Gen Relativ Gravit (2011) 43:2547–2553
DOI 10.1007/s10714-011-1212-8

ESSAY AWARDED BY THE GRAVITY RESEARCH FOUNDATION

Relative locality: a deepening of the relativity principle

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Published online: 28 June 2011

Abstract We describe a recently introduced principle of relative locality which we propose governs a regime of quantum gravitational phenomena accessible to experimental investigation. This regime comprises phenomena in which $\hbar$ and G_N may be neglected, while their ratio, the Planck mass $M_p = \sqrt{\hbar/G_N}$, is important. We propose that M_p governs the scale at which momentum space may have a curved geometry. We find that there are striking consequences for the concept of locality. The description of events in spacetime now depends on the energy used to probe it. But there remains an invariant description of physics in phase space. There is furthermore a reasonable expectation that the geometry of momentum space can be measured experimentally using astrophysical observations.

Keywords Relativity · Quantum gravity · Astrophysics

Second Award in the 2011 Essay Competition of the Gravity Research Foundation.

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How do we know we live in a spacetime? And, if so, how do we know we all share the same spacetime? According to the operational procedure introduced by Einstein [1], we infer the coordinates of a distant event by analyzing light signals sent between observer and the event. But when we do this we throw away information about the energy of the photons. This is clearly a good approximation, but is it exact? Suppose we use Planck energy photons or red photons in Einstein's localization procedure, can we be sure that the spacetimes we infer in the two cases are going to be the same? Also, how can we be sure that when two events are inferred to be at the same spacetime position by one observer, the same holds true for another, distant observer?

In special and general relativity the answer to these questions is yes. Simultaneity is relative but locality is absolute. This follows from the assumption that spacetime is a universal entity in which all of physics unfolds. However, all approaches to the study of the quantum-gravity problem suggest that locality must be weakened and that the concept of spacetime is only emergent and should be replaced by something more fundamental. A natural and pressing question is whether it is possible to relax the universal locality assumption in a controlled manner, such that it gives us a stepping stone toward the full theory of quantum gravity?

A natural guess is that the Planck length,[1] $\ell_p = \sqrt{\hbar G}$, sets an absolute limit to how precisely an event can be localized, $\Delta x \sim \ell_p$. However, the Planck length is non zero only if G and $\hbar$ are non zero, so this hypothesis requires a full fledged quantum gravity theory to elaborate it. But there is an alternative, which is to explore a "classical-non gravitational" regime of quantum gravity which still captures some of the key delocalising features of quantum gravity. In this regime, $\hbar$ and G are both neglected, while their ratio is held fixed (Fig. 1):

$$\hbar \to 0\,, \quad G_N \to 0\,, \quad \text{but with fixed } \sqrt{\frac{\hbar}{G_N}} = M_p \tag{1}$$

In this regime of quantum gravity, which is labeled the "relative-locality regime" in the recent Ref. [2], both quantum mechanics and gravity are switched off, but we still keep effects due to the presence of the Planck mass. Remarkably, as we will describe, this regime includes effects on very large scales which can be explored in astrophysical experiments [3,4]. Furthermore, since $\hbar$ and G_N are both zero it can be investigated in simple phenomenological models.

In Ref. [2] we show that the hypothesis of universal locality is equivalent to the statement that momentum space is a linear space. It is natural then to propose that the mass scale M_P parameterizes non linearities in momentum space. Remarkably, these non linearities can be understood as introducing on momentum space a *non trivial geometry*. In [2] we introduced a precise formulation of the geometry of momentum space from which the consequences for the questions we opened with can be exactly derived.

The idea that momentum space should have a non trivial geometry when quantum gravity effects are taken into account was originally proposed by Max Born, as

[1] We work in units such that the speed-of-light scale c is set to 1.

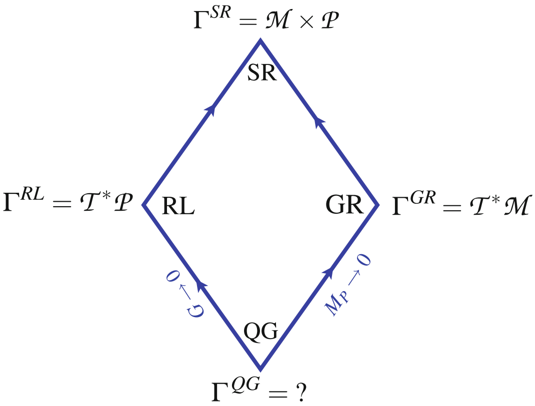

Fig. 1 We show here that general relativity and relative locality are two ways of deepening the relativity principle. In general relativity spacetime is curved but momentum space is flat. The opposite is the case in relative locality. This has consequences for the phase space description as is shown, and elaborated below. Alternatively, starting from an unknown quantum theory of gravity, one can ascend to special relativity through two paths. Taking $\hbar \to 0$ but keeping G_N fixed (so that M_p also goes to 0) one ascends on the *right* to general relativity. But there is an alternative. Keep M_p fixed while taking $G \to 0$ (and hence also $\hbar \to 0$) leads to the relative locality regime on the *left*

early as 1938 [5]. He argued that the validity of quantum mechanics implies there is in physics an equivalence between space and momentum space, which we now call Born reciprocity. The introduction of gravity breaks this symmetry between space and momentum space because space is now curved while momentum space is a linear space-and hence flat. Allowing the momentum space geometry to be curved is a natural way to reconcile gravity with quantum mechanics from this perspective.

Remarkably, this is exactly what has been shown to happen in a very illuminating toy model of quantum gravity, which is quantum gravity in $2+1$ dimensions coupled to matter. There Newton's constant G has dimensions of inverse mass, and indeed it turns out [6,7] that in $2+1$ dimensions the momentum space of particles and fields is a manifold of constant curvature G^2, while spacetime is (locally) flat [8].

There are two kinds of non-trivial geometry (metric and connection) any manifold, including momentum space, can have. Each of these has, as shown in [2], a characterization in terms of observable properties for the dynamics of particles. A metric in momentum space $ds^2 = g^{\mu\nu}(p)\mathrm{d}p_\mu \mathrm{d}p_\nu$ is needed in order to write energy-momentum on-shell relation

$$m^2 = D^2(p) \tag{2}$$

where $D(p)$ is the distance of the point p_μ from the origin $p_\mu = 0$. A non-trivial affine connection is needed in order to produce non-linearities in the law of composition of momenta, which is used in formulating the conservation of momentum

$$(p \oplus q)_\mu \simeq p_\mu + q_\mu - \frac{1}{M_p}\Gamma^{\alpha\beta}_\mu\, p_\alpha\, q_\beta + \cdots \tag{3}$$

where on the right-hand side we assumed momenta are small with respect to the Planck mass M_p and $\Gamma_\mu^{\alpha\beta}$ are the (M_p-rescaled) connection coefficients on momentum space evaluated at $p_\mu = 0$.

We can show that the geometry of momentum space has a profound effect on localisation through an elementary argument. To do this we look at the role that the special-relativistic linear law of conservation of momenta has in ensuring that locality is absolute. Suppose x_I^μ are the positions of several particles that coincide at the event e in the coordinates of a given observer. The total-momentum conservation law generates the transformation from that observer to another separated from the first observer by a vector, b^μ. In the special relativistic case the total momentum is the linear sum $P_\nu^{\text{tot}} = \sum_J p_\nu^J$ and one finds

$$\delta x_I^\mu = \left\{x_I^\mu, b^\nu P_\nu^{tot}\right\} = \left\{x_I^\mu, b^\nu \sum_J p_\nu^J\right\} = b^\mu \tag{4}$$

so that all the worldlines are translated together, independent of the momentum they carry.

This is the familiar notion of absolute locality afforded by the special-relativistic setting. If instead momentum space has a non-trivial connection, in the sense discussed above, then P_μ^{total} is nonlinear, i.e.,

$$P_\mu^{\text{total}} = \sum_I p_\mu^I + \frac{1}{M_p} \sum_{I<J} \Gamma_\mu^{\nu\rho} p_\nu^I p_\rho^J \tag{5}$$

Then

$$\delta x_I^\mu = \left\{x_I^\mu, b^\nu P_\nu^{\text{total}}\right\} = b^\mu + \frac{1}{M_p} b^\nu \sum_{J>I} \Gamma_\nu^{\mu\rho} p_\rho^J. \tag{6}$$

Thus we see that how much a worldline of a particle is translated depends on the momenta carried by it and the particles it interacts with. The net result is the feature we call "relative locality", illustrated in Fig. 2. Processes are still described as local in the coordinatizations of spacetime by observers close to them, but those same processes are described as nonlocal in the coordinates adopted by distant observers.

These novel phenomena have a consistent mathematical description in which the notion of spacetime gives way to an invariant geometry formulated in a phase space. In special relativity, the phase space associated with each particle is a product of spacetime and momentum space, i.e. $\Gamma^{SR} = \mathcal{M} \times \mathcal{P}$.

In general relativity, the spacetime manifold $\mathcal{M}$ has a curved geometry. The particle phase space is no longer a product. Instead, there is a separate momentum space, $\mathcal{P}_x$ associated to each spacetime point $x \in \mathcal{M}$. This is identified with the cotangent space of $\mathcal{M}$ at x, so that $\mathcal{P}_x = \mathcal{T}_x^*(\mathcal{M})$. The whole phase space is the cotangent bundle of $\mathcal{M}$, i.e. $\Gamma^{GR} = \mathcal{T}^*(\mathcal{M})$

Within the framework of relative locality, it is the momentum space $\mathcal{P}$ that is curved. There then must be a separate spacetime, $\mathcal{M}_p$ for each value of momentum,

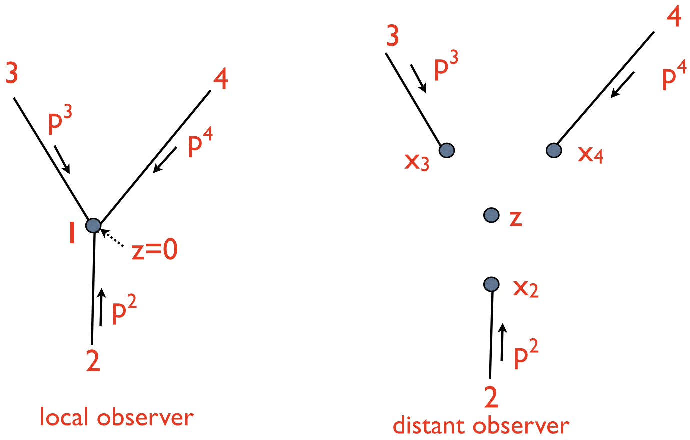

Fig. 2 Relative locality implies that the projection from the invariant phase space description to a description of events in spacetime leaves a picture of localization which is dependent on the relation of the observer (or origin of coordinates) to the event. If the event is at the origin of the observer's coordinate system, then the event is described as local, as on the *left*. But if the event is far from the origin of the observer's coordinates, the event is described as non-local, in the sense that the projections of the ends of the worldlines no longer meet at the point where the interaction takes place. This is not a weakening of the requirement that physics is local, it is instead a consequence of the energy dependence of the procedure by which the coordinates of distant events are inferred. There is an invariant description, but it is in a phase space

$\mathcal{M}_p = \mathcal{T}^*_p(\mathcal{P})$. The whole phase space is then the cotangent bundle over momentum space, i.e. $\Gamma^{RL} = \mathcal{T}^*(\mathcal{P})$.

If one wants to compare momenta of particles at different points of spacetime in general relativity, x and y, one needs to parallel transport the covector $p_a(x)$ along some path γ from x to y, using the spacetime connection. Now, suppose, within the dual framework of relative locality, we want to know if the worldlines of two particles, A and B, with different momenta, meet. We cannot assert that $x^\mu_A = x^\mu_B$ because, quite literally, they live in different spaces, as they correspond to particles of different momenta. What we can do is to ask that there is a parallel transport on momentum space that takes them to each other. If so, there will be a linear transformation, $[U_\gamma]^\mu_\nu$, which maps the spacetime coordinates associated with momenta p^A_μ to those associated with the momenta p^B_μ. This will be defined by the parallel transport along a path γ in momentum space, so that

$$x^\mu_B = [U_\gamma]^\mu_\nu x^\nu_A \tag{7}$$

This can be implemented very precisely from an action principle associated with every interaction process. The free part of the action associated with each worldline given by

$$S^{\text{free}} = \int \mathrm{d}s (x^\mu \dot{p}_\mu + N(D^2(p) - m^2)) \tag{8}$$

imposes the on-shell relation, while the interaction implement the conservation law $\mathcal{K}(p_I(0)) = 0$ at the interaction event

$$S^{\text{int}} = z^\mu \mathcal{K}_\mu(p_I(0)). \tag{9}$$

The relationship (7) follows from the variation of this action principle with respect to the momenta at the inreraction events. It turns out that the path γ, along which we parallel transport a spacetime coordinate in momentum space, is specified by the form of the conservation law at an interaction event between the two particles. This is very parsimonious, it says that the two particles need to interact if we are to assert whether their worldlines cross.

Notice that according to (7) one is still assured that if the event is such that, in the coordinates of a given observer, $x_A^\mu = 0$ then it is also the case that $x_B^\mu = 0$. This is why we assert that there are always observers, local to an interaction, who see it to be local. One also sees that if the connection vanishes then $(U_\gamma)_\mu^\nu = \delta_\mu^\nu$ and $x_A = x_B$ and we recover the usual picture where interaction are local.

Let us expand the parallel transport in terms of the connection:

$$[U_\gamma]_\nu^\mu = \delta_\mu^\nu + \frac{1}{M_p}\Gamma_\mu^{\nu\rho} p_\rho + \cdots \tag{10}$$

It will follow that the difference Δx^μ between x_A^μ and x_B^μ is proportional to x_A^μ and p_μ. It can therefore be said that the deviation of locality is at first order of the form

$$\Delta x \sim x \frac{E}{M_P}. \tag{11}$$

We see from this formula (11), that the smallness of M_p^{-1} can be compensated by a large distance x, so that over astrophysical distances values of Δx which are consequences of relative-locality effects take macroscopic values [4]. A more detailed analysis shows that there really are observable effects on these scales [4] which are relevant for current astrophysical observations of gamma ray bursts, in which precise measurements of arrival times are used to set bounds on the locality of distant events [3,9]. But this is not all. Other experiments which may measure or bound [10] the geometry of momentum space at order M_p^{-1} include tests of the linearity of momentum conservation using ultracold atoms [11] and the development of air showers produced by cosmic rays [12].

Such phenomena are very different in nature from the predictions of detailed quantum theories of gravity for the Planck length regime. It is unlikely we will ever detect a graviton [13–15], but it is reasonable to expect that relative locality can really be distinguished experimentally from absolute locality. By doing so the geometry of momentum space can be measured.

A nineteenth-century scientist conversant with Galilean relativity could have asked: do we “see” space? Einstein taught us that the answer is negative: there is a maximum speed and at best we “see” spacetime. We now argue that this too is wrong. What we really see in our telescopes and particle detectors are quanta arriving at different angles with different momenta and energies. Those observations allows us to infer the existence of a universal and energy-independent description of physics in a spacetime only if momentum space has a trivial, flat geometry. If, as Max Born argued, momentum space is curved, spacetime is just as observer dependent as space, and the invariant arena for classical physics is phase space.

So, look around. Do you "see" spacetime? or do you "see" phase space? It is up to experiment to decide.

Acknowledgment We are very grateful to Stephon Alexander, Michele Arzano, James Bjorken, Florian Girelli, Sabine Hossenfelder, Viqar Husain, Etera Livine, Seth Major, Djorje Minic, Carlo Rovelli, Frederic Schuller and William Unruh for conversations and encouragement. G.A.C. and J.K.G. thank Perimeter Institute for hospitality during their visits in September 2010, when the main idea of this paper was conceived. J.K.G. was supported in part by grant 182/N-QGG/2008/0. The work of G.A.C. was supported in part by grant RFP2-08-02 from The Foundational Questions Institute (fqxi.org). Research at Perimeter Institute for Theoretical Physics is supported in part by the Government of Canada through NSERC and by the Province of Ontario through MRI.

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