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RESEARCH ARTICLE

Radiation of charged black holes and modified dispersion relation

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Abstract We investigate the effects of a modified dispersion relation proposed by Majhi and Vagenas on the Reissner–Nordström black hole thermodynamics in a universe with large extra dimensions. It is shown that entropy, temperature and heat capacity receive new corrections and charged black holes in this framework have less degrees of freedom and decay faster compared to black holes in the Hawking picture. We also study the emission rate of black hole and compare our results with other quantum gravity approaches. In this regard, the existence of the logarithmic prefactor and the relation between dimensions and charge are discussed. This procedure is not only valid for a single horizon spacetime but it is also valid for the spacetimes with inner and outer horizons.

Keywords Black hole physics · Modified dispersion relations · Reissner–Nordström black hole

1 Introduction

A common feature of all promising candidates for quantum gravity such as string theory [1], loop quantum gravity [2], noncommutative geometry [3], and black hole physics is the existence of a minimum observable length [4,5]. This minimum measurable length gives rise to the modification of Heisenberg uncertainty principle, nowadays known as Generalized uncertainty principle (GUP) [6]. On the other hand, in the context of doubly special relativity (DSR) theories [7,8], in order to preserve the velocity of light and the planck energy as two invariant quantities, the existence of a

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maximal momentum is essentially required. So, it provides several novel and interesting features, some of which are studied in [9–13]. Also, DSR motivates the modified dispersion relation (MDR)[14,15], by the fact that all approaches to quantum gravity suggest that standard energy-momentum dispersion relation should be modified near the Planck energy. This deformation of the energy-momentum relation has been also suggested by the discreteness of the spacetime [16].

Recently, much attention has been devoted to resolving the quantum corrections to the black hole entropy. Identifying the microstates is one of the main problems in studying the entropy of black holes. Leading candidate theories of quantum gravity such as string theory and loop quantum gravity predict the following entropy [17–20]

$$S = \frac{A}{4l_p^2} + c_0 \ln\left(\frac{A}{4l_p^2}\right) + \sum_{n=1}^{\infty} c_n \left(\frac{A}{4l_p^2}\right)^{-n} + \text{const.}, \tag{1}$$

where the coefficients c_n can be regarded as model dependent parameters. Black holes are suitable examples of an extreme quantum gravity regime. Thus, study their thermodynamical behavior using MDR and comparing the results with other approaches may increase our understanding of their properties and structures. Indeed, the exact form of MDR could essentially lead us to a deeper understanding of the ultimate quantum gravity proposal.

Studying the thermodynamics of black holes in the presence of extra dimensions which is an interesting issue is the subject of this work [21–23]. Large extra dimension models (LED) offer exciting ways to solve the hierarchy problem and to study low scale quantum gravity effects [24–26]. From a theoretical point of view, one can expect that the properties of black holes may play an important role in understanding the nature of gravity in higher dimensions. The black hole and brane production in the LHC is also studied in Ref. [27]. Thus, it is important to investigate the effects of extra dimensions on the various properties of black holes.

In this paper, we intend to extend above analysis to the Reissner–Nordström (RN) black holes and we choose a specific form of MDR proposed by Majhi and Vagenas, in which both energy and momentum of particles are bounded. The organization of this work is as follows: In Sect. 2, we introduce briefly the modified dispersion relation (MDR*) which admits minimal length and maximal momentum. In Sect. 3, we investigate a charged black hole thermodynamics in universes with large extra dimensions. In Sects. 4–7, we obtain entropy, temperature and heat capacity of charged black hole in the presence of (MDR*). Also, we find new corrections in the emission rate of charged black holes. Finally, we present our conclusions in Sect. 8.

2 The modified dispersion relation (MDR*)

The idea of modified energy-momentum dispersion relation which is known as MDR is popular among those are interesting in effective approach to quantum gravity problems. The modified dispersion relations are usually in the form [28,29]

$$p^2 \simeq E^2 - \mu^2 + \alpha_1 l_p E^3 + \alpha_2 l_p^2 E^4 + \cdots , \tag{2}$$

where μ is the mass parameter and corresponds to the rest energy of the particle. These modified dispersion relations have been previously used to calculate the black hole entropy (see Ref. [28] for a brief discussion).

Recently, a new form of MDR [30] (so called MDR*), has been introduced which implies a minimum measurable length and a maximum measurable momentum.

$$p^0 = k^0, \qquad p^i = k^i(1 - \alpha k + 2\alpha^2 k^2), \tag{3}$$

where $k = |\mathbf{k}|$, p^a is the momentum at high energies, and k^a is the momentum at low energies which satisfies the ordinary dispersion relation. The gravitational background metric can be considered as ($g_{0i} = 0$)

$$ds^2 = g_{AB}dx^A dx^B = g_{00}c^2 dt^2 + g_{ij}dx^i dx^j, \tag{4}$$

and the square of the four-momentum in this background is

$$p^A p_A = g_{00}(k^0)^2 + g_{ij}k^i k^j (1 - \alpha\mathbf{k} + 2\alpha^2\mathbf{k}^2)^2 . \tag{5}$$

The energy of a particle can be expressed in terms of high energy momentum as follows [30]

$$E^2 = \left(-g_{AB}\xi^A p^B\right)^2 = -g_{00}\left(m^2c^4 + c^2p^2(1 + 2\alpha p)\right), \tag{6}$$

where $\xi^A = (1, 0, 0, \ldots)$ is the killing vector. Now, the energy of a particle is $\frac{E}{c} = -g_{AB}\xi^A p^B$ and the energy in the gravitational background with metric (4) is given as $E = -g_{00}cp^0$. Here, we work in the Minkowski spacetime in which $g_{00} = -1$.

Notice that, this procedure is similar to the modification of the Peierls–Landau relativistic uncertainty relation which is first proposed by Amelino-Camelia et al. [31]. To this end, after simple calculation (neglecting the rest mass), we obtain

$$\frac{dE}{dp} \simeq c\sqrt{-g_{00}}\left(1 + 2\alpha p - \frac{3}{2}\alpha^2 p^2\right), \tag{7}$$

to $\mathcal{O}(\alpha^2)$. Following the heuristic argument of Refs. [28,31,32], based on MDR , and using $p \simeq \frac{E}{c\sqrt{-g_{00}}}\left(1 - \frac{\alpha E}{c\sqrt{-g_{00}}}\right)$, we have

$$\delta E = \left(c\sqrt{-g_{00}} + 2E\alpha + \frac{7E^2}{2c\sqrt{-g_{00}}}\alpha^2 + \mathcal{O}(\alpha^3)\right)\delta p. \tag{8}$$

Now, taking $\delta E \simeq E$ and $E \geq \frac{1}{\delta x}$ which is suggested by quantum field theory we find

$$E\delta x \geq c\sqrt{-g_{00}}\left(1 + \frac{2\alpha}{c\sqrt{-g_{00}}\delta x} - \frac{7\alpha^2}{2c^2 g_{00}(\delta x)^2}\right). \tag{9}$$

There are two points should be considered here. First, according to Eq. (3) we keep all terms up to order of α^2 such that considering more generalized form of MDR* will not change these results. Second, if we consider all natural cut offs such as minimal length and maximal momentum, not only even powers of energy but also odd powers of energy should be present [21–23]. Notice that, in the absence of quantum gravity corrections, i.e., $\alpha = 0$, we obtain the standard dispersion relation $E^2 = m^2c^4 + c^2k^2$. Also, in the following sections we set $\hbar = c = k_B = 1$.

3 Reissner–Nordström (RN) black holes in extra dimensions

The RN black hole is a solution of the Einstein equation coupled to the Maxwell field [33,34]. Let us now consider the RN black hole thermodynamics in universes with large extra dimensions. There are many scenarios of LED such as Randall–Sundrum [24] , Arkani-Hamed–Dimopoulos–Dvali (ADD) [25] and Dvali–Gabadadze–Porrati [26].

In LED scenario, RN metric can be written as follows [35]

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega^2_{D-2}, \tag{10}$$

where

$$F(r) = 1 - \frac{2M}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}, \tag{11}$$

and $d\Omega^2_{D-2}$ is the line element on the $(D-2)$-dimensional unit sphere and the volume of the $(D-2)$-dimensional unit sphere is given by $\Omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-1}{2})}$. The mass and electric charge of the black hole are given by

$$M = \frac{8\pi G_D}{(D-2)\Omega_{D-2}} m, \qquad Q = \sqrt{\frac{8\pi G_D}{(D-2)(D-3)}}\, q. \tag{12}$$

Here, G_D is gravitational constant in D-dimensional spacetime such that in ADD model is given by

$$G_D = \frac{(2\pi)^{D-4}}{\Omega_{D-2}} M_{Pl}^{2-D}, \tag{13}$$

where M_{Pl} is the D-dimensional Planck mass and there is an effective 4-dimensional Newton constant related to M_{Pl} by

$$M_{Pl}^{2-D} = 4\pi G_4 R^{D-4}, \tag{14}$$

where R is the size of extra dimensions. It is necessary to note that in this work, the conventions for definition of the fundamental Planck scale M_{Pl} are the same as which have been used by ADD. The location of the outer and inner horizons, determined by $F(r) = 0$, are given by

$$r_{\pm} = \left(M \pm \sqrt{M^2 - Q^2}\right)^{\frac{1}{D-3}}, \qquad M^2 \geq Q^2, \tag{15}$$

and the horizon area is given by $A_D = \Omega_{D-2} r_+^{D-2}$. Moreover, the entropy reads $S = \frac{A_D}{4}$.

4 MDR* and the entropy of RN black holes

We are now interested to calculate the microcanonical entropy of RN black hole. Following heuristic considerations due to Bekenstein, the minimum increase of the area of a BH absorbing a classical particle of energy E and size R is given by [36,37] (After correcting the calibration factor)

$$(\Delta A)_{min} \geq 4\ln(2) L_{Pl}^{D-2} E R, \tag{16}$$

where $R \sim \delta x \sim r_+$ and $\delta x = \left(\frac{A}{\Omega_{D-2}}\right)^{\frac{1}{D-2}}$. If we set $(\Delta S)_{min} = \ln 2$, then we find

$$\frac{dS}{dA} \simeq \frac{(\Delta S)_{min}}{(\Delta A)_{min}} \simeq \frac{1}{4L_{Pl}^{D-2} E \delta x} = \frac{1}{4L_{Pl}^{D-2} \Phi(\delta x)}, \tag{17}$$

and

$$S_{MDR^*} = \int_{A_p}^{A} \frac{dA}{4L_{pl}^{D-2}\left(1 + 2\alpha\left(\frac{A}{\Omega_{D-2}}\right)^{\frac{1}{D-2}} - \frac{7\alpha^2}{2}\left(\frac{A}{\Omega_{D-2}}\right)^{\frac{2}{D-2}}\right)}. \tag{18}$$

The existence of a minimal length and a maximal momentum leads to the presence of a minimum event horizon area, $A_p = \Omega_{D-2}(\delta x)_{min}^{D-2} = \Omega_{D-2}(\alpha L_{Pl})^{D-2}$ [13]. After some calculations, the RN black hole entropy reads

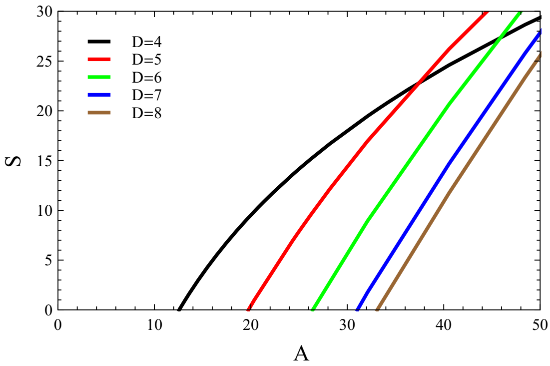
$$D = 4 \rightarrow S_4 = \frac{A}{4} - 2\alpha\sqrt{\pi}\sqrt{A} + \frac{15}{2}\alpha^2\pi \ln(A) + \frac{88\pi^{3/2}\alpha^3}{\sqrt{A}} - \frac{281\pi^2\alpha^4}{A} + \text{const.}, \tag{19}$$

$$D = 5 \rightarrow S_5 = \frac{A}{4} - \frac{3}{4}\alpha\sqrt[3]{2\pi^2}A^{2/3} + \frac{45}{8}\alpha^2\left(2\pi^2\right)^{2/3}\sqrt[3]{A} - 11\pi^2\alpha^3\ln(A) - \frac{843}{8}\alpha^4\sqrt[3]{\frac{2\pi^8}{A}} + \text{const.}, \tag{20}$$

$$D = 6 \rightarrow S_6 = \frac{A}{4} - \frac{2}{9}\alpha\sqrt{\pi}(6A)^{3/4} + \frac{5}{2}\alpha^2\pi\sqrt{6A} - \frac{22}{3}\alpha^3\sqrt[4]{3\pi^6}\sqrt[4]{512A} + \frac{281}{6}\pi^2\alpha^4\ln(A) + \text{const.} \tag{21}$$

There are many discussions concerning logarithmic corrections to the entropy area relation [22,23,38,39]. The logarithmic corrections to black hole have been also

Fig. 1 Modified Reissner–Nordström black hole's entropy as a function of event horizon area for different numbers of spacetime dimensions in the presence of MDR* for $\alpha = 1$



obtained in the tunneling formalism [40–45]. The logarithmic prefactor contains some information about the details of the underlying quantum gravity proposal. Here, the logarithmic prefactor is given by $c_0 = \frac{15}{2}\alpha^2\pi$ for the RN black hole with double horizons. Thus, we find that this procedure as mentioned in Ref. [46] is not only valid for single horizon spacetime but also valid for spacetimes with outer and inner horizons.

Now we easily conclude that the logarithmic prefactor will be appeared for all number of dimensions. In addition, for positive values of α, the sign of the logarithmic factor is positive for even number of dimensions but is negative for odd number of dimensions. This result is the main difference between new form of MDR with other quantum gravity approaches. In addition, we conclude that the existence of the logarithmic prefactor is independent of the dimensionality of the spacetime but depends on the used statistical ensemble.

Figure 1 shows the relation between the event horizon area and the entropy of the RN black hole. In scenarios with extra dimensions, black hole entropy decreases. The classical picture breaks down since the degrees of freedom of the black hole are small. In this situation one can use the semiclassical entropy to measure the validity of the semiclassical approximation. Also black holes in extra dimensional models have less entropy than black holes in four dimensions.

Figure 2 displays the black hole entropy versus its mass for 4-dimensional Schwarzschild and Reissner–Nordström black holes in the presence and absence of MDR*. Therefore, higher dimensional black hole remnants have less classical features relative to their four dimensional counterparts, and the mass of the black hole remnant usually increases with the spacetime dimension D [38,39]. It is worth mentioning that in the classical viewpoint, at the end of the evaporation process, the black hole contains zero remnant mass, zero final entropy and infinite finial temperature. However, as we will show, we obtain a nonzero remnant mass, nonzero final entropy, and finite final temperature.

5 MDR* and the temperature of RN black holes

The Hawking temperature for the spherically symmetric black holes has been obtained in several ways using MDR [28,38,47]. According to the first law of the RN black

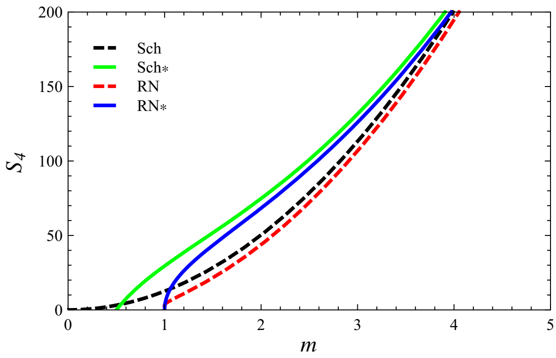


Fig. 2 Entropy of Schwarzschild (Sch), Reissner–Nordström (RN), modified Schwarzschild ($Sch*$), and modified Reissner–Nordström ($RN*$) black holes for $D = 4$

hole, we have

$$dM = \frac{\kappa}{8\pi} dA + \phi dQ = T dS + \phi dQ. \tag{22}$$

In more general situations, the entropy of black hole is assumed to be a function of its area, namely, $S = S(M, Q)$. The temperature is expressed as

$$T_{MDR^*} = \left(\frac{\partial M}{\partial S}\right)_Q = \frac{dA}{dS} \times \left(\frac{dM}{dA}\right)_Q = \frac{dA}{dS} \times \frac{\kappa}{8\pi}. \tag{23}$$

The surface gravity $\kappa(M, Q)$ can be obtained in the usual manner as [35,48]

$$\kappa = \frac{1}{2} |\partial_r F(r)|_{r=r_+}. \tag{24}$$

Now, using $T_{BH} = \frac{\hbar\kappa}{2\pi}$, the modified temperature of RN black hole reads

$$T_{MDR^*} = \frac{(D-3)}{4\pi r_+} \left(1 - \frac{\chi_-}{\chi_+}\right) \Phi(r_+), \tag{25}$$

where

$$\chi_+ = M + \sqrt{M^2 - Q^2}; \quad \chi_- = M - \sqrt{M^2 - Q^2}. \tag{26}$$

This relation shows implicitly that the black hole's temperature increases with the spacetime's dimension D. The higher temperature leads to faster decay and less classical properties of the black hole. As a result, both the temperature and the entropy of the RN black hole receive important corrections such that the temperature is bounded from above. Such remnants of black holes may be considered as a candidate for cold dark matter.

For $D = 4$ we have

$$T_4 = \frac{\sqrt{m^2 - q^2}}{2\pi\,(m + \sqrt{m^2 - q^2})^2}\left(1 + \frac{2\alpha}{m + \sqrt{m^2 - q^2}} - \frac{7\alpha^2}{2(m + \sqrt{m^2 - q^2})^2}\right). \tag{27}$$

If we set $\alpha = 0$, temperature reduces to result that has been reported in Ref. [34]. Also, for $\alpha = 0$ and $q = 0$, we obtain the well-known Hawking temperature, i.e., $T_{BH} = \frac{1}{8\pi m}$. Indeed, our result contains all limiting cases properly.

As Fig. 3 shows, the black hole radiates until it reaches to the minimum mass. During this process, its effective temperature reaches a maximum value. However, as the figure exhibits, when the radiation stops, the temperature goes to zero. We can say that, when the radiation reaches to its endpoint and the entropy becomes zero, and the temperature is in its maximum value, there is a remnant of black hole. Note that, remnants do not need horizon structure [5,39].

Figure 4 shows the comparison between the temperature of the Schwarzschild black hole, RN black hole and their modified temperature in the presence of MDR* for $D = 4$. It shows that the temperature of RN black hole remnant is smaller than the temperature of Schwarzschild black hole remnant. So, the RN black hole remnant is colder than the Schwarzschild black hole remnant and the natural cut offs become

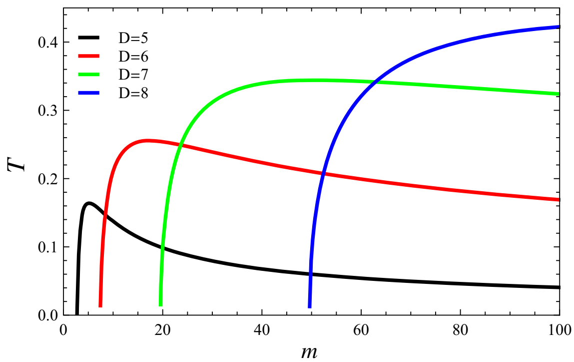


Fig. 3 Modified Reissner–Nordström Black hole's temperature as a function of mass for different numbers of spacetime dimensions in the presence of MDR* for $\alpha = 1$ and $q = 1$

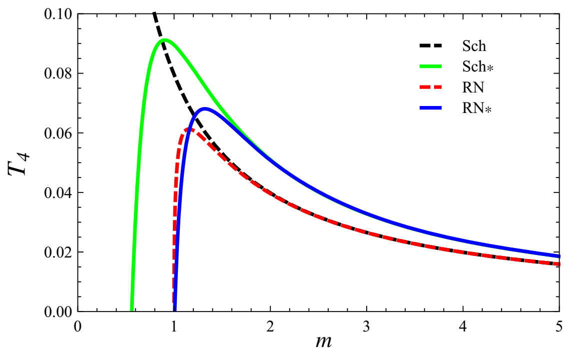


Fig. 4 Temperature of Schwarzschild (Sch), Reissner–Nordström (RN), modified Schwarzschild ($Sch*$) and modified Reissner–Nordström ($RN*$) black holes for $D = 4$, $\alpha = 1$ and $q = 1$

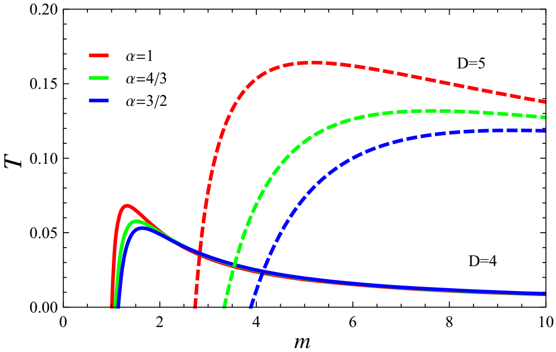


Fig. 5 Modified Reissner–Nordström black hole's temperature as a function of mass for different α ($L_{pl} = 1$)

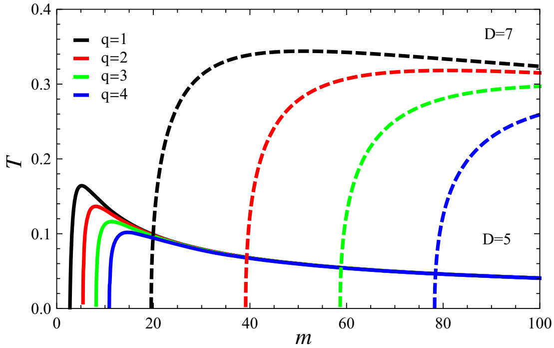


Fig. 6 Modified Reissner–Nordström black hole's temperature as a function of mass for different charges. ($q = 1$ and $L_{pl} = 1$)

more effective for small charges. Also, the temperature of the RN black hole and the modified RN black hole are distinct.

As another important outcome, according to [49,50] the noncommutative Schwarzschild black hole has features very similar to a commutative RN black hole. Indeed, it is shown that there is a close connection between charge and noncommutativity [49,50]. Here, using MDR* and comparing evaporation process of the standard and modified RN black hole, we conclude that there is a nontrivial connection between charge and the dimensionality of the spacetime near the Planck scale.

Figure 5 shows that when α increases, the minimum mass increases and the maximum temperature decreases. Figure 6 shows that the modified RN black hole temperature as a function of mass for different values of charge. As the figure shows, the final state temperature decreases as the black hole charge increases. We note that black hole evaporates through the radiation of charged particle-antiparticle pairs until it reaches a remnant with maximal temperature.

6 MDR* and the heat capacity of the RN black holes

The heat capacity is calculated from the entropy via the relation

$$C = T\left(\frac{\partial S}{\partial T}\right) = \left(\frac{\partial m}{\partial T}\right). \tag{28}$$

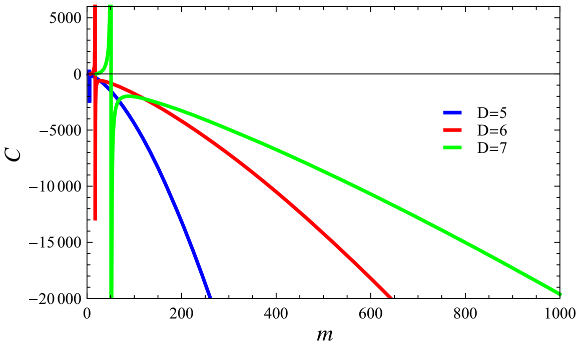


Fig. 7 Modified Reissner–Nordström black hole's heat capacity as a function of mass for different numbers of spacetime dimensions in the presence of MDR* for $\alpha = 1$ and $q = 1$

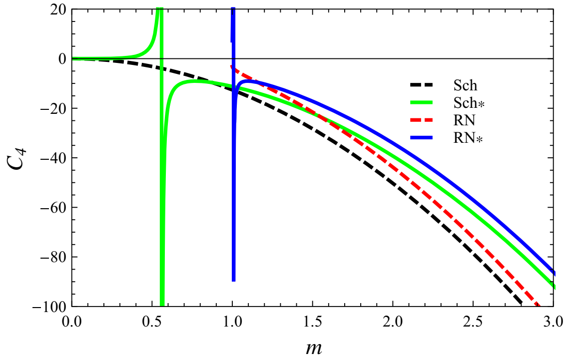


Fig. 8 Heat capacity of Schwarzschild (Sch), Reissner–Nordström (RN), modified Schwarzschild ($Sch*$) and modified RN ($RN*$) black holes for $D = 4$, $q = 1$, and $\alpha = 1$

The high energy corrections may prevent the black hole from total evaporation since the heat capacity vanishes as the temperature reaches its maximal value. Now, we find the heat capacity of the RN black hole as a function of its mass. For $D = 4$ we have

$$C_{MDR^*} = \frac{4\sqrt{m^2-q^2}\left(m+\sqrt{m^2-q^2}\right)^4}{(28\alpha^2-8\alpha m-4m^2+4q^2)\sqrt{m^2-q^2}-4m^3-8\alpha m^2+(-7\alpha^2+6q^2)m+12\alpha q^2}. \tag{29}$$

As it can be seen from Fig. 7, the negative heat capacity shows that the thermodynamical system is unstable and tends to decay. When heat capacity reaches to zero, the system goes to stability. Indeed, the black hole cannot radiate further and becomes an inert remnant, possessing only gravitational interactions.

The heat capacity as a function of mass for $D = 4$ is represented in Fig. 8. There is a discontinuity point for the heat capacity in the modified RN black hole so-called m_{ext} that leads to a catastrophic evaporation. When the mass of the RN black hole is above m_{ext}, the heat capacity is positive and it tends to a finite value when mass goes to infinity. It seems this behavior arises from failure of the standard thermodynamics near the Planck scale. So, a solution to this problem will also correct the catastrophic behavior [38,39,51].

7 MDR* and the emission rate of the RN black holes

The energy radiated per unit time (the emission rate) can be calculated using the Stefan–Boltzmann law by assuming that the energy loss is dominated by photons. In n_i-dimensional brane, the energy radiated by a black body of temperature T and surface area $A(M, n_i, D)$ is given by [52–56]

$$\frac{dE_{n_i}}{dt} = \sigma_{n_i} A(M, n_i, D) T^{n_i}. \tag{30}$$

We assume that the RN black hole induced area depends on n_i, M, and the dimension of the spacetime D. So, the RN geometric area which is induced on the n_i-dimensional subspace is

$$A_i(M, n_i, D) = \Omega_{n_i-2} r_c^{n_i-2}, \tag{31}$$

where Ω_{n_i-2} is the area of the unit $(n_i - 2)$-dimensional sphere and $r_c = \left(\frac{D-1}{2}\right)^{\frac{1}{D-3}} \left(\frac{D-1}{D-3}\right)^{\frac{1}{2}} r_+$ is the radius of the D-dimensional RN black hole of radius r_+. Also, σ_{n_i} is the n_i-dimensional Stefan-Boltzmann constant defined as $\sigma_{n_i} = \frac{\Omega_{n_i-3}\Gamma(n_i)\xi(n_i)}{(n_i-2)(2\pi)^{n_i-1}}$.

The thermal emission in the bulk of the brane can be neglected and the RN black hole is supposed to radiate mainly on the brane [53], i.e., $\frac{\frac{dE_4}{dt}}{\frac{dE_{11}}{dt}} \simeq 1$. Thus, the emission rate on the brane is given by

$$\left(\frac{dm}{dt}\right)_{MDR^*} \propto -\lambda T^4_{MDR^*}, \tag{32}$$

where $\lambda = \frac{\Omega_1\Omega_2\Gamma(4)\xi(4)}{2(2\pi)^3} \left(\frac{D-1}{D-3}\right) \left(\frac{D-1}{2}\right)^{\frac{2}{D-3}} r_+^2$.

The emission rate of black hole is shown in Fig. 9. This means that the emission rate of the RN black hole vanishes when the black hole reaches its minimal value. In the standard framework, the emission rate goes to infinity as the mass of the RN black hole tends to zero. In the MDR* picture, the modified emission rate of the RN black hole never diverges, and it just goes to zero when the black hole's mass reaches its minimal value. Also, Fig. 10 shows the relation between the emission rate of the Schwarzschild black hole, RN black hole and their modified temperature in the presence of MDR* for $D = 4$.

8 Conclusions

In this paper, we have studied the effects of a recently proposed MDR* on the charged black holes. We showed that the presence of a minimal length and a maximal momen-

Fig. 9 Emission rate of modified Reissner–Nordström black hole as a function of mass for different numbers of spacetime dimensions in the presence of MDR* for $\alpha = 1$ and $q = 1$

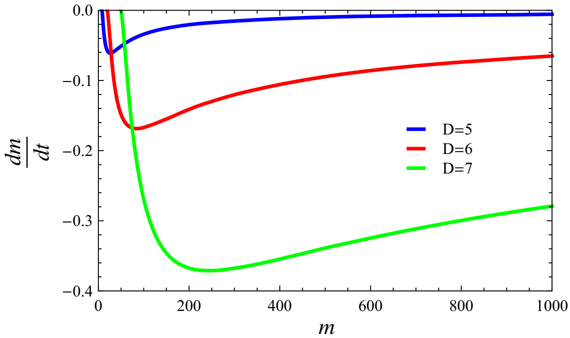
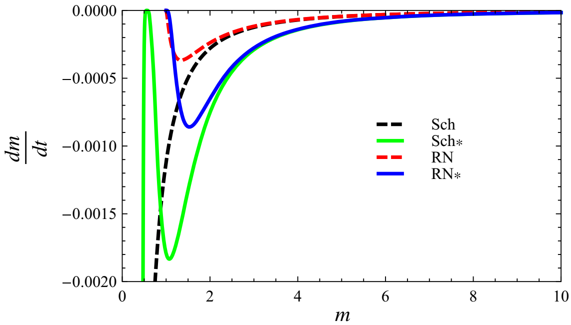


Fig. 10 Emission rate of Schwarzschild (Sch), Reissner–Nordström (RN), modified Schwarzschild ($Sch*$) and modified RN ($RN*$) black holes for $D = 4$, $q = 1$, and $\alpha = 1$



tum results in the modification of RN thermodynamics. Indeed, it leads to faster decay and less classical behaviors for black holes. We discussed the existence of the logarithmic prefactor and the relation between the dimensionality and the entropy. Also, we obtained the temperature, heat capacity and emission rate of the RN black hole in the presence of the extra dimensions and compared our result with the standard formalism. In addition, we discussed the failure of standard thermodynamics near the Planck energy scale. We showed that in the modified formalism, the RN black hole has a remnant. The existence of the remnant has also been predicted in the context of noncommutative geometry, Rainbow gravity, GUP and MDR in [32,38,49,50,57,58]. Thus, it seems that not only the MDR* but also above approaches of quantum gravity predict the absence of an effective horizon and the existence of the remnant for all black holes. This method is valid for both the single horizon spacetimes and symmetric spacetimes with double horizons (outer and inner horizons) and it offers a new way for studying the entropy corrections of the complicated spacetimes.

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