, thanks to the fundamental result (8) that the cosmological constant has a non-vanishing temperature, we can describe Λ as composed of N_h interacting gravitons with energy $\hbar\omega$ and $T_h = T_\Lambda$. The supposed interaction between gravitons justifies the assumption that their phase velocity slow down, as happens for photons propagating in a medium. The phase velocity v_g is now $v_g = c\gamma$ with $\gamma \sim 1/N_h^{1/3}$. As a consequence, we can calculate the mean frequency for gravitons $\overline{\omega}$ and an interesting formula, namely the (17), relating the mean wavelength $\overline{\lambda}$, the cosmological constant Λ and the gravitons number N_h at the apparent horizon. Thanks to the actual very low value for Λ, we obtain $\overline{\omega} \sim 10^{-18}$ Hz. This very low value for $\overline{\omega}$, together with the fact that practically $v_g \sim c/N_h^{1/3} \simeq 0$ motivates the view that the present day dominant cosmological constant is constituted by gravitons very near a BEC state. In practice, the temperature $T_\Lambda = T_h$ is a critical temperature for BEC and all the gravitons are very near the ground state with $\overline{\omega} = 0$. The ground state $\overline{\omega} = 0$ is rigorously obtained in the thermodynamic limit by sending $L_h \to \infty$ ($\Lambda \to 0$) and $N_h \to \infty$ but with $N_h/V_h = \rho_g$ held fixed. In this limit, $\{\overline{\omega}, T_h, v_g\} \to 0$, with $\overline{\lambda}$ finite, motivating our physical interpretation of the actual cosmological constant as composed of gravitons near the ground state of a BEC.

Gravitons are supposed massless. However, in massive gravity (see for example [37] and references therein) a very small mass m_g ($<10^{-62}$ g) is allowed. Our tractation is still substantially valid by taking a small value for m_g such that $m_g \leq \hbar\overline{\omega}/c^2$. For $\Lambda \sim 10^{-52}/\mathrm{m}^2$, we have $m_g \leq 10^{-65}$ g.

These results of this paper have been obtained considering a de Sitter universe. Although our universe is composed by the $\simeq$68 % of dark energy, dark matter is also present ($\simeq$28 %). It is thus interesting to wonder the possible modifications of the present paper calculations in presence of dark matter or other kinds of matter.

To this purpose, consider a Friedmann flat universe filled with a positive cosmological constant and usual dust matter with density ρ_m and radiation ρ_r with temperature T_r. First of all, it is rather physically reasonable to assume that gravitons decoupled from radiation during or before primordial inflation (see for example [38] and references therein). With the actual Λ made of gravitons, they are reasonable decoupled from the CMBR (and also dark matter starting from the recombination era) and as a consequence gravitons cannot be thermalized with photons. Instead, we can reasonably assume that $T_g \sim T_\Lambda \sim T_h$ in our universe dominated by Λ and practically the results of Sects. 3–4 still hold in our Friedmann flat universe where matter, radiation and gravitons are decoupled. In particular, gravitons composing Λ are very near to a BEC state with $\{\overline{\omega}, v_g\} \simeq 0$.

As an useful example [33], we could consider ρ_m and $\rho_\Lambda = \Lambda c^2/(8\pi G)$ as a mixture at the temperature $T_u > T_h = T_\Lambda$[5] given by

$$T_u = \frac{\rho_\Lambda T_\Lambda + \rho_m T_m + \rho_r T_r}{\rho_\Lambda + \rho_m + \rho_r}. \tag{20}$$

In this case [33], $T_u > T_h$ and $L_{h,t} > 0$. Only asymptotically $T_u \to T_h$ and a pure de Sitter phase emerges. We can suppose again interacting gravitons with formulas (8), (12), (13), (15), (16) still hold but with $T_h = \frac{\hbar H(t)}{4\pi k_B}$, where $H(t)$ is the Hubble flow. Hence, the mean value $\overline{\omega}$ becomes time dependent with $\overline{\omega} \sim H(t)$. When a de Sitter phase is reached with $H(t) \to c\sqrt{\Lambda/3}$ we regain the 'stationary' formula (15). Generally, we also expect that $N_h = N_h(t)$ and $\overline{\lambda} = \overline{\lambda}(t)$. By considering, for example, a time independent internal energy U_h, we must have $N_h(t) \sim 1/H(t)$. Concerning formula (17), we obtain

$$\overline{\lambda}(t) H(t) N_h^{\frac{1}{3}}(t) = 2.37c. \tag{21}$$

Note that, if we associate N_h to the number of gravitons of Λ at the apparent horizon, formula (21) holds for $N_h \neq 0$, i.e. for $\Lambda \neq 0$. However, with the hypothesis that gravitons are the 'quanta' of the gravitational field, we could suppose that formula includes gravitons constituting Λ and the ones N_{d_h} related to the intrinsic non static nature of a Friedmann universe, provided that they are thermalized with Λ, i.e. $T_{d_h} \simeq T_\Lambda$. In this case formula (21) still holds with $N_h \to N_h + N_{d_h} = N$.

With respect to (21), a de Sitter phase ($T_u = T_h = T_\Lambda,\ L_{h,t} = 0$) arises when $\overline{\lambda}(t) N^{\frac{1}{3}}(t) \simeq const$. Stated in other words, by denoting with ρ the density of the universe excluded Λ and with p the pressure with equation of state $p = k\rho c^2$, Friedmann equations become

$$\frac{\left(\overline{\lambda}(t) N^{\frac{1}{3}}(t)\right)_{,t}}{\overline{\lambda}(t) N^{\frac{1}{3}}(t)} = \frac{\overline{\lambda}_{,t}}{\overline{\lambda}} + \frac{N_{,t}(t)}{3N(t)} = \frac{4\sqrt{3}\pi G\rho(1+k)}{\sqrt{8\pi G\rho + \Lambda c^2}}, \tag{22}$$

$$\overline{\lambda}^{-2} N^{\frac{2}{3}} \left(\frac{8}{3}\pi G\rho + \frac{\Lambda c^2}{3}\right) = sc^2, \quad s \simeq 5.61, \tag{23}$$

[5] It is rather natural to suppose that Λ, thanks to (8), is thermalized with the apparent horizon.

where only thermodynamic quantities are included, together with the (21). For $\rho = 0$ we have obviously the de Sitter solution with the relation (17). Note that for $k \to -1$ a de Sitter phase emerges: in this case we have $\overline{\lambda}^3(t) \sim 1/N(t)$ but with $\overline{\lambda}$ and N, differently from the de Sitter universe, time dependent quantities.

As a final consideration, note that Eq. (17) implies that the value of Λ depends on the product $\overline{\lambda}^{2} N_h^{\frac{2}{3}}$, i.e. $\Lambda \sim 1/(\overline{\lambda}^{2} N_h^{\frac{2}{3}})$. Although with the formula (17) we have not further constraints to obtain the actual very low estimated value for Λ, it is certainly true that a very low value is natural in our approach. For example, for wavelengths $\overline{\lambda} \geq 10^6$ m, we have $N_h \leq 10^{60}$. In any case, thanks to the expected very large value for N_h, the issue of a very small Λ can be certainly alleviated.

Summarizing, the thermodynamic (8), (9) at the apparent horizon L_h together with the hypothesis that dark energy is made of interacting bosons very near to a BEC state due to massless gravitons or very light bosons, is a physically viable possibility and must be certainly further investigated.

References

1. Zlatev, I., Wang, L., Steinhardt, P.J.: Phys. Rev. Lett. **82**(5), 896 (1999)
2. Peebles, P.J.E., Ratra, B.: Rev. Mod. Phys. **75**(2), 559 (2003)
3. Steinhardt, P.J., Turok, N.: Science **312**, 1182 (2006)
4. Hogan, J.: Nature **448**(7151), 240 (2007)
5. Wiltshire, D.L.: Phys. Rev. Lett. **99**(25), 251101 (2007)
6. Caldwell, R.R.: Phys. Lett. B **545**, 23 (2002)
7. Alam, U., Sahni, V., Saini, T.D., Starobinsky, A.A.: Mon. Not. R. Astron. Soc. **344**, 1057 (2003)
8. Chiba, T., Okabe, T., Yamaguchi, M.: Phys. Rev. D **62**, 023511 (2000)
9. Zlatev, I., Wang, L., Steinhardt, P.J.: Phys. Rev. Lett. **82**, 896 (1999)
10. Wetterich, C.: Nucl. Phys. B **302**, 302 (1988)
11. Li, M.: Phys. Lett. B **603**, 1 (2004)
12. Gong, Y.: Phys. Rev. D **70**, 064029 (2004)
13. Gao, C., Wu, F.Q., Cheng, X., Shen, Y.G.: Phys. Rev. D **79**, 043511 (2009)
14. Bhatt, J.R., Desai, B.R., Ma, E., Rajasekaran, G., Sarkar, U.: Phys. Lett B. **687**, 75 (2010)
15. Basilakos, S., Lima, J.A.S., Solá, J.: Int. J. Mod. Phys. D **22**, 1342008 (2013)
16. Das, S., Bhaduri, R.K.: Class. Quantum Gravity **32**, 105003 (2015)
17. Susskind, L.: J. Math. Phys. **36**, 6377 (1995)
18. Jacobson, T.: Phys. Rev. Lett. **75**, 1260 (1995)
19. Corda, C.: Int. J. Mod. Phys. D **18**, 2275 (2009)
20. Abbott, B.P., LIGO Scientific Collaboration and Virgo Collaboration, et al.: Phys. Rev. Lett. **116**, 061102 (2016)
21. Akbar, M., Cai, R.G.: Phys. Rev. D **75**, 084003 (2007)
22. Padmanabhan, T.: Rep. Prog. Phys. **73**, 046901 (2010)
23. Padmanabhan, T.: Gen. Relativ. Gravit. **46**, 1673 (2014)
24. Akbar, M.: Chin. Phys. Lett. **25**, 4199 (2008)
25. Cai, R.G., Kim, S.P.: JHEP **0502**, 050 (2005)
26. Saridakis, E.N., Diaz, P.F.G., Siguenza, C.L.: Class. Quantum Gravity **26**, 165003 (2009)
27. Tian, D.W., Booth, I.: Phys. Rev. D **92**, 024001 (2015)
28. Hayward, S.A.: Phys. Rev. D **49**, 6467 (1994)
29. Faraoni, V.: Phys. Rev. D **84**, 024003 (2011)
30. Viaggiu, S.: Mod. Phys. Lett. A **17**, 1450091–1 (2014)
31. Viaggiu, S.: Gen. Relativ. Gravit. **47**(8), 86 (2015). doi:10.1007/s10714-015-1928-y
32. Viaggiu, S.: Mod. Phys. Lett. A **4**, 1650016 (2016)
33. Viaggiu, S.: Int. J. Mod. Phys. D **2**, 1650033 (2016)
34. Tryon, E.P.: Nature **246**, 396 (1973)

35. Klaers, J., Schmitt, J., Vewinger, F., Veitz, M.: Nature **468**, 545 (2010)
36. Ford, L.H.: Int. J. Mod. Phys. A **25**, 2355 (2010)
37. De Felice, A., Gumrukcuoglu, A.E., Chunshan, L., Mukohyama, S.: Class. Quantum Gravity **30**, 184004 (2013)
38. Bhattacharya, K., Mohanty, S., Nautiyal, A.: Phys. Rev. Lett. **97**(25), 251301 (2006)