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RESEARCH ARTICLE

Constraining condensate dark matter in galaxy clusters

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Abstract We constrain scattering length parameters in a Bose–Einstein condensate dark matter model by using galaxy clusters radii, with the implementation of a method previously applied to galaxies. At the present work, we use a sample of 114 clusters radii in order to obtain the scattering lengths associated with a dark matter particle mass in the range 10^{-6}–10^{-4} eV. We obtain scattering lengths that are five orders of magnitude larger than the ones found in the galactic case, even when taking into account the cosmological expansion in the cluster scale by means of the introduction of a small cosmological constant. We also construct and compare curves for the orbital velocity of a test particle in the vicinity of a dark matter cluster in both the expanding and the non-expanding cases.

Keywords Dark matter · Bose–Einstein condensate · Galaxy clusters · Rotation curves

1 Introduction

It has long been observed that almost 27 % of the energy density in the Universe is in the form of a rather mysterious entity dubbed dark matter [1,2]. So far, investigations on the nature of this sort of matter have presented no definitive conclusions.

When it comes to the dark matter present in structures such as galaxies and clusters, many proposals have been put forward. We can mention *Weakly Interacting Massive*

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Particles (WIMP's) as one of the most popular suggestions. These particles are very massive ($\mathcal{O}$(GeV)) and present very small coupling constants to the baryonic matter [3]. Experimental searches for WIMP's are being presently performed.

Recently, it has been proposed that the dark matter composing structures in the Universe is in the form of a Bose–Einstein condensate (BEC) [4–7] which suffered a gravitational collapse [8,9]. This condensate allows for the construction of suitable rotation curves for galaxies. The particles suggested to compose this condensate are sub-eV in mass and have little interaction with baryonic matter. They have been grouped in the *Weakly Interacting Slim Particles* (WISP's) [10] category, which includes the QCD axion [11,12] as the most prominent member.

Considering axion-like particles (particles in the mass range of the axion but with the possibility of having spin 0 or 1), the parameters of the condensate have been constrained by using galaxies' radii data [13]. In the present work, we apply the same procedure to galaxy clusters. We use a data set of 114 clusters radii and perform a statistical analysis to obtain the most representative scattering length value for a specific particle mass. Moreover, since cluster sizes may be in a scale which can be affected by the cosmological expansion, we repeat this procedure including a small valued cosmological constant in a Newtonian approximation [14,15], in the attempt to obtain noticeably distinct results from the non-expanding case (for a relativistic version of BEC dark matter, see, e.g, [16]).

This paper is organized as follows: Sect. 2 presents a brief review of the theoretical background on Bose–Einstein condensate dark matter. Section 3 shows the density profiles in both static and expanding cases along with the corresponding cluster radii. In Sect. 4, we perform the statistical analysis that allows us to obtain the scattering lengths for the dark matter particle, in the mass range 10^{-6}–10^{-4} eV. We show the orbital velocity of a test particle under the influence of the cluster mass in Sect. 5. Our conclusions appear in Sect. 6.

2 Bose–Einstein condensate dark matter

We recall here the theoretical description of the galactic Bose–Einstein condensate composed of axionlike particles.

At zero temperature, the dynamics of the field destruction operator $\hat{\psi}(\mathbf{r}, t)$ (representing each dark matter particle) in the Heisenberg picture, $-i\hbar\partial_t\hat{\psi}(\mathbf{r}, t) = [\hat{H}, \hat{\psi}(\mathbf{r}, t)]$, yields the time-independent Gross–Pitaevskii equation (GPE) for the BEC wavefunction $\psi(\mathbf{r})$ [17]

$$\mu\psi(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\mathbf{r})\psi(\mathbf{r}) + \frac{4\pi\hbar^2 a}{m}|\psi(\mathbf{r})|^2\psi(\mathbf{r}), \tag{1}$$

where m is the mass of the particle, a is the s-wave scattering length which characterizes two-body collisions between particles, $V(\mathbf{r})$ is the trapping potential and μ is the chemical potential.

When the potential $V(\mathbf{r})$ obeys the Poisson's equation (which is the case of a self-gravitating condensate),

$$\nabla^2 V = 4\pi G m \rho_{DM}, \tag{2}$$

where ρ_{DM} is the particle number density of the dark matter concentration, and we consider a large number of particles (this is the Thomas–Fermi (TF) approximation [18]), it has been demonstrated [5,13] that (1) has the solution

$$|\psi_{BH}(r)|^2 = \rho(r) = \begin{cases} \rho_0 \dfrac{\sin kr}{kr} & \text{for} \quad r \leq R \\ 0 & \text{for} \quad r > R \end{cases}, \tag{3}$$

with $k = \sqrt{Gm^3/\hbar^2 a}$, $R = \pi/k$ and ρ_0 is the central particle number density of the condensate. This is the Boehmer–Harko (BF) solution. It results in a halo radius given by

$$R = \pi \sqrt{\frac{\hbar^2 a}{Gm^3}}. \tag{4}$$

Using this relation and considering the dark matter particle mass range 10^{-6}–10^{-4} eV, the lower bound of the scattering length has been constrained to 10^{-29} m in galaxies [13].

The same results apply to the case of a particle with spin-1, with the important difference that now the condensate may assume two distinct states, polar (when the particles spins are antiparallel) and ferromagnetic (parallel spins) [19].

Hence, for the polar state, the radius is given by

$$R_p = \pi \sqrt{\frac{\hbar^2 (a_0^p + 2a_2^p)}{3Gm^3}}, \tag{5}$$

where a_0^p and a_2^p are the scattering lengths related to this phase.

For the ferromagnetic phase, one obtains

$$R_f = \pi \sqrt{\frac{\hbar^2 a_2^f}{Gm^3}}, \tag{6}$$

where a_2^f is a new scattering length for this particular state.

The central mass density $\varrho_0 = m\rho_0$ will be assumed throughout this paper to be the one of a typical cluster with mass $M \sim 10^{14}\, M_\odot$ and radius $R \sim 1$ Mpc, yielding $\varrho_0 \approx 10^{-24}\,\mathrm{kg\,m^{-3}}$.

3 Density profile and radii of galaxy clusters with a cosmological constant

Following the assumptions made in [14,15], we can consider the cluster to be embedded in an expanding spacetime background, with the expansion rate given by the Hubble parameter $H = \sqrt{\Lambda/3}$ and Λ being a small cosmological constant.

For the purpose of describing the cluster dark matter as a condensate, the effect of the expansion is equivalent to the introduction of an additional repulsive radial potential

$$V_\Lambda(r) = -\frac{m}{6}\Lambda r^2 \tag{7}$$

in the GPE (1). The total potential which confines the cluster is now

$$V'(r) = V(r) + V_\Lambda(r), \tag{8}$$

where $V(r)$ is the gravitational potential. When $V'(r)$ obeys the Poisson equation

$$\nabla^2 V' = 4\pi G m \rho_\Lambda, \tag{9}$$

where ρ_Λ is the particle density in the presence of the cosmological constant, differentiation of equation (1) results in

$$2\pi G m \rho - m\Lambda + \frac{2\pi \hbar^2 a}{m}\nabla^2 \rho = 0. \tag{10}$$

With the use of the identification $\rho_\Lambda = \frac{m^2}{2\pi\hbar^2 a}(2\pi G\rho - \Lambda)$, equation (10) can be recast in the form of the usual Lane–Emden equation

$$\frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left(\xi^2 \frac{\partial \theta}{\partial \xi}\right) + \theta^n = 0, \tag{11}$$

where we have used $\rho_\Lambda = \rho_0 \theta^n$, and θ being a function of the dimensionless coordinate ξ defined by $r = [(n+1)K\rho_c^{1/n-1}/4\pi G]^{1/2}\xi$. We recall that for a static condensate we have the polytropic equation of state, relating the density and the density and the pressure of the fluid, $p = K\varrho_\Lambda^{1+\frac{1}{n}}$, with K a constant and n the polytropic index.

In the present case, $n = 1$ and $K = 2\pi\hbar^2 a/m^3$, making it possible to obtain the analytical solution for the Lane–Emden equation as

$$\theta(\xi) = \frac{\sin(\xi)}{\xi}. \tag{12}$$

With the appropriate boundary conditions for the condensate, the particle number density profile obtained from (1) and (11) for a cluster under the action of a cosmological constant is thus

$$\rho_\Lambda(r) = \begin{cases} \left(\rho_0 - \frac{\Lambda}{4\pi m G}\right)\frac{\sin(kr)}{kr} + \frac{\Lambda}{4\pi m G} & \text{for} \quad r \le \bar{R} \\ 0 \quad \text{for} \quad r > \bar{R} \end{cases}, \tag{13}$$

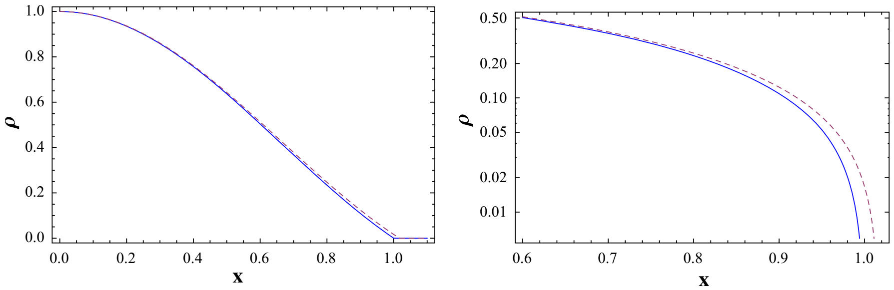

Fig. 1 Particle number density profile ρ_{BH} (in units of (ρ_0/π)) corresponding to the Boehmer–Harko solution in the Thomas–Fermi approximation with and without a cosmological constant (*dashed and solid lines*, respectively). The *right panel* shows a zoom in logarithmic scale in order to stress the difference between the functions. In both cases $x = r/R$. A central cluster mass density $\varrho_0 = 10^{-24}\,\mathrm{kg\,m^{-3}}$ and a cosmological constant $\Lambda = 1.4 \times 10^{-35}\,\mathrm{s^{-2}}$ [2] have been used in both plots

where $\bar{R}$ is the cluster radius in the expanding environment. We can see that the density has been rescaled and shifted by a small quantity that depends on the magnitude of Λ. For the sake of comparison, the density profiles in the expanding and non-expanding cases are depicted in Fig. 1.

For a spherically symmetric mass distribution, the total mass inside a radius r, resulting from (13), is calculated as

$$M_\Lambda(r) = 4\pi m \int_0^r \rho_\Lambda(r')(r')^2 dr' \\ = 4\pi m \left\{ \left(\rho_0 - \frac{\Lambda}{4\pi G m}\right) \frac{1}{k^2} \left(\frac{\sin(kr)}{k} - r\cos(kr)\right) + \frac{\Lambda}{12\pi G m} r^3 \right\}. \quad (14)$$

For the distance value $\bar{R}$ for which $\rho_\Lambda(\bar{R}) = 0$, we have

$$\frac{\sin(k\bar{R})}{k\bar{R}} = -\frac{\Lambda}{(4\pi m \rho_0 G - \Lambda)}. \quad (15)$$

For physically relevant results, the right hand side (r.h.s.) of (15) is supposed to be strictly negative. Hence, we have the lower bound for the dark matter density $\varrho_0 = m\rho_0 > \frac{\Lambda}{4\pi G}$. An upper bound, of course, is obtained when the dark matter density is large enough to render the r.h.s. of (15) negligible.

The solution of (15) provides the cluster radius with a cosmological constant

$$\bar{R}_\Lambda = 3.19587 \times \sqrt{\frac{\hbar^2 a}{G m^3}}. \quad (16)$$

The same solution applies for the spin-1 particle, with the appropriate substitution of the scattering lengths associated with each spin phase.

4 Statistical analysis

In order to constrain the values for the scattering lengths in cluster condensates, we use the data obtained from [20] to construct the Likelihood function $\mathcal{L}$ for the halo radius $R(a)$ through

$$\mathcal{L} \propto \prod_{i=1}^{N} \exp\left\{-\frac{1}{2\,\sigma_i^2}\left[R(a) - r_i\right]^2\right\}, \tag{17}$$

where $R(a)$ represents the theoretical radius obtained from Eqs. (4), (5) or (16), r_i are the data taken from observations and σ_i are the errors associated with these measurements (not available from [20], and therefore overestimated to half the value of each measurement). As usual, the maximum value of the probability density function derived from (17) gives the best fit value for the scattering length parameter a. The data set consists of 114 galaxy clusters R_{500} radii (the ones which encompass 500 times the critical density at each cluster's redshift) in the range 0.48–1.91 Mpc obtained by X-ray measurements.

Figures 2, 3 and 4 show the probability density functions obtained by this method for the scattering length a, considering a dark matter particle with mass ranging from 10^{-6} eV to 10^{-4} eV, and with spin-0 and spin-1. Also, the cosmological expansion has been taken into account in the calculations. These functions enable us to identify the most probable values for the scattering length, given the data set used. We point that, for the spin-1 case, we assume that $a_2^f = a_2^p = a$, and therefore we only constrain the value for a_0^p [13].

Using the previous analysis and the one presented in [13], we summarize the results obtained for the galactic and the cluster condensate in Table 1. We note that there is a difference of five orders of magnitude between the scattering lengths, even when the cosmological expansion is taken into account.

Considering the upper bound estimated in [21], which implies $a < 10^{-21}$ m, we conclude that the mass value $m = 10^{-6}$ eV is favoured in the present analysis. However, we can speculate that the significant difference in magnitude between the galactic

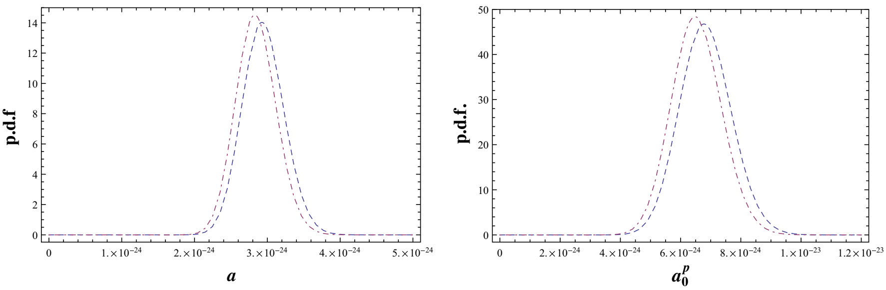

Fig. 2 Probability density function for the scattering length a of a particle with mass $m = 10^{-6}$ eV. The *left panel* refers to a spin-0 particle. The *right panel* refers to a spin-1 particle in the polar state. The *dashed* (*dot-dashed*) *curve* in *each panel* represents the non-expanding (expanding) case

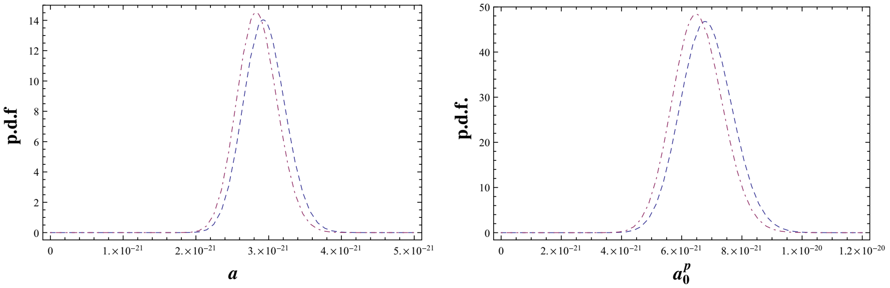

Fig. 3 Probability density function for the scattering length a of a particle with mass $m = 10^{-5}$ eV. The *left panel* refers to a spin-0 particle. The *right panel* refers to a spin-1 particle in the polar state. The *dashed* (*dot-dashed*) *curve* in *each panel* represents the non-expanding (expanding) case

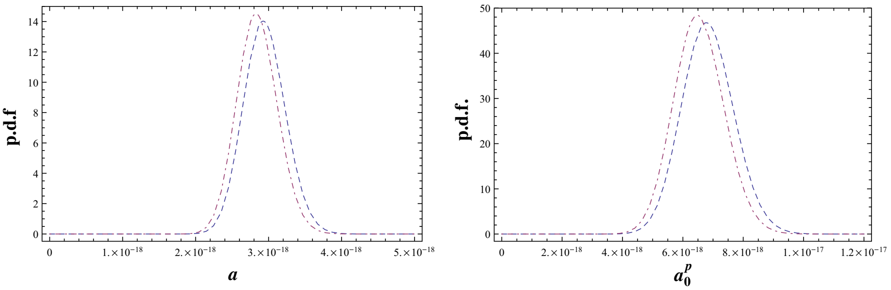

Fig. 4 Probability density function for the scattering length a of a particle with mass $m = 10^{-4}$ eV. The *left panel* refers to a spin-0 particle. The *right panel* refers to a spin-1 particle in the polar state. The *dashed* (*dot-dashed*) *curve* in *each panel* represents the non-expanding (expanding) case

Table 1 Values for the scattering lengths for galaxies (a_{gal}) and clusters (a_{clu}) for the particle masses constrained in [13]

m (eV)	a_{gal}(m)	a_{clu}(m)
10^{-6}	10^{-29}	10^{-24}
10^{-5}	10^{-26}	10^{-21}
10^{-4}	10^{-23}	10^{-18}

and the cluster cases seems to indicate that the scattering length may present some scale dependency, perhaps related to the gravitational potential or the total mass of the structure being analysed.

The analysis performed in this section complements the one performed in [13] for galactic radii.

5 Orbital velocity

In a Newtonian approximation, the attractive force exerted by a large scale concentration of mass $M(r)$ on a test particle with mass m is given simply by

$$F = \frac{GmM(r)}{r^2}, \tag{18}$$

which causes the centripetal acceleration on the orbiting body. Using (18), the velocity $v(r) = \sqrt{rF/m}$ of the test particle around the more massive object (which we can consider to be a cluster) is given by

$$v(r) = \sqrt{\frac{GM(r)}{r}}. \tag{19}$$

In the case of a cluster that is expanding due to a cosmological constant Λ, the centripetal force on the orbiting body is written as [15]

$$F = \frac{GmM(r)}{r^2} - \frac{1}{3}\Lambda mr. \tag{20}$$

From (20), the velocity of a test particle around a cluster expanding through the influence of a cosmological constant Λ is

$$v(r) = \sqrt{\frac{GM(r)}{r} - \frac{1}{3}\Lambda r^2}. \tag{21}$$

Substituting the mass function $M(r)$ in (19) by the one obtained from the BH density profile (3), the orbital velocity becomes

$$v_{BH}(r) = \left[\frac{4\pi G\varrho_0}{k^2}\left(\frac{\sin(kr)}{kr} - \cos(kr)\right)\right]^{1/2}. \tag{22}$$

We can consider the BH density profile modified by the addition of a cosmological constant, as shown in equation (13). With the input of the mass function derived from this profile in (21), one obtains

$$\begin{aligned} v_\Lambda(r) = \Bigg[& 4\pi Gm\left\{\left(\rho_0 - \frac{\Lambda}{4\pi Gm}\right)\frac{1}{k^2}\left(\frac{\sin(kr)}{kr} - \cos(kr)\right) + \frac{\Lambda}{12\pi Gm}r^2\right\} \\ & -\frac{1}{3}\Lambda r^2\Bigg]^{1/2}. \end{aligned} \tag{23}$$

As in [15], we can also keep the BH profile unmodified and add an expansion term such that the velocity takes the form

$$v_{BH\Lambda}(r) = \left[\frac{4\pi G\varrho_0}{k^2}\left(\frac{\sin(kr)}{kr} - \cos(kr)\right) - \frac{\Lambda}{3}r^2\right]^{1/2}. \tag{24}$$

For the case of a typical cluster, we can use the values $\varrho_0 = 10^{-24}\,\mathrm{kg/m^3}$, $m = 10^{-6}\,\mathrm{eV} = 1.78 \times 10^{-42}\,\mathrm{kg}$, $k = 1.8 \times 10^{-22}\,\mathrm{m^{-1}}$, $\Lambda = 1.4 \times 10^{-35}\,\mathrm{s^{-2}}$ [2] to plot the orbital velocity for the test particle, for both the expanding and non-expanding situations. The plot showing velocity curves is presented in Fig. 5.

In that plot, the point in which the velocity reaches zero value corresponds to null centripetal force, i.e., the particle ceases to be gravitationally influenced by the cluster

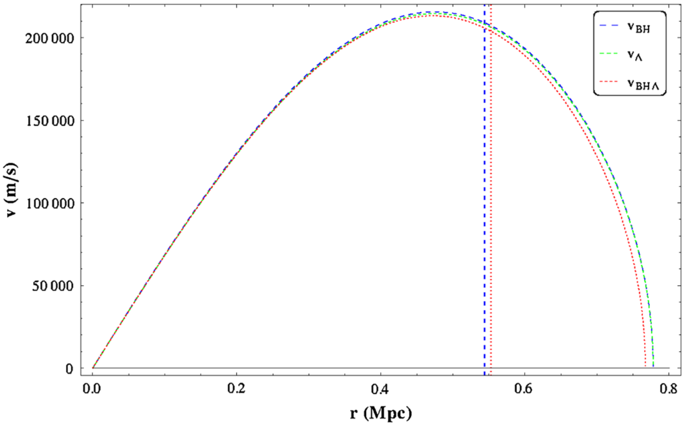

Fig. 5 Orbital velocity v (in $\mathrm{m\,s^{-1}}$), of a test particle around a cluster, for $m = 10^{-6}$ eV. The *dashed curve* represents the non-expanding case, and the *dotted curve*, the expanding one. The *vertical lines mark* the cluster radii obtained by the expressions (4) and (16) (*dashed and dotted lines*, respectively marking $R = 0.544$ Mpc and $R_\Lambda = 0.553$ Mpc)

and is carried away by the cosmological expansion. The vertical lines mark the radii obtained by the BH density profile, for the specific parameters chosen to draw the figure.

We can see that $v_{BH}(r)$ and $v_\Lambda(r)$ present no significant differences, specially at large r. This is consequence of the cancellation of the last two terms in (23). On the other hand, $v_{BH\Lambda}(r)$ is notably different for larger values of the radius r. This is in accordance with the findings of [15]. Nevertheless, we point out that the approximation used in (24) does not include an expansion in the mass distribution itself.

As expected, the maximum distance r for which the particle is still under the force of the cluster is bigger in the case of an expanding cluster [15].

6 Conclusions

In this work we have considered a Bose–Einstein condensate framework for dark matter in clusters. We have assumed an expanding cluster embedded in a de Sitter background to include a cosmological constant in a simpler Newtonian approximation. This approach allowed us to obtain a modified matter density profile for the cluster, slightly distinct from the Boehmer–Harko profile in the non-expanding situation, and to set the lower bound $\varrho_0 > (\Lambda/4\pi G)$ for the central dark matter density ϱ_0 with respect to the cosmological constant Λ.

The cluster radius resulting from this density profile has also been derived for both a spin-0 and a spin-1 particle. Using that information we were able to use a set of galaxy clusters radii data to constrain the scattering length of the Bose–Einstein condensate of a particle with masses in the range $10^{-6} - 10^{-4}$ eV, in a non-expanding as well as in an expanding case (by the inclusion of a small cosmological constant in a Newtonian approximation). The values obtained for this parameter are typically five orders of magnitude larger than what is obtained in the galactic case. This result poses the question whether it makes sense to use the same value of scattering length parameter for the galactic and the cluster scales. Apparently, at least for the mass range we are considering, some still undefined scale dependency may take place to describe the condensate in galaxies and clusters by using its microscopic parameters. Another speculative possibility is the dominance of a different kind of dark matter fluid in clusters, composed of particles endowed with distinct parameters from the ones which form galactic haloes. This possibility could be further explored in the mixed dark matter scenario [22].

We have used the Newtonian approximation for the gravitationally bound system in order to calculate the orbital velocity of a test particle around an expanding cluster. The maximal radius in which the velocity is null sets the greatest distance of influence of the dark matter cluster. This distance shows no appreciable difference in comparison to the non-expanding case when the new derived density profile is used, due to the smallness of the cosmological constant. This result is in contrast with the one found in [15], which did not consider a modified density profile for the cluster.

The issue of the most adequate values of the parameters of condensate dark matter in large scale structures has been occupying a considerable space in the literature, and we hope that the considerations presented here may guide us in future works on this subject.

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