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RESEARCH ARTICLE

Interacting entropy-corrected holographic dark energy with apparent horizon as an infrared cutoff

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Abstract In this work we consider the entropy-corrected version of interacting holographic dark energy (HDE), in the non-flat universe enclosed by apparent horizon. Two corrections of entropy so-called logarithmic 'LEC' and power-law 'PLEC' in HDE model with apparent horizon as an IR-cutoff are studied. The ratio of dark matter to dark energy densities u, equation of state parameter w_D and deceleration parameter q are obtained. We show that the cosmic coincidence problem is solved for interacting models. By studying the effect of interaction in EoS parameter of both models, we see that the phantom divide may be crossed and also understand that the interacting models can drive an acceleration expansion at the present and future, while in non-interacting case, this expansion can happen only at the early time. The graphs of deceleration parameter for interacting models, show that the present acceleration expansion is preceded by a sufficiently long period deceleration at past. Moreover, the thermodynamical interpretation of interaction between LECHDE and dark matter is described. We obtain a relation between the interaction term of dark components and thermal fluctuation in a non-flat universe, bounded by the apparent horizon. In limiting case, for ordinary HDE, the relation of interaction term versus thermal fluctuation is also calculated.

Keywords Logarithmic entropy correction · Power law entropy correction · Holographic dark energy · Coincidence problem · Thermodynamics

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1 Introduction

The dark energy scenario has attracted a great deal of attention in the last decades. Many cosmological observations reveal that our universe evolves under an acceleration expansion [1–4]. This expansion may be driven by an unknown energy component with negative pressure, so called, dark energy (DE), which fills ~ 70 percent of energy content of our universe with an effective equation of state (EoS) parameter $-1.48 < w_{eff} < -0.72$ [5–7]. Despite of much effort in this subject, the nature of DE is the most mysterious problem in modern cosmology. The first and simplest candidate of dark energy is ΛCDM model, in which $w_\Lambda = -1$ is constant. Although this model is consistent very well with all observations, it faces the fine tuning and cosmic coincidence problem. After this, the dynamical DE models have been proposed to solve the DE problems. Among many dynamical models of DE, in which w_D is not constant, the entropy-corrected dark energy models based on quantum field theory and gravitation have been widely extended by many authors in recent years [8–15]. The motivation of these corrections has been based on black hole physics, where some gravitational fluctuations and field anomalies can affect the entropy-area law of black holes. The logarithmic and power-law corrections of entropy are two procedures in dealing with this fluctuations. First correction has been given by logarithmic fluctuations at the spacetime, in the context of loop quantum gravity (LQG) [16–23]. The entropy-area relationship leads to the curvature correction in the Einstein-Hilbert action and vice versa [24–26]. In this case the corrected entropy is given by [27–35]

$$S_{BH} = \frac{A}{4G} + \tilde{\gamma} \ln \frac{A}{4G} + \tilde{\beta}, \tag{1}$$

where $\tilde{\gamma}$ and $\tilde{\beta}$ are dimensionless constants of order unity. By considering the entropy correction, the energy density of logarithmic entropy-corrected holographic dark energy (LECHDE) can be given as [36]

$$\rho_D = 3n^2 M_p^2 L^{-2} + \gamma L^{-4} \ln\left(M_p^2 L^2\right) + \beta L^{-4}. \tag{2}$$

Three parameters n, β and γ are parameters of model and M_P is the reduced Planck mass. The correction terms (two last terms of (2)) are effective only at the early stage of the universe and they will vanish when the universe becomes large, in which $\rho_D^{EC} \to \rho_D^O$, where $\rho_\Lambda^O = 3n^2 M_p^2 L^{-2}$ is the dark energy density of ordinary HDE model (more discussion of HDE model is referred to [37–46]). In this model, the IR-cutoff 'L' plays an essential role. If L is chosen as particle horizon, the HDE can not produce an acceleration expansion [47], while for future event horizon, Hubble scale '$L = H^{-1}$', and apparent horizon (AH) as an IR-cutoff, the HDE can simultaneously drive accelerated expansion and solve the coincidence problem [48–50]. More recently, a model of interacting HDE (i.e. a non gravitational interaction between DE and dark matter (DM)) at Ricci scale, in which $L = (\dot{H}+2H^2)^{-1/2}$ has been proposed. The authors performed a detailed discussion on the cosmic coincidence problem, age problem and obtained some observational constraints on their model [51].

The second class of ECHDE, power-law correction of entropy (PLEC), is appeared in dealing with the entanglement of quantum fields in and out of the horizon [52]. In this model, the corrected-entropy is given by [8]

$$S = \frac{A}{4G}[1 - K_\alpha A^{1-\alpha/2}], \tag{3}$$

where α is a dimensionless positive constant and

$$K_\alpha = \frac{\alpha}{4-\alpha}\left(4\pi r_c^2\right)^{\alpha/2-1}. \tag{4}$$

Here r_c is the crossover scale. Detailed discussion is referred to [8,52–54]. It is worthwhile to mention that in the most acceptable range of $4 > \alpha > 2$ [8,52], the correction term (i.e. the second term of (3)), is effective only at small A's and it falls off rapidly at large values of A. Therefore, for large horizon area, the ordinary entropy-area law (first term of (3)) is recovered. However the thermodynamical considerations predict that the case $\alpha \le 2$ may be acceptable, but as it will be shown in Sect. 3, this range should be removed by cosmic coincidence considerations. Due to entropy corrections to the Bekenstein–Hawking entropy (S_{BH}), the Friedmann equation should be modified [8]. In comparison with ordinary Friedman equation, the energy density of PLECHDE, has been given by [55]

$$\rho_D = 3n^2 M_p^2 L^{-2} - \delta M_p^2 L^{-\alpha}, \tag{5}$$

where δ and α are the parameters of PLECHDE model. it must be mentioned that the ordinary HDE is recovered for $\delta = 0$ or $\alpha = 2$.

In historical point of view, laws of black hole thermodynamics have made some relations between thermodynamics and a self gravitating system bounded by a horizon. In this theory, some thermodynamical quantities such as entropy and temperature are purely geometrical quantities which have been obtained from area and surface gravity of horizon, respectively. In the Friedmann–Robertson–Walker (FRW) universe with horizon, like future event horizon in black hole physics, by studying the thermodynamical quantities and generalized second law (GSL) [56–62], the best DE model or horizon can be chosen. For example, it has been shown that in a non-flat FRW universe, enclosed by apparent horizon, the GSL is governed irrespective of any DE model [50]. The investigation of GSL for LECHDE and PLECHDE models has been performed in [8].

Recently, the HDE and agegraphic/new-agegraphic DE models have been extended regarding the entropy corrections (LECHDE, PLECHDE, PLECNADE) and a thermodynamical description of the LECHDE model has been studied [9–15,36,55,63]. Also at Ref. [50], thermodynamics interpretation of interacting holographic dark energy with AH-IR-cutoff, enclosed by apparent horizon, has been studied. These papers give us a strong motivation to study the LECHDE and PLECHDE models with AH-IR-cutoff in a non-flat universe, enclosed by apparent horizon, which is a generalization of earlier works of Sheykhi et.al. [50,55]. It should be mentioned that, the motivation of a closed universe has been also shown in a suite CMB experiments [64–67] and of the cubic correction to the luminosity-distance of supernova measurements [68,69].

The outline of our paper is as follows: In Sect. 2, the interacting LECHDE model with AH-IR-cutoff is studied and the evolution of dark energy, deceleration parameter and EoS parameter are calculated. Also these calculations are performed for PLEC-HDE model with AH-IR-cutoff in Sect. 3. In Sect. 4, the thermodynamical quantities such as entropy and Hawking temperature of apparent horizon are obtained only for LECHDE model and then the interaction term due to thermal fluctuation is obtained in Sect. 5. We finish Our paper with some concluding remarks.

2 Interacting "LECHDE" model with AH-IR-cutoff

The line element of the homogenous and isotropic FRW universe is given by

$$ds^2 = h_{ab}dx^a dx^b + \widetilde{r}^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{6}$$

where $\widetilde{r} = a(t)r$, two non-angular metric $(x^0, x^1) = (t, r)$ and two dimensional metric h_{ab} is $diag[-1, a^2/(1-Kr^2)]$. Here K is the curvature parameter corresponding to a closed ($k = 1$), flat ($k = 0$) and open ($k = -1$) universe. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is $\widetilde{r}_A = (H^2 + K/a^2)^{-1/2}$ which has been derived by the relation $h^{ab}\partial_a\widetilde{r}\partial_b\widetilde{r} = 0$ [70]. This relation implies that the vector $\nabla\widetilde{r}$ is null on the apparent horizon surface. The apparent horizon may be considered as a causal horizon for a dynamical spacetime. Thus one can associate a gravitational entropy and surface gravity to it [71–73].

From Eq. (2), the energy density of LECHDE with apparent horizon, $\widetilde{r}_A$, as an IR-cutoff can be written as

$$\rho_D = 3n^2M_P^2\widetilde{r}_A^{-2} + \gamma\widetilde{r}_A^{-4}\ln\left(M_P^2\widetilde{r}_A^2\right) + \beta\widetilde{r}_A^{-4}. \tag{7}$$

The first Friedmann equation is

$$\frac{1}{\widetilde{r}_A^2} = H^2 + \frac{K}{a^2} = \frac{1}{3M_P^2}(\rho_m + \rho_D), \tag{8}$$

where $H = \dot{a}/a$ is the Hubble parameter. In a FRW universe, the total energy density $\rho = \rho_D + \rho_m$ is satisfied in a conservation equation

$$\dot{\rho} + 3H(1 + w)\rho = 0, \tag{9}$$

where $w = p/\rho$ is the EoS parameter. Due to interaction between dark components, two energy densities ρ_D and ρ_m are not conserved separately and the conservation equation is replaced by

$$\dot{\rho}_D + 3H(1 + w_D)\rho_D = -Q, \tag{10}$$

$$\dot{\rho}_m + 3H\rho_m = Q. \tag{11}$$

Here Q is the interaction term which has been usually considered in three forms as [74]

$$Q = \Gamma \rho_D = \left\{ \begin{array}{l} 3Hb^2\rho_D \\ 3Hb^2\rho_m \\ 3Hb^2(\rho_m + \rho_D) \end{array} \right\}. \tag{12}$$

In this equation, b^2 is coupling constant. Although a theoretical interpretation of this interaction has not been performed yet, as we see from Eqs. (10, 11), the interaction term Q should be given as a function of H multiplied to energy density. The interaction term indicates the decay rate of DE to CDM similar to the standard ΛCDM model, where vacuum fluctuations can decay into matter. In many models, the interaction term is necessary to solve the coincidence problem. It has been shown that this interaction can influence the perturbation dynamics, cosmic microwave background (CMB) spectrum and structure formation [75].

Differentiating Eq. (7) with respect to cosmic time and using the differentiation of apparent horizon with respect to cosmic time, we have

$$-\dot{\widetilde{r}}_A \widetilde{r}_A^{-3} = H\left(\dot{H} - \frac{K}{a^2}\right) = \frac{1}{6M_P^2}(\dot{\rho}_D + \dot{\rho}_m), \tag{13}$$

where from Eqs. (10, 11) we obtain

$$\dot{\widetilde{r}}_A = \frac{H}{2M_P^2}\widetilde{r}_A^3 \rho_D (1 + u + w_D), \tag{14}$$

$$\dot{\rho}_D = -\frac{H\rho_D \widetilde{r}_A^2}{M_P^2}(1 + u + w_D)\left[2\rho_D - \gamma \widetilde{r}_A^{-4} - 3n^2 M_P^2 \widetilde{r}_A^{-2}\right]. \tag{15}$$

Here $u = \rho_m/\rho_D$ is the ratio of energy densities. Also from Eq. (8), we find that $3M_P^2\widetilde{r}_A^{-2} = (1 + u)\rho_D$ where u is governed by

$$u = \frac{3M_P^2}{3n^2 M_P^2 + \gamma \widetilde{r}_A^{-2} \ln\left(M_P^2 \widetilde{r}_A^2\right) + \beta \widetilde{r}_A^{-2}} - 1. \tag{16}$$

From Eq. (16), we see that at sufficient large $\widetilde{r}_A$, where $\rho_D \approx 3n^2 M_P^2 \widetilde{r}_A^{-2}$, the ratio of energy densities will tend to a constant value $u \to 1/n^2 - 1$. In Fig. 1, the function u is plotted versus $\widetilde{r}_A$ for fixed γ, n and various β in the Planck mass unit in which $M_P = 1/\sqrt{8\pi G} = 1$. From this figure, we see that the coincidence problem gets alleviated, because for some values of model parameters, $u \sim \mathcal{O}(1)$ for wide range of $\widetilde{r}_A$ (including the present time), and it finally reches to a fixed value of order unity.

The deceleration parameter $q = -1 - \dot{H}/H^2$ is calculated by using the Friedmann equation and continuity equation as follows [49,50]:

$$q = -(1 + \Omega_K) + \frac{3}{2}\Omega_D(1 + u + w_D), \tag{17}$$

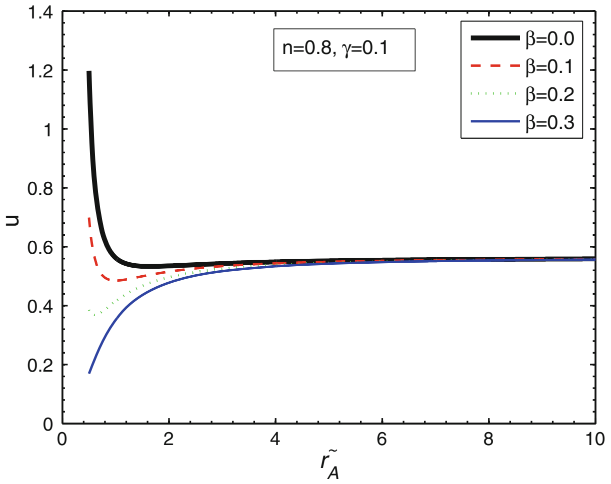

Fig. 1 The evolution of u versus $\widetilde{r}_A$ in LECHDE model. The asymptotic value is $u = 0.56$

where $\Omega_K = K/(a^2H^2)$, $\Omega_D = \rho_D/(3M_P^2H^2)$ and $\Omega_m = \rho_m/(3M_P^2H^2)$ are the energy density parameters. From these dimensionless parameters, the first Friedmann equation can be rewritten as: $1 + \Omega_K = \Omega_D + \Omega_m$. Using the third form of interacting term, in which $\Gamma/3H = b^2(1 + u)$ and combining Eq. (15) with (10), the EoS parameter w_D is given by

$$w_D = -1 - \frac{u\left(2\rho_D - 3n^2M_P^2\widetilde{r}_A^{-2} - \gamma\widetilde{r}_A^{-4}\right) - b^2(1+u)^2\rho_D}{(1-u)\rho_D - 3n^2M_P^2\widetilde{r}_A^{-2} - \gamma\widetilde{r}_A^{-4}}. \tag{18}$$

From this equation and Eq. (16), we find

$$\widetilde{r}'_A = \frac{3M_P^2\widetilde{r}_A}{2}\left[3n^2M_P^2\widetilde{r}_A^2 + \gamma\ln\left(M_P^2\widetilde{r}_A^2\right) + \beta + 3M_P^2\widetilde{r}_A^2(b^2-1)\right]\Big/ \left[3M_P^2\widetilde{r}_A^2(n^2-1) + 2\gamma\ln\left(M_P^2\widetilde{r}_A^2\right) + 2\beta - \gamma\right], \tag{19}$$

where "prime" denotes the differentiation with respect to $x = \ln\ a = -\ln(1+z)$, in which $H(d/dx) = d/dt$.

On the other hand, by using Eqs. (8) and (12), the evolution of dark energy density can be also obtained as

$$\rho'_D = -3\rho_D\left[1 + w_D + b^2(1+u)\right], \tag{20}$$

and then the evolution of Ω_D is calculated as:

$$\Omega'_D = -3\Omega_D\left[(1+w_D)(1-\Omega_D) + b^2(1+u) - \Omega_D u + \frac{2}{3}\Omega_K\right]. \tag{21}$$

Using Eq. (17), the deceleration parameter is given by

$$q = -(1+\Omega_K) - \frac{3}{2}\frac{\Omega_D(1+u)[u-b^2(1+u)]\rho_D}{(1-u)\rho_D - 3n^2M_P^2\widetilde{r}_A^{-2} - \gamma\widetilde{r}_A^{-4}}. \tag{22}$$

It is worthwhile to mention that Ω_K and Ω_D is related by

$$\frac{\Omega_K}{\Omega_m} = a\frac{\Omega_{K_0}}{\Omega_{m_0}} \quad \therefore \quad \Omega_K = \frac{e^x\Gamma(1-\Omega_D)}{1-e^x\Gamma}, \tag{23}$$

where $\Gamma = \Omega_{K_0}/\Omega_{m_0}$ is a constant value, which is given as $\Gamma \approx 0.04$, out of the recent data. Here the subscript '0', is used for the present time.

In the limiting case of ordinary HDE with $\gamma = \beta = 0$, Eqs. (16, 18, 22) reduce to the following simple forms

$$u = 1/n^2 - 1, \tag{24}$$

$$w_D = -\left(1+\frac{1}{u}\right)\frac{\Gamma}{3H}, \tag{25}$$

$$q = -(1+\Omega_K) - \frac{3}{2}\Omega_D(1+u)\left(\frac{\Gamma}{3Hu} - 1\right), \tag{26}$$

which had been also calculated by Sheykhi [50]. In this limit, from Eq. (19), the radius of apparent horizon, $\widetilde{r}_A$, can be obtained as

$$\widetilde{r}_A = \widetilde{r}_{A_0} e^{\frac{3M_P^2}{2}\left(\frac{n^2-1+b^2}{n^2-1}\right)x} = \widetilde{r}_{A_0}(1+z)^{\frac{3M_P^2}{2}\left(\frac{n^2-1+b^2}{1-n^2}\right)}. \tag{27}$$

Here we can choose $\widetilde{r}_{A_0} = 1$ at present time, in which $x = 0$ or vanishing redshift ($z = 0$). Therefore $\widetilde{r}_A$ may be considered as a normalized horizon radius. From Eq. (27), we find that the radius of apparent horizon is increased by cosmic time provided that $|n| > 1$ or $|n| < \sqrt{1-b^2}$. Moreover from Eq. (25), we see that, in the absence of interaction, we have $w_D = 0$, but in LECHDE model, the EoS parameter may cross the phantom divide ($w_D < -1$) even in the absence of interaction. In Fig. 2, the evolution of the EoS parameter of LECHDE versus $\widetilde{r}_A$ is studied, both in interacting and non-interacting cases for positive values of β, in the Planck mass unit. We specially focuss on the effect of coupling constant on the behavior of w_D. As it is shown in Fig. 2, by choosing the typical value of parameters of LECHDE model as '$\gamma = 0.1,\ \beta = 0.2,\ n = 0.8$', two distinct regions of $\widetilde{r}_A$ will be given as:

- **a**: ($0.22 > \widetilde{r}_A > 0$), Fig. 2a. Since $w_D > 0$, the LECHDE can not drive acceleration expansion at very early time, irrespective of any interaction term.
- **b**: ($\widetilde{r}_A > 0.23$), Fig. 2b. Both of interacting and non-interacting LECHDE, may accelerate the expansion and the phantom divide might be crossed. Interacting case always remains under the quintessence wall, while in non-interacting mode, the EoS parameter grows from phantom regime, $w_D < -1$, to positive values at

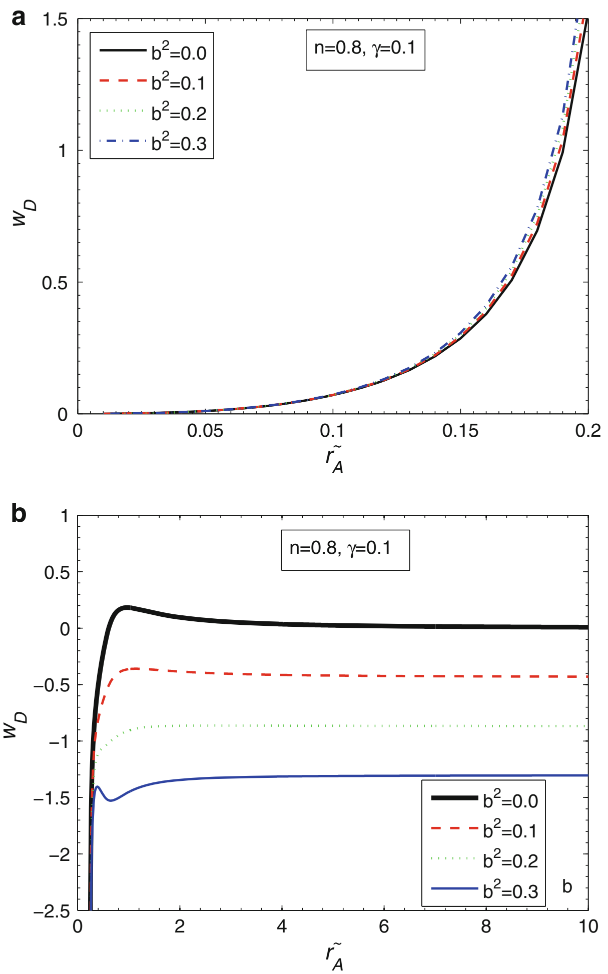

Fig. 2 The evolution of EoS parameter, w_D, versus $\widetilde{r}_A$ in LECHDE model, **a** $0.22 > \widetilde{r}_A > 0.0$. **b** $\widetilde{r}_A > 0.23$. "Q" denotes the Quintessence barrier ($w_D = -1/3$)

small $\widetilde{r}_A$. Therefore the non-interacting case can not drive the late time acceleration expansion in our universe.

By solving Eqs. (19, 21, 22, 23) numerically, the behavior of deceleration parameter q with respect to $x = \ln(a)$ can be studied. In Fig. 3, as we see, the present ($x \approx 0$) accelerated stage ($q < 0$) is preceded by a sufficiently long period deceleration at the early time ($x < 0$, far from $x = 0$). This is compatible with cosmic structure formation at matter dominated era and the present accelerated expansion.

The typical values of γ, β , n have been chosen in order to solve the cosmic coincidence problem.

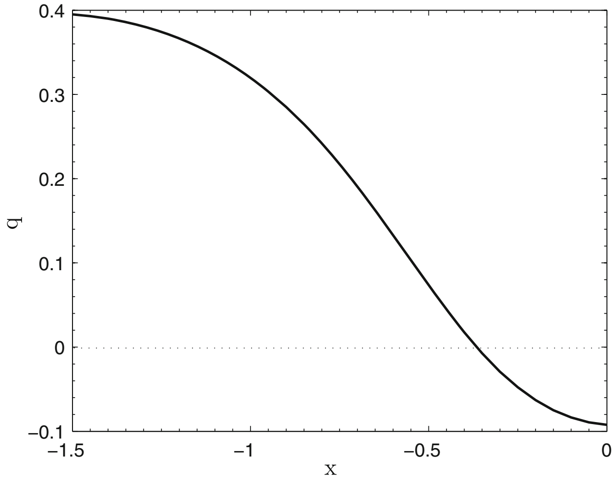

Fig. 3 The evolution of q versus $x = \ln(a)$ in LECHDE model for ($n = 0.8,\ \gamma = 0.1,\ \beta = 0.2,\ b^2 = 0.1,\ \Gamma = 0.04$)

3 Interacting "PLECHDE" model with AH-IR-cutoff

From Eq. (5), the energy density of PLECHDE with apparent horizon, $\widetilde{r}_A$, as an IR-cutoff, is written as

$$\rho_D = 3n^2 M_P^2 \widetilde{r}_A^{-2} - \delta M_P^2 \widetilde{r}_A^{-\alpha}, \tag{28}$$

where using (14, 28), the energy density evolution is given by

$$\dot{\rho}_D = -3H\rho_D(1 + u + w_D)\left[n^2 - \frac{\alpha\delta}{6}\widetilde{r}_A^{2-\alpha}\right]. \tag{29}$$

From Eqs. (8) and (28), the ratio of energy densities, u, is given by

$$u = \frac{1}{n^2 - \frac{\delta}{3}\widetilde{r}_A^{2-\alpha}} - 1. \tag{30}$$

Also from Eqs. (28) and (30), as same as Sect. 2, we see that at late time, for $\alpha > 2$, when $\widetilde{r}_A$ is large, we have $\rho_D \approx 3n^2 M_P^2 \widetilde{r}_A^{-2}$ and the ratio of energy densities u, will tend to a constant value $u \to 1/n^2 - 1$, while this is not valid for $\alpha < 2$. In Fig. 4, we study the behavior of u versus $\widetilde{r}_A$, for various positive values of δ and fixed value of α. From this figure, we see that the function u is descending for $\delta > 0$ and the present value of $u \sim 0.4$ is satisfied for a typical set ($\alpha = 3,\ n = 0.89, \delta = 0.2$) at $\widetilde{r}_A = 1$ (present time). In this case $u \sim \mathcal{O}(1)$, for $\widetilde{r}_A > 0.3$. Also the coincidence problem can be solved.

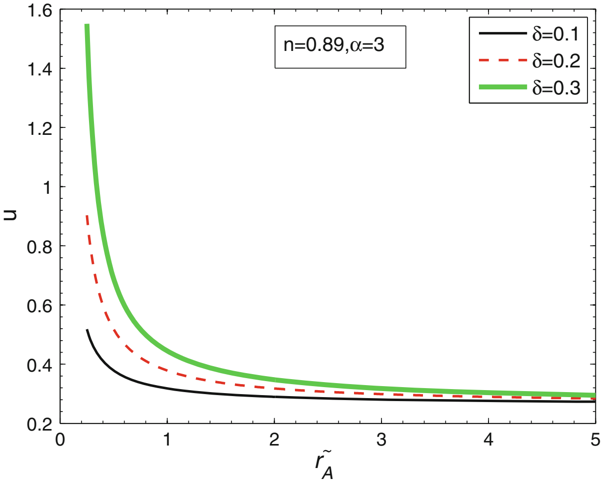

Fig. 4 The evolution of u versus $\widetilde{r}_A$ in PLECHDE model

Similar to the previous section, the EoS parameter $w_D, \widetilde{r}'_A, \Omega'_D$ and deceleration parameter q are calculated as

$$w_D = -\frac{1-(1+u)\left(n^2 - \frac{\alpha\delta}{6}\widetilde{r}_A^{2-\alpha} - b^2\right)}{1-\left(n^2 - \frac{\alpha\delta}{6}\widetilde{r}_A^{2-\alpha}\right)}, \tag{31}$$

$$\widetilde{r}'_A = \frac{3\widetilde{r}_A}{2}\left[\frac{1+b^2-\left(n^2-\frac{\delta}{3}\widetilde{r}_A^{2-\alpha}\right)}{1-\left(n^2-\frac{\alpha\delta}{6}\widetilde{r}_A^{2-\alpha}\right)}\right], \tag{32}$$

$$\Omega'_D = -\Omega_D\left[(1+u+w_D)\left(3n^2 - \frac{\alpha\delta}{2}\widetilde{r}_A^{2-\alpha} - 3\Omega_D\right) + 2\Omega_K\right], \tag{33}$$

$$q = -(1+\Omega_K) + \frac{3\Omega_D}{2}\left[\frac{u-b^2(1+u)}{1-\left(n^2-\frac{\alpha\delta}{6}\widetilde{r}_A^{2-\alpha}\right)}\right]. \tag{34}$$

The limiting case of Eqs. (30, 31, 34), with $\delta = 0$ or large $\widetilde{r}_A$, has been given by Eqs. (24, 25, 26). Also in this case the Eq. (32) reaches to Eq. (27) in the previous section. In PLECHDE model, the EoS parameter may cross the phantom divide ($w_D < -1$) even in the absence of interaction. In Fig. 5, the EoS parameter of PLECHDE is studied both in various interacting and non-interacting modes. As it is shown in Fig. 5, by choosing the typical value of parameters of PLECHDE as: ($\alpha = 3,\ \delta = +0.2,\ n = 0.89$), we encounter with two distinct regions of $\widetilde{r}_A$ as follow:

a: ($0.08 > \widetilde{r}_A > 0$), Fig. 5a. Since ($w_D > 0$), the acceleration expansion is not expectable at very early stage of the universe.

b: ($\widetilde{r}_A > 0.09$), Fig. 5b. Both of interacting and non-interacting DE models, may drive the acceleration expansion and the phantom divide will be crossed. Interacting case always remains under the quintessence regime ($w_D < -1/3$), while in

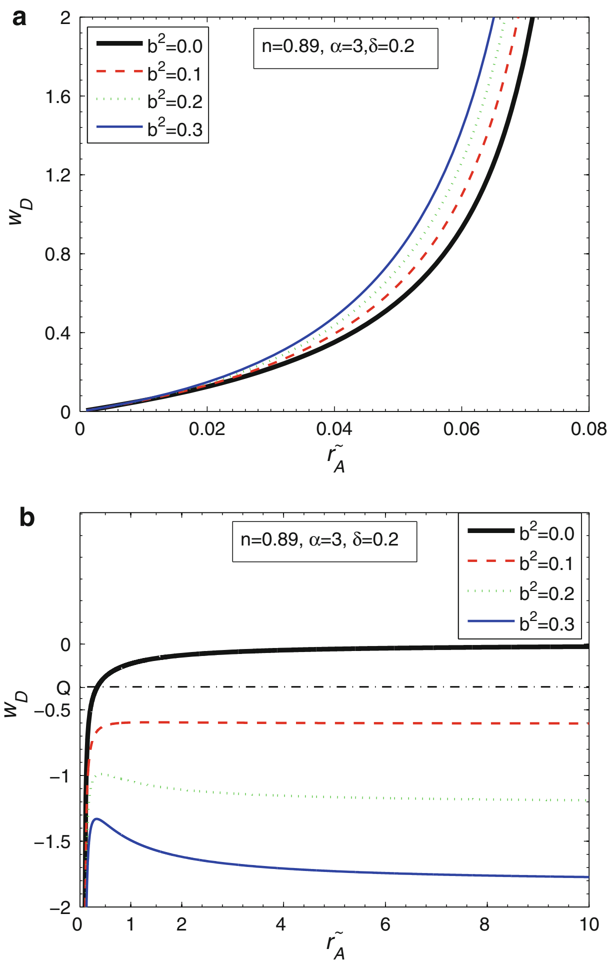

Fig. 5 The evolution of EoS parameter, w_D, versus $\widetilde{r}_A$ in PLECHDE model. **a** $0.08 > \widetilde{r}_A > 0.0$ and $\delta = 0.2$. **b** $\widetilde{r}_A > 0.09$ and $\delta = 0.2$. "Q" denoted the Quintessence barrier ($w_D = -1/3$)

non-interacting mode, the EoS parameter grows from phantom regime, $w_D < -1$, to above the quintessence regime ($w_D > -1/3$) very soon. Therefore, like as previous section, the non-interacting case can not drive the late time acceleration.

Now we want to study the deceleration parameter of PLECHDE model. By solving Eqs. (32, 33, 34, 23), numerically, the behavior of q with respect to x can be studied. In Fig. 6, similar to previous case, the present ($x \approx 0$) acceleration has been supported by a long period deceleration phase at past ($x < 0$).

4 Thermodynamics of non-interacting LECHDE with AH-IR-cutoff

In this section we want to associate a thermodynamical description to cosmological horizons, similar to black hole physics. In a FRW universe enclosed by an apparent

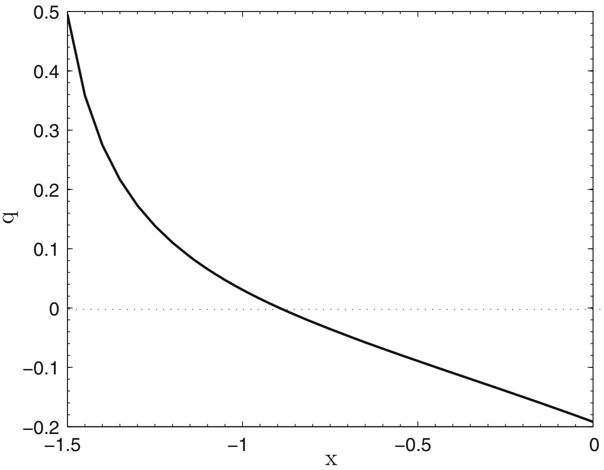

Fig. 6 The evolution of q versus $x = \ln(a)$ in PLECHDE model for ($n = 0.89,\ \alpha = 3,\ \delta = 0.2,\ b^2 = 0.1,\ \Gamma = 0.04$)

horizon, one can associate the Hawking temperature to the horizon, which is inversely proportional to size of the apparent horizon. We know that the FRW universe may consist of several cosmic ingredients including dark energy, dark matter, radiation and baryonic matter. However a lot of cosmological evidences reveal that the dark energy and matter are two dominant components in our universe. In the following, we will consider LECHDE and CDM components of energy in a non-flat FRW universe enclosed by apparent horizon. In a local thermal equilibrium, where there is not any heat flow from the apparent horizon, the temperature of the energy content of the universe (T) should be equal to the temperature which is associated with apparent horizon (T_h). Although this assumption is not rigorously established now, the thermal equilibrium state can be accessed at a finite time and therefore we will consider a unit temperature for the whole of spacetime (contain DE, CDM and AH). In the non-equilibrium universe, the heat will flow outside (inside) the apparent horizon if the temperature of cosmic fluid is hotter (colder) than the apparent horizon. The equilibrium entropy of the LECHDE is connected with its energy and pressure, p_D, through the Gibbs law of thermodynamics

$$TdS_D = dE_D + p_D dV, \tag{35}$$

where $V = (4\pi/3)r_A^3$ is the volume of whole space up to horizon surface and S_D is the entropy of DE component. The equilibrium temperature T, can be obtained from the surface gravity (κ_H) of horizon as follow [70]

$$T = \frac{|\kappa_H|}{2\pi} = \frac{1}{4\pi\sqrt{-h}} \left|\partial_a(\sqrt{-h}h^{ab}\partial_b\widetilde{r})\right|. \tag{36}$$

From this equation, the temperature of apparent horizon is calculated as

$$T = \frac{1}{2\pi\widetilde{r}_A}\left(1 - \frac{\dot{\widetilde{r}}_A}{2H\widetilde{r}_A}\right). \tag{37}$$

Following Cai and Kim [70], the apparent horizon radius $\widetilde{r}_A$ should be regarded to have a fixed value in thermal equilibrium. It means that $\dot{\widetilde{r}}_A \approx 0$. Thus the temperature is given by

$$T = 1/\left(2\pi\widetilde{r}_A^{(0)}\right). \tag{38}$$

Now from Eq. (35), we have

$$TdS = \rho_D(1 + w_D)dV + Vd\rho_D, \tag{39}$$

and by using Eq. (7), we can obtain

$$\frac{dS_D^0}{d\widetilde{r}_A^0} = \frac{8}{3}\pi^2\left(\widetilde{r}_A^0\right)^3\left[6n^2M_P^2\left(\widetilde{r}_A^0\right)^{-2} + 2\gamma(\widetilde{r}_A^0)^{-4} - \rho_D^0\left(1 - 3w_D^0\right)\right], \tag{40}$$

where superscript (0) denotes that the universe is in a stable thermodynamical equilibrium state.

5 Thermodynamics of interacting LECHDE with AH-IR-cutoff

In the presence of interaction, ($Q \neq 0$), the thermal equilibrium is no further maintain due to thermal fluctuation which has been arisen from decaying of dark energy to dark matter. The conservation equations for ρ_m and ρ_D, have been given by Eqs. (10, 11). In this case, however the Gibbs law of thermodynamics may hold only approximately for dynamical apparent horizon, the entropy is affected under a first order logarithmic correction ($S_D^{(1)}$) involving temperature T and the heat capacity C, as follow [76]

$$S_D^{(1)} = -\frac{1}{2}\ln(CT^2). \tag{41}$$

Hence, the entropy should be modified as: $S_D = S_D^{(0)} + S_D^{(1)}$. The heat capacity in thermal equilibrium has been defined as: $C = T\partial S_D^{(0)}/\partial T$. Using (38), the heat capacity can be rewritten as: $C = -(\widetilde{r}_A^0)\partial S_D^{(0)}/\partial\widetilde{r}_A^0$. Using Eq. (40) in thermal equilibrium, the corrected term $S_D^{(1)}$ is calculated as

$$S_D^{(1)} = -\frac{1}{2}\ln\left[\rho_D^0\left(\widetilde{r}_A^0\right)^2\left(1 - 3w_D^0\right) - 6n^2M_P^2 - 2\gamma\left(\widetilde{r}_A^0\right)^{-2}\right] - \frac{1}{2}\ln\left(\frac{2}{3}\right). \tag{42}$$

Similar to Eq. (40) with interaction, one obtains

$$dS_D = \frac{8}{3}\pi^2 \widetilde{r}_A^3 \left[6n^2 M_P^2 \widetilde{r}_A^{-2} + 2\gamma \widetilde{r}_A^{-4} - \rho_D(1-3w_D)\right] d\widetilde{r}_A, \tag{43}$$

where from $dS_D = dS_D^{(0)} + dS_D^{(1)}$, we find

$$1 - 3w_D = \left[6n^2 M_P^2 \widetilde{r}_A^{-2} + 2\gamma \widetilde{r}_A^{-4} - \frac{3}{8\pi^2 \widetilde{r}_A^3}\left(\frac{dS_D^{(0)}}{d\widetilde{r}_A} + \frac{dS_D^{(1)}}{d\widetilde{r}_A}\right)\right]\rho_D^{-1}. \tag{44}$$

By using Eqs. (40, 42), we can calculate

$$\begin{aligned}
\frac{dS_D^{(0)}}{d\widetilde{r}_A} &= \frac{dS_D^{(0)}}{d\widetilde{r}_A^0}\frac{d\widetilde{r}_A^0}{d\widetilde{r}_A} \\
&= \frac{8}{3}\pi^2 \left(\widetilde{r}_A^0\right)^3 \left[6n^2 M_P^2 \left(\widetilde{r}_A^0\right)^{-2} + 2\gamma \left(\widetilde{r}_A^0\right)^{-4} - \rho_D^0 \left(1 - 3w_D^0\right)\right]\frac{d\widetilde{r}_A^0}{d\widetilde{r}_A}, \qquad (45) \\
\frac{dS_D^{(1)}}{d\widetilde{r}_A} &= \frac{dS_D^{(1)}}{d\widetilde{r}_A^0}\frac{d\widetilde{r}_A^0}{d\widetilde{r}_A} \\
&= \frac{2\rho_D^0 \left(\widetilde{r}_A^0\right)\left(1-3w_D^0\right) + 4\gamma \left(\widetilde{r}_A^0\right)^{-3} + \left(\widetilde{r}_A^0\right)^2 \frac{d}{d\widetilde{r}_A^0}\left[\rho_D^0\left(1-3w_D^0\right)\right]}{2\left[-\rho_D^0 \left(\widetilde{r}_A^0\right)^2\left(1-3w_D^0\right) + 6n^2 M_P^2 + 2\gamma \left(\widetilde{r}_A^0\right)^{-2}\right]}\frac{d\widetilde{r}_A^0}{d\widetilde{r}_A},
\end{aligned} \tag{46}$$

where from (18) and (16), we have

$$\begin{aligned}
1 - 3w_D &= 4 + 3\frac{u\left(2\rho_D - 3n^2 M_P^2 \widetilde{r}_A^{-2} - \gamma \widetilde{r}_A^{-4}\right) - \frac{\Gamma}{3H}(1+u)\rho_D}{(1-u)\rho_D - 3n^2 M_P^2 \widetilde{r}_A^{-2} - \gamma \widetilde{r}_A^{-4}}, \qquad (47) \\
1 - 3w_D^0 &= 4 + 3u^0 \frac{2\rho_D^0 - 3n^2 M_P^2 \left(\widetilde{r}_A^0\right)^{-2} - \gamma \left(\widetilde{r}_A^0\right)^{-4}}{(1-u^0)\rho_D^0 - 3n^2 M_P^2 \left(\widetilde{r}_A^0\right)^{-2} - \gamma \left(\widetilde{r}_A^0\right)^{-4}}, \qquad (48) \\
\frac{du^0}{d\widetilde{r}_A^0} &= -(1+u^0)\left[\frac{2}{\widetilde{r}_A^0} + \frac{d}{d\widetilde{r}_A^0}\ln\left(\rho_D^0\right)\right]. \qquad (49)
\end{aligned}$$

Now, we can find a relation between the interaction term and the thermal fluctuation. For this purpose, by comparing two Eqs.(44) and (47), the interaction term can be calculated with respect to thermal fluctuation as

$$\begin{aligned}\frac{\Gamma}{3H} &= \frac{2}{3(1+u)\rho_D^2}\left\{\left(2\rho_D - 3n^2M_P^2\widetilde{r}_A^{-2} - \gamma\widetilde{r}_A^{-4}\right)\right.\\ &\quad\times\left(\left(1+\frac{u}{2}\right)\rho_D - 3n^2M_P^2\widetilde{r}_A^{-2} - \gamma\widetilde{r}_A^{-4}\right)\\ &\quad+\frac{3\widetilde{r}_A^0}{32\pi^2\widetilde{r}_A^3}\frac{(1-u)\rho_D - 3n^2M_P^2\widetilde{r}_A^{-2} - \gamma\widetilde{r}_A^{-4}}{6n^2M_P^2 + 2\gamma\left(\widetilde{r}_A^0\right)^{-2} - \rho_D^0\left(\widetilde{r}_A^0\right)^2\left(1-3w_D^0\right)}\\ &\quad\times\left[4\gamma\left(\widetilde{r}_A^0\right)^{-4} + 2\rho_D^0\left(1-3w_D^0\right)\right.\\ &\quad+\frac{16}{3}\pi^2\left(6n^2M_P^2 + 2\gamma\left(\widetilde{r}_A^0\right)^{-2} - \rho_D^0\left(\widetilde{r}_A^0\right)^2\left(1-3w_D^0\right)\right)^2\\ &\quad\left.\left.+\widetilde{r}_A^0\frac{d}{d\widetilde{r}_A^0}[\rho_D^0\left(1-3w_D^0\right)\right]\frac{d\widetilde{r}_A}{d\widetilde{r}_A^0}\right\}. \end{aligned} \tag{50}$$

In limiting case, from Eqs. (25, 50), for ordinary HDE ($\gamma = \beta = 0$), where $w_D^0 = 0$ and $\rho_D = 3n^2M_P^2\widetilde{r}_A^{-2}$, it can be obtained

$$\frac{\Gamma}{3H} = \frac{1-n^2}{3}\left[1 - \widetilde{r}_A^0\frac{d}{d\widetilde{r}_A^0}\ln(\widetilde{r}_A)\right]. \tag{51}$$

6 Conclusion

In this paper the logarithmic and power-law entropy-corrected version of interacting HDE with AH-IR-cutoff in a non-flat universe enclosed by apparent horizon have been studied. In fact we generalized the ordinary HDE model by considering the entropy correction due to fluctuation of spacetime and AH-IR-cutoff. The ratio of dark matter to dark energy densities u, EoS parameter w_D and deceleration parameter q have been calculated for both models. We showed that the cosmic coincidence problem is solved for appropriate model parameters. In dealing with cosmic coincidence problem, we found an appropriate set of values for LECHDE model as: ($\gamma = 0.1,\ \beta = 0.2,\ n = 0.8$) and for PLECHDE model as: ($n = 0.89\ \alpha = 3,\ \delta = 0.2$). By studying the effect of interaction in EoS parameter, we saw that the phantom divide may be crossed and also find that the interacting models can drive an acceleration expansion at the present and future, while in non-interacting case, this expansion can happen only at the early time. The graphs of deceleration parameter for interacting models, showed that the present acceleration expansion is preceded by a sufficiently long period deceleration at past.

Moreover, the thermodynamical interpretation of interaction between LECHDE and dark matter was described. Based on the Gibbs law of thermodynamics, for dark energy sector of the universe in non-interacting case, we calculated a differentiation of entropy of DE with respect to $\widetilde{r}_A$. Although in the absence of interaction between dark energy and dark matter, these two dark components conserved separately, while by imposing an interaction term, a stable fluctuation around equilibrium is expectable. Therefore, in the interacting case, where the entropy is affected under a first order logarithmic correction, we obtained a relation between the interaction term and thermal fluctuation in the non-flat universe enclosed by the apparent horizon. Also in limiting case for ordinary HDE, the relation of interaction term versus thermal fluctuation was calculated.

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