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RESEARCH ARTICLE

Entropic corrected Newton's law of gravitation and the loop quantum black hole gravitational atom

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Abstract One proposal by Verlinde is that gravity is not a fundamental, but an entropic force (Verlinde in JHEP 1104:029, 2011. arXiv:hep-th/1001.0785). Based on this new interpretation of the gravity, Verlinde has provide us with a way to derive the Newton's law of gravitation from the Bekenstein–Hawking entropy-area formula. On the other hand, since it has been demonstrated that this formula is susceptible to quantum gravity corrections, one may hope that such corrections could be inherited by Newton's law. In this sense, the entropic interpretation of Newton's law could be a prolific way in order to get verifiable or falsifiable quantum corrections to ordinary gravity in an observationally accessible regimes. On the other hand, loop quantum gravity is a theory that provide a scheme to approach the quantum properties of spacetime. From this theory, emerges a quantum corrected semiclassical black hole solution called loop quantum black hole or self-dual black hole. Among the interesting features of loop quantum black holes, is the fact that they give rise to a modified entropy-area relation where quantum gravity corrections are present. In this work, we obtain a quantum corrected Newton's law from the entropy-area relation given by loop quantum black holes *by using the nonrelativistic Verlinde's approach*. Moreover, in order to relate our results with the recent experimental activity, we consider the quantum mechanical properties of a huge gravitational atom consisting in a light neutral elementary particle in the presence of a loop quantum black hole.

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1 Introduction

Since the rising of black hole thermodynamics in the seventies, through the Hawking's demonstration that all black holes emit blackbody radiation [1], investigations about these objects break up the limits of astrophysics. In fact, black holes have been put in the heart of the debate of the most fascinating issues in theoretical physics. Among these issues, the search for a better understanding of the quantum nature of gravity, since the quantum behavior of spacetime must be revealed within the presence of a black hole strong gravitational field.

Among the most important lessons from black hole thermodynamics, arises the Bekenstein–Hawking formula which establishes that, in a different way from other usual thermodynamical systems, the entropy of a black hole is not given as proportional to its volume, but to its horizon area: $S = k_B c^3 A/4\hbar G$. A deep intersection between gravity, quantum mechanics, and thermodynamics could be contained in this formula, since it gives us one of the few situations in physics where Newton's gravitational constant G and the speed of light c meet the Planck constant $\hbar$ and the Boltzmann constant k_B. In fact, it has been shown by string theory and loop quantum gravity that black-hole thermodynamics must has its origin in the atomic structure of the spacetime [2–5]. Moreover, in [6–8] it has been argued that a topology change process due to the dynamics of the quantum spacetime could be the origin of black hole entropy and the Generalized Second Law of black hole thermodynamics.

In 1995, a surprising result by Jacobson has deepened the significance of the Bekenstein–Hawking formula. Assuming the proportionality between entropy and horizon area, Jacobson derived the Einstein's field equations by using the fundamental Clausius relation [9]. The procedure behind this result is to require that such relation, $\delta Q = T dS$, associating heat, temperature and entropy, holds for all the local Rindler causal horizon through each spacetime point, with δQ and T interpreted, respectively, as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. In this sense, the spacetime could be viewed as a kind of gas whose entropy is given by the Bekenstein–Hawking formula, and the Einstein's field equation as an equation of state describing this gas.

Following Jacobson's results, several authors, most notably Thanu Padmanabhan, have addressed the issue of the relation between gravity and thermodynamics (For a review and a voluminous list of references see [10]). More recently, Verlinde [11] conjectured that gravity is a non fundamental interaction but would be explained as an entropic force. In this way, the second law of Newton is obtained when one tie up the entropic force with the Unruh temperature. On the other hand, Newton's law of gravitation is obtained when associating these arguments with the holographic principle and using the equipartition law of energy. Verlinde's formalism has been used in several contexts including cosmological ones [12–16].

Controversially, by using the measurement result of quantum states of ultra-cold neutron under the Earth's gravity, Kobakhidze [17] presented an argument in opposition to Verlinde's proposal. The problem pointed by Kobakhidze comes from the fact that the entropy formula defined by Verlinde's formalism, in principle, leads to a quantum neutron mixed state. However, it disagrees with the results from the ultra-cold neutron experiment. Kobakhidze's criticism have been questioned in [18] and one resolution has been suggested by Abreu et al. [19]. This resolution can be found out by abandoning the implicit assumption in [17] that the entropy on the holographic screen is additive.

Verlinde's ideas, in this way, have provide us with a way to derive Newton's law of gravitation from Bekenstein–Hawking formula. However, from another standpoint, it is also known that, in other contexts than Einstein's gravity, this formula of black hole entropy may not be held. For example, when higher order curvature term appears in some gravity theory, the entropy-area formula has to be modified [20]. Modifications to Bekenstein–Hawking formula also appear when quantum gravity effects are included. For example, when a Generalized Uncertainty Principle (GUP) is taken into account [21,22]. In this way, deviations of the entropic force due to corrections imposed on the area law by quantum effects and extra dimensions have been investigated [23–31]. Quantum gravity corrections to Bekenstein–Hawking formula also appear in the context of loop quantum gravity. The most popular form to these corrections appear as logarithmic corrections which arises due to thermal equilibrium fluctuations and quantum fluctuations [32–35].

Another way to get quantum corrections to Bekenstein–Hawking formula arises in the context of loop quantum black holes. Actually, efforts in order to obtain black hole solutions in the context of Ashtekar's reformulation of general relativity and loop quantum gravity have been done since the 1980's by several authors [36–46]. However, in this work, we shall deal with a loop quantum black hole solution, that consists in a quantum gravity corrected Schwarzschild black hole that appears from a simplified model of loop quantum gravity and possess the interesting property of self-duality [47–52]. This property guarantees the black hole singularity resolution, since an asymptotic flat region corresponding to a Planck-sized wormhole arises in the place of the black hole singularity. The wormhole throat, in this scenario, is described by the Kantowski-Sachs solution.

The thermodynamical properties of loop black holes have been addressed in the references [48–52]. Moreover, in the reference [53], such thermodynamical properties were obtained by the use of a tunneling method with the introduction of back-reaction effects. On the other hand, in the reference [54], the tunneling formalism has been applied in order to include corrections due to a Generalized Uncertainty Principle to loop quantum black hole's thermodynamics. Among the results related with the thermodynamics of loop black holes, we have a quantum corrected Bekenstein–Hawking formula for the entropy of a black hole in which quantum gravity ingredients have been included. Phenomenological issues related with this scenario have also been addressed in the literature. In this way, gravitational lenses effects due to this kind of black holes have been investigated in [55]. On the other hand, loop quantum black hole's quasinormal modes have been calculated in [56], [57] and [58]. In the last, axial gravitational perturbations have been considered.

In the present work, we shall address how the Newton's law of gravitation would be modified in the presence of loop quantum black holes, when quantum properties of spacetime are taken into account. In order to do this, we shall use the Verlinde's entropic approach in the non-relativistic realm [11]. Moreover, motivated by the progress in the experimental activity (LIGO [59], GEO-600 [60], TAMA-300 [61] and VIRGO [62]), we shall consider the quantum mechanical system of a *non-relativistic* gravitational atom consisting in a light neutral elementary particle in the presence of a loop quantum black hole. In particular, we apply the Bohr–Sommerfeld formalism to this system, by the use of the modified Newton's potential, in order to obtain its energy levels.

2 Loop quantum black holes

The loop quantum black hole (LQBH) scenario that we shall deal in this work appeared at the first time from a simplified model of Loop Quantum Gravity(LQG) [47]. This scenario is described by a quantum gravitationally corrected Schwarzschild metric, which can be written in the form

$$ds^2 = -G(r)dt^2 + F^{-1}(r)dr^2 + H(r)d\Omega^2 \tag{1}$$

with

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2, \tag{2}$$

where, in the Eq. (1), the metric functions are given by

$$G(r) = \frac{(r-r_+)(r-r_-)(r+r_*)^2}{r^4+a_0^2}, \tag{3}$$

$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_0^2)}, \tag{4}$$

and

$$H(r) = r^2 + \frac{a_0^2}{r^2}, \tag{5}$$

with

$$r_+ = \frac{2mG}{c^2}; \quad r_- = \frac{2mP^2G}{c^2}\ .$$

In this way, two horizons appear in this LQBH's scenario—an event horizon at r_+ and a Cauchy horizon at r_-. Furthermore, we have that $r_* = \sqrt{r_+r_-} = 2mPG/c^2$, where P is the polymeric function given by

$$P = \frac{\sqrt{1+\epsilon^2}-1}{\sqrt{1+\epsilon^2}+1}, \tag{6}$$

and

$$a_0 = \frac{A_{min}}{8\pi}, \tag{7}$$

with A_{min} as the minimal value of area in LQG. Moreover, $\epsilon = \gamma\delta_b$, where γ is the Barbero–Immirzi parameter [5], and δ_b is the polymeric parameter.

On the other hand, we have that the ADM mass is the mass inferred by an observer at flat asymptotic infinity; it is determined solely by the metric in this realm. In this way, in the limit $r \to \infty$,

$$F(r) \to 1 - \frac{2m(1+P)^2}{r}, \tag{8}$$

in a way that we can read off the ADM mass of the system M as

$$M = m(1+P)^2. \tag{9}$$

In the metric (1), since $g_{\theta\theta}$ is not just r^2, r is only asymptotically the usual radial coordinate. From the form of the function $H(r)$, one obtains a more physical radial coordinate given by

$$R = \sqrt{r^2 + \frac{a_0^2}{r^2}}. \tag{10}$$

In this way, the proper circumferential distance is measured by R.

The Eq. (10) reveals important aspects of the LQBH's internal structure. From this expression, we have that, as r decreases from ∞ to 0, R first decreases from ∞ to $\sqrt{2a_0}$ at $r = \sqrt{a_0}$ and then increases again to ∞. The value of R associated with the event horizon is given by

$$R_{EH} = \sqrt{H(r_+)} = \sqrt{\left(\frac{2mG}{c^2}\right)^2 + \left(\frac{a_0c^2}{2mG}\right)^2}. \tag{11}$$

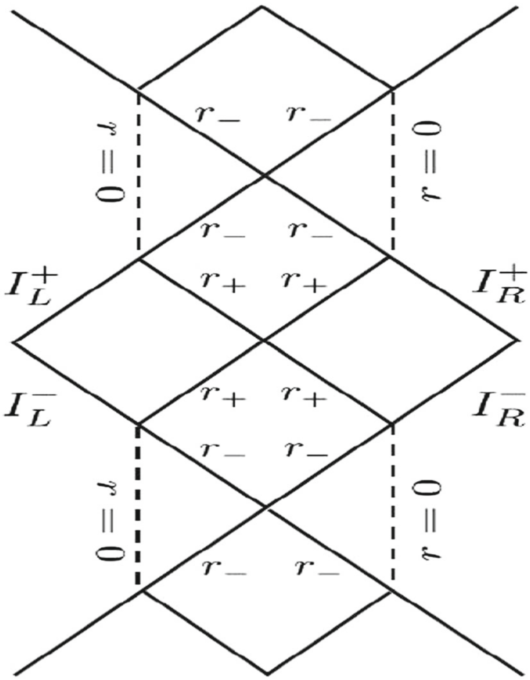
A peculiar feature in LQBH's scenario is the property of self-duality. This property says that if one introduces the new coordinates $\tilde{r} = a_0/r$ and $\tilde{t} = tr_*^2/a_0$, with $\tilde{r}_\pm = a_0/r_\mp$, the metric preserves its form. The dual radius is given by $r_{dual} = \tilde{r} = \sqrt{a_0}$ and corresponds to the minimal possible surface element. Moreover, since the equation (10) can be written as $R = \sqrt{r^2 + \tilde{r}^2}$, it is clear that, in the LQBH's scenario, we have another asymptotically flat Schwazschild region in the place of the singularity in the limit $r \to 0$. This new region corresponds to a Planck-sized wormhole. Figure (1) shows the Carter–Penrose diagram for the LQBH.

The derivation of the black hole's thermodynamical properties from the metric (1) proceeds in the usual way. The Bekenstein–Hawking temperature T_{BH} can be obtained by the calculation of the surface gravity κ by $T_{BH} = \hbar\kappa/2\pi ck_B$, with

$$\kappa^2 = -g^{\mu\nu}g_{\rho\sigma}\nabla_\mu\chi^\rho\nabla_\nu\chi^\sigma = -\frac{1}{2}g^{\mu\nu}g_{\rho\sigma}\Gamma^\rho_{\mu 0}\Gamma^\sigma_{\nu 0}, \tag{12}$$

where $\chi^\mu = (1, 0, 0, 0)$ is a timelike Killing vector and $\Gamma^\mu_{\sigma\rho}$ are the connections coefficients.

Fig. 1 Carter–Penrose diagram for the LQBH's metric. The *diagram* shows two asymptotic flat regions, one localized at infinity and the other near the origin, which can not be reached by an observer in a finite time



By connecting with the metric, one obtains that the LQBH's temperature is given by

$$T_H = \frac{\hbar c^3}{G k_B} \frac{(2m)^3(1-P^2)}{4\pi\left[(2m)^4 + a_0^2\right]}. \tag{13}$$

It is easy to see that one can recover the usual Hawking's temperature in the limit of large masses. However, differently from the Hawking's case, the temperature (13) goes to zero for $m \to 0$, as have been shown in the Fig. (2). In this point, we remind that the black hole's ADM mass $M = m(1+P)^2 \approx m$, since $P \ll 1$.

The black hole's entropy can be found out by making use of the thermodynamical relation $S_{BH} = \int dm/T(m)$,

$$S = \frac{4\pi(1+P)^2}{(1-P^2)} \frac{k_B G}{c\hbar} \left[\frac{16m^4 - a_0^2}{16m^2}\right], \tag{14}$$

which can be expressed in terms of the black hole horizon area as [52]

$$S = \pm k_B \frac{\sqrt{A^2 - A_{min}^2}}{4L_P^2} \frac{(1+P)}{(1-P)}, \tag{15}$$

where we have set the possible additional constant to zero, and $L_P = \sqrt{G\hbar/c^3}$ is the Planck length. S is positive for $m > \sqrt{a_0}/2$ and negative otherwise.

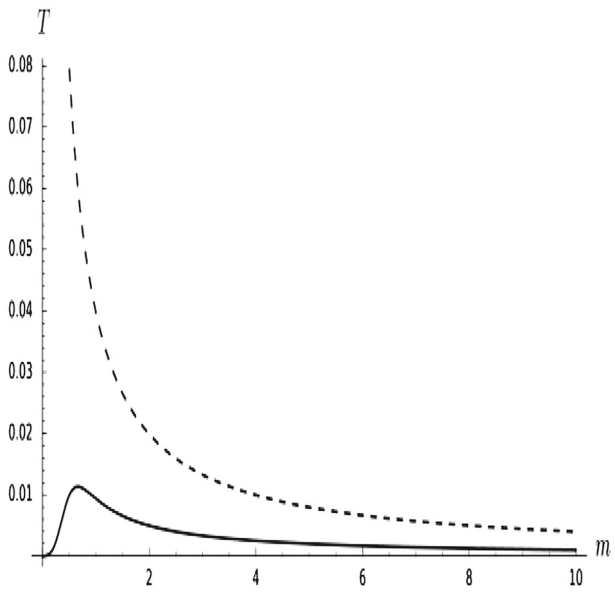


Fig. 2 The LQBH's temperature (*solid line*) in contrast with the Schwarzschild black hole temperature (*dashed line*)

The double possibility in the signal of the LQBH's entropy is related with the two possible physical situations that arise from its structure [51]. In the first of these possibilities, the event horizon stays outside the wormhole throat. In order to have this situation, the condition $r_+ > \sqrt{a_0}$ is necessary, which implies that $m > \sqrt{a_0}/2$. In this case, the bounce takes place after the black hole forms for a super-Planckian LQBH and the exterior is similar, in a qualitative way, to that would be produced by a Schwarzschild black hole with the same mass. In this way, outside the event horizon, the LQBH's scenario is different from the Schwarzschild's one only by Planck-scale corrections. On the other hand, in the sub-Planckian regime, we have a more instigating situation. In this case, the event horizon becomes the other side of the wormhole throat. Moreover, the deviations from the Schwarzschild metric are very expressive and the bounce takes place before the event horizon forms. Consequently, even large event horizons (what would happen for $m \ll m_P$), it will be invisible to observers at $r > \sqrt{a_0}$.

The LQBH's solution (1) could be conceived, in an alternative way, as having been generated by an effective matter fluid that simulates the loop quantum gravity corrections (in analogy with [63,64]). In this case, the effective gravity-matter system satisfies by definition the Einstein equations $G = 8\pi T$, where T is the effective energy tensor. However, for the LQBH's case, $T \neq 0$ contrarily to the classical Schwarzschild solution [48].

The thermodynamics properties of LQBHs has also been obtained through the Hamilton–Jacobi version of the tunneling formalism [53]. By the use of this formalism, back-reaction effects could be included. Moreover, extensions of the LQBH solution to scenarios where charge and angular momentum are preset can be found in [65]. The issue of information loss has been also addressed in the context of loop black holes. In this case, it has been pointed that the problem of information loss by black holes could be relieved in this framework [50,53,66]. This result may be related with the absence of a singularity in the loop black hole interior, and consists in a forceful result in benefit of this approach. Another interesting result in the realm of LQBHs is the fact that, as have been demonstrated in [67], it can been seen as the building blocks of loop quantum cosmology (LQC), in the sense that, starting from the LQBH's entropy expression, LQC equations can been obtained through the use of Jacobson's formalism to obtain the Einstein's gravitational equations.

In the next section, following the formalism developed by Verlinde [11], we shall derive the quantum corrected Newton's law from the modified entropy-area relation given by the Eq. (15).

3 Quantum corrected Newton's law from loop quantum black holes

In this section, we shall derive the quantum corrected Newton's gravitational law of gravitation which arises in the presence of LQBHs. In order to do this, we shall use the nonrelativistic Verlinde's entropic force formalism.

In this point, an explanation about the choice of this method is suitable. In fact, since we have the LQBH's metric on our hands, one could derive straightforwardly a quantum corrected form to the Newtonian potential by taking into account the point of view of an observer located at infinity (weak field approximation). However, by the use of this method, in the case of LQBH, only the quantum correction that depends on the polymeric function P are revealed, while the high energy corrections which depend on the minimal area a_0 become hidden. In the nonrelativistic regime, for example, the weak field approximation would imply, only the redefinition of the ADM mass by a factor $(1 + P)^2$(see Eq. 8). Actually, one can verify that, in the relativistic regime, a similar result will be obtained. It is due to the fact that, in the case of LQBHs, the quantum correction which depends on the minimal area occurs in the $H(r)$ function, which differs from r^2 only for short distances. On the other hand, the Verlinde's method is based on the entropy functional, where the minimal area corrections are present independently from the distance range. In this way, one hopes to find out, even for large distances, additional contributions to Newton's gravitational law, which depends on the minimal area, by applying the Verlinde's formalism in the LQBH's case.

In this way, we have that Verlinde has conjectured that gravity is not fundamental but can be explained as an entropic force. In this section, following the Verlinde's entropic force approach to gravity, we shall derive a quantum corrected Newton's law of gravitation from LQBH's entropy-area relation (15).

We have that in thermodynamics, if the number of states depends on position Δx, an entropic force F arises as the thermodynamical conjugate of Δx. In this case, the first law of thermodynamics can be written as

$$F\Delta x = T\Delta S. \tag{16}$$

Based on the Bekenstein's entropy bound, Verlinde postulated that when a test particle moves approaching a holographic screen, the change of entropy on this screen is proportional to the mass m of the particle, and the distance Δx between the test particle and the screen

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x. \tag{17}$$

In order to make use of the entropic force hypothesis, (17) should hold at least when Δx is smaller than or comparable with the Compton wave-length of the particle.

The temperature that appears in (16) can be understood in two ways: one can relate temperature and acceleration using Unruh's rule

$$k_B T = \frac{1}{2\pi} \frac{\hbar a}{c}, \tag{18}$$

or relate temperature, energy and the number of used degrees of freedom using the equipartition rule

$$E = \frac{1}{2} N k_B T. \tag{19}$$

It is necessary to point that the temperature T in Eqs. (18) and (19) have different meaning. In the first equation, the temperature is defined in the bulk. However, in the second, the temperature is defined on the holographic screen. To admit these two temperatures to be equal is a further supposition in Verlinde's paper. In despite of this, one can get two important results. The first one is obtained from the Eqs. (16), (17) and (18) and corresponds to the second Newton's law of motion $F = ma$. The second result corresponds to the Newton's law of gravitation.

In this point, we reach a core question in this paper. In order to obtain the Newton's law of gravitation, one must have a way to relate the number of bits on the holographic screen with the black hole horizon area. Following the Shannon's definition of entropy [68], we have that the number of bits on the screen is proportional to the horizon entropy. In this way, in the Verlinde's treatment to a classical black hole, where the black hole's entropy is proportional to its horizon area, it has been assumed that the number of bits is proportional to the black hole event horizon area [11]. However, in the present case, quantum gravity corrections must adjust the number of bits encoded on the black hole horizon [29–31]. In this way, from the Eq. (15), we shall write

$$N = \frac{(1+P)}{(1-P)} \frac{\sqrt{A^2 - A_{min}^2}}{L_P^2}. \tag{20}$$

Putting it together with the Eqs. (16), (19) and $E = Mc^2$, we get, for $R > \sqrt{2a_0}$,

$$F = -\frac{GMm}{R^2} \frac{(1+P)}{(1-P)} \times \frac{1}{\sqrt{1 - A_{min}^2/16\pi^2 R^4}}. \tag{21}$$

Moreover, for the gravitational potential $V(r) = -\int F(R)dR$, we obtain

$$V(r) = -\frac{GMm}{R}\frac{(1+P)}{(1-P)}\left(1 - \frac{A_{min}^2}{160\pi^2 R^4} - \frac{A_{min}^4}{18432\pi^4 R^8} + \cdots\right). \tag{22}$$

In this way, corrections to Newton's gravitational law can be obtained from LQBH's entropy-area relation. As we can see, the deviations on the Newton's law depend on the value of minimal area A_{min} in LQG, as well as on the polymeric parameter P. As a consequence, the corrections found out are important in the case of submilimeter distances, even though it could be realized in the context of large distances through the dependence on the P parameter, as well as a_0, since the Barbero–Immirzi parameter does not suffer with the problem of mass suppression, as have been pointed in [55].

Moreover, note that the leading term in the gravitational potential found out in this work no longer depends on $a_0 \sim l_P^2$ as in other models for quantum corrections to Newton's law of gravitation as perturbation quantum gravity [69] and GUP approaches [29–31], but on $a_0^2 \sim l_P^4$.

In the next section, the quantum corrections to Newton's law of gravitation found out here will pave the way to the study of a system composed by a LQBH with a particle orbiting it, which could be a suitable candidate to dark matter.

4 The loop quantum black hole atom

In the seventies, Hawking introduced the possibility that a free charged particle could be capture by a primordial charged black hole forming neutral and non-relativistic ultra-heavy black hole atoms [70]. After, the term gravitational atom was coined by Flambaum and Berengut in [71] for a gravitationally bound neutral black hole and a charged particle.

An interesting fact about gravitational atoms is that they have been pointed as an important constituent of dark matter. In fact, primordial black hole remnants left after the Hawking evaporation have been considered as a source of dark matter by several authors for more than two decades [72–81] (for a review see [82–84]). However, a central question is whether some remnants could leave after the Hawking evaporation, forming a stable nucleus for the gravitational atom. In other words, in order to have a gravitational atom system as a suitable candidate to describe dark matter, it would be necessary that, at some point of its evolution, the black hole nucleus establish a thermal stable equilibrium with its neighborhood.

In the Schwarzschild scenario, this kind of situation is possible for a black hole to be in equilibrium with the Cosmic Microwave Background (CMB) for a black hole mass of 4.50×10^{22} kg. However, this equilibrium is not stable because, for a Schwarzschild black hole, the temperature always increases as its mass decreases and vice versa [(see the dashed line in the Fig. (2)]. On the other hand, a new phenomenon emerges in the LQBH's scenario. From Eq. (13), all light enough LQBHs would radiate, and their temperature cools, until the point they would be in thermal equilibrium with the CMB. In fact, a stable thermal equilibrium occurs for a black hole mass given by $m_{\text{stable}} \approx 10^{-19}$ kg. Based on this feature of LQBHs, Modesto et al have yet pointed to the possibility that these objects could be an important component of dark matter [48]. In this way, one could think about the possibility of gravitational atoms, where a LQBH could appear as the atomic nucleus.

In order to give a first glance on these kind of system, let us use the expression for the gravitational force between a LQBH and a neutral particle orbiting it, which will be given by the Eq. (21):

$$F = \frac{GMm}{R^2}\frac{(1+P)}{(1-P)} \times \frac{1}{\sqrt{1 - A_{min}^2/16\pi^2 R^4}} = \frac{mv^2}{R}, \tag{23}$$

where v is the particle velocity in the orbit.

In this way, we shall have:

$$v = \left[\frac{(1+P)}{(1-P)}\frac{GM}{R}\right]^{\frac{1}{2}} \times \left(1 - A_{min}^2/16\pi^2 R^4\right)^{-\frac{1}{4}}. \tag{24}$$

Now, using the Bohr–Sommerfeld quantization method, $mvR = j\hbar$, we shall get the following equation

$$R^6 - \frac{(1-P)}{(1+P)}\frac{(j\hbar)^4}{(GMm^2)^2}R^4 + \frac{(1-P)}{(1+P)}\frac{(j\hbar)^4 A_{min}^2}{(GMm^2)^2} = 0, \tag{25}$$

whose only real solution is given by

$$R_j = \Bigg[\frac{\hbar^4 j^4 (P-1)}{3m^4 G^2 M^2 (P+1)} + \left(\frac{\hbar^4 j^4 A_{min}(P-1)\sqrt{27m^8 A_{min}^2 G^4 M^4 (P+1)^2 - 4h^8 j^8 (P-1)^2}}{2(3^{3/2})m^8 G^4 M^4 (P+1)^2} + \frac{\hbar^4 j^4 (1-P)[2\hbar^8 j^8 (1-P)^2 - 27m^8 A_{min}^2 G^4 M^4 (P+1)^2]}{54 m^{12} G^6 M^6 (P+1)^3}\right)^{1/3} + \frac{\hbar^{12} j^{12} (P-1)^3}{9m^8 G^4 M^4 (P+1)^2} \times \left(\frac{\sqrt{27m^8 A_{min}^2 G^4 M^4 (P+1)^2 - 4h^8 j^8 (P-1)^2}}{2(3^{3/2})m^8 G^4 M^4 (P+1)^2} - \frac{2\hbar^8 j^8 (P-1)^2 - 27m^8 A_{min}^2 G^4 M^4 (P+1)^2}{54 m^{12} G^6 M^6 (P+1)^3}\right)^{1/3}\Bigg]^{1/2},$$

which can be expanded as

$$R_j = \frac{\sqrt{(P+1)(1-P)}\hbar^2 j^2}{m^2 GM(P+1)} - \frac{\sqrt{(P+1)(1-P)}m^6 G^3 M^3 (P+1)}{2\hbar^6 j^6 (P-1)^2} A_{min}^2 + \cdots \tag{26}$$

where the first term corresponds to the usual gravitational atomic radius, unless the P parameter factors.

The energy levels E_j of the LQBH gravitational atom are obtained from the expressions (22), (24) and (26),

$$\begin{aligned} E_j &= \frac{1}{2}mv^2 + V \\ &= -\frac{m^3 G^2 M^2 (P+1)^{3/2}}{2\hbar^2 j^2 (1-P)^{3/2}} + \frac{m^{11} G^6 M^6 (P+1)^{7/2}}{64\hbar^{10} j^{10} \pi^2 (1-P)^{7/2}} A_{min} \\ &\quad + \frac{m^{11} G^6 M^6 (P+1)^{7/2} \left[15 m^8 G^4 M^4 (P+1)^2 - (5120\pi^2 - 128)\hbar^8 j^8 \pi^2 (1-P)^2\right]}{20480\hbar^{18} j^{18} \pi^4 (1-P)^{11/2}} A_{min}^2 \\ &\quad + \cdots . \end{aligned} \tag{27}$$

The first therm in the expression above corresponds to the usual expression to the gravitational atom energy levels (unless the dependence on the polymeric function), which can be obtained in the limit where the quantum gravity corrections goes to zero.

In order to measure the deviation from the classical results, let us calculate the relation

$$\frac{\nu_j}{\nu_j^{class}} = \frac{\delta E_j}{\delta E_j^{class}}, \tag{28}$$

between the frequencies emitted by a gravitational atom in the quantum (ν_j) and classical (ν_j^{class}) cases for the $j \to j-1$ transition. If we consider the case of a gravitational atom composed by a LQBH in the nucleus and a particle with mass of order of neutron's mass orbiting it, the values shown in the Table (1) are obtained. As we can see from the table, the deviation from the classical values in the energy levels assumes values for which the experimental devices could be sensible, approaching 16 % for $P \approx 0.05$.

5 Conclusions and remarks

We have derived quantum corrected Newton's gravitation law from the LQBH's entropy-area relation using the Verlinde's entropic force interpretation to gravity. Our results point to some quantum deviation from classical Newton's law that must have an important rule in sub-millimeter distances where Newton's gravitation theory has not been tested yet, even though these deviations could be perceived in the realm of large distances through the dependence on the parameter P. Such dependence on the P parameter, by the quantum corrections to Newton's gravitation law that appear in

Table 1 The first four ν_j/ν_j^{class} for several values of the polymeric parameter P

P	$j = 2$	$j = 3$	$j = 4$	$j = 5$
0.01	1.030455564470397655796319	1.030455564470397655796319	1.030455564470397877840924	1.030455564470398321930134
0.02	1.061845043311036018707227	1.061845043311035796662622	1.061845043311036240751832	1.061845043311036018707227
0.03	1.094203842773455503945002	1.094203842773455503945002	1.094203842773455281900397	1.094203842773455725989606
0.04	1.127569083044155151540622	1.127569083044155151540622	1.127569083044155151540622	1.127569083044155373585227
0.05	1.161979699346818994953878	1.161979699346818994953878	1.161979699346818994953878	1.161979699346818772909273

the LQBH's framework, offers some advantages under the experimental point of view front others scenarios. It is because the P parameter, as well as a_0, which are defined in terms of the Barbero–Immirzi parameter [5], does not suffer with the problem of mass suppression as occurs with the parameters of other quantum gravity theories like superstring theory or noncommutative theory, as has been pointed in [55].

On the other hand, due to its self-duality property, LQBHs can have a mass lower than the Planck one. Particularly, for $m_{\text{stable}} \approx 10^{-19}$ kg, a LQBH would assume a stable thermal equilibrium with the CMB, which makes possible that this kind of black holes can be seen as a good candidate for dark matter. In this way, impelled by the current experimental activity, we have investigated the energy spectrum of a huge gravitational atom composed by a neutral particle orbiting a LQBH, by the use of the Bohr–Sommerfeld formalism. As have been demonstrated, the energy levels depend on the quantum gravitational corrections inherited from the LQBH's metric. By the way, if one takes a particle with mass of order of neutron's mass, the deviation from the classical frequencies assumes values that approach 16 % for $P = 0.05$, which could be captured by experiments.

The leading term in the gravitational potential found out in this work no longer depends on $a_0 \sim l_P^2$ as in other models for quantum corrections to Newton's law of gravitation as perturbation quantum gravity [69] and GUP approaches [29–31], but on $a_0^2 \sim l_P^4$. It is due to the fact that, as one can obtain by expand the LQBH's entropy in terms of a_0, the leading quantum correction on the entropy depends on a_0^2. In this way, further investigations could be done by consideration of LQBH's entropy lower order corrections in l_P. These kind of corrections have been found out in [54] and could reconcile the results found out in the present work with the other approaches.

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