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RESEARCH ARTICLE

Regular models with quadratic equation of state

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Abstract We provide new exact solutions to the Einstein–Maxwell system of equations which are physically reasonable. The spacetime is static and spherically symmetric with a charged matter distribution. We utilise an equation of state which is quadratic relating the radial pressure to the energy density. Earlier models, with linear and quadratic equations of state, are shown to be contained in our general class of solutions. The new solutions to the Einstein–Maxwell are found in terms of elementary functions. A physical analysis of the matter and electromagnetic variables indicates that the model is well behaved and regular. In particular there is no singularity in the proper charge density at the stellar centre unlike earlier anisotropic models in the presence of the electromagnetic field.

Keywords Relativistic charged fluids · Equations of state · Einstein–Maxwell equations

1 Introduction

The study of charged relativistic objects in general relativity is achieved by solving the Einstein–Maxwell system of equations and imposing conditions for physical acceptability. This is not easy to achieve because of the nonlinearity of the field equations. The exact solutions found have many applications in relativistic astrophysics. The models generated have been used in the description of neutron stars and black hole formation by Ray et al. [1] and de Felice et al. [2]. Particular models have also helped in the establishment of the absolute stability limit for charged spheres by Giuliani et

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al. [3] and Bohmer and Harko [4]. Several models of charged relativistic matter have been used to study strange stars by Mak and Harko [5], Bombaci [6], Komathiraj and Maharaj [7] and Thirukkanesh and Maharaj [8]. Charged models have been also used in the description of strange quark matter by Discus et al. [9], hybrid protoneutron stars by Nicotra et al. [10], and bare quark stars by Usov et al. [11]. A geometric approach is to assume the existence of a group of conformal motions on spacetime; exact solutions have been found by Mak and Harko [12] for strange quark matter and Esculpi and Aloma [13] for anisotropic relativistic charged matter by assuming the existence of a conformal killing vector in static spherically symmetric spacetimes.

Models with an equation of state are desirable in the description of realistic astrophysical matter. However most explicit solutions of the Einstein–Maxwell system that have been found do not satisfy this property. There have been some attempts made recently to find exact analytic solutions of the Einstein–Maxwell system with a linear equation of state. These include the treatments of Ivanov [14], Sharma and Maharaj [15], and Thirukkanesh and Maharaj [8]. Particular solutions with a quadratic equation of state, relating the radial pressure to the energy, where found by Feroze and Siddiqui [16]. This is an important advance since the complexity of the model is greatly increased because of the nonlinearity of the radial pressure in terms of the energy density. However the investigations mentioned above all suffer from the undesirable property of possessing a singularity in the property charge density at the centre of sphere. An essential requirement for a well behaved electromagnetic field is regularity of the proper charge density throughout the matter distribution, particularly at the stellar centre. The importance of this feature has been highlighted in the analysis of Varela et al. [17] whose treatment offers a general approach of dealing with anisotropic charged matter with linear or nonlinear equations of state. It is desirable to eliminate the singularity in the charge density for a detailed and complete analysis of physical properties of charged compact objects.

Our results may be helpful in the study of compact stars and gravitational collapse relating to neutron stars and black holes. In this regard we refer to particular papers some of which have static spherical geometry and others are dynamical. Novikov [18] showed in the case of spherical geometry that collapse of electrically charged matter may replaced by expansion and infinite densities are avoided. A general treatment of collapsing charged matter was completed by Bekenstein [19] who showed that nonzero pressure plays a significant role. The analysis of Raychaudhuri [20] for charged dust distributions showed that conditions for collapse and oscillation depend on the ratio of matter density to charge density. If this ratio is large, corresponding to weakly charged dust spheres, then shell crossings cannot be avoided in gravitational collapse as proved by Ori [21]. Krasinski and Bolejko [22] showed that there exist initial conditions for a charged dust sphere with finite radius so that a full cycle of pulsation can be completed by the outer layer with no internal singularity. A full and comprehensive analysis of charged, dissipative collapse is provided by Di Prisco et al. [23] for the free-streaming and diffusion approximations. A related and detailed analysis in the gravitational collapse of a charged medium was performed by Kouretsis and Tsagas [24] where the role of Raychaudhuri equation is highlighted. Exact solutions with an equation of state, such the quadratic case considered in this paper, are helpful in such studies.

The objective of this paper is to find new exact solutions of the Einstein–Maxwell field equations with a charged anisotropic matter distribution and a quadratic equation of state. We indicate that particular models found in the past with an equation of state are part of our general analytical framework. Previous solutions with a linear or quadratic equation of state are regained in our treatment. We ensure that the charge density is regular at the centre of the compact body and the physical criteria are satisfied. In Sect. 2, we give the Einstein–Maxwell field equations for a static spherically symmetric line element as an equivalent system of differential equations utilizing a transformation due to Durgapal and Bannerji [25]. In Sect. 3, we present new exact solutions to the Einstein–Maxwell system with a quadratic equation of state. The solution is regular at the centre of the compact object. This analysis extends the treatment of Thirukkanesh and Maharaj [8], and Feroze and Siddiqui [16]. Known solutions with an equation of state are presented in Sect. 4, as particular cases of our new results. In Sect. 5, a physical analysis of the new solutions is performed; the matter variables and the electromagnetic quantities are plotted. We summarise the results obtained in this paper.

2 Field equations

In standard coordinates the line element for a static spherically symmetric spacetime, modelling the interior of the relativistic object, has the form

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{1}$$

We take the energy momentum tensor to be of the form

$$T_{ij} = \text{diag}\left(-\rho - \frac{1}{2}E^2,\ p_r - \frac{1}{2}E^2,\ p_t + \frac{1}{2}E^2,\ p_t + \frac{1}{2}E^2\right), \tag{2}$$

where the quantity p_t is the tangential pressure, p_r is the radial pressure, ρ is the density, and E is the electric field intensity. Then the Einstein–Maxwell equations can be written in the form

$$\frac{1}{r^2}\left[r(1-e^{-2\lambda})\right]' = \rho + \frac{1}{2}E^2, \tag{3}$$

$$-\frac{1}{r^2}(1-e^{-2\lambda}) + \frac{2\nu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2, \tag{4}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2, \tag{5}$$

$$\sigma = \frac{1}{r^2}e^{-\lambda}(r^2E)', \tag{6}$$

where primes represent differentiation with respect to r. The quantity σ represents the proper charge density. We are utilising units where the coupling constant $\frac{8\pi G}{c^2} = 1$

and the speed of light $c = 1$. The mass within a radius r of the sphere is

$$M(r) = \frac{1}{2}\int_0^r \omega^2 \rho(\omega) d\omega. \tag{7}$$

We now introduce a new independent variable x and define new functions y and Z so that

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)}, \quad A^2 y^2(x) = e^{2\nu(r)}, \tag{8}$$

where A and C are constants. We assume an equation of state of the general form $p_r = p_r(\rho)$ for the matter distribution. We take the quadratic form

$$p_r = \gamma\rho^2 + \alpha\rho - \beta, \tag{9}$$

relating the radial pressure p_r to the energy density ρ. In the above α, β, and γ are constants. Then the Einstein–Maxwell equations governing the gravitational behaviour of a charged anisotropic sphere, with a quadratic equation of state, are represented by

$$\frac{\rho}{C} = \frac{1-Z}{x} - 2\dot{Z} + \frac{E^2}{2C}, \tag{10}$$

$$p_r = \gamma\rho^2 + \alpha\rho - \beta, \tag{11}$$

$$p_t = p_r + \Delta, \tag{12}$$

$$\begin{aligned}\frac{\Delta}{C} &= 4xZ\frac{\ddot{y}}{y} + 2[x\dot{Z} + 2Z]\frac{\dot{y}}{y} - \alpha\left[\frac{(1-Z)}{x} - 2\dot{Z} - \frac{E^2}{2C}\right] \\ &\quad -C\gamma\left[\frac{(1-Z)}{x} - 2\dot{Z} - \frac{E^2}{2C}\right]^2 + \dot{Z} - \frac{E^2}{2C} + \frac{\beta}{C},\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\dot{y}}{y} &= \frac{(1-Z)(1+\alpha)}{4Z} - \frac{(1+\alpha)E^2}{8CZ} - \frac{\alpha\dot{Z}}{4Z} - \frac{\beta}{4CZ} \\ &\quad + \frac{C\gamma}{4Z}\left[\frac{(1-Z)}{x} - 2\dot{Z} - \frac{E^2}{2C}\right]^2,\end{aligned} \tag{14}$$

$$\frac{\sigma^2}{C} = \frac{4Z}{x}\left(x\dot{E} + E\right)^2, \tag{15}$$

where dots denote differentiation with respect to the variable x. Equations (10)–(15) are similar to the field equations of Thirukkanesh and Maharaj [8]; however in this case the equation of state is quadratic. The quantity $\Delta = p_t - p_r$ is called the measure of anisotropy and vanishes for isotropic pressures. The nonlinear system as given in (10)–(15) consists of six independent equations in six variables involving the matter and electromagnetic quantities $\rho, p_r, p_t, \Delta, E, \sigma$ and the two gravitational potentials y and Z. The nonlinearity of the Einstein–Maxwell system (10)–(15) has been increased, when compared with many earlier treatments, because of the appearance of the quadratic term in (11); when $\gamma = 0$ then there is a linear equation of state. In addition Eq. (14) now contains terms with E^4 increasing the complexity of system since $\gamma \neq 0$ in general.

3 New solutions

To integrate the Einstein–Maxwell system we make the particular choices

$$Z = \frac{1+bx}{1+ax}, \tag{16}$$

$$\frac{E^2}{C} = \frac{k(3+ax)+sa^2x^2}{(1+ax)^2}. \tag{17}$$

The gravitational potential Z is well behaved and finite at the origin. The electric field intensity E is continuous, regular at the origin and approaches a constant value for increasing values of x. The constants a, b, k and s are real. The general analytic functional forms for Z and E regain particular cases studied in the past with an equation of state.

On substituting (16) and (17) into (14) we obtain the first order equation

$$\begin{aligned} \frac{\dot{y}}{y} &= \frac{(1+\alpha)(a-b)}{4[1+(a-b)x]} + \frac{\alpha(a-b)}{2(1+ax)[1+bx]} - \frac{\beta(1+ax)}{4C[1+bx]} \\ &\quad -\frac{(1+\alpha)[k(3+ax)+sa^2x^2]}{8(1+ax)[1+bx]} + \frac{C\gamma[(3+ax)(2a-ab-k)-sa^2x^2]^2}{16(1+ax)(1+bx)}, \end{aligned} \tag{18}$$

for the metric function y. In spite of complexity of Eq. (18) it can be solved in general. On integrating (18) we get

$$y = D(1+ax)^m[1+bx]^n \exp[F(x)], \tag{19}$$

where D is the constant of integration. The function *F(x)* is given explicitly by

$$\begin{aligned} F(x) &= \gamma[2(a-b)-k]^2\left[\frac{2(2b-a)(1+ax)+(b-a)}{8(b-a)^2(1+ax)^2}\right] \\ &\quad -Cs\gamma\left[\frac{(a-b)^2(ax+2)-a(2a+s)(1+ax)}{4(a-b)(1+ax)^2}\right] \\ &\quad -Cs\gamma\left[\frac{(a-b)(4k+s)+(2k(b-3a)+3bs)(1+ax)}{32(a-b)^2(1+ax)^2}\right] \\ &\quad +\frac{ax}{16bC}[C^2s^2\gamma-2Cs(1+\alpha)-4\beta]. \end{aligned} \tag{20}$$

The constants m and n have the form

$$
\begin{aligned}
m =& -\frac{(1+\alpha)(s+2k)}{8(b-a)} + \frac{\alpha}{2} + \gamma[2(a-b)-k]^2\left[\frac{b^2}{(b-a)^3} + \frac{b}{(b-a)^2} + \frac{1}{4}\right] \\
& + \frac{Cs\gamma}{8(a-b)^3}\Big[(a-b)[2s(a-b)+a+b] + 3ab(k-2b) - b^2k \\
& + 2b^3(2a-1)\Big] \\
n =& \frac{(1+\alpha)}{8b}[2(a-b)-k] + \frac{(1+\alpha)k - 2\alpha(a-b)}{4(b-a)} + \frac{\beta(a-b)}{4Cb^2} \\
& + \gamma[2(a-b)-k]^2\left[\frac{b^2}{(b-a)^3} + \frac{b}{(b-a)^2} + \frac{1}{4}\right] + \frac{Csa^2(1+\alpha)}{8b^2(b-a)} \\
& + \frac{Cs\gamma}{16b^2(b-a)^3}\left[a^4(s+4b) + (k+2b)(6a^2b^2 - 2a^3b)\right].
\end{aligned}
$$

Then we can generate an exact model for the Einstein–Maxwell system (10)–(15) in the form

$$e^{2\lambda} = \frac{1+ax}{1+bx}, \tag{21}$$

$$e^{2\nu} = A^2D^2\,(1+ax)^{2m}\,[1+bx]^{2n}\exp[2F(x)], \tag{22}$$

$$\frac{\rho}{C} = \frac{(2a-2b-k)(3+ax) - sa^2x^2}{2(1+ax)^2}, \tag{23}$$

$$p_r = \gamma\rho^2 + \alpha\rho - \beta, \tag{24}$$

$$p_t = p_r + \Delta, \tag{25}$$

$$
\begin{aligned}
\frac{\Delta}{C} =& \frac{4x(1+bx)}{1+ax}\left[\frac{m(m-1)a^2}{(1+ax)^2} + \frac{2mnab}{(1+ax)(1+bx)} + \frac{2ma\dot{F}(x)}{1+ax}\right. \\
& \left. + \frac{b^2n(n-1)}{(1+bx)^2} + \frac{2nb\dot{F}(x)}{1+bx} + \ddot{F}(x) + \dot{F}(x)^2\right] + \left[-\frac{2(a-b)x}{(1+ax)^2} + \frac{4(1+bx)}{(1+ax)}\right] \\
& \left[\frac{am}{1+ax} + \frac{bn}{1+bx} + \dot{F}(x)\right] - C\gamma\left[\frac{C(2(a-b)-k)(3+ax) - Csa^2x^2}{2(1+ax)^2}\right]^2 \\
& - \frac{1}{2(1+ax)^2}\left[2C(a-b) + k(3+ax) + sa^2x^2 - \frac{2\beta}{C}(1+ax)^2\right] \\
& - \alpha\left[\frac{(2(a-b)-k)(3+ax) - sa^2x^2}{2(1+ax)^2}\right],
\end{aligned} \tag{26}
$$

$$\frac{E^2}{C} = \frac{k(3+ax) + sa^2x^2}{(1+ax)^2}, \tag{27}$$

$$\frac{\sigma^2}{C} = \frac{C[1+bx]\left(\sqrt{k}(a^2x^2+3ax+6) + 2\sqrt{s}ax\sqrt{3+ax}(2+ax)\right)^2}{x(3+ax)(1+ax)^5}, \tag{28}$$

where *F(x)* is given by (20). We observe that the exact solution (21)–(28) of the Einstein–Maxwell system has been written solely in terms of elementary functions. For this solution the mass function is given by

$$M(x) = \frac{1}{8C^{3/2}}\left[\frac{[(12a(a-b)-6ak)x+s(15+10ax-2a^2x^2)]x^{1/2}}{3a(1+ax)} - \frac{5s\arctan(\sqrt{ax})}{a^{3/2}}\right]. \tag{29}$$

The gravitational potentials, matter variables and electromagnetic variables are well behaved and regular in the stellar interior. However in general there is a singularity in the charge density at the centre which is evident in (28). This singularity is avoidable when $k=0$, so that we have

$$\frac{\sigma^2}{C} = \frac{4Csa^2x[1+bx](2+ax)^2}{(1+ax)^5}. \tag{30}$$

At the centre of the star $x=0$ and the charge density vanishes.

4 Known solutions

We have found a general class of exact solutions to the Einstein–Maxwell system with a quadratic equation of state. It is interesting to observe that for particular parameter values we can regain uncharged anisotropic and isotropic models ($k=0,\ s=0$) from our general solution (21)–(28). We regain the following particular cases of physical interest.

4.1 Feroze and Siddiqui model

This is a special case of our general solution with the quadratic equation of state $p_r = \gamma\rho^2+\alpha\rho-\beta$. If we set $s=0,\ C=1$ and $A^2D^2=B$, then we regain the line element

$$ds^2 = B(1+ar^2)^m(1+br^2)^n\exp[2F(r)]dt^2 + \frac{1+ar^2}{1+br^2}dr^2 + r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{31}$$

where

$$F(x) = -\frac{\beta ar^2}{4b} + \gamma[2(a-b)-k]^2\left[\frac{2(2b-a)(1+ar^2)+(b-a)}{8(b-a)^2(1+ar^2)^2}\right],$$

$$m = \frac{\alpha}{2} - \frac{(1+\alpha)k}{4(b-a)} + \gamma[2(a-b)-k]^2\left[\frac{b^2}{(b-a)^3}+\frac{b}{(b-a)^2}+\frac{1}{4}\right],$$

$$n = \frac{(1+\alpha)}{8b}[2(a-b)-k] + \frac{(1+\alpha)k - 2\alpha(a-b)}{4(b-a)} + \frac{\beta(a-b)}{4b^2}$$
$$+\gamma[2(a-b)-k]^2\left[\frac{b^2}{(b-a)^3} + \frac{b}{(b-a)^2} + \frac{1}{4}\right].$$

The line element (31) was found by Feroze and Siddiqui [16] which was the first model with quadratic equation of state. Some minor misprints in [16] have been corrected in our result. This solution may be used to model a compact body.

4.2 Thirukkanesh and Maharaj model

If we set $\gamma = 0$ then we have the linear equation of state $p_r = \alpha\rho - \beta$. Also setting $C = 1,\ s = 0$ and $b = a - \tilde{b}$, we get the line element

$$ds^2 = A^2D^2(1+ar^2)^{2m}[1+(a-\tilde{b})r^2]^{2n}\exp\left[\frac{-a\beta r^2}{2(a-\tilde{b})}\right]dt^2$$
$$+\frac{1+ar^2}{1+(a-\tilde{b})r^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad (32)$$

where

$$m = \frac{2\alpha\tilde{b} - (1+\alpha)k}{4\tilde{b}},$$
$$n = \frac{1}{8\tilde{b}(a-\tilde{b})^2}\Big[2a^2(k(1+\alpha) - 2\alpha\tilde{b}) - a\tilde{b}(5k(1+\alpha) - 2\tilde{b}(1+5\alpha))$$
$$+\tilde{b}^2(3k(1+\alpha) - 2\tilde{b}(1+3\alpha) + 2\beta)\Big].$$

The metric (32) was found by Thirukkanesh and Maharaj [8]. This solution may be used to model realistic charged compact spheres and strange stars with quark matter in the presence of electromagnetic field.

4.3 Sharma and Maharaj model

If we set $\gamma = 0,\ \beta = \alpha\tilde{\rho}$, then we regain the linear equation of state $p_r = \alpha(\rho - \tilde{\rho})$ where $\tilde{\rho}$ is the density at the surface. By setting $k = 0,\ s = 0,\ b = a - \tilde{b},\ C = 1$ and $A^2D^2 = B$ we find the following form of the line element

$$ds^2 = -B(1+ar^2)^{2m}[1+(a-\tilde{b})r^2]^{2n}\exp\left(\frac{-a\beta r^2}{2(a-\tilde{b})}\right)dt^2$$
$$+\frac{1+ar^2}{1+(a-\tilde{b})r^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad (33)$$

where

$$m = \frac{\alpha}{2},$$
$$n = \frac{5a\tilde{b}\alpha - 2a^2\alpha - 3\tilde{b}^2\alpha + a\tilde{b} - \tilde{b^2} + \tilde{b}\beta}{4(a-\tilde{b})^2}.$$

The line element (33) represents an uncharged anisotropic sphere and was found by Sharma and Maharaj [15]. It may be used to describe strange stars with a linear equation of state with quark matter.

4.4 Lobo model

If we set $\gamma = 0, \ \beta = 0$, then we obtain the linear equation of state $p_r = \alpha\rho$. On setting $k = 0, \ s = 0, \ b = a - \tilde{b}, \ a = 2\tilde{b}, \ C = 1$ and $A^2D^2 = B$ we regain the line element

$$ds^2 = -(1 + 2\tilde{b}r^2)^{2m}(1 + \tilde{b}r^2)^{2n}dt^2 + \left(\frac{1 + 2\tilde{b}r^2}{1 + \tilde{b}r^2}\right)dr^2$$
$$+ r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{34}$$

where

$$m = \frac{\alpha}{2},$$
$$n = \frac{1-\alpha}{4}.$$

The metric (34) was first found by Lobo [26] which represents uncharged anisotropic matter. This solution serves as a stellar interior with $\alpha < -\frac{1}{3}$ and may be matched to the Schwarzchild exterior for dark energy stars.

4.5 Isotropic models

We observe that $\Delta \neq 0$ in general and the model remains anisotropic. However, we can show for particular parameter values that $\Delta = 0$ in the general solution (21)–(28). If we set $b = (a - 1), \ k = 0, \ s = 0, \ a = 0$, then we obtain

$$m = \frac{\alpha}{2}$$
$$n = \frac{1}{4C}[\beta - (1 + 3\alpha)C]$$
$$\Delta = \frac{x}{4C(1-x)}[\beta - 3(1 + \alpha)C][\beta - (1 + 3\alpha)C]. \tag{35}$$

Two different cases arise from (35) by setting $\Delta = 0$. Firstly, we observe that when $\beta = 0$ and $\alpha = -1$ then $\Delta = 0$. The equation of state becomes $p_r (= p_t) = -\rho$. With the line element

$$ds^2 = -\left(1 + \frac{r^2}{R^2}\right) dt^2 + \left(1 + \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (36)$$

where we have set $A = D = 1$ and $C = \frac{1}{R^2}$. We mention that the metric (36) is the isotropic uncharged de Sitter model. Secondly, we observe that when $\beta = 0$ and $\alpha = -\frac{1}{3}$ then $\Delta = 0$. The equation of state becomes $p_r (= p_t) = -\frac{1}{3}\rho$ with the line element

$$ds^2 = -A^2 dt^2 + \left(1 + \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (37)$$

where $D = 1$ and $C = \frac{1}{R^2}$. The metric (37) is the isotropic uncharged Einstein model.

5 Physical analysis

We show that the exact solutions of the Einstein–Maxwell system found in Sect. 3 are well behaved by generating graphical plots of matter and electromagnetic variables. We used the software package Mathematica [27] and we make the particular choices $C = 1$, $a = 2.5$, $b = 2$, $\gamma = 0.01$, $\alpha = 0.33$, $s = 0.017$ and $k = \beta = 0$. We generated the plots for the energy density (Fig. 1), radial pressure (Fig. 2), electric field intensity (Fig. 3), charge density (Fig. 4), mass (Fig. 5), speed of sound (Fig. 6), tangential pressure (Fig. 7) and the measure of anisotropy (Fig. 8). The energy density ρ is a finite and monotonically decreasing function. The radial pressure p_r is similarly well behaved and continuous. The electric field intensity E is initially small and approaches a maximum value as the boundary is approached. The proper charge density σ is nonsingular at the origin, increases and then decreases after reaching a maximum value. The mass function is a strictly increasing function which is continuous and finite. The speed of sound is less than the speed of light and causality is maintained throughout the stellar interior. The radial pressure is decreasing and does reach a finite value of the radial coordinate. The tangential pressure is also a decreasing function. The measure of anisotropy is a decreasing function as the boundary is approached and remains finite in the interior. Thus all the matter variables, electromagnetic variables and the gravitational potentials are nonsingular and regular in the region containing the stellar centre. In particular the proper charge density σ is finite at the centre unlike earlier treatments.

It is desirable to study comprehensively the stability of our new models; this is a objective for the future reseach. The solutions generated may be matched to the

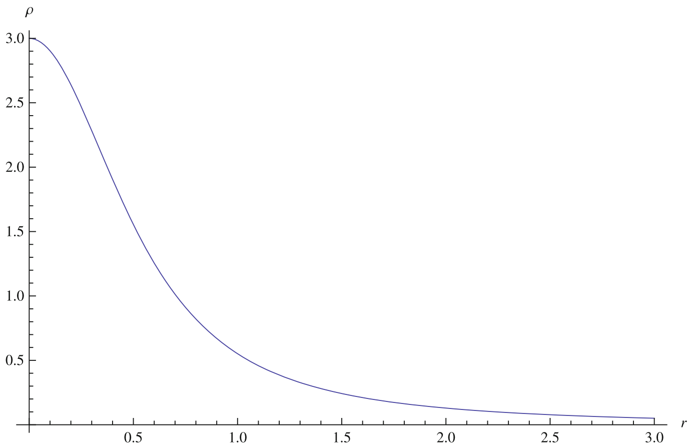


Fig. 1 Energy density

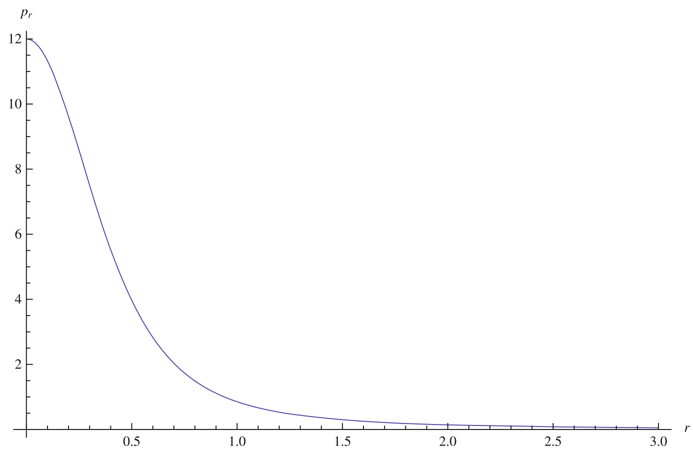


Fig. 2 Radial pressure

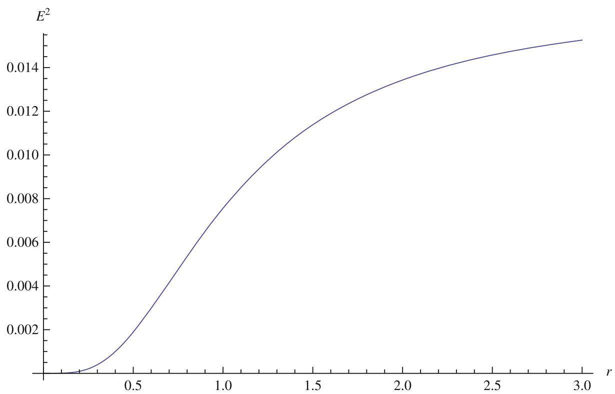


Fig. 3 Electric field intensity

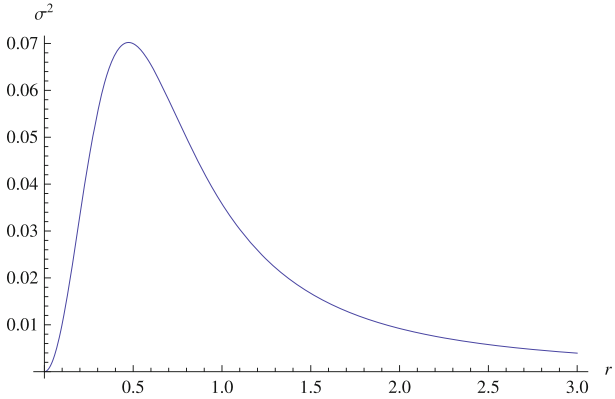
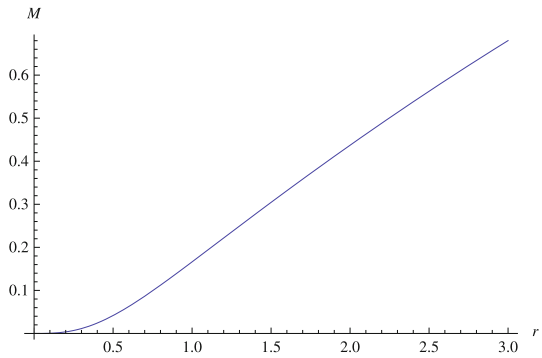


Fig. 4 Charge density

Fig. 5 Mass



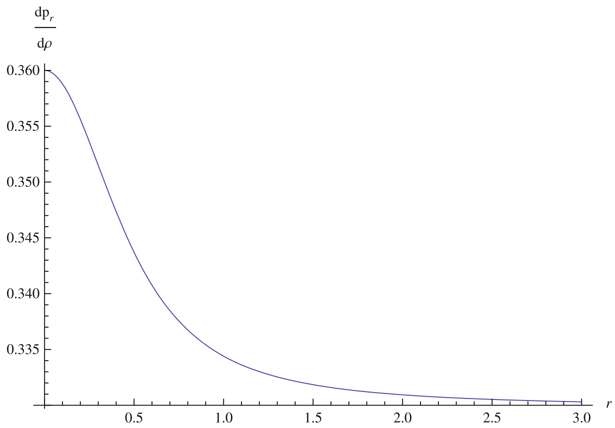


Fig. 6 Speed of sound

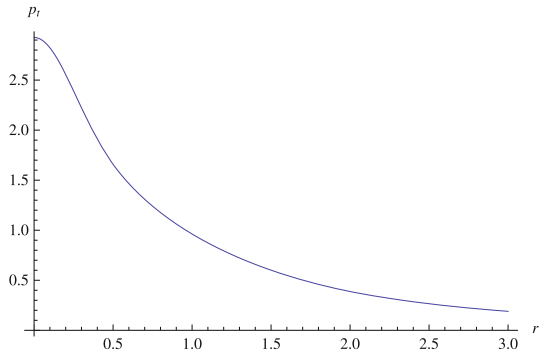


Fig. 7 Tangential pressure

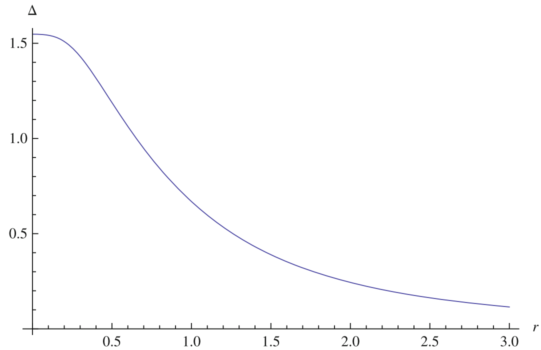


Fig. 8 Measure of anisotropy

exterior Reissner-Nordstrom spacetime

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 \\ +r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{38}$$

across the boundary $r = \Re$. This generates the following conditions

$$1 - \frac{2M}{\Re} + \frac{Q^2}{\Re^2} = A^2 y^2(C\Re^2), \tag{39}$$

and

$$\left(1 - \frac{2M}{\Re} + \frac{Q^2}{\Re^2}\right)^{-1} = \frac{1 + aC\Re^2}{1 + bC\Re^2}, \tag{40}$$

relating the constants $a, b, A, C, \alpha, \beta$ and γ. There are sufficients number of free parameters to ensure the continuity of the metric coefficients across the boundary of the star. It is possible to study the astrophysical significance of the exact solutions to the Einstein–Maxwell equations found in this paper. This is the object of future research. We point out that for suitable parameter values we regain the mass $M = 1.433M_{\odot}$

of Dey et al. [28–30] corresponding to a strange star model when there is no electromagnetic field. Therefore the solutions found in this paper may be used to generalise earlier results and to model charged relativistic strange and quark stars.

Our aim in this paper was to find new regular exact solutions to the Einstein–Maxwell system for spherically symmetric gravitational field with an equation of state. In particular we selected a quadratic equation of state relating the energy density to the radial pressure. The new models presented in this paper may be used to model relativistic compact objects in astrophysics.

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