

Estimating the parameters of the Sgr A* black hole

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Abstract The measurement of relativistic effects around the galactic center may allow in the near future to strongly constrain the parameters of the supermassive black hole likely present at the galactic center (Sgr A*). As a by-product of

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these measurements it would be possible to severely constrain, in addition, also the parameters of the mass-density distributions of both the innermost star cluster and the dark matter clump around the galactic center.

Keywords Black holes · Galactic center (Sgr A*) · Gravitational lensing · Strong gravitation field relativistic effects

1 Introduction

Wheeler was one of the greatest scientists of the twentieth century because of his many contributions to nuclear physics, theory of quantum measurement, general relativity and relativistic astrophysics. He had a talent to express complex concepts in simple and catchy form. In particular, he coined the term “black hole” which has gained enormous popularity even among the non-scientists. He has been called the greatest un-crowned Nobel laureate. His way of writing was very attractive for readers because he was a great scientist, teacher and writer in one personality. The last scientific paper by Wheeler was on retro-MACHOs (we prefer to call them retro-lensing or relativistic images), written in collaboration with Holz [1]. The authors took seriously something that was well known to be possible around a black hole (BH) in principle. They considered our Sun as the source of light rays that, if the impact parameter is right, may go around the BH and reach the observer forming a ring around the BH. Light rays with a slightly smaller impact parameter may go twice around the BH and then escape to the observer forming a slightly smaller ring, and so on. So, in the case of perfect alignment, an infinite series of rings should appear and therefore they suggested that a survey of the sky be made to look for these rings as a way to search for isolated BHs near the Sun.

In general, Earth moves on its orbit so that the retro-lensing image magnitude changes with time in a periodic way. The problem is that our Sun is not a very bright source and even using the HST, only a BH heavier than $10 M_{\odot}$ within 0.01 pc might be revealed in this way. Although it may be expected that sooner or later a survey of the sky to search for such rings will take place, it is for the moment much better to consider binary systems composed of massive BHs and luminous stars and look, therefore, towards well known BH candidates. This is exactly what has been suggested in [2] soon after the Holz and Wheeler paper appeared, that is the supermassive black hole at the galactic center (Sgr A*) is the most interesting retro-lens to look for.¹

A classical method for estimating the physical parameters (in particular mass and angular momentum) of BHs and in particular of the candidate supermassive black hole at Sgr A* is to look for the periastron or apoastron shift of the stars orbiting around it (the so-called S-stars). In Sect. 2 we shall briefly review this issue. However, the amount of the apoastron shift strongly depends also on the distribution of both the stars and the dark matter making it practically impossible to estimate the BH parameters.

¹ Similar results had already been pointed out in [3] (see also [4] as a recent paper on this issue).

A more direct way to estimate the central BH parameters, including in principle also its electric charge) is to measure the shape of the retro-lensing images of the brightest and innermost stars in the K-band of the electromagnetic spectrum (or also the shape of the BH shadow in the radio band). This issue, together with the information that can be obtained by measuring the spectrum (effect not considered in detail in the literature) of the stellar retro-lensing images will be discussed in Sects. 3 and 4.

By combining these measurements, also taking into account the proper motion of the supermassive BH, it will be possible in the near future to estimate both the BH parameters and those of the stellar and dark matter distributions.

2 S-stars apoastron advance: a laboratory to measure the supermassive BH parameters

The precise measurements of the velocity dispersion of the S-stars orbiting around SgrA* have allowed us, in the last decade, to constrain more and more strongly the mass enclosed in a smaller and smaller region around the galactic center. With the Keck 10m telescope, the proper motion of several stars orbiting the Galactic Center have been accurately monitored and the entire orbit of the S2 star has been measured allowing an unprecedented description of the Galactic Center region (for a review see [5]). The estimates of the supermassive BH mass, or more exactly of the amount of mass $M(< r)$ contained within a certain distance r from the galactic center, changed dramatically during the last decade from about $2.6 \times 10^6 M_{\odot}$ within about 10^{-2} pc [6], to $\simeq 4 \times 10^6 M_{\odot}$ within 10^{-2} pc [7]. Ghez et al. [8,9] have found that a mass of about $(3.67 \pm 0.19) \times 10^6 M_{\odot}$ must be present within the central 3×10^{-4} pc of the galactic center.

An important point to be mentioned is that the identification of the super massive BH at the galactic center is also based on the observations from radio to Near IR to X-ray wavelengths [10]. The bolometric luminosity of Sgr A* is $\simeq 2 \times 10^{36}$ erg s⁻¹, that is $\simeq 4 \times 10^{-9}$ of the Eddington luminosity (this is also known as the “blackness problem” and accounting for that is an important issue to be settled in all details). This means that Sgr A* is a very faint source, most luminous in the sub-millimeter wavelengths (explained naturally as synchrotron emission by thermal electrons near the BH). Also X and γ photons are detected from the direction of Sgr A* that, however, is not resolved at these wavelengths.² The identification of the supermassive BH at Sgr A* may be considered as not completely settled as yet, although the supermassive BH option seems to be that that more naturally explains the different observational evidences.

Further information may be obtained, in principle, from the measurement of the stellar periastron (or apoastron) shifts yielding Rosetta-type orbital shapes. Several authors have discussed the possibility of measuring the general relativistic corrections to Newtonian orbits for the BH at SgrA*, assuming either a Schwarzschild or a Kerr

² Recently, a $\simeq 20$ min variability observed by XMM-Newton in the 2004 X-ray flare towards Sgr A* has been interpreted as due to matter falling from the innermost stable orbit to the Schwarzschild radius of a $3.7 \times 10^6 M_{\odot}$ BH at the galactic center [11].

BH. For a test particle orbiting a Schwarzschild black hole of mass M_{BH} , the periastron shift is given by

$$\Delta\phi_S \simeq \frac{6\pi GM_{BH}}{d(1-e^2)c^2} + \frac{3(18+e^2)\pi G^2 M_{BH}^2}{2d^2(1-e^2)^2c^4}, \tag{1}$$

d and e being the semi-major axis and eccentricity of the test particle orbit, respectively. For a rotating black hole with spin parameter $a = |\mathbf{a}| = J/GM_{BH}$, in the most favorable case of equatorial plane motion ($\mathbf{a} \cdot \mathbf{v} = 0$), the shift is given by (Boyer and Price [12], but see also [13] and references therein as a recent paper on this issue)

$$\Delta\phi_K \simeq \Delta\phi_S + \frac{8a\pi M_{BH}^{1/2} G^{3/2}}{d^{3/2}(1-e^2)^{3/2}c^3} + \frac{3a^2\pi G^2}{d^2(1-e^2)^2c^4}, \tag{2}$$

which reduces to Eq. (1) for $a \rightarrow 0$. The apoastron (periastron) shifts (measured in mas/revolution) as seen from Earth (at the assumed distance of $R_0 = 8$ kpc from the galactic center) is $\Delta\phi_{E*^\pm} = d(1 \pm e)\Delta\phi/R_0$, where the sign $+$ holds for the apoastron and the $-$ for the periastron, respectively.

As discussed in [14], notice that the differences between the periastron shifts for the Schwarzschild and the maximally rotating Kerr black hole is at most 0.01 mas for the S2 star and 0.009 mas for the S16 star. In order to make these measurements with the required accuracy, one needs to measure S-stars orbits with a precision of at least $10\mu\text{as}$. At present³ this precision is not attainable by the available instruments, but in the near future it will certainly be reached. For example, there is a proposal to improve the angular resolution of VLTI with the PRIMA facility (see the web site <http://obswww.unige.ch/PRIMA/home/introduction>) which, by using a phase referenced imaging technique, will get $\sim 10\mu\text{as}$ angular resolution.

However, even if at least in principle the effect of a maximally spinning BH on the periastron shift of an S-star can be distinguished from that induced by a Schwarzschild BH with the same mass, actually that is, in practice, impossible due to the presence of the stellar cluster surrounding the supermassive BH. The effect of this stellar cluster distribution gives (for most of the possible configurations) a much larger effect on the periastron shift with respect to that of the central BH. In particular, if one describes the stellar cluster by a Plummer model with density profile

$$\rho_{CL}(r) = \rho_0 \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-\alpha/2}, \tag{3}$$

where the central cluster density ρ_0 , its core radius r_c and the power slope α are free parameters constrained by the condition that the total mass $M(r) = M_{BH} + M_{CL}(r)$, it can be shown that if r_c is smaller than the semimajor axis of a considered star, the apoastron shift induced by the cluster is in the same direction of that induced by the BH, while they are in opposite directions if r_c is larger than the stellar semimajor

³ For the case of relative astrometry, the present uncertainty in the position measurement of a star brighter than $K = 15.5$ is about 0.1 mas [15].

axis (for further details see [14]). A detailed analysis shows that, if one considers the S2 star, the transition from prograde shift (due to the BH) to retrograde shift (due to the stellar cluster) occurs for a cluster mass of only 0.1–0.3% of the BH mass for $r_c \simeq 5.5$ mpc, almost irrespectively of the value of α . This means that a small fraction of mass in the cluster drastically changes the overall shift.

3 Shadow, or gravitational retrolensing to measure the supermassive BH hairs

The calculation of the retro-lensing images by numerical methods is quite complicated because it is necessary to integrate null geodesics with very high accuracy (see, e.g. [3, 16, 17]). In the Schwarzschild case for example, obviously, as the impact parameter u becomes smaller and smaller the deflection angle grows (the weak deflection limit is no longer valid at this point) until it diverges at the limiting value of the impact parameter $u_m = 3\sqrt{3}GM/c^2$. Therefore, very large deflections are possible for u close to u_m . A deflection of order π means that the photon turns around the BH and goes back to the source (retro-lensing as named by Holz and Wheeler). Photons with impact parameter equal to u_m are injected in a circular orbit of radius $r_m = 1.5 r_s$ around the BH and photons with $u < u_m$ fall inside the BH and get lost. Let's note that the angular separation from the BH of the images formed by photons with impact parameter u is practically given by $\theta = \arcsin(u/D_{OL}) \simeq u/D_{OL}$ and the fact that no deflection is possible for $u < u_m$ implies the existence of a lower value for the image angular separation that is (in other words no source image can appear at angular separation less than) $\theta_m = u_m/D_{OL}$. This minimum angle θ_m defines the so-called shadow of the BH. The case for spinning and/or charged black holes has been also considered in the literature (see, e.g. [18–25]). The shadow of the BH is slightly larger than the BH horizon and the shape depends on the BH parameters, as for the retrolensing curves. Due to the strong gravitational field the light emitted behind the BH cannot reach us since it is deflected, trapped and absorbed by the BH. The effect manifests itself with the absence of radiation—the shadow—at the boundary of the event horizon. The dimension of the shadow should be about $30 \mu\text{as}$. It is like the shadow cast by the moon on the sun during a solar eclipse. The difference in the BH case is the strong gravitational field that makes the shadow larger than the event horizon.

The magnitude of the retrolensing images by the BH in Sgr A* has been discussed (see, e.g. [2, 20]) both for the Schwarzschild and Kerr BH. In the Schwarzschild case, for the S2 star, they are in the range 33.3–36.8 (depending on the star distance from the black hole) in the K-band, that turns out to be close to the limiting magnitude of the next generation of space-based telescopes. The BH spin effect gives only a minor correction on the retrolensing image brightness and so it cannot increase substantially the image amplification with respect to the Schwarzschild black hole case [20]. An even more important effect induced by the BH spin is the retrolensing image deformation [21, 25] since light rays co-rotating with the BH spin can get closer to the BH horizon, while counter-rotating photons get further away. Also the eventual BH electric charge has an influence on the shape of the retrolensing images in the sense that the higher the charge the smaller is the size of the retrolensing image that becomes $r_s/2$ for a maximally charged Reissner–Nordstrom BH [22].

The shape of the retrolensing images will be possibly measured in the future in the K-band or even in X-rays. In the last case the source would be an accreting neutron star or stellar mass BH (there are many of these objects around Sgr A*, as shown by CHANDRA). However, the first direct information about the parameters of the BH at Sgr A* will probably come by the detections of the shadow shape in the radio band. The present angular resolution obtained last year by Doeleman is about $37 \mu\text{as}$ at 1.3 mm [26]. However, at wavelengths higher than 1 mm the scattering by electrons surrounding the BH blurs the image making any progress in the resolution useless. Sub-millimeter data would probably be able to show the shadow rather soon and later on will be even possible to observe its shape.

The angular resolution of an instrument able to see the shape of the shadow is close to that necessary to read a newspaper open on the moon from Earth. In the next few years, it will be likely possible to identify the spin of the BH in Sgr A*, either by detecting the periodic signature of hot spots at the innermost stable circular orbit or by producing, for example by a (sub)millimeter VLBI “Event Horizon Telescope”, images of the galactic center emission to see the shadow silhouette. These techniques are also applicable to the BH in M87, where the BH spin may be key to understanding the jet-launching region.

Before closing this section we wish to emphasize that, while many of the available observations in the radio, IR and X-ray bands may or may not be interpreted as evidence for the supermassive BH at the galactic center, the only one that would unambiguously demonstrate the existence of such a BH is the observation of retrolensing images towards Sgr A*.

4 Retrolens spectrum

A novel effect—which is a consequence of the black hole spin—on the retro-lensing image: the frequency change of the observed light.

The effect we intend to exploit for our purposes was essentially found by Penrose and Floyd [27]. They demonstrated that rotational energy can be extracted from a spinning black hole by sending in a particle co-rotating with the hole. If the particle decays inside the ergosphere (the region inside which particles at rest would appear to be moving superluminally according to observers at infinity) the decay product falling into the black hole would have a spacelike momentum vector and so it would deposit a negative energy according to the observer at infinity. It was later shown by Christodoulou and Ruffini [28] that there is a limit on the amount of energy that can be so extracted. This fact leads one to consider what happens to a photon passing through the ergosphere, and close to the limit it is allowed to reach before it is pulled into the hole (the photosphere). At first one would be tempted to answer that there is no change, as the photon does not decay inside the ergosphere. However, this intuitive answer takes for granted that the photon is a test particle. One should really consider the back-reaction of the photon on the geometry. This may be too difficult to accomplish directly. Let us, therefore, consider the matter qualitatively.

We consider the photon as being absorbed and then emitted by the black hole. More correctly, what has happened is that the photon enters and then exits the region where frame dragging is significant. This speeds up the hole by a minuscule amount if the

photon is co-rotating with it (and slows it down when it is counter-rotating), as some angular momentum and energy are imparted to it while it carries the photon along. Now the photon will have somewhat more energy than when it was emitted by the source. The longer it is “carried along” the more energy is gained. The closer it gets to the hole the more the effect is. This is simply the analogue for photons, of the “gravitational slingshot” used to accelerate spacecraft, that gives us an estimate of the energy gained by the photon. In terms of the change of frequency $\delta\nu$ we then have

$$\frac{\delta\nu}{\nu} \simeq \frac{Gm a}{bc^2 b}, \tag{4}$$

which is ma/b^2 in gravitational units ($G = c = 1$), where m is the mass of the hole, b the distance of closest approach to the hole and a the spin parameter of the hole (being the angular momentum per unit mass). To be able to obtain the numerical factor and to be surer of the result, we need a more precise analysis.

We could also obtain the frequency change by invoking the formula: $\delta\nu/\nu = \delta t/t$, where δt is the time delay due to the rotation of the hole and t is the time spent in the strong field of the hole. If the light goes round the hole and is reflected, the time taken is the distance travelled, which is half way round the hole (at light speed), which is πb . The time delay is due to the cross term in the metric, responsible for frame dragging, $2ma/b$ and acts over an angle 2π . Thus, the total effect for the spectral shift for the n th retro-lensed ring is

$$\left(\frac{\delta\nu}{\nu}\right)_n = 4\frac{ma}{b^2}(2n - 1). \tag{5}$$

Indeed, $4ma/b^2$ is the effect that would be expected for the primary retro-lensing image. The secondary one would have an additional time delay of going once around, i.e. it would be three times this value and the next five times, so that for the n th ring it would be $(2n - 1)$ times this value.

One may worry whether the t in the denominator takes on the extra multiples and so $\delta\nu/\nu$ should be the same for all “rings”. To check this it is only necessary to Lie transport the momentum vector for the photon along the orbit around the hole. It is obvious from here that the $(2n - 1)$ factor is real. Another worry could be that the endless increase of photon energy by going around the hole many times may seem unreasonable. To see that this is not so, notice the analogy of the process used with the acceleration of charged particles in an accelerator. By taking a charged particle many times around in an appropriate electromagnetic field, one can go on accelerating it. Here we are taking a photon around a gravitational field and “accelerating” it in the sense of increasing its energy. We may view the spinning hole as a “giant *photon* synchrotron” with very low yield.

For a counter-rotating photon, since the angle ϕ is integrated in the reverse direction, the effect is negative and we get a red shift instead of a blue shift. Naively, we might expect a ring, blue-shifted on one side and red-shifted on the other. This will not be the case. There are two problems with this naive expectation. The first is that the distance of closest approach for a Schwarzschild hole is the *photosphere*, $3m/2$, but for the Kerr hole it changes. In the co-rotating direction it coincides with the horizon for

an extreme hole and in the counter-rotating direction it is 4 times that value. As such the ring should be off the center of the hole and the $\delta\nu/\nu$ decreased by the square of the factor increasing b . Roughly speaking then, in the case of the perfect alignment of Holz and Wheeler, we should expect—on account of the black hole spin—a ring-like pattern one side of which is bluer and the other redder, with the hole at the focus closer to the bluer side. The ring will be brighter on the bluer side and fade away from it as the redder side is approached.

Of course, this is only the *Doppler* shift due to the photon being “accelerated” or “decelerated”. There is also the gravitational red-shift to take into account. Since the photon approaches closer to the source when it is blue-shifted and is further away when it is red-shifted, the two gravitational red-shifts are not equal. A simple computation, for $a = 1$, gives the red-shift on closer approach to be $9/29$ and on further approach to be $8/13$. The net blue-shift for the first ring gives $\delta\nu/\nu \simeq 1.945$ and the red-shift to be $\delta\nu/\nu \simeq -0.950$. Since the gravitational red-shift remains the same for all the rings for the second (and fainter) ring the total blue shift would be $\delta\nu/\nu \simeq 7.065$ and the total red-shift would be $\delta\nu/\nu \simeq -2.230$.

The shape of the retro-lensing image in the perfect alignment case is shown in Fig. 1. The circular shape corresponds to a Schwarzschild black hole, the second image to a

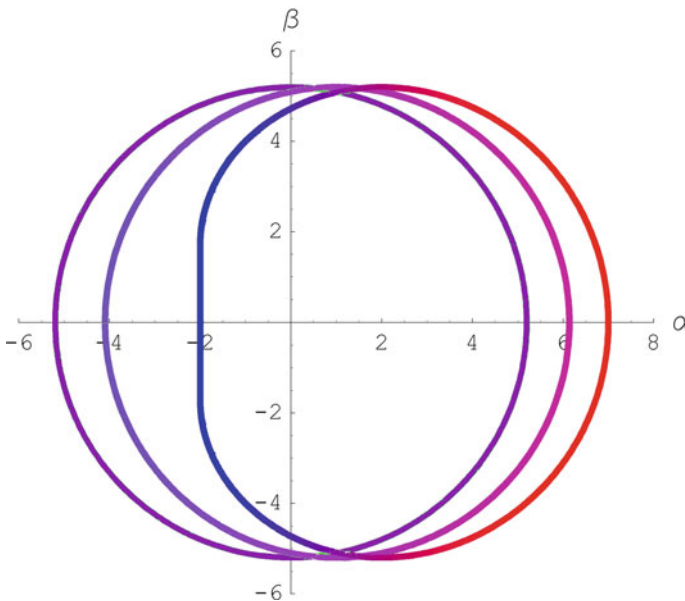


Fig. 1 The precise shape of the retro-lensing image in the case of perfect alignment, with the spin perpendicular to the line of sight, is shown. Here, α and β are the celestial coordinates of the image as seen by an observer at infinity. The circular image corresponds to the retro-lensing image for a Schwarzschild black hole. The second image is that expected for a Kerr black hole with rotation parameter $a = 0.5$. The third one corresponds to a maximally rotating Kerr black hole with $a = 1$. The *coloring of the rings* is given to indicate the frequency shift due to the spin effect (not giving the precise color shifts). Note that the actual ring colors have been simulated much more precisely for quantitative prediction [31]. Here the black hole is assumed to rotate counterclockwise as seen from above. The unit of length along the coordinate axes α and β is M , so that the image in the Schwarzschild case is at a distance of $3\sqrt{3}/2$ Schwarzschild radii from the black hole center

Kerr black hole with $a = 0.5$, the third one to a maximally rotating Kerr black hole ($a = 1$). Here, the black hole is assumed to rotate counterclockwise as seen from above. As is clear from the discussion above, in the Schwarzschild case the retrolensing image has everywhere the same color, since no spectral shift effects are present in this case. For a Kerr black hole, instead, the light rays co-rotating with the black hole form a closer image which is also blue-shifted with respect to that formed by counter-rotating photons. The maximal effect is obtained for a Kerr black hole with $a = 1$. In Fig. 1, α and β are the celestial coordinates of the image as seen by an observer at infinity which are function of the parameters ξ and η describing the null geodesics in the equatorial plane (see Chandrasekhar [29]).

The other problem is that photons going off the equatorial plane will not be reflected properly [24,25]. The further they are off the plane the more the path is altered. As such they will be distorted. Further, the photons for the extra rings will be shot off at even further removed angles and so will disappear even more sharply than would be the case for the Schwarzschild hole. Precise simulations of the large angle deflection images about rotating black holes have been made by using ray tracing techniques [30,31]. In particular, [30] have looked at the K_α line of iron and its Doppler shift. The inner regions of the images created in the simulations are given in [31].

Both the above papers ignore the higher order relativistic images discussed in the present paper. There would be a time delay from one image to the next due to the photon going around an extra time [32]. For Sgr A* the time delay would be as much as 10 min! The second ring, though much fainter (on account of the lower intensity of photons), would show much higher frequency shifts and would not be so contaminated by the other rays that broaden the primary rings. This is because only those rays would be able to go around the extra time that are adequately well aligned. (In fact that is what makes the image fainter.)

5 Concluding remarks

Let us now look at the possibility of really measuring in the near future the BH parameters at Sgr A*. At first let us consider the retrolensing method and in particular the shape of the retrolensed images. As can easily understood, this is a more difficult task than detecting simply the retro-lensing images. For simplicity let us confine ourselves to the perfectly aligned case and consider only the first of the infinite number of the retro-lensing rings. The first effect of the black hole spin is in fact that of deforming the circular rings expected for the Schwarzschild case. This effect is considered in [20], but it is obvious that detecting the deformation of the rings is not an easy task since the angular resolution of the instrument must be smaller with respect to the ring angular extent, which in turn is around $3r_s/2$. This distance is expected to be in most cases much smaller with respect to the angular resolution of available instruments.

As discussed in [2], with the next generation space telescopes like NGST [33] it will be possible to detect in the K-band the retro-lensing images of the S2 star orbiting around Sgr A*. However, NGST will not have the angular resolution necessary to make the spectrum of the two sides of the image to measure the spectral shift. Since the angular size of the Schwarzschild radius of the Sgr A* black hole is of about

10 μas as an instrument with at least that angular resolution is necessary to eventually observe the proposed effect. In the near-medium future, several space-based instruments may have the necessary angular resolution such as VLBI [34,35] in the radio band and MAXIM [36] and Constellation X [37] in the X-ray band, so that there is a hope of measuring the black hole spin by simply measuring the spectrum of a retrolensing image. Actually, it is very likely that the first real measurement of the central BH parameters will be made by using radio observations detecting the shadow of the BH. Radio observations have presently angular resolution of 37 μas at 230 GHz with a baseline of 4,500 km [26], that is very close to the size of the BH shadow (about 30 μas for a 4 million solar mass BH). Of course, to measure the shape of the shadow a much better angular resolution is needed, but progress in this field is rather fast and in the next one or two decades an angular resolution of a few μas in the radio band is attainable. Moreover, it would be also possible in future to use different bandwidths to determine the spectral effect discussed in the previous section and therefore estimate the supermassive BH parameters.

As discussed in Sect. 2, the measurement of the supermassive BH physical parameters by using retrolensing images (or the BH shadow), together with the measurement of the periastron (or apoastron) shift of some of the S-stars, offers a unique opportunity of estimating also the mass and density distribution of the star cluster present around the BH at Sgr A*. With an instrument able to get accurate position measurements (within 10 μas , the pericenter shift measurements of only three S-stars is enough to strongly constrain the Plummer model parameters of the stellar cluster.

A dark matter (DM) component (for example constituted by WIMPs) may be present around the galactic center, in addition to the BH and the stellar cluster, and this mass component could also modify the trajectories of the stars moving around Sgr A* significantly, depending on the DM mass distribution. One can therefore use a three component model for the central region of our galaxy constituted by a central BH, a stellar cluster and a DM sphere, i.e.

$$M(< r) = M_{BH} + M_*(< r) + M_{DM}(< r). \tag{6}$$

Following [38], the DM concentration can be described by a mass distribution of the form

$$M_{DM}(< r) = \begin{cases} M_{DM} \left(\frac{r}{R_{DM}} \right)^{3-\beta}, & r \leq R_{DM} \\ M_{DM}, & r > R_{DM} \end{cases} \tag{7}$$

where β is a free parameter and M_{DM} and R_{DM} are the total amount of DM in the form of WIMPs and the radius of the spherical mass distribution, respectively. In this case, for the central stellar cluster, in [38] the empirical mass profile obtained by the data has been used

$$M_*(< r) = \begin{cases} M_* \left(\frac{r}{R_*} \right)^{1.6}, & r \leq R_* \\ M_* \left(\frac{r}{R_*} \right)^{1.0}, & r > R_* \end{cases} \tag{8}$$

with a total stellar mass $M_* = 0.88 \times 10^6 M_\odot$ and a size $R_* = 0.3878 \text{ pc}$.

The present upper limit of 10 mas on the periastron shift of the S2 and S16 stars allows us to constrain the radius of the dark matter distribution (assumed, following [38], with a total mass $M_{DM} \simeq 2 \times 10^5 M_{\odot}$) more strongly than the results in [38] (where it is found that there are acceptable configurations only with size in the range 10^{-4} –1 pc). Consideration of the available upper limit of the stellar periastron shift allows instead the conclusion that DM configurations of the same mass are acceptable only for R_{DM} in the range between 10^{-3} and 10^{-2} pc, almost irrespectively of the DM β value [39,40].

Much stronger constraints on both the DM concentration and stellar cluster may be obtained in the future if a real measurement (and not only an upper limit) of the periastron shift of some S-stars will be available. With a measurement with angular resolution about $10 \mu\text{as}$ of the periastron shift of only two stars it would be possible to severely constrain the DM distribution if the stellar cluster distribution is known and provided that the BH parameters are known. With the S-stars presently known, and taking into account their periods [41,42], it would require about 19 years of measurements of the stellar orbits with a precision of about $10 \mu\text{as}$ to get the desired result. With the measurement of the periastron shift of five S-stars it would be possible to constrain the parameters of both the stellar and DM clusters. That would require about 50 years of observations with that precision.

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