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RESEARCH ARTICLE

Energy conditions in generalized teleparallel gravity models

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Abstract In this paper, we investigate the energy conditions (including null, weak, strong, dominant) in generalized teleparallel gravities including pure $F(T)$, teleparallel gravity with non-minimally coupled scalar field and $F(T)$ with non-minimally coupled scalar field models. In particular, we apply them to Friedmann–Robertson–Walker cosmology and obtain some corresponding results. Using two specific phenomenological forms of $F(T)$, we show that some of the energy conditions are violated.

Keywords Energy conditions · Torsion · Cosmology

1 Introduction

Numerous astrophysical observations [1–6] show that our Universe is undergoing a period of accelerated expansion and which started in the near past. There are two broad categories to classify candidates of cosmic acceleration. The first is to consider an exotic energy with negative pressure, termed 'dark energy'. The simplest candidate of dark energy is the cosmological constant satisfying an equation of state $w = -1$ [7–11]. It, however, suffers from two serious theoretical problems, i.e., the fine tuning

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problem and the cosmic coincidence problem. Also some scalar field models, including quintessence field [12–15] and phantom energy [16–27], are discussed. For single scalar field models, it has been shown in the literature that the equation of state cannot cross the phantom divide line ($w = -1$). Therefore, models with a combination of phantom and quintessence commonly termed 'quintom' [28], and scalar field models with scalar-dependent coupling in front of kinetic term [29–31] as well as fluid models [32] have also been constructed to realize the crossing of the phantom divide line, which still seems to be allowed by recent observations [33–40].

There are four candidates of the gravitational sector of model: (A) metric compatible and torsionless (Einstein gravity); (B) Weyl's type and torsionless (Weyl space); (C) metric compatible with torsion (Einstein–Cartan space); and (D) Weyl's type with torsion (Einstein–Cartan–Weyl theory). A recent version of torsion based gravity is $F(T)$ (commonly termed 'generalized teleparallel gravity') which is based on the Einstein–Cartan geometry [41–44], where T is the torsion scalar constructed from the tetrad. Choosing $F(T) = T$, leads to the pure teleparallel gravity [45–47] which is in good agreement with some standard tests of the general relativity at the solar system scale [45,46]. Numerous features of theoretical interest have been studied in this gravity already including Birkhoff's theorem [48], cosmological perturbations [49], cosmological attractor solutions [50], generalized second law in $F(T)$ [51] and phantom crossing of the state parameter [52]. Moreover, the local Lorentz invariance is violated which henceforth leads to violation of first law of thermodynamics [53,54]. Also the entropy-area relation in this gravity takes a modified form Bamba et al. [51]. The Hamiltonian structure of $F(T)$ gravity has been investigated and found that there are five degrees of freedom [55]. Recently Iorio and Saridakis [56] studied solar system constraints on the model $f(T) = T + \alpha T^2$ and found the bound $|\alpha| \leq 1.8 \times 10^4 m^2$. Further a T^2 term can cure all the four types of the finite-time future singularities in $f(T)$ gravity, similar to that in $F(R)$ gravity [57,58].

In teleparallel gravity, the equations of motion for any geometry are exactly the same as of general relativity. Due to this reason, the teleparallel gravity is termed as 'teleparallel equivalent of general relativity' (TEGR) [59–61]. In teleparallel gravity, the dark energy puzzle is studied by introducing a scalar field with a potential. If this field is minimally coupled with torsion, then this effectively describes quintessence dark energy. However if it is non-minimally coupled with torsion, than more rich dynamics of the field appears in the form of either quintessence or phantom like, or by experiencing a phantom crossing [62]. Xu et al [63,64] investigated the dynamics and stability of a canonical scalar field non-minimally coupled to gravity (arising from torsion). They found that the dynamical system has an attractor solution and rich dynamical behavior was found. In the context of general relativity, a scalar field non-minimally coupled with gravity has been studied in [66].

In this paper, we discuss the energy conditions in generalized teleparallel gravities including pure $F(T)$, teleparallel gravity with non-minimally coupled scalar field and $F(T)$ with non-minimally coupled scalar field models. In particular, we apply them to FRW cosmology and obtain some corresponding results. Using two specific phenomenological forms of $F(T)$, we show that some of the energy conditions are violated. We follow the plan: In Sect. 2 we review the energy conditions in any gravitational theory. In Sect. 3, we give a review of teleparallel and $f(T)$ gravity. In Sect. 4, we start with the energy conditions for the simplest modified teleparallel gravity. In Sect. 5, we study energy conditions for a teleparallel gravity non-minimally coupled with a scalar field. In Sect. 6, we discuss energy conditions for $F(T)$ theory coupled with a minimally coupled scalar field. In Sect. 7, we provide conclusion.

2 Energy conditions

The energy conditions are used in different contexts to derive variety of general results which hold for different situations. Under these conditions, one allows not just gravity to be attractive but also the energy density to be positive and flows not to be faster than light [67]. The notion of energy conditions arise from the Raychaudhuri equation, given by

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu, \tag{1}$$

where u^μ is a vector field representing the congruence of "timelike geodesics". Also $R_{\mu\nu}$, θ, $\sigma_{\mu\nu}$ and $\omega_{\mu\nu}$ represent Ricci tensor, the expansion parameter, the shear and the rotation associated with the congruence respectively. Similarly, in the case of a congruence of "null geodesics" defined by the vector field k^μ, the Raychaudhuri equation becomes

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu. \tag{2}$$

From both the Raychaudhuri equations, it is apparent that these are purely geometric and independent of any gravity theory. The origin of these energy conditions is independent of any gravity theory and that these are purely geometrical (for a review on the energy conditions, see the classic text [68]). Using the modified (effective) gravitational field equations the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and the dominant energy condition (DEC) are given by Garci'a et al. [69] and Gong et al. [70]

$$\text{NEC} \Longleftrightarrow \rho_{\text{eff}} + p_{\text{eff}} \geq 0. \tag{3}$$

$$\text{WEC} \Longleftrightarrow \rho_{\text{eff}} \geq 0 \text{ and } \rho_{\text{eff}} + p_{\text{eff}} \geq 0. \tag{4}$$

$$\text{SEC} \Longleftrightarrow \rho_{\text{eff}} + 3p_{\text{eff}} \geq 0 \text{ and } \rho_{\text{eff}} + p_{\text{eff}} \geq 0. \tag{5}$$

$$\text{DEC} \Longleftrightarrow \rho_{\text{eff}} \geq 0 \text{ and } \rho_{\text{eff}} \pm p_{\text{eff}} \geq 0. \tag{6}$$

Note that NEC implies WEC and WEC implies SEC and DEC. In all the subsequent models we will assume that the regular matter satisfies all the energy conditions separately i.e. $\rho_m \geq 0,\ \rho_m \pm p_m \geq 0,\ \rho_m + 3p_m \geq 0$. In literature, the $f(R)$ theory has been tested against the energy conditions [71]. Thus, we need to check the validity of these conditions for energy density and pressure for different versions of $F(T)$ gravity. In addition, the various energy conditions are on a different footing. For instance, the violation of the strong energy condition is needed to have an accelerating phase, on the other hand, dark energy with $w < -1$ violates the NEC.

3 Teleparallel and modified teleparallel gravity

In the teleparallel (TEGR) formulation of Einstein general relativity (GR), the independent dynamical variable is a set of the tetrad fields e^i_μ. The Greek indices (holonomic) denote the local coordinates of the manifold while the Latin indices (anholonomic) denote the locally Lorentzian frame. We use the same symbol to show the inverse of e^i_μ. Let us define

$$e^i_\mu e^\nu_i = \delta^\nu_\mu, \quad e^i_\mu e^\mu_j = \delta^i_j, \tag{7}$$

where the coordinate free representation of the metric is given by

$$g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu. \tag{8}$$

Here $\eta_{ij} = \mathrm{diag}(1, -1, -1, -1)$ is the standard Minkowski metric, which geometrically plays the role of the tangent space metric. The same metric g is used to do the "gymnastic" of indices and η for frame indices. In TEGR, there exists a coordinate system where the metric is globally Minkowskian. In this case the tetrad fields give rise to a new connection defined by

$$\Gamma^\sigma_{\mu\nu} = e^\sigma_i \partial_\nu e^i_\mu = -e^i_\mu \partial_\nu e^\sigma_i, \tag{9}$$

which is the so-called assymetric Weitzenböck connection. Clearly, this cannot be the Levi-Civita connection since its torsion is zero by definition. We define torsion and contortion by

$$T^\sigma_{\mu\nu} = \Gamma^\sigma_{\nu\mu} - \Gamma^\sigma_{\mu\nu} = e^\sigma_i \left(\partial_\mu e^i_\nu - \partial_\nu e^i_\mu\right), \tag{10}$$

$$K^{\mu\nu}_\sigma = -\frac{1}{2}\left(T^{\mu\nu}_\sigma - T^{\nu\mu}_\sigma - T^{\mu\nu}_\sigma\right), \tag{11}$$

respectively. The contortion tensor can also be defined in terms of the Weitzenböck and Levi-Civita connections. It turns out to be useful to define the totally assymetric tensor $S^{\mu\nu}_\sigma$ in the following way

$$S^{\mu\nu}_\sigma = \frac{1}{2}\left(K^{\mu\nu}_\sigma + \delta^\mu_\sigma T^{\rho\nu}_\rho - \delta^\nu_\sigma T^{\rho\mu}_\rho\right). \tag{12}$$

It's straightforward to define the scalar torsion T which is given by

$$T = S^{\mu\nu}_\sigma T^\sigma_{\mu\nu}, \tag{13}$$

In TEGR, T has the same role as R in GR. But there is no trivial interpretation of the $T = cte$ solutions as a de Sitter solution as $R = cte$ in GR. Indeed, from one simple checking, we observe, there is no de-Sitter solution in TEGR with the same (exactly the same) properties as the d(A)S solution in GR.

Let us consider the modified action in usual convent geometrized units $c = G = 1$)

$$\mathcal{S} = \int ef(T)d^4x + \int e\mathcal{L}_{\text{matter}}d^4x, \tag{14}$$

where e is the determinant of e^i_μ.

To derive the EOMs, by variations of the action with respect to the tetrads e^i_μ gives the field equations of $f(T)$ modified gravity which are given by

$$S_i^{\mu\nu} f_{TT}\partial_\mu T + e^{-1}\partial_\mu\left(eS_i^{\mu\nu}\right) f_T - T^\sigma_{\mu i}S^{\nu\mu}_\sigma f_T - \frac{1}{4}e^\nu_i f = -\mathcal{T}_i^\nu, \tag{15}$$

where $S_i^{\mu\nu} = e^\sigma_i S^{\mu\nu}_\sigma$, f_T and f_{TT} denote the first and second derivatives of f with respect to T. $\mathcal{T}_{\mu\nu}$ is the energy momentum tensor. Conservation of the energy momentum tensor is ensured by the field equations.

4 Energy conditions in modified teleparallel gravity

The Friedmann equations in effective notation are given by

$$\rho_{\text{eff}} = 3H^2, \;\; p_{\text{eff}} = -(2\dot{H} + 3H^2), \tag{16}$$

where

$$\begin{aligned}\rho_{\text{eff}} &= \rho_m + \rho_T \\ &= \rho_m + Tf_T - \frac{f}{2},\end{aligned} \tag{17}$$

$$\begin{aligned}p_{\text{eff}} &= p_m + p_T \\ &= p_m + (2\dot{H} - T)f_T + 4\dot{H}Tf_{TT} + \frac{f}{2}.\end{aligned} \tag{18}$$

To check the viability of this cosmological model, we check the energy conditions (3)–(6) using (17) and (18):

$$2\dot{H}(f_T + 2Tf_{TT}) \geq 0 \tag{19}$$

$$-f + 2Tf_T \geq 0 \tag{20}$$

$$f + 2Tf_T + 6\dot{H}(f_T + 2Tf_{TT}) \geq 0 \tag{21}$$

$$2\dot{H}(f_T + 2Tf_{TT}) + (f - 2Tf_T) \leq 0 \tag{22}$$

Now we use two recently proposed models of $f(T)$ gravity [73]

$$f_1(T) = \alpha(-T)^n \tanh\left(\frac{T_0}{T}\right) \quad \text{(Model-I)}$$

$$f_2(T) = \alpha(-T)^n\left[1 - \exp\left(-p\frac{T_0}{T}\right)\right] \quad \text{(Model-II)}$$

Here $T_0 = -6H_0^2$. These two models are able to give rise to crossing the phantom divide. In Model-I the exponent $n > \frac{3}{2}$, the parameter of the model [73]

$$\alpha = -\frac{1 - \Omega_{m0} - \Omega_{r0}}{\left(6H_0^2\right)^{n-1}\left[\frac{2}{\cosh(1)^2} + (1-2n)\tanh(1)\right]}$$

Similarly for Model-II, we know that $n > \frac{1}{2}$ and [73]

$$\alpha = \frac{\left(6H_0^2\right)^{1-n}(1 - \Omega_{m0} - \Omega_{r0})}{-1 + 2n + e^p(1 - 2n + 2p)}$$

Here Ω_{m0}, Ω_{r0} are the present values of the energy densities of the dark matter and radiation. Now we examine the energy conditions, based on these two viable models Fig. 1.

- NEC: First note that the NEC reduces to

$$-4\dot{H}\sqrt{-T}\frac{d}{dT}\left[\sqrt{-T}f_T\right] \leq 0 \tag{23}$$

For model-I in case $\dot{H} > 0$, the top left figure shows this model does not satisfy the NEC but in case $\dot{H} < 0$ this model satisfies NEC. For model-II similarly, when $\dot{H} > 0$ the NEC breaks down but when $\dot{H} < 0$, NEC is valid (top right figure).

- WEC: It is easy to show that we must check the following inequality

$$\frac{d}{dT}\left[\frac{f(T)}{\sqrt{-T}}\right] \leq 0 \tag{24}$$

For model-I, middle left figure shows that the WEC is satisfied. Also for model-II, this energy condition will be satisfied as can be seen in middle right figure.

- SEC: In regime $\dot{H} > 0$, we know from NEC $f_T + 2Tf_{TT} \geq 0$. It remains only to check whether $f + 2Tf_T \geq 0$. Note that $f + 2Tf_T = -2\sqrt{-T}\frac{d}{dT}\left(f\sqrt{-T}\right) \geq 0$. For models I and II, we observe that this SEC is satisfied as shown in bottom left and right figures.
- DEC: We can write this condition as

$$-4\dot{H}\sqrt{-T}\frac{d}{dT}\left(\sqrt{-T}f_T\right) - 2\left(-\frac{f}{2} + Tf_T\right) \leq 0.$$

Note that the last bracket is positive on account of validity of WEC. Also from NEC, we know that for both models f_1 and f_2, always $\frac{d}{dT}(\sqrt{-T}f_T) \geq 0$. If $\dot{H} > 0$ then DEC is satisfied.

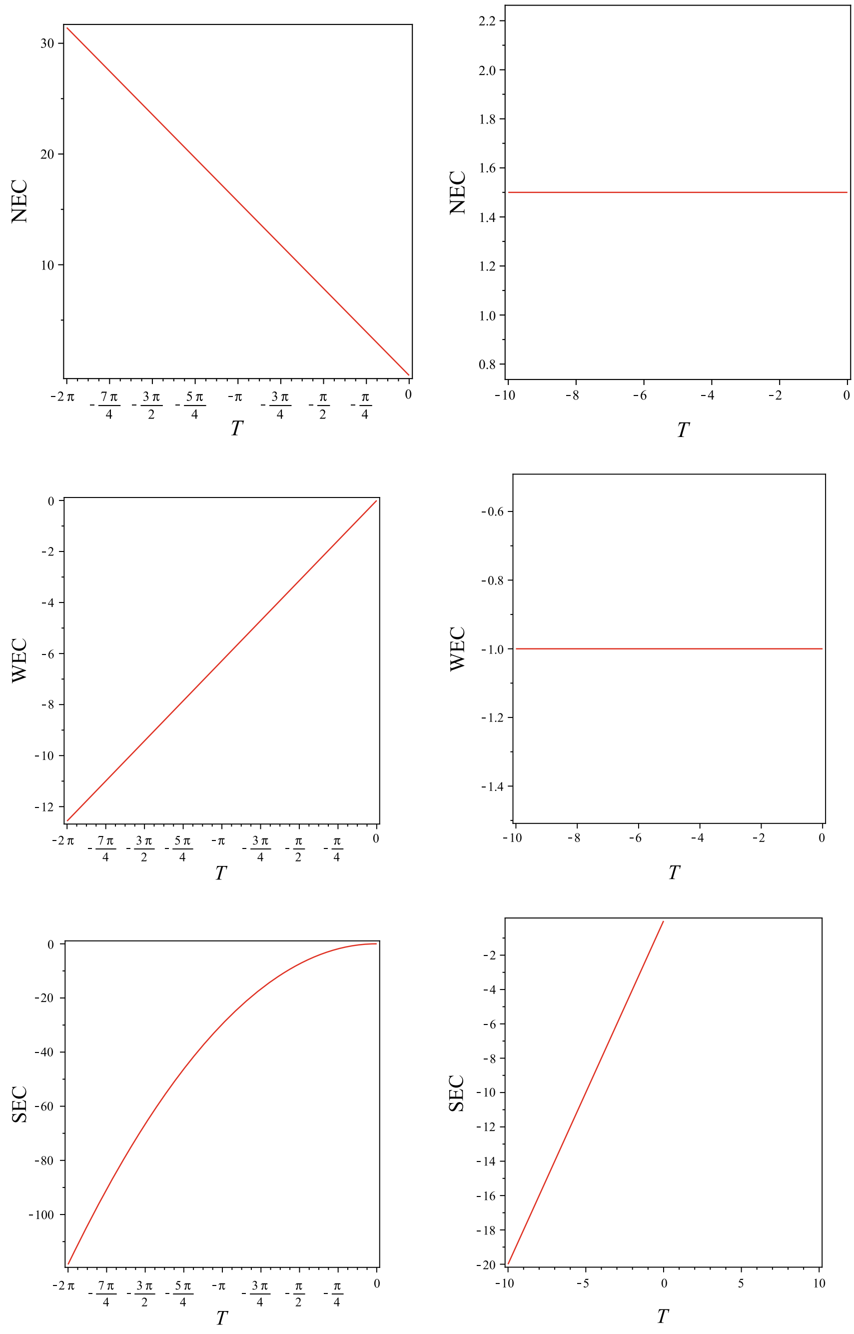

Fig. 1 (*Top left*) NEC for model 1. (*Top right*) NEC for model 2. (*Middle left*) WEC for model 1. (*Middle right*) WEC for model 2. (*Bottom left*) SEC for model 1. (*Bottom right*) SEC for model 2. The left and right panel figures correspond to f_1 and f_2, respectively

5 Teleparallel gravity with a non-minimally coupled scalar field

As a generalization of pure teleparallel gravity, Xu and collaborators [63,64] added a canonical scalar field, allowing for a non-minimal coupling with (teleparallel) gravity. Their proposed action with a little modification is

$$\mathcal{S} = \int d^4x\, a^3 \left[\frac{T}{2} + \frac{1}{2}\left(\epsilon \phi_{,\mu}\phi^{,\mu} + \xi T \phi^2 \right) - V(\phi) + \mathcal{L}_m \right]. \tag{25}$$

The symbol $\epsilon = +1, -1$ refer to quintessence and phantom dark energy respectively. ξ represents the coupling between the scalar torsion and the scalar field. $\mathcal{L}_m$ is the Lagrangian density for the matter component. Recently the present authors have investigated the more general case of (25) by replaced T with $f(T)$. The authors showed that by choosing a cosmologically viable function $f(T)$, the dynamical system of equations do not admit any attractor solution, however some evidence of phantom crossing does appear [65].

In this gravity (25), the effective energy density and effective pressure for a flat FRW universe is [63,64]

$$\rho_{\text{eff}} = \rho_m + \frac{\epsilon}{2}\dot{\phi}^2 + V(\phi) - 3\xi H^2 \phi^2, \tag{26}$$

$$p_{\text{eff}} = p_m + \frac{\epsilon}{2}\dot{\phi}^2 - V(\phi) + 4\xi H \phi \dot{\phi} + \xi \phi^2 \left(3H^2 + 2\dot{H} \right). \tag{27}$$

- NEC: $\epsilon\dot{\phi}^2 + 4\xi H\phi\dot{\phi} + 2\xi \dot{H}\phi^2 \geq 0$. We discuss two cases:
  Case-1: $\epsilon = +1,\ \dot{H} < 0$: In this case NEC is obeyed when $\dot{H} \geq -\frac{1}{2\xi\phi^2}\Big(\dot{\phi}^2 + 4\xi\phi\dot{\phi}H\Big)$.
  Case-2: $\epsilon = -1,\ \dot{H} > 0$: In this case NEC is justified when $\dot{\phi}^2 \leq 4\xi H\phi\dot{\phi} + 2\xi\dot{H}\phi^2$.
- WEC: If $\epsilon = +1,\ V(\phi) \geq 0,\ 3\xi H^2\phi^2 \leq \frac{\epsilon}{2}\dot{\phi}^2 + V(\phi)$.
  If $\epsilon = -1,\ V(\phi) \geq 0,\ V(\phi) \geq 3\xi H^2\phi^2 + \frac{\epsilon}{2}\dot{\phi}^2$.
- SEC: We must check whether $\epsilon\dot{\phi}^2 - V(\phi) + 3\xi H^2\phi^2 + 6\xi H\phi\dot{\phi} + 3\xi\dot{H}\phi^2 \geq 0$.
  Case-1: $\epsilon = +1,\ \dot{H} < 0$: In this quintessence model, we must have

$$\dot{\phi} \geq \frac{1}{6\xi H\phi}\Big(-\dot{\phi}^2 + V(\phi) - 3\xi H^2\phi^2 - 3\xi\dot{H}\phi^2 \Big).$$

  Case-2: $\epsilon = -1,\ \dot{H} > 0$: We conclude that $\dot{\phi} \geq \frac{1}{6\xi\phi H}\Big(\dot{\phi}^2 + V(\phi) - 3\xi H^2\phi^2 - 3\xi\dot{H}\phi^2\Big)$.
- DEC: First we check the condition $\rho_{\text{eff}} \geq 0$. It means that $\frac{\epsilon}{2}\dot{\phi}^2 + V(\phi) - 3\xi H^2\phi^2 \geq 0$. We have two special cases
  Case-1: $\epsilon = +1$. In this case $\dot{\phi}^2 \geq 2(3\xi H^2\phi^2 - V(\phi))$.
  Case-2: $\epsilon = -1$. Here $\dot{\phi}^2 \leq 2(3\xi H^2\phi^2 - V(\phi))$.
  Further the condition $\rho_{\text{eff}} \geq p_{\text{eff}}$ we have

$$\dot{\phi} \leq \frac{2V(\phi) - 6\xi H^2\phi^2 - 2\xi\phi^2\dot{H}}{4\xi H\phi}$$

6 $F(T)$ model with a minimally coupled scalar field

In a recent paper, we investigated the Noether symmetries of $F(T)$ cosmology involving matter and dark energy, the later being represented by a canonical scalar field with a potential [74]. We showed that $F(T) \sim T^{3/4}$ and $V(\phi) \sim \phi^2$, both of which are cosmologically viable functions. It was shown that this model closely mimics the Λ CDM model and allows phantom crossing. The total action of such a theory with contributions from torsion, matter and a minimal coupled scalar field component reads [74]

$$\mathcal{S} = \int d^4xa^3\Big[F(T) - \rho_m + \frac{1}{2}\epsilon\phi_{,\mu}\phi^{,\mu} - V(\phi)\Big]. \tag{28}$$

where $F(T) = T + f(T)$. The scalar field ϕ has the potential energy $V(\phi)$ and $\rho_m = \rho_{m0}a^{-3}$ is the energy density of matter with vanishing pressure and ρ_{m0} is a constant energy density at some initial time.

The forms of effective energy density and pressure are

$$\begin{aligned}\rho_{\text{eff}} &= \rho_m + \rho_\phi + \rho_T\\ &= \rho_m + \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi) + Tf_T - \frac{f}{2},\end{aligned} \tag{29}$$

$$\begin{aligned}p_{\text{eff}} &= p_m + p_\phi + p_T\\ &= p_m + \frac{1}{2}\epsilon\dot{\phi}^2 - V(\phi) + (2\dot{H} - T)f_T + 4\dot{H}Tf_{TT} + \frac{f}{2}.\end{aligned} \tag{30}$$

The analysis of energy conditions for this model is given below:

- NEC: $\epsilon\dot{\phi}^2 + 2\dot{H}(f_T + 2Tf_{TT}) \geq 0$. Notice that the last term in bracket is positive as demonstrated in section A. If $\epsilon = +1$, then NEC holds but violated otherwise.
- WEC: If $\epsilon = +1$ and $V(\phi) > 0$, than from section A, we have $-\frac{f}{2} + Tf_T \geq 0$. Thus WEC holds. If $\epsilon = -1$ and $V(\phi) > 0$, then WEC is satisfied if $V \geq \dot{\phi}^2 + \frac{f}{2} - Tf_T$.
- SEC: If $\epsilon = +1$, $\dot{H} < 0$ and $V(\phi) > 0$, than $2\dot{\phi}^2 + f \geq 2V(\phi) - 2Tf_T - 6\dot{H}f_T$. If $\epsilon = -1$, $\dot{H} > 0$ and $V(\phi) > 0$, than $f + 6\dot{H}f_T \geq 2\dot{\phi}^2 + 2V(\phi) - 2Tf_T$.
- DEC: If $\epsilon = +1$, $\dot{H} < 0$ and $V(\phi) > 0$, than $\frac{1}{2}\dot{\phi}^2 + V(\phi) \geq \frac{f}{2} - Tf_T$. If $\epsilon = -1$, $\dot{H} > 0$ and $V(\phi) > 0$, than $V(\phi) \geq \frac{f}{2} - Tf_T + \frac{1}{2}\dot{\phi}^2$.

7 Conclusion

In this paper, we discussed the energy conditions in three different models of generalized teleparallel gravities. These energy conditions are stronger test to check the viability of these theories. Here we examined three torsion based models with two phenomenological forms of $f(T)$. In the case of pure $f(T)$ gravity, we showed that all the energy conditions can be satisfied for both kind of dark energy models. Then by adding a non-minimally scalar interaction, we showed that these energy conditions can be fulfilled for some specific values of the dynamical quantities of the model. Further we showed that given a minimally coupled dark energy component depending on the value of interaction, the energy conditions can be valid.

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