Gen Relativ Gravit (2014) 46:1689
DOI 10.1007/s10714-014-1689-z

REVIEW ARTICLE

Numerical relativity: the role of black holes in gravitational wave physics, astrophysics and high-energy physics

Ulrich Sperhake

Received: 30 November 2013 / Accepted: 2 January 2014 / Published online: 17 April 2014

Abstract Black holes play an important role in many areas of physics. Their modeling in the highly-dynamic, strong-field regime of general relativity requires the use of computational methods. We present a review of the main results obtained through numerical relativity simulations of black-hole spacetimes with a particular focus on the most recent developments in the areas of gravitational-wave physics, astrophysics, high-energy collisions, the gauge-gravity duality, and the study of fundamental properties of black holes.

Keywords Black holes · Numerical relativity · Gravitational waves · Higher dimensions

1 Introduction

Throughout the history of Einstein's theory of general relativity, black holes (BH) have played a very important role in the study of the theory. Even though the term *black hole* was not coined until the 1960s by John Wheeler, corresponding solutions of the Einstein equations have been known ever since 1916 when Schwarzschild discovered the metric that bears his name

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right). \qquad (1)$$

This article belongs to the Topical Collection: The First Century of General Relativity: GR20/Amaldi10.

U. Sperhake (✉)
Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK
e-mail: sperhake@tapir.caltech.edu; u.sperhake@damtp.cam.ac.uk

Analytic solutions such as that of Schwarzschild have contributed enormously to the understanding of general relativity [88] but their status as real physical objects had been in doubt for a long time. This viewpoint changed drastically in the 1960s when astrophysical observations started accumulating evidence of highly concentrated massive objects in X-ray binaries [159] or as sources of accretion processes in quasi-stellar radio sources [139]. Astrophysical BHs are commonly classified as either *stellar mass* BHs with masses $M \sim \mathcal{O}(10\ M_{\odot})$ or supermassive BHs with $M \approx 10^6 \dots 10^{10}\ M_{\odot}$. Whether the mass gap in between is filled by so-called *intermediate-mass* BHs remains unclear. It should be noted, however, that our evidence of the existence of BHs is of indirect nature. The current efforts to directly observe the universe through gravitational waves (GW) may change this picture and provide direct evidence in the rather near future. BHs are indeed expected to form one of the strongest sources of detectable GWs and their observation is likely to provide us with new and unexpected insight into the dynamics of the universe. In more recent years, the range of physics where BHs play an important role has expanded further, including for example high-energy physics, fluid analogs and, through the gauge gravity duality, nuclear and condensed matter physics. In view of these developments, the modeling of BHs has become an increasingly urgent issue in contemporary physics.

The theoretical description of BH spacetimes is provided by general relativity or, in some cases, a modified version thereof describing an alternative theory of gravity. The task at hand, therefore, is to find solutions to the Einstein equations

$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta}. \tag{2}$$

In many applications, we consider a vanishing cosmological constant Λ and energy-momentum tensor $T_{\alpha\beta}$. The vacuum Einstein equations for this case simplify to $R_{\alpha\beta} = 0$.

Approaches to obtaining solutions to these equations can roughly be classified into the following groups. (i) For BH spacetimes with a sufficient degree of symmetry, the Einstein equations simplify enormously, so that analytic solutions as for example the Schwarzschild metric (1) are available in closed form; for a review see [69]. (ii) By expanding the Einstein equations either around a known background solution or in the form of a post-Newtonian (PN) series expansion in the velocity parameter v/c, one can derive solutions using perturbation theory [143] or PN calculations [30]. (iii) In the dynamic, fully non-linear regime the only approach currently available to generate solutions to the Einstein equations is the use of numerical methods on supercomputers, a field often referred to as *numerical relativity* (NR).

The research field of NR saw major breakthroughs in the year 2005 when Pretorius obtained the first evolution of a BH binary through inspiral, merger and ringdown [132] and shortly thereafter, the Brownsville and Goddard groups independently achieved similar results using a method now often referred to as *moving punctures* [18,45]. Details about the numerical methods employed in the current generation of BH evolution codes can be found in the books by Alcubierre [8], Baumgarte and Shapiro [23] and Bona et al. [36] or the reviews by Centrella et al. [50] and Lehner [106]. Numerical relativity has generated a wealth of results in recent years. Purpose of this article is to provide a brief review of the most recent developments of numerical relativity applied to BH studies and provide some guidance among the plentiful literature NR has generated in the various areas mentioned above. The reader may find it helpful to also consult earlier reviews on the topic given in [131,134,150,151]. For reasons of space limitation, this review is largely focussed on BHs in vacuum and we will not be able to cover in detail applications such as BH binaries in non-vacuum spacetimes, core-collapse supernovae or cosmology.

This article is organized as follows. In Sect. 2 we briefly discuss recent developments in the formulation of the Einstein equations as an initial value problem. In Sects. 3–6 we review results generated by numerical relativity investigations on the role of BHs in GW physics, astrophysics, high-energy collisions, and the gauge-gravity duality. Fundamental properties of BHs are discussed in Sect. 7 and we present conclusions and an outlook to the future in Sect. 8.

2 Formulations of numerical relativity

Numerical simulations of spacetimes with specific symmetries often start from a simplified line element that employs coordinates adapted to these symmetries. One derives the Einstein equations for this line element which determine the evolution of the metric functions in either a space-time or characteristic formulation. As an example of such applications, see for example the simulations done by Chesler and Yaffe [53]. For generic, 3+1 dimensional spacetimes, however, the vast majority of numerical simulations is based on either the *Generalized Harmonic Gauge* (GHG) formulation [75,133] or the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation [22,145] which form the basis of the breakthrough simulations mentioned in the introduction. It is interesting to note the different strengths of these two formulations. The GHG system does not generically contain zero speed modes[1] and strongly benefits from the addition of constraint damping terms [83]. Furthermore, the wave-equation-type principal part allows for a straightforward construction of constraint preserving boundary conditions [140,141]. The BSSN system, on the other hand, has proven an exceptionally robust method that is capable of handling even the most extreme types of configurations as for example high-energy collisions with little if any modifications in the numerical parameters or gauge conditions. It is tempting, therefore, to formulate the Einstein equations in a way that combines these advantages. Recent work has identified a conformal version of the $Z4$ system, originally developed by Bona et al. [35], as a highly promising candidate to achieve this goal.

The $Z4$ formalism extends the Einstein equations to a wider class of equations given by

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta} - \nabla_\alpha Z_\beta - \nabla_\beta Z_\alpha + g_{\alpha\beta}\nabla_\mu Z^\mu + \kappa_1 \left[n_\alpha Z_\beta + n_\beta Z_\alpha + \kappa_2 g_{\alpha\beta} n_\mu Z^\mu \right], \tag{3}$$

where Z_α is a vector field of constraints which is decomposed into space and time components according to $\Theta \equiv -n^\mu Z_\mu$ and $Z_i = \perp^\mu{}_i Z_\mu$, where n^μ is the timelike unit

[1] Strictly speaking, this depends on the choice of gauge conditions.

normal field, $\perp^\alpha{}_\mu = \delta^\alpha{}_\mu + n^\alpha n_\mu$ the projection operator onto the spatial hypersurface and κ_1, κ_2 are parameters. Equation (3) reduces to the Einstein equations if $Z_\mu = 0$ and the choice of κ_1, κ_2 is made such that any deviations from $Z_\mu = 0$ are damped away. The conformal version of the $Z4$ system is obtained in a manner analogous to the conformal decomposition applied in the BSSN formulation and results in a set of equations strikingly similar to the BSSN system but augmented by an extra equation for the constraint variable Θ. Two slightly different conformal versions of the $Z4$ system have been presented in Refs. [9,49], either of which can be implemented rather straightforwardly in existing BSSN codes. Applications to BH and neutron-star evolutions have indeed demonstrated that the active enforcement of constraint damping leads to a reduction in the constraint violations as compared with the BSSN system [9,94].

3 Black holes in gravitational wave physics

Astrophysical BH binary systems are one of the strongest expected sources of GWs expected to be observed with the current generation of ground based detectors as well as a future space-based mission of LISA type. The weak interaction of GWs with any type of matter makes the identification of signals of physical origin in the noisy data stream a highly non-trivial task. In practice, the search for physical signals relies heavily on a method known as *matched filtering* [72], where the instrumental data is cross-correlated with theoretical predictions for a family of expected sources. In simple terms, the "best-matching" theoretical template is then identified as the best candidate for the GW source. The key challenge for the BH modeling community in this context is to cover the BH binary parameter space sufficiently densely with accurate theoretical waveforms. A detailed discussion on the accuracy standards thus required for theoretical waveform predictions can be found in [109,110].

BH binaries are characterized as sources of gravitational waves by a number of parameters which are commonly divided into so-called *intrinsic* and *extrinsic* parameters [42]. Intrinsic parameters describe the physical properties of the binary systems as for example the mass ratio and the individual holes' spins. Extrinsic parameters, such as the sky location, distance or inclination of the orbital plane with respect to the line of sight, on the other hand, relate the location of the source with respect to the observer. From the perspective of the theoretical modeling of BH binaries, it is therefore sufficient to consider only intrinsic parameters. By the time BH binaries enter the sensitivity regime of GW detectors, their orbits are commonly expected to have vanishing eccentricity due to the circularizing effect of GW emission [130]. Furthermore, vacuum spacetimes containing BHs are invariant under a rescaling of the total mass of the system. Whereas the total mass M must be taken into account in the analysis of GW data, because the detectors sensitivity introduces a characteristic length or time scale to the problem, in the theoretical modeling, a change in the total mass corresponds to a trivial rescaling of the waveform predictions and therefore does not require a separate calculation. In summary the effective intrinsic parameter space of BH binaries as sources of GWs is seven dimensional: one mass ratio and six parameters for the individual hole's spin.

For illustration of the GW signal and the inspiral, we show in Fig. 1 the trajectory and the quadrupole of the GW strain h_{22} for the last 11 orbits of the inspiral of a non-spinning BH binary with mass ratio $q \equiv m_2/m_1 = 1/4$; cf. [153] for a more in-depth discussion of this system. The generation of GW catalogues now faces two serious obstacles. (i) Even a rather modest coverage of the BH binary parameter space with, say, 10 templates per dimension requires 10^7 simulations which is computationally too expensive; (ii) waveforms to be used in GW data analysis must contain a larger number of orbits or GW cycles than the example shown in Fig. 1. Furthermore, the evolution time per orbit increases rapidly with the separation of the binary members, so that a significant increase in the number of orbits beyond $\mathcal{O}(10)$ becomes very costly [111].

Overcoming these difficulties requires the combination of numerical relativity results with (semi-)analytic predictions. The methods developed for this purpose can be classified into two main approaches.

So-called *phenomenological waveform models* construct relatively simple functions determined by some model parameters to describe the phase and amplitude of the GW signal. A map between the model parameters and the physical ones characterizing the BH binary can then be calibrated by comparison for a set of specific binary configurations. For these configurations, so-called *hybrid waveforms* are constructed out of stitching numerical relativity results for the final $\mathcal{O}(10)$ orbits onto a PN prediction that covers a vast number of orbits up to the late inspiral, plunge and ringdown stage; cf. [7]. The construction of such phenomenological models has so far mostly focused on either non-spinning binaries or configurations where the BH spin is aligned with the orbital angular momentum [3–6,142]. Most recent work in Refs. [85,156] considers the extension to the general case of precessing binaries.

An alternative approach is provided by the *effective-one-body* (EOB) method which combines PN predictions with the description of a particle in an effective metric and BH ringdown [43,44]. The parameters appearing in this model are partially determined by PN calculations and remaining free parameters can be calibrated using numerical relativity calculations. EOB models for GW template banks have also concentrated first on non-spinning or non-precessing binaries (see e.g. Refs. [59–61,125,126,157]) while the extension to general precessing binaries is the subject of ongoing work; cf. [158].

Investigations also explore the possibility of reducing the effective dimensionality of the parameter space of BH binary systems to be modeled. This may be achieved by designing approximate descriptions of the gravitational waveforms using a single-spin approximation or model waveforms of precessing binary systems in terms of the signals of non-precessing binaries in a non-inertial frame; see [38,85,128,136] and references therein.

The use of numerically generated waveforms for the construction of template banks and use in GW data analysis has been explored in two community wide projects, the NRAR [175] and Ninja [174] collaborations. The main goal of the NRAR effort is to pool efforts from nine groups, standardize the analysis tools applied to the numerical results and establish comparison with analytic models. The waveforms generated inside this effort for non-precessing binaries demonstrate good agreement with previously derived EOB models for non-precessing systems. Not unexpectedly, newly

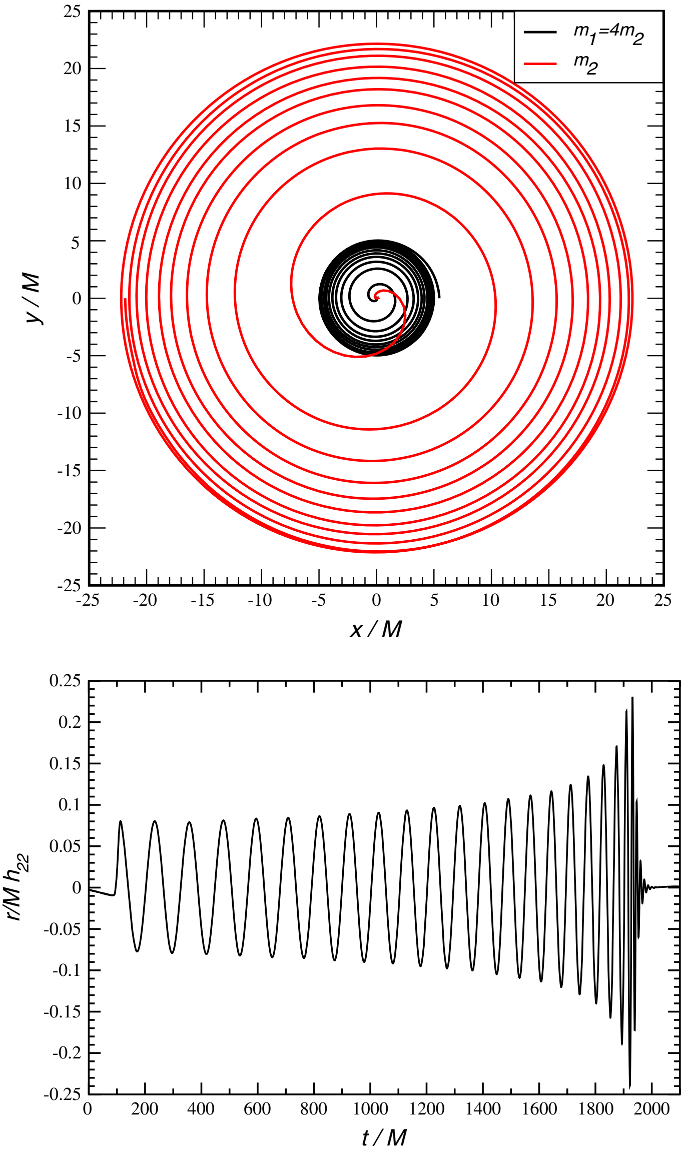

Fig. 1 *Upper* Trajectory of the BHs in the inspiral of a binary with mass ratio $q = 1/4$ and vanishing spins. *Lower* The (real part of the) quadrupole of the GW strain h_{22}

generated waveforms of precessing systems display larger deviations, measured in terms of the unfaithfulness, from these EOB models [95]. As the number of precessing waveforms is continuously increasing (see for example [118] for a recent catalog of 171 waveforms), both phenomenological and EOB models are being improved. Focus of the Ninja project is the use of GW signals generated by numerical methods in the analysis of detector data. In its first stage [16,17], purely numerical waveforms were injected into a simulated data stream which was analysed with existing GW search algorithms. While most signals could be detected, more work is needed to optimize waveforms and search pipelines for parameter estimation [16,17]. The second stage of the Ninja project therefore focused on complete inspiral-merger-ringdown waveforms obtained through the above mentioned hybridization procedure. The corresponding procedures were detailed in Ref. [7] together with a verification of sufficient accuracy of the waveforms. The injection of this improved set of waveforms into a simulated data stream is presently under way [1].

Most recently, numerical relativity has started to explore the dynamics of BH binaries in alternative theories of gravity, namely scalar-tensor theory of gravity. In order to obtain a non-trivial impact of the additional scalar field on the inspiral of the BHs, it is necessary to have a mechanism that circumvents the *no-hair* theorem. This has been achieved by Healey et al. [89] by prescribing initial data in the form of a scalar field bubble that implodes on the orbiting binary and gives rise to a non-circular orbital motion. Simulations performed by Berti et al. [27], instead generate initial data containing a spatial gradient in the scalar field as suggested by Horbatsch and Burgess [96]. This gradient interacts non-trivially with the inspiraling binary which leads to a scalar wave dipole signal oscillating at twice the orbital frequency. It is encouraging to see that existing numerical codes for Einstein's theory of general relativity can be modified in a straightforward manner to open up wider classes of theories of gravity.

4 Black holes in astrophysical systems

One of the most exciting results of numerical simulations of BH binaries concerns the gravitational recoil or kick generated in the merger of astrophysical BHs. It has been known since the 1960s that the gravitational waves emitted by compact objects can carry momentum and thus impart, by conservation of momentum, a recoil on its source [26,37,129], but the magnitude of this effect remained uncertain until the breakthroughs in numerical relativity opened up for source modeling the fully non-linear regime of general relativity.

By symmetry, the net momentum carried away in the inspiral and merger of non-spinning equal-mass BH binaries vanishes, so that a non-zero effect requires some deviation from this symmetry. The attention of numerical simulations first focused on a breaking of this symmetry by unequal mass ratios. The resulting kick velocity as a function of the mass ratio q for non-spinning binaries is well approximated by Fitchett's [73] formula

$$v = A\eta^2\sqrt{1-4\eta}(1+B\eta), \quad \eta = \frac{q}{(1+q)^2}, \tag{4}$$

with $A = 1.20 \times 10^4$ and $B = -0.93$, and predicts a maximum recoil of 175 km/s realized for $q = 0.36$ [78]. To put this number into perspective, we note that estimates for the escape velocities from globular clusters and small galaxies are in the range of a few tens to the order of 100 km/s while those from large elliptic galaxies can be as large as ∼1,000 km/s [116]. Kicks resulting from the merger of non-spinning BH binaries are thus unlikely to eject BHs from large galactic hosts which is in agreement with the observation that large galaxies appear to ubiquitously harbor BHs [112].

It came as quite a surprise when simulations of spinning BHs were found to result in much larger kicks of ≈2,500 km/s, with an extrapolation to maximal spin magnitude reaching ≈ 4,000 km/s, for specific spin orientations commonly referred to as *superkick* configurations [47,48,77]. These superkicks are realized for equal mass-ratio and BH spins oriented in the orbital plane opposite to each other. A recent study by Lousto and Zlochower observed even larger kicks up to 5,000 km/s for configurations where the spins' projection onto the orbital plane still points in opposite directions but the spins also have a component aligned with the orbital angular momentum. Spins aligned with the orbital angular momentum have been known to result in particularly large amounts of GW energy [46,93] but no kick (due to symmetry). The configurations resulting in this particularly large recoil are therefore referred to as *hang-up* kicks and can be interpreted as a combination of maximizing the total amount of GW emission while still maintaining the spin-orbit interaction arising from the opposite spin components in the orbital plane.

Superkicks or hang-up kicks of such a magnitude clearly have the potential to eject BHs from even their most massive host galaxies. This ejection may result in observational signatures (e.g. [51,82,105]) and represents a potential obstacle for BH growth via merger, and thus puts constraints on merger-history models, which must be able to explain the assembly of supermassive BHs by redshifts $z \geq 6$ [84, 108,160]. As mentioned above, however, frequent ejection of BHs would be at odds with astrophysical observations which identify BHs residing in most galaxies. It thus appears that super or hang-up kicks, while theoretically possible, are not frequently realized in nature. Mechanisms that would result in the suppression of the specific spin orientations necessary for these large kicks have been suggested in the form of partial alignment of spins due to the presence of torques from accretion disks and resonance effects due to spin-orbit coupling in the inspiral [28,34,102,103,144].

Recent years have seen an increasing number of numerical simulations of BH systems in the presence of matter. Most of this work is motivated by the identification of potential electromagnetic counterparts resulting from the merger of BH binaries. Such counterparts may, for example, be realized through accretion of matter, relativistic beaming or shocks generated in the matter by recoiling binaries [11,32,70,115]. In comparison with single BHs, BH binaries may result in an enhanced accretion rate and luminosity [71]. While numerical simulations indicate that circumbinary disks may not produce detectable electromagnetic counterparts [10,33,117], the interaction of the disk's magnetic field provides a way to tap into the rotational energy of the binary in analogy to the Blandford and Znajek [31] effect. This may result in observable electromagnetic emission along single or double jets [121–124].

The extraction of rotational energy from BHs also plays a key role in a particularly intriguing feature in the interaction of BHs with bosonic matter, the *superradiant instability* [25,166,167]. The scattering of oscillating scalar or vector fields with a real part of the frequency Re[ω] below the angular horizon velocity Ω_H of the BH results in an amplification of the field amplitude and energy in a potentially runaway manner for the case of massive fields. This mechanism should result in a reduction of the BHs spin rate which has been suggested as a possibility to obtain bounds on the photon mass through observations of the BH at the center of the Milky Way [127]. Numerical relativity has just started exploring the interaction of scalar and vector fields with rotating BHs. For fixed background spacetimes, the numerical simulations identify that vector fields get amplified through the superradiant instability on significantly shorter timescales than scalar fields [64,162]. A main question to be explored in future numerical work is the impact of the non-linear back reaction on the BH's rotation rate.

5 High-energy collisions of BHs

The so-called *hierarchy problem* of physics is one of the main open issues that is not explained within the framework of the standard model of particle physics. The hierarchy problem consists in the extraordinarily low strength of the gravitational interaction which is 30 to 40 orders of magnitude weaker than the other fundamental interactions. One manifestation of the hierarchy problem is the large discrepancy between the electroweak energy scale $\mathcal{O}(10^3)$ GeV and the Planck scale $\mathcal{O}(10^{19})$ GeV where all interactions are expected to be of comparable strength. One possible explanation of the hierarchy problem is given by so-called *Tera-electron Volt* (TeV) *gravity* models which evoke extra dimensions. In a model often referred to as the ADD model, the weakness of the gravitational force at length scales $\gtrsim$1 mm is due to $n \geq 2$ compact extra dimensions; the extra dimensions are accessible to gravitons leading to a faster fall-off than $1/r^2$, whereas all other fields of the standard model are confined to a four-dimensional brane [13–15]. In consequence, the effective Planck scale where the fundamental interactions become comparable in strength may be reduced from $\mathcal{O}(10^{19})$ GeV to the TeV regime. The alternative Randall-Sundrum model provides a similar explanation in terms of extra dimensions with a warp factor [137,138]. An intriguing possibility arising from an effective Planck scale much below its four-dimensional value is the creation of BHs in parton-parton collisions at the Large Hadron Collider (LHC) [63,76]; for a review see Kanti [101].

Formation of BHs in such collision experiments is expected to manifest itself in specific signatures in the decay products such as their jet multiplicity and transverse energy. For the detection of such signatures, theoretical predictions from Monte-Carlo generators such as BlackMax [58] or Charybdis [74,87] are compared with the experimental data. The scattering cross section for BH formation and the initial mass and spin functions of the BHs form vital input parameters for these generators. In the context of numerical relativity, it is of particular interest that at ultrarelativistic energies, the structure of the colliding particles should become irrelevant for the collision dynamics and the collision should be well modeled by two point particles or BHs in d-dimensional general relativity [20,76].

Numerical simulations have been able to confirm this hypothesis for collisions of concentrated boson fields, perfect fluid balls and BHs with internal structure in the form of spins. By colliding boson fields with varying initial boost factor, Choptuik and Pretorius [57] have demonstrated that a BH is formed above a threshold Lorentz factor $\gamma_{\rm thr} \approx 2.9$ in agreement with expectations from the Hoop conjecture. Likewise, East and Pretorius [68] confirmed this observation for collisions of perfect fluids. In

Sperhake et al. [152], the impact of spins on the dynamics of BH collisions disappears at Lorentz boosts $\gtrsim 3$.

BH collisions at velocities near the speed of light are currently best understood in four spacetime dimensions. While this case does not contain extra dimensions as evoked in the TeV gravity models, these studies have revealed exciting features in the collision dynamics. The first, and simplest, type of collisions studied was that of a head-on collision of two equal-mass, non-spinning BHs [154]. These collisions are characterized by a single parameter, the initial boost γ. The amount of energy radiated away in the form of gravitational waves increases enormously relative to the non-boosted case and reaches $14 \pm 3\,\%$ of the total center-of-mass (CoM) energy, about half of the upper limit obtained from Penrose's construction quoted in [67]. Even larger amounts of GW energy have been found by Sperhake et al. [155] for grazing collisions, where a second parameter is introduced in the form of the impact parameter b. Emission of 35 % of the total energy of the system in GWs has been observed in these grazing collisions. In addition to scattering and promptly merging configurations, these simulations revealed a third possible outcome, a *delayed merger* where the BHs separate after their first encounter but loose enough kinetic energy in GWs to form a bound system and eventually merge. These delayed mergers occur for impact parameters close to a critical value b^* identified by Pretorius and Khurana [135] as the *threshold of immediate merger*. In this regime, the binary completes a number of orbits proportional to the logarithmic distance of the impact parameter b from the critical value. A remarkably simple functional relation for the scattering threshold b_{scat} has been found by Shibata et al. [146]: a merger into a single BH occurs for impact parameters $b \lesssim 2.5\ M/v$, where v represents the initial velocity in units of the speed of light.

The enormous amounts of energy released in GWs in high-energy collisions has prompted the question whether it is possible for the binary to loose all kinetic energy through radiation. This would result in much reduced amounts of energy available for the formation of BHs in the collision. The above mentioned study of high-energy collisions of spinning BHs [152], however, demonstrates that there exists an upper limit of about 50 % of the total energy that can be lost in GWs. The other half inevitably is absorbed and ends up as BH rest mass in merging as well as scattering configurations.

The main challenge for the numerical relativity community is to extend these results to collisions in d dimensions. For this purpose, two formulations have been developed that allow for the modeling of higher-dimensional spacetimes with $SO(D-3)$ isometry on a computational domain of three (spatial) dimensions: a reduction by isometry [173] and a *modified Cartoon* method [147]; for more extended discussions of these methods see [165, 168]. For the numerical construction of initial data, a spectral solver developed by Ansorg [12] has been generalized to an arbitrary number of spacetime dimensions by Zilhão et al. [169].

Collisions of two equal-mass, non-spinning BHs starting from rest have been studied in Witek et al. [161, 163] and found to radiate 0.089 % of the CoM energy in GWs in $d = 5$ dimensions, just under twice the value found for $d = 4$. For unequal masses, this amount decreases in good agreement with point particle predictions. Boosted collisions of BHs in $d = 5$ dimensions have been studied by Okawa et al. [120] who find the scattering threshold to decrease from 3.6 to 3.3 Schwarzschild radii as the velocity is increased from $v = 0.4$ to 0.6. Due to numerical stability issues, a determination of the scattering threshold at higher velocities has not yet been possible.

Collisions of charged BHs starting from rest have been simulated in Zilhão et al. [171,172]. As expected, for equal charges, the collision is slowed down due to the repulsive electromagnetic force which results in a decreasing amount of energy E_{GW} radiated in GWs as the charge-to-mass ratio Q/M is increased. The energy E_{EM} radiated through electromagnetic waves reaches a maximum at about $Q/M = 0.6$. For the case of opposite charges, in contrast, both E_{GW} and E_{EM} increase monotonically with $|Q/M|$ reaching values of about 0.1 and 0.5 %, respectively, in the extreme limit $|Q| = M$.

6 BHs and the gauge-gravity duality

The gauge-gravity duality conjectures the mathematical equivalence or *duality* between field theories including gravity in d dimensions and field theories without gravity in $d-1$ dimensions. Because of Maldacena's prototypical example [113] of the duality between type IIb string theory on five dimensional Anti-de Sitter (AdS) times the S^5 sphere, and $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) conformal field theory (CFT) in four dimensions, the duality is often referred to as the AdS/CFT correspondence even though the principle covers a wider range of theories. A particularly attractive feature of the correspondence is that it relates the strong coupling regime of the field theory to the weakly coupled regime of string theory where classical general relativity should provide a good approximation. The duality thus opens up a new technique to study field theories in the strongly coupled regime through the modeling of asymptotically AdS spacetimes.

The AdS spacetime in d dimensions is the maximally symmetric solution of the Einstein equations $G_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta}$ with negative cosmological constant $\Lambda < 0$. It can be represented as a d dimensional hyperboloid embedded in a $d+1$ dimensional flat spacetime of signature $- - + \cdots +$. The induced metric on this hyperboloid can be transformed to two particular coordinate systems that have been used frequently in the modeling of asymptotically AdS spacetimes. In *global coordinates*, the AdS metric is given by

$$ds^2 = \frac{L^2}{\cos^2 \rho} \left(-d\tau^2 + d\rho^2 + \sin^2 \rho \, d\Omega^2_{d-2} \right), \tag{5}$$

where $\Lambda = -(d-1)(d-2)/(2L^2)$, $d\Omega^2_{d-2}$ is the metric on a $d-2$ dimensional unit hyper sphere and the coordinate ranges are $0 \le \rho < \pi/2$, $-\pi < \tau \le \pi$. The AdS boundary is obtained in the limit $\rho \to \pi/2$. An alternative description is given in terms of Poincaré coordinates

$$ds^2 = \frac{L^2}{z^2} \left[-dt^2 + dz^2 + \sum_{i=1}^{d-2} \left(dx^i \right)^2 \right], \tag{6}$$

where $z > 0$ and $t \in \mathbb{R}$ and the boundary corresponds to $z \to 0$. It can be shown that the Poincaré patch merely covers half of the AdS spacetime; the other half corresponds to $z < 0$ [24].

The exploration of field theories through the gauge-gravity duality requires a translation of properties of the fields on the gravity side into physical quantities of the field theory. This so-called *dictionary* is established by the equivalence of the gravitational action of the bulk taken in the limit of the boundary and the action of the field theory on the boundary [81,113,164]. More specifically, the vacuum expectation values of the field theory $\langle T_{ij} \rangle$ are given by the quasi-local Brown and York [39] stress-energy tensor and, thus directly related to the bulk spacetime metric. These relations have been worked out for both global and Poincaré coordinates; see for example [19,21,86] and references therein. Detailed reviews of the AdS/CFT correspondence can be found in [2,104,119].

Many numerical applications of the AdS/CFT correspondence have addressed the rapid thermalization of quark-gluon plasma generated in heavy-ion collisions. While the plasma is in a far-from-equilibrium state immediately after the collision, its behavior is well modeled by a hydrodynamic description after short times of the order of 1 fm$/c$. Even though this process is governed by quantum chromo dynamics (QCD), many features appear to be captured by $\mathcal{N} = 4$ SYM theory through the duality. Perturbative studies predict that departures from the equilibrium state decay exponentially and correspond to quasi-normal modes of AdS BHs [97].

The use of numerical methods to model BH systems in the context of the gauge-gravity duality has been pioneered by Chesler and Yaffe's [52,53] simulations of anisotropic sources with boost invariance as well as rotational and translational symmetry. By monitoring the energy density as well as the longitudinal and transverse pressure components extracted from the asymptotically AdS bulk metric, they demonstrate that the anisotropies decay on timescales inversely proportional to the local temperature at the onset of the hydrodynamic regime which translates to 0.5 fm$/c$ assuming a temperature of 350 MeV. The short thermalization timescales were confirmed by Heller et al. for a variety of initial far-from-equilibrium configurations [90,91] and also in linearized calculations [92]. In Ref. [54], Chesler and Yaffe relax their symmetry assumptions and simulate two colliding shock waves with translational symmetry in the transverse direction and observe isotropization about 0.35 fm$/c$ after the waves start overlapping. A more detailed description of their numerical infrastructure together with a discussion of turbulent fluid flow in two spatial dimensions is given in [55].

A numerical code based on the GHG formulation has been developed by Bantilan et al. [21]. By assuming an $SO(3)$ symmetry, they reduce their computational domain to 2+1 dimensions, but otherwise maintain the structure of the GHG formulation as used for generic spacetimes without symmetry. Their evolutions start with a concentrated scalar field which promptly collapses to form a distorted BH. Through quasi-normal ringdown, the BH rapidly settles down into a stationary configuration. Right from the outset of their simulations, they observe that the energy momentum tensor evolves consistently with a thermalized $\mathcal{N} = 4$ SYM fluid, in agreement with the previously observed short thermalization time scales. Their work furthermore discusses in detail the choice of variables and gauge conditions required to provide numerically stable simulations.

In Refs. [98,99], Horowitz et al. use numerical simulations of BHs in AdS spacetimes for the modeling of the conductivity of so-called *cuprates* or *strange metals*. Expressed in frequency space, the conductivity of these materials has been known for a long time to obey Drude's law [65,66] at low frequencies whereas quantum effects lead to a plateau at high frequencies. Experimental results have suggested a power-law dependency $\sim \omega^{-2/3}$ at intermediate frequencies which has been theoretically confirmed in the AdS/CFT based study of Horowitz et al..

7 Fundamental properties of black holes

Black holes have for many decades proved invaluable tools to deepen our insight into the theory of general relativity and also alternative theories of gravity that arise from modifications of general relativity. The focus in this section is therefore not so much on the role of BHs from the perspective of other areas of physics, but rather on general properties of BHs, in particular on their stability properties and their formation from initially regular matter fields. BHs reveal a much richer structure in higher-dimensional and non-asymptotically flat spacetimes [69] and, as we shall see, it is for those types of spacetimes where numerical relativity has generated particularly interesting results in recent years.

One of the earliest and most influential discovery obtained through numerical relativity is the critical behaviour identified by Choptuik [56] in the collapse of spherically symmetric, massless scalar fields minimally coupled to Einstein gravity in four dimensions. By studying the behaviour of one-parameter families of scalar fields characterized by a measure p for the strength of the initial pulse, Choptuik found a critical value p^* above which a BH forms and below which the field eventually scatters off to infinity. Furthermore, for values of p near the critical p^*, the evolutions exhibit *universal* behaviour in the strong field limit: (i) if a BH forms, its mass scales as $(p - p^*)^\gamma$ with a universal $\gamma \approx 0.37$. (ii) For a universal value $\Delta t \approx 3.4$, the scalar field profile at time $t + \Delta t$ is identical to that at time t up to a rescaling by a factor $e^{\Delta t}$. Sorkin and Oren [149] generalized Choptuik's study to higher dimensions up to $d = 11$; the values of γ and Δt are dimension dependent but retain their otherwise universal character.

The corresponding setup in asymptotically AdS spacetime has been investigated by Bizón and Rostworowski [29] and revealed a strikingly different pattern due to the nature of the AdS outer boundary. As in Choptuik's case, initially strong pulses promptly collapse into a BH whereas below a critical value p^*, the field instead scatters off to infinity. In contrast to the asymptotically flat case, however, the field reaches spatial infinity in finite time, bounces back and collapses to a BH upon the second implosion. By further reducing p, the field scatters back to infinity a second time, but forms a BH upon the third implosion and so on. No matter how small p is chosen, a BH eventually forms after some possibly large number of reflections from the outer AdS boundary. The same behaviour was found by Buchel et al. [40] and, for the case of higher-dimensional spacetimes, by Jałmużna et al. [100]. These simulations reveal a transfer of energy within the pulse to shorter wavelengths and, thus, to an increasingly higher concentration of energy density. Asymptotically AdS spacetimes, however, have also been found to admit many stable configurations as for example boson stars or time-periodic solutions [41,62,114].

The stability of fast rotating Myers-Perry BHs has been investigated by Shibata and Yoshino [147,148]. Here the perturbation is given in the form of a bar-mode superposed on the rotating BH. The simulations demonstrate that above a spin parameter of 0.87 (0.74, 0.73, 0.77) for $d = 5$ (6, 7, 8) spacetime dimensions, the perturbed BH sheds angular momentum through the emission of GWs and settles down into a stationary state with a rotation rate below the critical value.

Lehner and Pretorius [107] evolved infinite black strings in $d = 5$ dimensions. These strings are known to be subject to the Gregory–Laflamme instability [79,80]. The numerical results demonstrate a cascade of a segmentation of the string into nearly spherical BHs connected by ever thinner string elements which reach zero width in finite asymptotic time. This observation indicates a violation of Penrose's cosmic censorship conjecture in five dimensions as a naked singularity may be formed in finite time. Cosmic censorship has also been investigated in four dimensional, asymptotically de Sitter spacetimes by Zilhão et al. [170]. BHs in de Sitter are characterized by two parameters, the BH mass M and the cosmological expansion rate H. For sufficiently large M, these spacetimes represent a naked singularity. The question addressed in the numerical study is whether the collision of two "large" BHs from small initial separation can lead to a merger and, thus, the formation of a single BH with mass M above the threshold for a naked singularity. The answer obtained is no; either a naked singularity is already present in the initial data, or the cosmological expansion will prevent the BHs from merging into a potentially censorship-violating single BH.

8 Conclusions

Numerical relativity has been a highly productive field of research in recent years. Especially following the breakthroughs of the year 2005, many exciting results have been obtained from the numerical modeling of BHs in the non-linear strong field regime of general relativity. With the exception of the big bang itself, dynamical systems containing BHs probably generate the most energetic events in our universe. For example, a binary system of two non-spinning stellar-mass BHs in quasi-circular inspiral converts about 3 % of its rest mass into gravitational radiation over a time interval of the order of a millisecond during the last orbit, merger and ringdown. That roughly corresponds to the energy released by the sun over its entire lifetime of the order of 10^{10} years. Anisotropic GW emission can generate recoil velocities of thousands of km/s, enough to eject BHs from their most massive host galaxies. High-energy collisions of BHs are capable of converting about half of the center-of-mass energy into radiation.

Arguably the most surprising development of the past decade, however, is the wide range of physics where BHs have started playing an important if not central role. As mentioned in the introduction, BHs started being recognized as important astrophysical objects in the 1960s. It is highly likely that they will appear as one of the most important sources of GWs whose observations are expected to provide us with a qualitatively new view of the universe within the present decade. In high-energy physics, BHs enable us to experimentally test physics beyond the standard model of particle physics. And through the gauge-gravity duality, BHs even provide a tool for the modeling of heavy-ion collisions, the electric conductivity of matter or turbulence phenomena. At the same time, BHs do not stop surprising us with their behaviour and about 100 years after Einstein's discovery of the theory, the community still gains new, and sometimes surprising, insights into general relativity.

In spite of the invaluable access to studying strong-field dynamics that has been opened up by numerical relativity, it has been a common theme throughout this review that a comprehensive understanding of the multitude of BH phenomena is obtained through a close interplay between numerical and analytic techniques. The generation of GW template banks clearly requires the combination of numerical relativity with post-Newtonian and related methods. Perturbation theory and point-particle calculations often allow us to gain an intuitive understanding of the processes or to extrapolate results to regions of the parameter space where purely computational methods are prohibitively costly. Last but not least, the diagnostic tools employed in the analysis of numerical simulations are almost universally based on analytic calculations or approximations.

We conclude this review with a brief list of some of the most important outstanding tasks to be completed by the numerical relativity community. In GW physics, the generation of complete waveform catalogues represents a most urgent goal to optimize the analysis of observational GW data once these become available. Closely related to this topic is an improved understanding of the type of electromagnetic counterparts we should expect from astrophysical sources of GWs. High-energy collisions of BHs are now well understood in four spacetime dimensions. Extension of these results to higher-dimensional spacetimes is vital to support the search for signatures of TeV gravity in the data taken from collision experiments. The range of applications of the gauge-gravity duality has probably merely been scratched so far. In contrast to the other fields discussed, there does not yet exist a generic computational framework suitable for the modeling of essentially arbitrary BH systems in asymptotically AdS spacetimes. Our understanding of the stability of BHs has progressed enormously over the years, but there remain many BH solutions, especially in higher dimensions, whose stability properties remain unknown. The formation of naked singularities in the time evolution of black strings indicates that cosmic censorship may not hold in arbitrary spacetime dimensions. More studies of a wider class of BHs will help improving our understanding of the role of censorship. The reader is encouraged to remain on the watch for new developments in these areas. Judging by the past few years (s)he may not have to wait for too long.

Acknowledgments The author thanks Emanuele Berti, Vitor Cardoso, Pau Figueras, Leonardo Gualtieri, Carlos Herdeiro, Luis Lehner, Christian Ott, Frans Pretorius, Harvey Reall, Christian Reisswig, Carlos Sopuerta, Helvi Witek, and Miguel Zilhão for many fruitful discussions. This work was supported by the FP7-PEOPLE-2011-CIG CBHEO Grant No. 293412, the FP7-PEOPLE-2011-IRSES NRHEP Grant No. 295189, the STFC Grant No. ST/I002006/1, the XSEDE Grant No. PHY-090003 by the National Science Foundation, the Cosmos supercomputer infrastructure, part of the DiRAC HPC Facility funded by STFC and BIS, the Centro de Supercomputación de Galicia (CESGA) under Grant No. ICTS-2013-249, and the European Union's FP7 ERC Starting Grant DyBHo-256667.

References

1. Aasi, J., et al.: The NINJA-2 project: Detecting and characterizing gravitational waveforms modelled using numerical binary black hole simulations. Class. Quantum Grav. LIGO-P1300199(2014). ArXiv:1401.0939 [gr-qc]
2. Aharony, O., Gubser, S.S., Maldacena, J.M., Ooguri, H., Oz, Y.: Large N field theories, string theory and gravity. Phys. Rep. **323**, 183–386 (2000). doi:10.1016/S0370-1573(99)00083-6. Hep-th/9905111
3. Ajith, P.: Gravitational-wave data analysis using binary black-hole waveforms. Class. Quantum Grav. **25**, 114033 (2008). ArXiv:0712.0343 [gr-qc]
4. Ajith, P., Hannam, M., Husa, S., Chen, Y., Brügmann, B., et al.: Inspiral-merger-ringdown waveforms for black-hole binaries with non-precessing spins. Phys. Rev. Lett. **106**, 241101 (2011). doi:10.1103/PhysRevLett.106.241101. ArXiv:0909.2867 [gr-qc]
5. Ajith, P., et al.: Phenomenological template family for black-hole coalescence waveforms. Class. Quantum Grav. **24**, S689–S700 (2007). doi:10.1088/0264-9381/24/19/S31. ArXiv:0704.3764 [gr-qc]
6. Ajith, P., et al.: A template bank for gravitational waveforms from coalescing binary black holes: I. Non-spinning binaries. Phys. Rev. D **77**, 104017 (2008). ArXiv:0710.2335 [gr-qc]
7. Ajith, P., et al.: The NINJA-2 catalog of hybrid post-Newtonian/numerical-relativity waveforms for non-precessing black-hole binaries. Class. Quantum Grav. **29**, 124001 (2012). doi:10.1088/0264-9381/29/12/124001. ArXiv:1201.5319 [gr-qc]
8. Alcubierre, M.: Introduction to 3+1 Numerical Relativity. Oxford University Press, Oxford (2008)
9. Alic, D., Bona-Casas, C., Bona, C., Rezzolla, L., Palenzuela, C.: Conformal and covariant formulation of the Z4 system with constraint-violation damping. Phys. Rev. D **85**, 064040 (2012). doi:10.1103/PhysRevD.85.064040. ArXiv:1106.2254 [gr-qc]
10. Alic, D., Mösta, P., Rezzolla, L., Zanotti, O., Jaramillo, J.L.: Accurate simulations of binary black-hole mergers in force-free electrodynamics. Astrophys. J. **754**, 36 (2012). doi:10.1088/0004-637X/754/1/36. ArXiv:1204.2226 [gr-qc]
11. Anderson, M., Lehner, L., Megevand, M., Neilsen, D.: Post-merger electromagnetic emissions from disks perturbed by binary black holes. Phys. Rev. D **81**, 044004 (2010). doi:10.1103/PhysRevD.81.044004. ArXiv:0910.4969 [astro-ph]
12. Ansorg, M., Brügmann, B., Tichy, W.: A single-domain spectral method for black hole puncture data. Phys. Rev. D **70**, 064011 (2004). doi:10.1103/PhysRevD.70.064011. Gr-qc/0404056
13. Antoniadis, I.: A Possible new dimension at a few TeV. Phys. Lett. B **246**, 377–384 (1990). doi:10.1016/0370-2693(90)90617-F
14. Antoniadis, I., Arkani-Hamed, N., Dimopoulos, S., Dvali, G.R.: New dimensions at a millimeter to a Fermi and superstrings at a TeV. Phys. Lett. B **436**, 257–263 (1998). doi:10.1016/S0370-2693(98)00860-0. Hep-ph/9804398
15. Arkani-Hamed, N., Dimopoulos, S., Dvali, G.R.: The hierarchy problem and new dimensions at a millimeter. Phys. Lett. B **429**, 263–272 (1998). doi:10.1016/S0370-2693(98)00466-3. Hep-ph/9803315
16. Aylott, B., et al.: Status of NINJA: the Numerical INJection Analysis project. Class. Quantum Grav. **26**, 114008 (2009). doi:10.1088/0264-9381/26/11/114008. ArXiv:0905.4227 [gr-qc]
17. Aylott, B., et al.: Testing gravitational-wave searches with numerical relativity waveforms: results from the first Numerical INJection Analysis (NINJA) project. Class. Quantum Grav. **26**, 165008 (2009). doi:10.1088/0264-9381/26/16/165008. ArXiv:0901.4399 [gr-qc]
18. Baker, J.G., Centrella, J., Choi, D.I., Koppitz, M., van Meter, J.: Gravitational-wave extraction from an inspiraling configuration of merging black holes. Phys. Rev. Lett. **96**, 111102 (2006). doi:10.1103/PhysRevLett.96.111102. Gr-qc/0511103
19. Balasubramanian, V., Kraus, P.: A stress tensor for anti-de Sitter gravity. Commun. Math. Phys. **208**, 413–428 (1999). doi:10.1007/s002200050764. Hep-th/9902121
20. Banks, T., Fischler, W.: A model for high energy scattering in quantum gravity (1999). Report number RU-99–23, UTTP-03-99. Hep-th/9906038
21. Bantilan, H., Pretorius, F., Gubser, S.S.: Simulation of asymptotically AdS5 spacetimes with a generalized harmonic evolution scheme. Phys. Rev. D **85**, 084038 (2012). doi:10.1103/PhysRevD.85.084038. ArXiv:1201.2132 [hep-th]
22. Baumgarte, T.W., Shapiro, S.L.: On the numerical integration of Einstein's field equations. Phys. Rev. D **59**, 024007 (1998). doi:10.1103/PhysRevD.59.024007. Gr-qc/9810065
23. Baumgarte, T.W., Shapiro, S.L.: Numerical Relativity. Cambridge University Press, Cambridge (2010)

24. Bayona, C.A., Braga, N.R.: Anti-de Sitter boundary in Poincare coordinates. Gen. Relativ. Gravit. **39**, 1367–1379 (2007). doi:10.1007/s10714-007-0446-y. Hep-th/0512182
25. Bekenstein, J.D.: Extraction of energy and charge from a black hole. Phys. Rev. D **7**, 949–953 (1973). doi:10.1103/PhysRevD.7.949
26. Bekenstein, J.D.: Gravitational-radiation recoil and runaway black holes. Astrophys. J. **183**, 657–664 (1973). doi:10.1086/152255
27. Berti, E., Cardoso, V., Gualtieri, L., Horbatsch, M., Sperhake, U.: Numerical simulations of single and binary black holes in scalar-tensor theories: circumventing the no-hair theorem. Phys. Rev. D **87**, 124020 (2013). doi:10.1103/PhysRevD.87.124020. ArXiv:1304.2836 [gr-qc]
28. Berti, E., Kesden, M., Sperhake, U.: Effects of post-Newtonian spin alignment on the distribution of black-hole recoils. Phys. Rev. D **85**, 124049 (2012). doi:10.1103/PhysRevD.85.124049. ArXiv:1203.2920 [astro-ph]
29. Bizoń, P., Rostworowski, A.: On weakly turbulent instability of anti-de Sitter space. Phys. Rev. Lett. **107**, 031102 (2011). doi:10.1103/PhysRevLett.107.031102. ArXiv:1104.3702 [gr-qc]
30. Blanchet, L.: Gravitational radiation from post-Newtonian sources and inspiralling compact binaries. Living Rev. Rel. **9**(4) (2006). http://www.livingreviews.org/lrr-2006-4
31. Blandford, R.D., Znajek, R.L.: Electromagnetic extractions of energy from Kerr black holes. Mon. Not. Roy. Astron. Soc. **179**, 433–456 (1977)
32. Bode, T., Haas, R., Bogdanović, T., Laguna, P., Shoemaker, D.: Relativistic mergers of supermassive black holes and their electromagnetic signatures. Astrophys. J. **715**, 1117–1131 (2010). doi:10.1088/0004-637X/715/2/1117. ArXiv:0912.0087 [gr-qc]
33. Bode, T., et al.: Mergers of supermassive black holes in astrophysical environments. Astrophys. J. **744**, 45 (2011). doi:10.1088/0004-637X/744/1/45. ArXiv:1101.4684 [gr-qc]
34. Bogdanović, T., Reynolds, C.S., Miller, M.C.: Alignment of the spins of supermassive black holes prior to coalescence. Astrophys. J. **661**, L147–L150 (2007). Astro-ph/0703054
35. Bona, C., Ledvinka, T., Palenzuela, C., Žáček, M.: General-covariant evolution formalism for numerical relativity. Phys. Rev. D **67**, 104005 (2003). doi:10.1103/PhysRevD.67.104005. Gr-qc/0302083
36. Bona, C., Palenzuela-Luque, C., Bona-Casas, C.: Elements of Numerical Relativity and Relativistic Hydrodynamics. Springer, London, New York (2009)
37. Bonnor, W.B., Rotenberg, M.A.: Transport of momentum by gravitational waves: the linear approximation. Proc. R. Soc. Lond. A. **265**, 109–116 (1961)
38. Brown, D.A., Lundgren, A., O'Shaughnessy, R.: Nonspinning searches for spinning binaries in ground-based detector data: amplitude and mismatch predictions in the constant precession cone approximation. Phys. Rev. D **86**, 064020 (2012). doi:10.1103/PhysRevD.86.064020. ArXiv:1203.6060 [gr-qc]
39. Brown, J.D., York Jr, J.W.: Quasilocal energy and conserved charges derived from the gravitational action. Phys. Rev. D **47**, 1407–1419 (1993). doi:10.1103/PhysRevD.47.1407. Gr-qc/9209012
40. Buchel, A., Lehner, L., Liebling, S.L.: Scalar collapse in AdS. Phys. Rev. D **86**, 123011 (2012). doi:10.1103/PhysRevD.86.123011. ArXiv:1210.0890 [gr-qc]
41. Buchel, A., Liebling, S.L., Lehner, L.: Boson stars in AdS. Phys. Rev. D **87**, 123006 (2013). doi:10.1103/PhysRevD.87.123006. ArXiv:1304.4166 [gr-qc]
42. Buonanno, A., Chen, Y., Valisneri, M.: Detection template families for gravitational waves from the final stages of binary-black-hole inspirals: Nonspinning case. Phys. Rev. D **67**, 024016 (2003). [Erratum-ibid. 74, 029903 (2006)] gr-qc/0205122.
43. Buonanno, A., Damour, T.: Effective one-body aproach to general relativistic two-body dynamics. Phys. Rev. D **59**, 084006 (1999)
44. Buonanno, A., Damour, T.: Transition from inspiral to plunge in binary black hole coalescences. Phys. Rev. D **62**, 064015 (2000)
45. Campanelli, M., Lousto, C.O., Marronetti, P., Zlochower, Y.: Accurate evolutions of orbiting black-hole binaries without excision. Phys. Rev. Lett. **96**, 111101 (2006). doi:10.1103/PhysRevLett.96.111101. Gr-qc/0511048
46. Campanelli, M., Lousto, C.O., Zlochower, Y.: Spinning-black-hole binaries: the orbital hang up. Phys. Rev. D **74**, 041501 (2006). doi:10.1103/PhysRevD.74.041501. Gr-qc/0604012
47. Campanelli, M., Lousto, C.O., Zlochower, Y., Merritt, D.: Large merger recoils and spin flips from generic black-hole binaries. Astrophys. J. **659**, L5–L8 (2007). doi:10.1086/516712. Gr-qc/0701164
48. Campanelli, M., Lousto, C.O., Zlochower, Y., Merritt, D.: Maximum gravitational recoil. Phys. Rev. Lett. **98**, 231102 (2007). Gr-qc/0702133

49. Cao, Z., Hilditch, D.: Numerical stability of the Z4c formulation of general relativity. Phys. Rev. D **85**, 124032 (2012). doi:10.1103/PhysRevD.85.124032. ArXiv:1111.2177 [gr-qc]
50. Centrella, J.M., Baker, J.G., Kelly, B.J., van Meter, J.R.: Black-hole binaries, gravitational waves, and numerical relativity. Rev. Mod. Phys. **82**, 3069 (2010). doi:10.1103/RevModPhys.82.3069. ArXiv:1010.5260 [gr-qc]
51. Chang, P., Strubbe, L.E., Menou, K., Quataert, E.: Fossil gas and the electromagnetic precursor of supermassive binary black hole mergers. MNRAS **407**, 2007–2016 (2010). ArXiv:0906.0825 [astro-ph]
52. Chesler, P.M., Yaffe, L.G.: Horizon formation and far-from-equilibrium isotropization in supersymmetric Yang-Mills plasma. Phys. Rev. Lett. **102**, 211601 (2009). doi:10.1103/PhysRevLett.102.211601. ArXiv:0812.2053 [hep-th]
53. Chesler, P.M., Yaffe, L.G.: Boost invariant flow, black hole formation, and far-from-equilibrium dynamics in N = 4 supersymmetric Yang-Mills theory. Phys. Rev. D **82**, 026006 (2010). doi:10.1103/PhysRevD.82.026006. ArXiv:0906.4426 [hep-th]
54. Chesler, P.M., Yaffe, L.G.: Holography and colliding gravitational shock waves in asymptotically AdS_5 spacetime. Phys. Rev. Lett. **106**, 021601 (2011). doi:10.1103/PhysRevLett.106.021601. ArXiv:1011.3562 [hep-th]
55. Chesler, P.M., Yaffe, L.G.: Numerical solution of gravitational dynamics in asymptotically anti-de Sitter spacetimes (2013). ArXiv:1309.1439 [hep-th]
56. Choptuik, M.W.: Universality and scaling in graviational collapse of a massless scalar field. Phys. Rev. Lett. **70**, 9–12 (1993). doi:10.1103/PhysRevLett.70.9
57. Choptuik, M.W., Pretorius, F.: Ultra relativistic particle collisions. Phys. Rev. Lett. **104**, 111101 (2010). doi:10.1103/PhysRevLett.104.111101. ArXiv:0908.1780 [gr-qc]
58. Dai, D.C., Starkman, G., Stojkovic, D., Issever, C., Rizvi, E., et al.: BlackMax: a black-hole event generator with rotation, recoil, split branes, and brane tension. Phys. Rev. D **77**, 076007 (2008). doi:10.1103/PhysRevD.77.076007. ArXiv:0711.3012 [hep-ph]
59. Damour, T., Nagar, A.: Comparing effective-one-body gravitational waveforms to accurate numerical data. Phys. Rev. D **77**, 024043 (2008). ArXiv:0711.2628 [gr-qc]
60. Damour, T., Nagar, A., Hannam, M.D., Husa, S., Brügmann, B.: Accurate effective-one-body waveforms of inspiralling and coalescing black-hole binaries. Phys. Rev. D **78**, 044039 (2008). ArXiv:[0803.3162]
61. Damour, T., Trias, M., Nagar, A.: Accuracy and effectualness of closed-form, frequency-domain waveforms for non-spinning black hole binaries. Phys. Rev. D **83**, 024006 (2011). doi:10.1103/PhysRevD.83.024006. ArXiv:1009.5998 [gr-qc]
62. Dias, O.J.C., Horowitz, G.T., Marolf, D., Santos, J.E.: On the nonlinear stability of asymptotically anti-de sitter solutions. Class. Quantum Grav. **29**, 235019 (2012). doi:10.1088/0264-9381/29/23/235019. ArXiv:1208.5772 [gr-qc]
63. Dimopoulos, S., Landsberg, G.: Black Holes at the LHC. Phys. Rev. Lett. **87**, 161602 (2001). doi:10.1103/PhysRevLett.87.161602. Hep-th/0106295
64. Dolan, S.R.: Superradiant instabilities of rotating black holes in the time domain. Phys. Rev. D **87**, 124026 (2013). doi:10.1103/PhysRevD.87.124026. ArXiv:1212.1477 [gr-qc]
65. Drude, P.: Zur elektronentheorie der metalle. Annalen der Physik **306**, 566 (1900). doi:10.1002/andp.19003060312
66. Drude, P.: Zur elektronentheorie der metalle: II. Teil. galvanomagnetische und thermomagnetische effekte. Annalen der Physik **308**, 369 (1900). doi:10.1002/andp.19003081102
67. Eardley, D.M., Giddings, S.B.: Classical black hole production in high-energy collisions. Phys. Rev. D **66**, 044011 (2002). doi:10.1103/PhysRevD.66.044011. Gr-qc/0201034
68. East, W.E., Pretorius, F.: Ultrarelativistic black hole formation. Phys. Rev. Lett. **110**, 101101 (2013). doi:10.1103/PhysRevLett.110.101101. ArXiv:1210.0443 [gr-qc]
69. Emparan, R., Reall, H.S.: Black holes in higher dimensions. Living Rev. Rel. **11**(6) (2008). http://www.livingreviews.org/lrr-2008-6
70. Farris, B.D., Gold, R., Paschalidis, V., Etienne, Z.B., Shapiro, S.L.: Binary black hole mergers in magnetized disks: simulations in full general relativity. Phys. Rev. Lett. **109**, 221102 (2012). doi:10.1103/PhysRevLett.109.221102. ArXiv:1207.3354 [astro-ph]
71. Farris, B.D., Liu, Y.T., Shapiro, S.L.: Binary black hole mergers in gaseous environments: 'Binary Bondi' and 'Binary Bondi-Hoyle-Lyttleton' accretion. Phys. Rev. D **81**, 084008 (2010). doi:10.1103/PhysRevD.81.084008. ArXiv:0912.2096 [asttro-ph]

72. Finn, L.S.: Detection, measurement and gravitational radiation. Phys. Rev. D **46**, 5236–5249 (1992). Gr-qc/9209010
73. Fitchett, M.J.: The influence of gravitational wave momentum losses on the centre of mass motion of a newtonian binary system. MNRAS **203**, 1049–1062 (1983)
74. Frost, J.A., Gaunt, J.R., Sampaio, M.O.P., Casals, M., Dolan, S.R., et al.: Phenomenology of Production and Decay of Spinning Extra- Dimensional Black Holes at Hadron Colliders. JHEP **10**, 014 (2009). doi:10.1088/1126-6708/2009/10/014. ArXiv:0904.0979 [hep-th]
75. Garfinkle, D.: Harmonic coordinate method for simulating generic singularities. Phys. Rev. D **65**, 044029 (2002). doi:10.1103/PhysRevD.65.044029. Gr-qc/0110013
76. Giddings, S.B., Thomas, S.: High energy colliders as black hole factories: the end of short distance physics. Phys. Rev. D **65**, 056010 (2002). doi:10.1103/PhysRevD.65.056010. Hep-ph/0106219
77. González, J.A., Hannam, M.D., Sperhake, U., Brügmann, B., Husa, S.: Supermassive kicks for spinning black holes. Phys. Rev. Lett. **98**, 231101 (2007). doi:10.1103/PhysRevLett.98.231101. Gr-qc/0702052
78. González, J.A., Sperhake, U., Brügmann, B., Hannam, M.D., Husa, S.: The maximum kick from nonspinning black-hole binary inspiral. Phys. Rev. Lett. **98**, 091101 (2007). doi:10.1103/PhysRevLett.98.091101. Gr-qc/0610154
79. Gregory, R., Laflamme, R.: Black strings and p-branes are unstable. Phys. Rev. Lett. **70**, 2837–2840 (1993). doi:10.1103/PhysRevLett.70.2837. Hep-th/9301052
80. Gregory, R., Laflamme, R.: The Instability of charged black strings and p-branes. Nucl. Phys. B **428**, 399–434 (1994). doi:10.1016/0550-3213(94)90206-2. Hep-th/9404071
81. Gubser, S.S., Klebanov, I.R., Polyakov, A.M.: Gauge theory correlators from non-critical string theory. Phys. Lett. B **428**, 105–114 (1998). doi:10.1016/S0370-2693(98)00377-3. Hep-th/9802109
82. Guedes, J., Madau, P., Mayer, L., Callegari, S.: Recoiling massive black holes in gas-rich galaxy mergers. Astrophys. J. **729**, 125 (2011). ArXiv:1008.2032 [astro-ph]
83. Gundlach, C., Calabrese, G., Hinder, I., Martín-García, J.M.: Constraint damping in the Z4 formulation and harmonic gauge. Class. Quantum Grav. **22**, 3767–3773 (2005). doi:10.1088/0264-9381/22/17/025
84. Haiman, Z., Loeb, A.: What is the highest plausible redshift of luminous quasars? Astrophys. J. **552**, 459–463 (2001)
85. Hannam, M., et al.: Twist and shout: a simple model of complete precessing black-hole-binary gravitational waveforms (2013). ArXiv:1308.3271 [gr-qc]
86. de Haro, S., Solodukhin, S.N., Skenderis, K.: Holographic reconstruction of space-time and renormalization in the AdS/CFT correspondence. Commun. Math. Phys. **217**, 595–622 (2001). doi:10.1007/s002200100381. Hep-th/0002230
87. Harris, C.M., Richardson, P., Webber, B.R.: CHARYBDIS: a black hole event generator. JHEP **0308**, 033 (2003). Hep-ph/0307305
88. Hawking, S.W., Ellis, G.F.R.: The Large Scale Structure of Space-Time. Cambridge University Press, Cambridge (1973)
89. Healy, J., Bode, T., Haas, R., Pazos, E., Laguna, P., Shoemaker, D.M., Yunes, N.: Late inspiral and merger of binary black holes in scalar-tensor theories of gravity (2011). ArXiv:1112.3928 [gr-qc]
90. Heller, M.P., Janik, R.A., Witaszczyk, P.: A numerical relativity approach to the initial value problem in asymptotically anti-de Sitter spacetime for plasma thermalization—an ADM formulation. Phys. Rev. D **85**, 126002 (2012). doi:10.1103/PhysRevD.85.126002. ArXiv:1203.0755 [hep-th]
91. Heller, M.P., Janik, R.A., Witaszczyk, P.: The characteristics of thermalization of boost-invariant plasma from holography. Phys. Rev. Lett. **108**, 201602 (2012). doi:10.1103/PhysRevLett.108.201602. ArXiv:1103.3452 [hep-th]
92. Heller, M.P., Mateos, D., van der Schee, W., Trancanelli, D.: Strong coupling isotropization of non-abelian plasmas simplified. Phys. Rev. Lett. **108**, 191601 (2012). doi:10.1103/PhysRevLett.108.191601. ArXiv:1202.0981 [hep-th]
93. Hemberger, D.A., Lovelace, G., Loredo, T.J., Kidder, L.E., Scheel, M.A., Szilágyi, B., Taylor, N.W., Teukolsky, S.A.: Final spin and radiated energy in numerical simulations of binary black holes with equal masses and equal, aligned or anti-aligned spins. Phys. Rev. D **88**, 064014 (2013). doi:10.1103/PhysRevD.88.064014. ArXiv:1305.5991 [gr-qc]
94. Hilditch, D., Bernuzzi, S., Thierfelder, M., Cao, Z., Tichy, W., Brügmann, B.: Compact binary evolutions with the Z4c formulation. Phys. Rev. D **88**, 084057 (2013). doi:10.1103/PhysRevD.88.084057. ArXiv:1212.2901 [gr-qc]

95. Hinder, I., et al.: Error-analysis and comparison to analytical models of numerical waveforms produced by the NRAR collaboration . Class. Quant. Grav. **31**, 025012 (2014). ArXiv:1307.5307 [gr-qc]
96. Horbatsch, M.W., Burgess, C.P.: Cosmic black-hole hair growth and quasar OJ287. J. Cosmol. Astropart. Phys. **1205**, 010 (2012). doi:10.1088/1475-7516/2012/05/010. ArXiv:1111.4009 [gr-qc]
97. Horowitz, G.T., Hubeny, V.E.: Quasinormal modes of AdS black holes and the approach to thermal equilibrium. Phys. Rev. D **62**, 024027 (2000). doi:10.1103/PhysRevD.62.024027. Hep-th/9909056
98. Horowitz, G.T., Santos, J.E., Tong, D.: Further evidence for lattice-induced scaling. JHEP **1211**, 102 (2012). doi:10.1007/JHEP11(2012)102. ArXiv:1209.1098 [hep-th]
99. Horowitz, G.T., Santos, J.E., Tong, D.: Optical conductivity with holographic lattices. JHEP **1207**, 168 (2012). doi:10.1007/JHEP07(2012)168. ArXiv:1204.0519 [hep-th]
100. Jałmużna, J., Rostworowski, A., Bizoń, P.: A Comment on AdS collapse of a scalar field in higher dimensions. Phys. Rev. D **84**, 085021 (2011). doi:10.1103/PhysRevD.84.085021. ArXiv:1108.4539 [gr-qc]
101. Kanti, P.: Black holes at the LHC. Lect. Notes Phys. **769**, 387–423 (2009). doi:10.1007/978-3-540-88460-6_10. ArXiv:0802.2218 [hep-th]
102. Kesden, M., Sperhake, U., Berti, E.: Final spins from the merger of precessing binary black holes. Phys. Rev. D **81**, 084054 (2010). ArXiv:1002.2643 [astro-ph]
103. Kesden, M., Sperhake, U., Berti, E.: Relativistic suppression of black hole recoils. Astrophys. J. **715**, 1006–1011 (2010). ArXiv:1003.4993 [astro-ph]
104. Klebanov, I.R.: TASI lectures: introduction to the AdS/CFT correspondence. pp. 615–650 (2000). Hep-th/0009139
105. Komossa, S., Zhou, H., Lu, H.: A recoiling supermassive black hole in the quasar SDSSJ092712.65+294344.0? Astrophys. J. **678**, L81 (2008). ArXiv:0804.4585 [astro-ph]
106. Lehner, L.: Numerical relatvity: a review. Class. Quantum Grav. **18**, R25–R86 (2001). doi:10.1088/0264-9381/18/17/202. Gr-qc/0106072
107. Lehner, L., Pretorius, F.: Black strings, low viscosity fluids, and violation of cosmic censorship. Phys. Rev. Lett. **105**, 101102 (2010). doi:10.1103/PhysRevLett.105.101102. ArXiv:1006.5960 [hep-th]
108. Li, Y., et al.: Formation of $z\tilde{}6$ quasars from hierarchical galaxy mergers. Astrophys. J. **665**, 187–208 (2007). Astro-ph/0608190
109. Lindblom, L.: Use and abuse of the model waveform accuracy standards. Phys. Rev. D **80**, 064019 (2009). ArXiv:0907.0457 [gr-qc]
110. Lindblom, L., Baker, J.G., Owen, B.J.: Improved time-domain accuracy standards for model gravitational waveforms. Phys. Rev. D **82**, 084020 (2010). ArXiv:1008.1803 [gr-qc]
111. MacDonald, I., Nissanke, S., Pfeiffer, H.P.: Suitability of post-Newtonian/numerical-relativity hybrid waveforms for gravitational wave detectors. Class. Quantum Grav. **28**, 134002 (2011). doi:10.1088/0264-9381/28/13/134002. ArXiv:1102.5128 [gr-qc]
112. Magorrian, J., Tremaine, S., Richstone, D., Bender, R., Bower, G., Dressler, A., Faber, S.M., Gebhardt, K., Green, R., Grillmair, C., Kormendy, J., Lauer, T.: The demography of massive dark objects in galaxy centers. Astron. J **115**, 2285–2305 (1998). Astro-ph/9708072
113. Maldacena, J.M.: The large N limit of superconformal field theories and supergravity. Adv. Theor. Math. Phys. **2**, 231 (1997). Hep-th/9711200
114. Maliborski, M., Rostworowski, A.: Time-periodic solutions in Einstein AdS—massless scalar field system. Phys. Rev. Lett. **111**, 051102 (2013). doi:10.1103/PhysRevLett.111.051102. ArXiv:1303.3186 [gr-qc]
115. Megevand, M.: Perturbed disks get shocked. Binary black hole merger effects on accretion disks. Phys. Rev. **D80**, 024012 (2009). doi:10.1103/PhysRevD.80.024012. ArXiv:0905.3390 [astro-ph]
116. Merritt, D., Milosavljević, M., Favata, M., Hughes, S., Holz, D.: Consequences of gravitational radiation recoil. Astrophys. J. **607**, L9–L12 (2004). Astro-ph/0402057
117. Mösta, P., et al.: Vacuum electromagnetic counterparts of binary black-hole mergers. Phys. Rev. D **81**, 064017 (2010). doi:10.1103/PhysRevD.81.064017. ArXiv:0912.2330 [gr-qc]
118. Mroué, A.H., et al.: A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy. Phys. Rev. Lett. **111**, 241104 (2013). ArXiv:1304.6077 [gr-qc]
119. Nastase, H.: Introduction to AdS-CFT (2007). Report number TIT-HEP-578. ArXiv:0712.0689 [hep-th].
120. Okawa, H., Nakao, K.I., Shibata, M.: Is super-Planckian physics visible? Scattering of black holes in 5 dimensions. Phys. Rev. D **83**, 121501 (2011). ArXiv:1105.3331 [gr-qc]

121. Palenzuela, C., Anderson, M., Lehner, L., Liebling, S.L., Neilsen, D.: Stirring, not shaking: binary black holes' effects on electromagnetic fields. Phys. Rev. Lett. **103**, 081101 (2009). doi:10.1103/PhysRevLett.103.081101. ArXiv:0905.1121 [astro-ph]
122. Palenzuela, C., Garrett, T., Lehner, L., Liebling, S.L.: Magnetospheres of black hole systems in force-free plasma. Phys. Rev. D **82**, 044045 (2010). doi:10.1103/PhysRevD.82.044045. ArXiv:1007.1198 [gr-qc]
123. Palenzuela, C., Lehner, L., Liebling, S.L.: Dual jets from binary black holes. Science **329**, 927 (2010). doi:10.1126/science.1191766. ArXiv:1005.1067 [astro-ph]
124. Palenzuela, C., Lehner, L., Yoshida, S.: Understanding possible electromagnetic counterparts to loud gravitational wave events: binary black hole effects on electromagnetic fields. Phys. Rev. D **81**, 084007 (2010). doi:10.1103/PhysRevD.81.084007. ArXiv:0911.3889 [gr-qc]
125. Pan Yi. and Buonanno, A., Boyle, M., Buchman, L.T., Kidder L, E., Pfeiffer, H.P., Scheel, M.A.: Inspiral-merger-ringdown multipolar waveforms of nonspinning black-hole binaries using the effective-one-body formalism. Phys. Rev. D **84**, 124052. doi:10.1103/PhysRevD.84.124052. ArXiv:1106.1021 [gr-qc]
126. Pan, Y., Buonanno, A., Buchman, L.T., Chu, T., Kidder, L.E., Pfeiffer, H.P., Scheel, M.A.: Effective-one-body waveforms calibrated to numerical relativity simulations: coalescence of nonprecessing, spinning, equal-mass black holes. Phys. Rev. D **81**, 084041 (2010). doi:10.1103/PhysRevD.81.084041. ArXiv:0912.3466 [gr-qc]
127. Pani, P., Cardoso, V., Gualtieri, L., Berti, E., Ishibashi, A.: Black hole bombs and photon mass bounds. Phys. Rev. Lett. **109**, 131102 (2012). doi:10.1103/PhysRevLett.109.131102. ArXiv:1209.0465 [gr-qc]
128. Pekowsky, L., O'Shaughnessy, R., Healy, J., Shoemaker, D.: Comparing gravitational waves from nonprecessing and precessing black hole binaries in the corotating frame. Phys. Rev. D **88**, 024040 (2013). doi:10.1103/PhysRevD.88.024040. ArXiv:1304.3176 [gr-qc]
129. Peres, A.: Classical radiation recoil. Phys. Rev. **128**, 2471–2475 (1962)
130. Peters, P.C.: Gravitational radiation and the motion of two point masses. Phys. Rev. **136**, B1224–B1232 (1964)
131. Pfeiffer, H.P.: Numerical simulations of compact object binaries. Class. Quantum Grav. **29**, 124004 (2012). doi:10.1088/0264-9381/29/12/124004. ArXiv:1203.5166 [gr-qc]
132. Pretorius, F.: Evolution of binary black-hole spacetimes. Phys. Rev. Lett. **95**, 121101 (2005). doi:10.1103/PhysRevLett.95.121101. Gr-qc/0507014
133. Pretorius, F.: Numerical relativity using a generalized harmonic decomposition. Class. Quantum Grav. **22**, 425–452 (2005). doi:10.1088/0264-9381/22/2/014. Gr-qc/0407110
134. Pretorius, F.: Binary black hole coalescence. In: Colpi, M., et al. (ed.) Physics of relativistic objects in compact binaries: from birth to coalescence. Springer, New York (2009). ArXiv:0710.1338 [gr-qc]
135. Pretorius, F., Khurana, D.: Black hole mergers and unstable circular orbits. Class. Quantum Grav. **24**, S83–S108 (2007). doi:10.1088/0264-9381/24/12/S07. Gr-qc/0702084
136. Pürrer, M., Hannam, M., Ajith, P., Husa, S.: Testing the validity of the single-spin approximation in inspiral-merger-ringdown waveforms. Phys. Rev. D **88**, 064007 (2013). doi:10.1103/PhysRevD.88.064007. ArXiv:1306.2320 [gr-qc]
137. Randall, L., Sundrum, R.: A large mass hierarchy from a small extra dimension. Phys. Rev. Lett. **83**, 3370–3373 (1999). doi:10.1103/PhysRevLett.83.3370. Hep-ph/9905221
138. Randall, L., Sundrum, R.: An alternative to compactification. Phys. Rev. Lett. **83**, 4690–4693 (1999). doi:10.1103/PhysRevLett.83.4690. Hep-th/9906064
139. Rees, M.J.: Accretion and the quasar phenomenon. Phys. Scr. **17**, 193–200 (1978)
140. Rinne, O., Lindblom, L., Scheel, M.A.: Testing outer boundary treatments for the Einstein equations. Class. Quantum Grav. **24**, 4053–4078 (2007). doi:10.1088/0264-9381/24/16/006. ArXiv:0704.0782 [gr-qc]
141. Ruiz, M., Rinne, O., Sarbach, O.: Outer boundary conditions for Einstein's field equations in harmonic coordinates. Class. Quantum Grav. **24**, 6349–6378 (2007). doi:10.1088/0264-9381/24/24/012. ArXiv:0707.2797 [gr-qc]
142. Santamaria, L.: L.: Matching post-newtonian and numerical relativity waveforms: systematic errors and a new phenomenological model for non-precessing black hole binaries. Phys. Rev. D **82**, 064016 (2010). ArXiv:1005.3306
143. Sasaki, M., Tagoshi, H.: Analytic black hole perturbation approach to gravitational radiation. Living Rev. Rel. **6**(6) (2003). http://www.livingreviews.org/lrr-2003-6

144. Schnittman, J.D.: Spin-orbit resonance and the evolution of compact binary systems. Phys. Rev. D **70**, 124020 (2004). Astro-ph/0409174
145. Shibata, M., Nakamura, T.: Evolution of three-dimensional gravitational waves: harmonic slicing case. Phys. Rev. D **52**, 5428–5444 (1995). doi:10.1103/PhysRevD.52.5428
146. Shibata, M., Okawa, H., Yamamoto, T.: High-velocity collisions of two black holes. Phys. Rev. D **78**, 101501(R) (2008). doi:10.1103/PhysRevD.78.101501. ArXiv:0810.4735 [gr-qc]
147. Shibata, M., Yoshino, H.: Bar-mode instability of rapidly spinning black hole in higher dimensions: numerical simulation in general relativity. Phys. Rev. D **81**, 104035 (2010). doi:10.1103/PhysRevD.81.104035. ArXiv:1004.4970 [gr-qc]
148. Shibata, M., Yoshino, H.: Nonaxisymmetric instability of rapidly rotating black hole in five dimensions. Phys. Rev. D **81**, 021501 (2010). doi:10.1103/PhysRevD.81.021501. ArXiv:0912.3606 [gr-qc]
149. Sorkin, E., Oren, Y.: On Choptuik's scaling in higher dimensions. Phys. Rev. D **71**, 124005 (2005). doi:10.1103/PhysRevD.71.124005. Hep-th/0502034
150. Sperhake, U.: Numerical relativity in higher dimensions. Int. J. Mod. Phys. D **22**, 1330005 (2013). doi:10.1142/S021827181330005X. ArXiv:1301.3772 [gr-qc]
151. Sperhake, U., Berti, E., Cardoso, V.: Numerical simulations of black-hole binaries and gravitational wave emission. C. R. Phys. **14**, 306–317 (2013). doi:10.1016/j.crhy.2013.01.004. ArXiv:1107.2819 [gr-qc]
152. Sperhake, U., Berti, E., Cardoso, V., Pretorius, F.: Universality, maximum radiation and absorption in high-energy collisions of black holes with spin. Phys. Rev. Lett. **111**, 041101 (2013). doi:10.1103/PhysRevLett.111.041101. ArXiv:1211.6114 [gr-qc]
153. Sperhake, U., Brügmann, B., Müller, D., Sopuerta, C.F.: 11-orbit inspiral of a mass ratio 4:1 black-hole binary. Class. Quantum Grav. **28**, 134004 (2011). doi:10.1088/0264-9381/28/13/134004. ArXiv:1012.3173 [gr-qc]
154. Sperhake, U., Cardoso, V., Pretorius, F., Berti, E., González, J.A.: The high-energy collision of two black holes. Phys. Rev. Lett. **101**, 161101 (2008). doi:10.1103/PhysRevLett.101.161101. ArXiv:0806.1738 [gr-qc]
155. Sperhake, U., Cardoso, V., Pretorius, F., Berti, E., Hinderer, T., Yunes, N.: Cross section, final spin and zoom-whirl behavior in high-energy black hole collisions. Phys. Rev. Lett. **103**, 131102 (2009). doi:10.1103/PhysRevLett.103.131102. ArXiv:0907.1252 [gr-qc]
156. Sturani, R., et al.: Complete phenomenological gravitational waveforms from spinning coalescing binaries. J. Phys. Conf. Ser. **243**, 012007 (2010). ArXiv:1005.0551 [gr-qc]
157. Taracchini, A.: A.: A prototype effective-one-body model for non-precessing spinning inspiral-merger-ringdown waveforms. Phys. Rev. D **86**, 024011 (2012). doi:10.1103/PhysRevD.86.024011. ArXiv:1202.0790 [gr-qc]
158. Taracchini, A, et al.: Effective-one-body model for black-hole binaries with generic mass ratios and spins (2013). ArXiv:1311.2544 [gr-qc]
159. van Paradijs, J.: Gamma-ray Bursts. In: American Astronomical Society Meeting Abstracts. Bulletin of the American Astronomical Society, vol. 30, pp. 1291-+ (1998). ArXiv:astro-ph/9802177.
160. Volonteri, M., Gültekin, K., Dotti, M.: Gravitational recoil: effects on massive black hole occupation fraction over cosmic time. MNRAS **404**, 2143–2150 (2010). ArXiv:1001.1743 [astro-ph]
161. Witek, H., Cardoso, V., Gualtieri, L., Herdeiro, C., Sperhake, U., Zilhão, M.: Head-on collisions of unequal mass black holes in $d = 5$ dimensions. Phys. Rev. D **83**, 044017 (2011). ArXiv:1011.0742 [gr-qc]
162. Witek, H., Cardoso, V., Ishibashi, A., Sperhake, U.: Superradiant instabilities in astrophysical systems. Phys. Rev. D **87**, 043513 (2013). doi:10.1103/PhysRevD.87.043513. ArXiv:1212.0551 [gr-qc]
163. Witek, H., Zilhão, M., Gualtieri, L., Cardoso, V., Herdeiro, C., Nerozzi, A., Sperhake, U.: Numerical relativity for D dimensional space-times: head-on collisions of black holes and gravitational wave extraction. Phys. Rev. D **82**, 104014 (2010). doi:10.1103/PhysRevD.82.104014. ArXiv:1006.3081 [gr-qc]
164. Witten, E.: Anti-de Sitter space and holography. Adv. Theor. Math. Phys. **2**, 253–291 (1998). Hep-th/9802150
165. Yoshino, H., Shibata, M.: Higher-dimensional numerical relativity: current status. Prog. Theor. Phys. Suppl. **189**, 269–310 (2011). doi:10.1143/PTPS.189.269
166. Zel'dovich, Y.B.: Pis'ma. Zh. Eksp. Teor. Fiz. **14**, 270 (1971)
167. Zel'dovich, Y.B.: Zh. Eksp. Teor. Fiz **62**, 2076 (1972)

168. Zilhão, M.: New frontiers in Numerical Relativity. Ph.D. thesis, University of Porto (2012). ArXiv:1301.1509 [gr-qc]
169. Zilhão, M., Ansorg, M., Cardoso, V., Gualtieri, L., Herdeiro, C., Sperhake, U., Witek, H.: Higher-dimensional puncture initial data. Phys. Rev. D **84**, 084039 (2011). ArXiv:1109.2149 [gr-qc]
170. Zilhao, M., Cardoso, V., Gualtieri, L., Herdeiro, C., Sperhake, U., Witek, H.: Dynamics of black holes in de Sitter spacetimes. Phys. Rev. D **85**, 104039 (2012). doi:10.1103/PhysRevD.85.104039. ArXiv:1204.2019 [gr-qc]
171. Zilhao, M., Cardoso, V., Herdeiro, C., Lehner, L., Sperhake, U.: Collisions of charged black holes. Phys. Rev. D **85**, 124062 (2012). doi:10.1103/PhysRevD.85.124062. ArXiv:1205.1063 [gr-qc].
172. Zilhão, M., Cardoso, V., Herdeiro, C., Lehner, L., Sperhake, U.: Collisions of oppositely charged black holes. Phys. Rev. D **89**, 044008 (2014). ArXiv:1311.6483 [gr-qc]
173. Zilhão, M., Witek, H., Sperhake, U., Cardoso, V., Gualtieri, L., Herdeiro, C., Nerozzi, A.: Numerical relativity for D dimensional axially symmetric space-times: formalism and code tests. Phys. Rev. D **81**, 084052 (2010). doi:10.1103/PhysRevD.81.084052. ArXiv:1001.2302 [gr-qc]
174. Ninja homepage: https://www.ninja-project.org/doku.php
175. NRAR homepage: https://www.ninja-project.org/doku.php?id=nrar:home