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REVIEW ARTICLE

Summary of session B3 at GR20/Amaldi10

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Abstract A wide variety of results was presented in session B3, the "non-astrophysical" numerical relativity parallel session. some results included improved numerical methods for such things as asymptotically flat spacetimes, generation of initial data, and characterization of binary black hole systems. Others included the propagation of various types of matter fields in the presence of black holes, naked singularities, and wormholes. There were also several simulations of spacetimes asymptotic to anti-de Sitter space. These simulations are of interest both for general relativity and (through the AdS/CFT correspondence) for the behavior of quantum field theories.

Keyword Numerical relativity · AdS/CFT · Numerical methods · Black holes

1 Introduction

Parallel session B3 at GR20/Amaldi10 was titled "Numerical relativity: methods, theoretical gravity and high energy applications." However, it would have been just as accurate to call the session "non-astrophysical numerical relativity." Numerical relativity is the computer simulation of the Einstein field equations. An important use of numerical relativity is in astrophysical situations where gravity is strong, especially those involving black holes or neutron stars. Presentations on the astrophysical uses of numerical relativity were done in parallel session B2, and "all the rest" of numerical relativity took place in parallel session B3. Nonetheless, it is possible to categorize

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"all the rest," and for that purpose the categories "methods," "theoretical gravity" and "high energy applications" are very useful.

As with any numerical field, in numerical relativity one is always in search of new methods to make the simulations more accurate, or more stable, or more efficient, or simply to cover a physical situation that has never been simulated before and for which the standard methods do not suffice. Numerical simulations are useful to help us understand the nature of strong gravity, both in vacuum and in the presence of various types of matter. For the purposes of this article, such results will be classified as "theoretical gravity." Through the conjectured AdS/CFT correspondence, the properties of gravity in an asymptotically anti-de Sitter $n + 1$ dimensional spacetime correspond to the properties of a strongly coupled n dimensional quantum field theory. Thus classical relativity simulations have applications to high energy physics.

2 Methods

Numerical simulations take place on a finite grid, but the physical system simulated is usually of infinite spatial extent. One way to handle this difficulty is to introduce compactified coordinates, where at a finite value of the radial coordinate one gets all the way out to null infinity. This is also helpful for reading off the properties of radiation. Spatial slices that get out to null infinity are called hyperboloidal. One tricky issue is how to handle the apparently singular behavior of the field equations at null infinity. Oliver Rinne gave a talk on "Hyperboloidal evolution of the Einstein-Yang-Mills system." In this work they treat the spherically symmetric Einstein-Yang-Mills system and in particular critical gravitational collapse. Rodrigo Macedo gave a talk on "Fully spectral code for linear axi-symmetric wave equations on hyperboloidal foliations." This code also uses hyperboloidal slices and a compactified coordinate. However, rather than using finite difference methods, the code is spectral in space and time. This work treats the Teukolsky equation for perturbations of the Kerr spacetime, and provides a very accurate result for the tail decay of the perturbations. Jörg Frauendiener also treats the field equations using a finite grid. However, rather than treat the Einstein equation, he uses the conformal field equations in which the variables include a metric that is conformal to the physical metric and the corresponding conformal factor. In a talk titled "Linearized gravitational fields near spacelike and null infinity" [1] he presented work on the linearized Einstein equation. Here initial data that reaches spatial infinity is evolved and used to provide data that reaches null infinity.

In order to simulate the time evolution of a gravitating system, one must start with initial data. But in general relativity initial data cannot be freely specified: it must satisfy a set of nonlinear elliptic constraint equations. The usual approach to numerically solving nonlinear elliptic equations is to start with a guess for the solution and then iterate to get a series of guesses that converges to a solution. But this only works if the iterative method is convergent. Charalampos Markakis presented a talk titled "Iterative stability of the Einstein constraint equations." The result is that there are several different ways one could perform the iteration, some of which are stable and some of which are unstable. Much of the effort in numerical relativity goes towards simulating the collision of black holes, and one of the most used methods is the BSSN variables for the Einstein equations combined with the “moving puncture” method for chosing the initial data and gauge. Tim Dietrich gave a talk on “Black hole spin within the moving puncture method: Numerical experiments.” He shows that with this choice of gauge there is a simple way to read off the black hole spin from the behavior of the spatial metric and extrinsic curvature. Binary black hole simulations depend on the masses and spins of the black holes, so it would take much too long to do an exhaustive search of parameter space to find all the waveforms that could be produced by such collisions. Instead one would like to sample points in parameter space and use those to estimate the waveforms produced for all parameters. Chad Galley in his talk on “Reduced order modeling of binary black hole coalescences” shows how to do this in an efficient and accurate way [2].

3 Theoretical gravity

Olivier Sarbach gave a talk on “Resonances of massive scalar fields propagating on a black hole spacetime.” [3] They find, numerically and analytically, quasi-bound modes of a scalar field in the presence of a black hole. Peter Diener gave a talk on “Numerical challenges in the evolution of non-singular ’quantum universes”’ [4]. He shows that loop quantum cosmology gives rise to finite difference equations that can be treated numerically. The simulations gives a bouncing universe that avoids a big crunch singularity and instead turns it into a big bang. The collapse of dust can give rise to a naked singularity. In order to obtain an understanding of the nature of this singularity, Nestor Ortiz performed numerical simulations of the behavior of test fields in the presence of the collapse. The results of these simulations were presented in his talk “Stability of test fields propagating on a collapsing spherical dust cloud.” Anna Nakonieczna gave a talk on “Formation of wormholes in gravitational collapse.” The system simulated was an electrically charged scalar field and a phantom dilaton field in spherical symmetry [5]. The collapse resulted in dynamical wormholes. Hirotada Okawa and Helvi Witek gave talks on what they called “The black hole bomb mechanism” [6]. This is essentially a “boson star” type scalar field in the presence of a black hole with superradiant scattering and mirror type boundary conditions to reflect the scalar field back towards the black hole. They perform numerical simulations and find that the scalar field settles into an accretion disk around the black hole. Hirotaka Yoshino gave a talk on “Axion bosenova and gravitational waves” [7]. They simulate the superradiant scatering of axions in the presence of a black hole and find the gravitational radiation produced by this process.

4 High energy applications

Though Minkowski spacetime and de-Sitter spacetime are stable, it has recently been shown [8] that anti-de Sitter spacetime is unstable. This leads to the question of whether arbitrary perturbations of AdS lead to instability and black hole formation. Maciej Maliborski, in his talk “Time periodic solutions in Einstein AdS massless scalar field system” [9] shows that this is not the case. Their solutions are “islands of stability” in the largely unstable “ocean” of perturbed AdS. Alex Buchel gave a talk on “Quantum quenches in strongly coupled systems from AdS/CFT correspondence" [10]. They model quantum quenches by introducing time dependent behavior of a scalar field near the AdS boundary and seeing how the result propagates through the spacetime and back to the boundary. They find types of universal behavior of the quenches. The use of more than 4 spacetime dimensions has given rise to a multitude of black holes and related objects, such as black strings. Many of these are not known in closed form, so it is reasonable to see whether they can be found numerically. Pau Figueras uses harmonic coordinates to find these solutions [11]. He presented these results in a talk entitled "Numerical methods for stationary non-Killing horizons and applications to holography." One type of "black object" is a black hole on the boundary of AdS that also extends into the bulk spacetime. Sebastian Fischetti studies these objects as models of heat flow in the corresponding field theory. He presented this study in a talk titled "Black funnels and black droplets: heat transport in strongly coupled CFTs from holography" [12].

5 Conclusion

There is a wide range of topics treated in non-astrophysical numerical relativity. With the recent addition of numerical studies of asymptotically AdS spacetimes, the field has changed markedly in the last three years. It will be interesting to see what numerical work is presented in 2016 at GR21.

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