Gen Relativ Gravit (2009) 41:1835–1851
DOI 10.1007/s10714-008-0755-9

RESEARCH ARTICLE

Star models with dark energy

R. Chan · M. F. A. da Silva · Jaime F. Villas da Rocha

Received: 9 July 2008 / Accepted: 31 December 2008 / Published online: 29 January 2009

Abstract We have constructed star models consisting of four parts: (i) a homogeneous inner core with anisotropic pressure (ii) an infinitesimal thin shell separating the core and the envelope; (iii) an envelope of inhomogeneous density and isotropic pressure; (iv) an infinitesimal thin shell matching the envelope boundary and the exterior Schwarzschild spacetime. We have analyzed all the energy conditions for the core, envelope and the two thin shells. We have found that, in order to have static solutions, at least one of the regions must be constituted by dark energy. The results show that there is no physical reason to have a superior limit for the mass of these objects but for the ratio of mass and radius.

Keywords Dark energy · Star model · Gravastar

1 Introduction

Over the past decade, one of the most remarkable discoveries is that our universe is currently accelerating. This was first observed from high red shift supernova Ia [1–7],

R. Chan (✉)
Coordenação de Astronomia e Astrofísica, Observatório Nacional,
Rua General José Cristino 77, São Cristóvão, Rio de Janeiro, RJ CEP 20921-400, Brazil
e-mail: chan@on.br

M. F. A. da Silva
Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro,
Rua São Francisco Xavier 524, Maracanã, Rio de Janeiro, RJ CEP 20550-013, Brazil
e-mail: mfasnic@gmail.com

J. F. Villas da Rocha
Coordenadoria de Educação em Ciências, Museu de Astronomia e Ciências Afins,
Rua General Bruce 586, São Cristóvão, Rio de Janeiro, RJ CEP 20921-030, Brazil
e-mail: jaime@mast.br

and confirmed later by cross checks from the cosmic microwave background radiation [8,9] and large scale structure [10–15].

In Einstein's general relativity, in order to have such an acceleration, one needs to introduce a component to the matter distribution of the universe with a large negative pressure. This component is usually referred as dark energy. Astronomical observations indicate that our universe is flat and currently consists of approximately $2/3$ dark energy and $1/3$ dark matter. The nature of dark energy as well as dark matter is unknown, and many radically different models have been proposed, such as, a tiny positive cosmological constant, quintessence [16–18], DGP branes [19,20], the non-linear $F(R)$ models [21–23], and dark energy in brane worlds, among many others [24–43]; see also the review articles [44,45], and references therein.

On the other hand, another very important issue in gravitational physics is black holes and their formation in our universe. Although it is generally believed that on scales much smaller than the horizon size the fluctuations of dark energy itself are unimportant [46], their effects on the evolution of matter overdensities may be significant [47,48]. Then, a natural question is how dark energy affects the process of the gravitational collapse of a star. It is known that dark energy exerts a repulsive force on its surrounding, and this repulsive force may prevent the star from collapse. Indeed, there are speculations that a massive star does not simply collapse to form a black hole, instead, to the formation of stars that contain dark energy. In a recent work, Mazur and Mottola [49] have suggested a solution with a final configuration without neither singularities nor horizons, which they called "gravastar" (gravitational vacuum star). In this case, the gravastar is a system characterized by having a thin shell but not infinitesimal made of stiff matter, which separates an inner region with de Sitter spacetime from the Schwarzschild exterior spacetime. The elimination of the apparent horizon is done using suitable choice of the inner and outer radius of the thin shell, in such way that the inner radius be shorter than the horizon radius of de Sitter spacetime and the outer radius longer than the Schwarzschild horizon radius. In a later work, Visser and Wiltshire [50] have shown that the gravastar is dynamically stable. The possibility of the existence of objects like the gravastar brings all the discussions about the fact that is unavoidable that gravitational collapse always forms a black hole. As a result, black holes may not exist at all [51,52]. Another related issue is that how dark energy affects already-formed black hole is related to the fact that it was shown that the mass of a black hole decreases due to phantom energy accretion and tends to zero when the Big Rip approaches [53,54]. Gravitational collapse and formation of black holes in the presence of dark energy were first considered by several works [55–58].

Based on the discussions about the gravastar picture some authors have proposed alternative models [59–63]. Among them, we can find a Chaplygin dark star [64], a gravastar supported by non-linear electrodynamics [65], a gravastar with continuous anisotropic pressure [66]. Beside these ones, Lobo [67] has studied two models for a dark energy fluid. One of them describes a homogeneous energy density and another one which uses an ad-hoc monotonic decreasing energy density, both of them with anisotropic pressure. In order to match an exterior Schwarzschild spacetime he has introduced a thin shell between the interior and the exterior spacetimes.

Since all the works cited previously are based in particular solutions, the investigation of more general solutions or others particular ones is important in order to establish the generality of these results. Our aim with this work is to construct another alternative model to black holes considering the possibility of gravitational trapping of dark energy by standard energy, i.e., not dark energy. In this context we could have the dark energy sustaining the collapse of the standard energy, while the standard matter would trap the dark energy. In our model the mass function is a natural consequence of the Einstein's field equations and the energy density as well as the pressure decreases with the radial coordinate (envelope), as expected for known stellar models. In order to eliminate the central singularity present in this model, we have considered a core with a homogeneous energy density, described by the Lobo's first solution [67]. The junction between the envelope and the Schwarzschild exterior spacetime has imposed a presence of a thin shell, as well as, the junction between the core and the envelope.

The paper is organized as follows. In Sect. 2 we show the Einstein field equations. In Sect. 3 we present a particular solution that represents an isotropic and inhomogeneous dark energy fluid. In order to construct a more realistic model we consider it as the envelope of the star with a core constituted by a regular anisotropic and homogeneous fluid and a Schwarzschild exterior. In Sect. 4 we show the junction conditions between the core and envelope regions and between the envelope and the exterior spacetime. Finally, in Sect. 5 we present our final considerations.

2 The field equations

We consider here a static spherically symmetric spacetime given by the following metric

$$ds^2 = -\exp\left[2\int g(\tilde{r})d\tilde{r}\right]dt^2 + \frac{dr^2}{1-2m(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where $g(r)$ and $m(r)$ are arbitrary functions of the radial coordinate, r.

The stress-energy tensor for an isotropic distribution of matter is given by

$$T_{\mu\nu} = (\rho + p)v_\mu v_\nu + p g_{\mu\nu}, \quad (2)$$

where v^μ is the four-velocity, ρ is the energy density, p is the radial pressure measured in the radial direction.

Thus, the Einstein field equation, $G_{\mu\nu} = 8\pi T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein tensor, provides the following relationships

$$m' = 4\pi r^2 \rho, \quad (3)$$

$$g = \frac{m + 4\pi r^3 p}{r(r-2m)}, \quad (4)$$

$$p' = -\frac{(\rho + p)(m + 4\pi r^3 p)}{r(r-2m)}, \quad (5)$$

where the prime denotes a derivative with respect to the radial coordinate, r. Equation (5) corresponds to the Tolman–Oppenheimer–Volkoff (TOV) equation.

The standard cosmology considers the Universe constituted of a perfect fluid described by an equation of state of the type $p = \omega\rho$. Using this type of equation of state with a sufficiently negative pressure could explain the positive acceleration of the Universe in the context of the General Relativity theory. We consider this kind of equation of state in order to investigate the conditions of existence of dark energy star. Assuming an equation of state of the type $p = \omega\rho$ and substituting it in Eq. (5) we have

$$\omega\rho' + (\rho + \omega\rho)\frac{(m + 4\pi r^3\omega\rho)}{r(r - 2m)} = 0. \tag{6}$$

Solving Eq. (6) in terms of m we get

$$m = \frac{\omega r^2\left(\rho' + 4\pi r\omega\rho^2 + 4\pi r\rho^2\right)}{2\omega\rho' r - \omega\rho - \rho}. \tag{7}$$

Differentiating Eq. (7) and substituting into Eq. (3) we obtain

$$\begin{aligned}\frac{r}{(2\omega\rho' r - \omega\rho - \rho)^2}&\Big(-24\pi r^2\omega^2\rho^2\rho' - 12\pi r^2\rho^2\omega\rho' + 28\pi r\omega^2\rho^3 + 20\pi r\omega\rho^3\\ &+ 4\pi r\rho^3 - 3\omega^2\rho'^2 r + 2\rho'\omega^2\rho + 2\rho'\omega\rho - \omega\rho'^2 r + 12\pi r\omega^3\rho^3 + r\rho''\omega^2\rho\\ &+ 8\pi r^3\omega^3\rho^2\rho'' + 8\pi r^3\omega^2\rho^2\rho'' - 16\pi r^3\omega^3\rho\rho'^2 - 12\pi r^2\omega^3\rho^2\rho' + r\rho''\omega\rho\Big) = 0.\end{aligned} \tag{8}$$

Below we work with a particular solution of this equation.

3 Solution of the physical equations

Solving Eq. (8) in terms of ρ we can obtain a particular solution written as

$$\rho = \frac{\omega}{2\pi(\omega^2 + 6\omega + 1)r^2}, \tag{9}$$

where we can note that the values of ω cannot be $-3+2\sqrt{2}$ or $-3-2\sqrt{2}$, in order to avoid the singularities for these values. Since $\rho \geq 0$ thus $-3-2\sqrt{2} < \omega < -3+2\sqrt{2}$ or $\omega \geq 0$. Besides, our envelope with $\omega = 1$ reproduces the same rigid fluid used by Mazur and Mottola [49], with the pressure proportional to r^{-2}.

Thus, using Eqs. (3) and (4) we obtain that

$$m(r) = \frac{2\omega r}{\omega^2 + 6\omega + 1}, \tag{10}$$

$$g(r) = \frac{2\omega}{(\omega + 1)r}. \tag{11}$$

The metric can be rewritten as

$$ds_+^2 = -r^{4\omega/(\omega+1)}dt^2 + \frac{\omega^2+6\omega+1}{(\omega+1)^2}dr^2 + r^2\left(d\theta^2+\sin^2\theta d\phi^2\right), \tag{12}$$

where the subscript (+) denotes the envelope spacetime as we consider it (see below). Since the metric function g_{rr} is negative for $-3-2\sqrt{2} < \omega < -3+2\sqrt{2}$, this interval is not allowed. Thus, this solution implies $\omega \geq 0$.

The Kretschmann scalar for (12) is given by

$$K = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{16\omega^2(3\omega^2+2\omega+7)}{r^4(\omega^2+6\omega+1)}, \tag{13}$$

where $R_{\alpha\beta\mu\nu}$ is the Riemann tensor. We can note that $\lim_{r\to\infty} K = 0$, i.e., this solution is asymptotically flat. But, we can see also that $\lim_{r\to 0} K \to +\infty$, the solution is singular at the origin. Then, we consider the metric (12) as an envelope solution. Below we present a core solution.

3.1 Core solution

In order to avoid the singularity at $r = 0$ we cut the spacetime (12) around its origin and fit another one, an anisotropic fluid with the density of energy μ constant. We have chosen an anisotropic solution for the core since it was shown that gravastars solutions must exhibit anisotropic pressures to be finite-sized objects [66]. Using the results from [67] we have

$$\mu = \mu_0 = \text{constant}, \tag{14}$$

$$p_r = k\mu_0, \tag{15}$$

and

$$p_t = p_r\left[1 + \frac{(1+k)(1+3k)M(\mathbf{r})}{2k(\mathbf{r}-2M(\mathbf{r}))}\right]. \tag{16}$$

In this case, we have as isotropic pressure limits, $k = -1/3$ and $k = -1$, otherwise we have anisotropic pressures. Here we point out that the anisotropy in the pressure changes the ranges where the energy conditions are satisfied, which depend on the values of $M(\mathbf{r})/\mathbf{r}$ [68].

The core metric can be written as

$$ds_-^2 = -\left[1 - \frac{2M(\mathbf{r})}{\mathbf{r}}\right]^{-(1+3k)/2} dv^2 + \left[1 - \frac{2M(\mathbf{r})}{\mathbf{r}}\right]^{-1} d\mathbf{r}^2 + \mathbf{r}^2(d\theta^2+\sin^2\theta d\phi^2), \tag{17}$$

where $M(\mathbf{r}) = 4\pi\mu_0\mathbf{r}^3/3$ and the subscript (-) means the core spacetime (see Fig. 1).

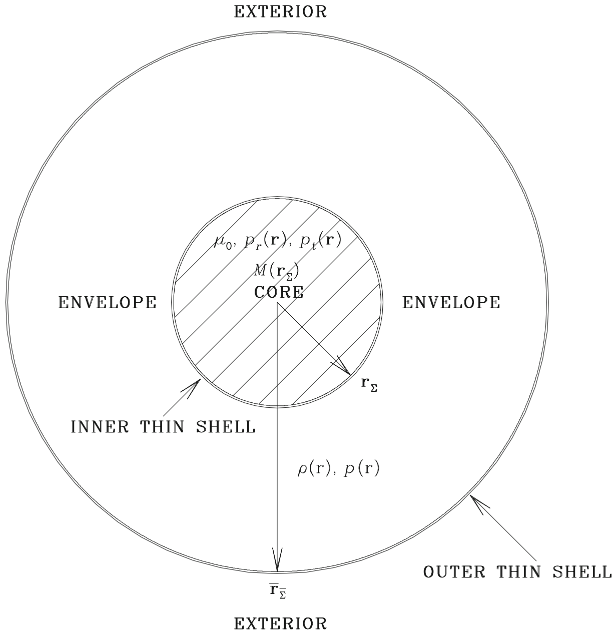

Fig. 1 The regions of the star: the core, the envelope and the exterior. The radius of the core is $\mathbf{r}_\Sigma$ and the total mass inside $\mathbf{r}_\Sigma$ is $M(\mathbf{r}_\Sigma)$. The density of the core is homogeneous and the pressure can be isotropic or anisotropic. The outer radius of the envelope is $\bar{\mathbf{r}}_{\bar{\Sigma}}$

Note from Eq. (16) that the anisotropy of the core $|p_t - p_r|$ decreases with the $\mathbf{r}$ coordinate. Then, the supposition of an isotropic envelope is physically reasonable.

3.2 Exterior solution

In order to limit the matter of the star we match the exterior spacetime with a Schwarzschild solution. The Schwarzschild exterior metric can be written as

$$ds_e^2 = -\left(1 - \frac{2\bar{M}}{\bar{\mathbf{r}}}\right) du^2 + \left(1 - \frac{2\bar{M}}{\bar{\mathbf{r}}}\right)^{-1} d\bar{\mathbf{r}}^2 + \bar{\mathbf{r}}^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (18)$$

where $\bar{M}$ is the total mass of the star, including the mass of the core, and the subscript (e) denotes the exterior spacetime.

4 Junction conditions

The metric of the hypersurface Σ at the frontier of the core and the envelope is given by

$$ds_\Sigma^2 = -d\tau^2 + R(\tau)^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (19)$$

where τ is the time coordinate define only on Σ.

The metric of the hypersurface $\bar{\Sigma}$ at the frontier of the envelope and the exterior is given by

$$ds^2_{\bar{\Sigma}} = -d\bar{\tau}^2 + \bar{R}(\bar{\tau})^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{20}$$

where $\bar{\tau}$ is the time coordinate define only on $\bar{\Sigma}$.

From the conditions $(ds^2_-)_\Sigma = (ds^2_+)_\Sigma$ and $(ds^2_+)_{\bar{\Sigma}} = (ds^2_e)_{\bar{\Sigma}}$ and using Eqs. (12), (17) and (18), we get the following relations

$$\mathbf{r}_\Sigma = r_\Sigma = R(\tau), \tag{21}$$

$$\bar{\mathbf{r}}_{\bar{\Sigma}} = r_{\bar{\Sigma}} = \bar{R}(\bar{\tau}), \tag{22}$$

$$\left(\frac{dv}{d\tau}\right)^2 = \left[1 - \frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{(1+3k)/2}, \tag{23}$$

$$\left(\frac{dt}{d\tau}\right)^2 = r_\Sigma^{-4\omega/(\omega+1)}, \tag{24}$$

$$\left(\frac{du}{d\bar{\tau}}\right)^2 = \left(1 - \frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right)^{-1}, \tag{25}$$

$$\left(\frac{dt}{d\bar{\tau}}\right)^2 = r_{\bar{\Sigma}}^{-4\omega/(\omega+1)}. \tag{26}$$

The core extrinsic curvature is given by

$$K^-_{\tau\tau} = -\frac{1}{2\mathbf{r}_\Sigma}\left[\frac{-2M(\mathbf{r}_\Sigma) + \mathbf{r}_\Sigma}{\mathbf{r}_\Sigma}\right]^{-3k/2}\left(\frac{dv}{d\tau}\right)^2 \times \frac{1}{2M(\mathbf{r}_\Sigma) - \mathbf{r}_\Sigma}\left[M(\mathbf{r}_\Sigma) - M'(\mathbf{r}_\Sigma)\mathbf{r}_\Sigma\right](3k+1), \tag{27}$$

$$K^-_{\theta\theta} = -\left[\frac{-2M(\mathbf{r}_\Sigma) + \mathbf{r}_\Sigma}{\mathbf{r}_\Sigma}\right]^{-1/2}[2M(\mathbf{r}_\Sigma) - \mathbf{r}_\Sigma], \tag{28}$$

$$K^-_{\phi\phi} = -\left[\frac{-2M(\mathbf{r}_\Sigma) + \mathbf{r}_\Sigma}{\mathbf{r}_\Sigma}\right]^{-1/2}[2M(\mathbf{r}_\Sigma) - \mathbf{r}_\Sigma]\sin^2\theta, \tag{29}$$

and the envelope extrinsic curvature is given by

$$K^+_{\tau\tau} = -2r_\Sigma^{-4/(\omega+1)} r_\Sigma^3\sqrt{\omega^2 + 6\omega + 1}\left(\frac{dt}{d\tau}\right)^2\frac{1}{\omega^2 + 6\omega + 1}\omega, \tag{30}$$

$$K^+_{\theta\theta} = \sqrt{\omega^2 + 6\omega + 1}(\omega + 1)\frac{1}{\omega^2 + 6\omega + 1}r_\Sigma, \tag{31}$$

$$K^+_{\phi\phi} = \sqrt{\omega^2 + 6\omega + 1}(\omega + 1)\frac{1}{\omega^2 + 6\omega + 1}r_\Sigma\sin^2\theta. \tag{32}$$

The core extrinsic curvature can be rewritten as

$$K^{-}_{\tau\tau} = \frac{(1+3k)}{2\mathbf{r}_\Sigma^2}\left[1-\frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{-\frac{1}{2}}\left[M(\mathbf{r}_\Sigma)-M'(\mathbf{r}_\Sigma)\mathbf{r}_\Sigma\right]$$

$$= -(1+3k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma^2}\left[1-\frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{-\frac{1}{2}}, \tag{33}$$

$$K^{-}_{\theta\theta} = \mathbf{r}_\Sigma\left[1-\frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{\frac{1}{2}}, \tag{34}$$

$$K^{-}_{\phi\phi} = K^{-}_{\theta\theta}\sin^2\theta, \tag{35}$$

the envelope extrinsic curvature can be rewritten as

$$K^{+}_{\tau\tau} = -\frac{2\omega}{r_\Sigma}\left(\omega^2+6\omega+1\right)^{-\frac{1}{2}}, \tag{36}$$

$$K^{+}_{\theta\theta} = r_\Sigma(\omega+1)\left(\omega^2+6\omega+1\right)^{-\frac{1}{2}}, \tag{37}$$

$$K^{+}_{\phi\phi} = K^{+}_{\theta\theta}\sin^2\theta, \tag{38}$$

and the Schwarzschild exterior extrinsic curvature can be written as

$$K^{e}_{\bar\tau\bar\tau} = -\frac{\bar M}{\bar{\mathbf{r}}_{\bar\Sigma}^2}\left[1-\frac{2\bar M}{\bar{\mathbf{r}}_{\bar\Sigma}}\right]^{-\frac{1}{2}}, \tag{39}$$

$$K^{e}_{\theta\theta} = \bar{\mathbf{r}}_{\bar\Sigma}\left[1-\frac{2\bar M}{\bar{\mathbf{r}}_{\bar\Sigma}}\right]^{\frac{1}{2}}, \tag{40}$$

$$K^{e}_{\phi\phi} = K^{e}_{\theta\theta}\sin^2\theta. \tag{41}$$

4.1 Junction conditions between core-envelope: without a thin shell

Using the junction conditions $K^{-}_{\tau\tau} = K^{+}_{\tau\tau}$ and $K^{-}_{\theta\theta} = K^{+}_{\theta\theta}$ we get two equations in terms of $M(\mathbf{r}_\Sigma)$ and $\mathbf{r}_\Sigma$

$$-(1+3k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma^2}\left[1-\frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{-\frac{1}{2}} + \frac{2\omega}{\mathbf{r}_\Sigma}\left(\omega^2+6\omega+1\right)^{-\frac{1}{2}} = 0, \tag{42}$$

and

$$\mathbf{r}_\Sigma\left[1-\frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{\frac{1}{2}} - \mathbf{r}_\Sigma(\omega+1)\left(\omega^2+6\omega+1\right)^{-\frac{1}{2}} = 0. \tag{43}$$

We can solve these two equations, obtaining the total mass of the core and the radius of the core as a function of ω and k giving

$$\omega = 3k, \tag{44}$$

and

$$\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} = \frac{2\omega}{(\omega^2 + 6\omega + 1)} \leq \frac{1}{4}. \tag{45}$$

Note that although there is a negative interval for ω where the mass is positive, this same interval is forbidden by the metric's signature, Eq. (12). Consequently, k is also non-negative. We note that the solution can represent a star model with an upper limit for the ratio of mass to radius of the core.

4.2 Junction conditions between core-envelope: with a thin shell

So, we match a thin shell of energy density σ and pressure P at the frontier between the core and the envelope. Thus, we can write

$$\sigma = -\frac{1}{4\pi}\kappa_\theta^\theta, \tag{46}$$

$$P = \frac{1}{8\pi}\left(\kappa_\theta^\theta + \kappa_\tau^\tau\right), \tag{47}$$

where

$$\kappa_{ij} = K_{ij}^+ - K_{ij}^-. \tag{48}$$

Since

$$\kappa_\tau^\tau = \frac{2\omega}{\mathbf{r}_\Sigma}\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} - (1+3k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma^2}\left[1 - \frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{-\frac{1}{2}}, \tag{49}$$

$$\kappa_\theta^\theta = \frac{1}{\mathbf{r}_\Sigma}(\omega+1)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} - \frac{1}{\mathbf{r}_\Sigma}\left[1 - \frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{\frac{1}{2}}, \tag{50}$$

then

$$\sigma = -\frac{1}{4\pi\mathbf{r}_\Sigma}\left\{(\omega+1)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} - \left[1 - \frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{\frac{1}{2}}\right\}, \tag{51}$$

$$\begin{aligned} P = \frac{1}{8\pi\mathbf{r}_\Sigma}\Bigg\{&(3\omega+1)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} \\ &- \left[1 + (1+3k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]\left[1 - \frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{-\frac{1}{2}}\Bigg\}, \end{aligned} \tag{52}$$

where $\omega < -3 - 2\sqrt{2}$ or $\omega > -3 + 2\sqrt{2}$ (in order to have σ and P real) and $\mathbf{r}_\Sigma > 2M(\mathbf{r}_\Sigma)$. Since $\sigma \geq 0$ and $\rho \geq 0$ then $\omega \geq 0$, which represents a not dark energy envelope, but no restriction is imposed to the core, i.e., we can have any values for k.

4.2.1 Energy conditions for the thin shell: core-envelope

The energy conditions [69] for a thin shell can be written as

1. Weak: $\sigma \geq 0$ and $\sigma + P \geq 0$
2. Dominant: $\sigma + P \geq 0$ and $\sigma - P \geq 0$
3. Strong: $\sigma + P \geq 0$ and $\sigma + 2P \geq 0$

The characterization of dark energy fluid is the violation of one of the strong energy conditions, more specifically, that one related the Raychaudhuri equation [68]. If the second of the weak energy conditions is violated, we have a phantom dark energy fluid.

For the inner thin shell, that is, between the core and the envelope, we have

$$\sigma \geq 0, \tag{53}$$

$$\sigma + P = \frac{1}{8\pi \mathbf{r}_\Sigma}\left\{(\omega - 1)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} + \left[1 - (5+3k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]\left[1 - \frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{-\frac{1}{2}}\right\}, \tag{54}$$

$$\sigma - P = \frac{1}{8\pi \mathbf{r}_\Sigma}\left\{-(5\omega + 3)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} + 3\left[1 - (1-k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]\left[1 - \frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{-\frac{1}{2}}\right\}, \tag{55}$$

$$\sigma + 2P = \frac{1}{4\pi \mathbf{r}_\Sigma}\left\{2\omega\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} - 3\left[(1+k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]\left[1 - \frac{2M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right]^{-\frac{1}{2}}\right\}. \tag{56}$$

In Table 1, we summarize the results of the energy conditions of the inner thin shell for different limits and $k \geq -1/3$. We analyzed only the case where $k \geq -1/3$ because we are interested to check if it is possible to have all the structures constituted by standard energy.

4.3 Junction conditions between envelope-exterior: without thin shell

Using the junction conditions $K^{+}_{\bar\tau\bar\tau} = K^{e}_{\bar\tau\bar\tau}$ and $K^{+}_{\theta\theta} = K^{e}_{\theta\theta}$ we get two equations in terms of $\bar{M}$ and $\bar{\mathbf{r}}_{\bar\Sigma}$

$$-\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar\Sigma}^2}\left[1 - \frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar\Sigma}}\right]^{-\frac{1}{2}} + \frac{2\omega}{\bar{\mathbf{r}}_{\bar\Sigma}}\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} = 0, \tag{57}$$

Table 1 This table summarizes the results of the energy conditions of the inner thin shell for different limits and $k \geq -1/3$

Case	$8\pi \mathbf{r}_\Sigma(\sigma + P)$	$8\pi \mathbf{r}_\Sigma(\sigma - P)$	$4\pi \mathbf{r}_\Sigma(\sigma + 2P)$
$\omega \to 0$ $M(\mathbf{r}_\Sigma)/\mathbf{r}_\Sigma \ll 1$	$-(4+3k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \leq 0$	$3k\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \geq 0$	$-3(1+k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \leq 0$
$\omega \to 0$ $M(\mathbf{r}_\Sigma)/\mathbf{r}_\Sigma \to 1/2$	$-1 + \frac{1-(5+3k)/2}{\sqrt{1-2\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}}} \leq 0$	$-3 + \frac{3[1-(1-k)/2]}{\sqrt{1-2\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}}} \geq 0$	$\frac{-3(1+k)/2}{\sqrt{1-2\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}}} \leq 0$
$\omega \to 1$ $M(\mathbf{r}_\Sigma)/\mathbf{r}_\Sigma \ll 1$	$1-(4+3k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \geq 0$ $\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \leq \frac{1}{4+3k}$	$-\frac{4}{\sqrt{2}} + 3\left[1 + k\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}\right] \geq 0$ $\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \leq \frac{1}{3k}\left(\frac{4}{\sqrt{2}} - 3\right)$	$\frac{1}{\sqrt{2}} - 3(1+k)\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \geq 0$ $\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \leq \frac{1}{3\sqrt{2}(1+k)}$
$\omega \to 1$ $M(\mathbf{r}_\Sigma)/\mathbf{r}_\Sigma \to 1/2$	$\frac{1-(5+3k)/2}{\sqrt{1-2\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}}} \leq 0$	$-\frac{4}{\sqrt{2}} + \frac{3[1-(1-k)/2]}{\sqrt{1-2\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}}} \geq 0$	$\frac{1}{\sqrt{2}} - \frac{3(1+k)/2]}{\sqrt{1-2\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}}} \leq 0$

and

$$\bar{\mathbf{r}}_{\bar{\Sigma}}\left[1 - \frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right]^{\frac{1}{2}} - \bar{\mathbf{r}}_{\bar{\Sigma}}(\omega+1)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}} = 0. \tag{58}$$

Considering these two equations we have the equation

$$(\omega+1)^2 = 1, \tag{59}$$

which gives us

$$\begin{aligned} \omega &= 0, \\ \omega &= -2, \end{aligned} \tag{60}$$

where only the first one is solution of the original equations (57) and (58). This solution implies that

$$\bar{M} = M(\mathbf{r}_\Sigma) = 0, \tag{61}$$

which means that all the spacetime is Minkowski. Then we can conclude that for this kind of fluid distribution, it is not possible to have structures surrounded by a Schwarzschild spacetime without the introduction of a thin shell.

However, in a Lobo's work [67] it was necessary the introduction of a thin shell on the junction hypersurface. The authors have suggested that it is possible to match the interior and exterior spacetime without a thin shell. However, eliminating the thin shell, vanishing its energy density and pressure (equations 25 and 26 in the original work), we get the same junction conditions considered in this present work, i.e., giving a Minkowski spacetime. In order to do that we assume $m = M$ and $\omega = 0$ or $m' = 0$ with also $\dot{a} = 0$. Thus, the unique interior solutions that admits a matching with Schwarzschild spacetime, without the introduction of a thin shell, is the Minkowski ($m' = 0$) and the dust ($\omega = 0$) solution.

4.4 Junction Conditions between envelope-exterior: with a thin shell

Since as seen in the above subsections, we cannot match an exterior Schwarzschild spacetime directly with the envelope, we build a thin shell of energy density $\bar{\sigma}$ and pressure $\bar{P}$ in order to do this match. Thus,

$$\bar{\sigma} = -\frac{1}{4\pi}\bar{\kappa}_{\theta}^{\theta}, \tag{62}$$

$$\bar{P} = \frac{1}{8\pi}\left(\bar{\kappa}_{\theta}^{\theta} + \bar{\kappa}_{\bar{\tau}}^{\bar{\tau}}\right), \tag{63}$$

where

$$\bar{\kappa}_{ij} = K_{ij}^{e} - K_{ij}^{+}. \tag{64}$$

Since

$$\bar{\kappa}_{\bar{\tau}}^{\bar{\tau}} = \frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}^2}\left(1 - \frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right)^{-\frac{1}{2}} - \frac{2\omega}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}}, \tag{65}$$

$$\bar{\kappa}_{\theta}^{\theta} = \frac{1}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\left(1 - \frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right)^{\frac{1}{2}} - \frac{1}{\bar{\mathbf{r}}_{\bar{\Sigma}}}(\omega + 1)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}}, \tag{66}$$

then

$$\bar{\sigma} = -\frac{1}{4\pi\bar{\mathbf{r}}_{\bar{\Sigma}}}\left[\left(1 - \frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right)^{\frac{1}{2}} - (\omega + 1)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}}\right], \tag{67}$$

$$\bar{P} = \frac{1}{8\pi\bar{\mathbf{r}}_{\bar{\Sigma}}}\left[\left(1 - \frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right)\left(1 - \frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right)^{-\frac{1}{2}} - (3\omega + 1)\left(\omega^2 + 6\omega + 1\right)^{-\frac{1}{2}}\right], \tag{68}$$

where $\omega < -3-2\sqrt{2}$ or $\omega > -3+2\sqrt{2}$ (in order to have σ and P real) and $\bar{\mathbf{r}}_{\bar{\Sigma}} > 2\bar{M}$. Since $\rho \geq 0$ then the unique physical system must have $\omega \geq 0$.

Since $\bar{\sigma} \geq 0$ then we must have

$$\frac{2\omega}{\omega^2 + 6\omega + 1} \leq \frac{\bar{M}}{\bar{\mathbf{r}}_{\Sigma}} \leq \frac{1}{2}, \tag{69}$$

Table 2 This table summarizes the results of the energy conditions of the outer thin shell for different limits

Case	$8\pi \mathbf{r}_{\Sigma}(\bar{\sigma}+\bar{P})$	$8\pi \mathbf{r}_{\Sigma}(\bar{\sigma}-\bar{P})$	$4\pi \mathbf{r}_{\Sigma}(\bar{\sigma}+2\bar{P})$
$\omega \rightarrow 0$ $\bar{M}/\bar{\mathbf{r}}_{\bar{\Sigma}} \ll 1$	$-2\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \leq 0$	$-2\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \leq 0$	$-\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \leq 0$
$\omega \rightarrow 0$ $\bar{M}/\bar{\mathbf{r}}_{\bar{\Sigma}} \rightarrow 1/2$	$-3+\frac{1/2}{\sqrt{1-2\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}}} \geq 0$	$-1-\frac{1/2}{\sqrt{1-2\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}}} \leq 0$	$-2-\frac{1/2}{\sqrt{1-2\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}}} \geq 0$
$\omega \rightarrow 1$ $\bar{M}/\bar{\mathbf{r}}_{\bar{\Sigma}} \ll 1$	$\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \leq \frac{3\sqrt{2}-4}{2\sqrt{2}}$	$\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \leq \frac{1}{2}$	$\frac{2\sqrt{2}-3}{\sqrt{2}}-\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \leq 0$
$\omega \rightarrow 1$ $\bar{M}/\bar{\mathbf{r}}_{\bar{\Sigma}} \rightarrow 1/2$	$-\frac{4}{\sqrt{2}}+\frac{1/2}{\sqrt{1-2\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}}} \geq 0$	$-\frac{1/2}{\sqrt{1-2\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}}} \leq 0$	$-\frac{3}{\sqrt{2}}+\frac{1/2}{\sqrt{1-2\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}}} \geq 0$

for $\omega \geq 0$, and where

$$\frac{2\omega}{\omega^2+6\omega+1} \leq \frac{1}{4}. \tag{70}$$

4.4.1 Energy conditions for the thin shell: envelope-exterior

The energy conditions [69] for the thin shell between the envelope and the exterior can be written as

$$\bar{\sigma} \geq 0 \tag{71}$$

$$\bar{\sigma}+\bar{P}=\frac{1}{8\pi \bar{\mathbf{r}}_{\bar{\Sigma}}}\left\{-(5\omega+3)\left(\omega^2+6\omega+1\right)^{-\frac{1}{2}}+\left[3-5\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right]\left[1-\frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right]^{-\frac{1}{2}}\right\}, \tag{72}$$

$$\bar{\sigma}-\bar{P}=\frac{1}{8\pi \bar{\mathbf{r}}_{\bar{\Sigma}}}\left\{(\omega-1)\left(\omega^2+6\omega+1\right)^{-\frac{1}{2}}+\left[1-3\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right]\left[1-\frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right]^{-\frac{1}{2}}\right\}, \tag{73}$$

$$\bar{\sigma}+2\bar{P}=\frac{1}{4\pi \bar{\mathbf{r}}_{\bar{\Sigma}}}\left\{-2(2\omega+1)\left(\omega^2+6\omega+1\right)^{-\frac{1}{2}}+\left[2-3\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right]\left[1-\frac{2\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}\right]^{-\frac{1}{2}}\right\}. \tag{74}$$

In Table 2 we summarize the results of the energy conditions of the outer thin shell for different limits.

Comparing the Tables 1 and 2 we can have the following conclusions:

1. Limits $\omega \rightarrow 0$, $\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \ll 1$, $\frac{\bar{M}}{\bar{\mathbf{r}}_\Sigma} \ll 1$: both (inner and outer) thin shells are phantom;
2. Limits $\omega \rightarrow 0$, $\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \rightarrow 1/2$, $\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \rightarrow 1/2$: the inner thin shell is made of dark energy and the outer thin shell violates the dominant energy condition;
3. Limits $\omega \rightarrow 1$, $\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \ll 1$, $\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \ll 1$: the inner thin shell is constituted by dark and not dark energy, depending on $\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma}$ and $\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}}$, and the outer thin shell is made of dark energy;
4. Limits $\omega \rightarrow 1$, $\frac{M(\mathbf{r}_\Sigma)}{\mathbf{r}_\Sigma} \rightarrow 1/2$, $\frac{\bar{M}}{\bar{\mathbf{r}}_{\bar{\Sigma}}} \rightarrow 1/2$: the inner thin shell is made of dark energy and the outer thin shell is constituted by not dark energy, violating the dominant energy condition.

We can conclude with these limits that we always have one or both thin shells constituted by dark energy.

5 Conclusions

We have constructed a star model consisting of four parts: (i) a homogeneous inner core with anisotropic pressure (ii) an infinitesimal thin shell separating the core and the envelope; (iii) an envelope of inhomogeneous density and isotropic pressure; (iv) an infinitesimal thin shell matching the envelope boundary and the exterior Schwarzschild spacetime. We have analyzed all the energy conditions for the core, envelope and the two thin shells.

In our model the mass function is a natural consequence of the Einstein's field equations and the energy density as well as the pressure decreases with the radial coordinate (envelope), as expected for known stellar models. In order to eliminate the central singularity present in this model, we have considered a core with a homogeneous energy density, described by the Lobo's first solution [67].

We have proposed in this work an alternative model and a generalization to gravastars. Note that for $k = -1$ we get a vacuum core with a cosmological constant $\Lambda = 8\pi\mu_0$, i.e., a de Sitter solution. Thus, we have constructed models with the same structures of the Mazur and Mottola's one [49], with five regions (an interior de Sitter solution + an infinitesimal shell + a non-infinitesimal shell + infinitesimal shell + an exterior Schwarzschild solution). However, in our models each one of these regions is made of more general kind of fluids.

In Table 3 we summarize the results of the match of the core's spacetime with envelope's spacetime and the results of the match of the envelope's spacetime with the exterior spacetime.

Combining the results of Tables 1, 2 and 3 we can see that in the analyzed cases we always have the presence of the dark energy at least in one of the thin shell or in the core. Note also that from the results that there is no physical reason to have a superior limit for the mass of these objects but for the ratio of mass and radius, in order to find out which one is made of dark energy.

Table 3 This table summarizes the results of the match of the core's spacetime with envelope's spacetime and the results of the match of the envelope's spacetime with the exterior spacetime

Core	Thin shell: core-envelope	Envelope	Thin shell: envelope-exterior	Exterior spacetime
Minkowski	None	Minkowski	None	Minkowski
Not dark energy	$\omega \to 0, \frac{\bar{M}}{\bar{r}_\Sigma} \ll 1$ Phantom $\omega \to 0, \frac{\bar{M}}{\bar{r}_\Sigma} \to 1/2$ Dark energy $\omega \to 1, \frac{\bar{M}}{\bar{r}_\Sigma} \ll 1$ Dark or Not dark energy $\omega \to 1, \frac{\bar{M}}{\bar{r}_\Sigma} \to 1/2$ Dark energy	$\omega > 0$ Not dark energy	$\omega \to 0, \frac{\bar{M}}{\bar{r}_\Sigma} \ll 1$ Phantom $\omega \to 0, \frac{\bar{M}}{\bar{r}_\Sigma} \to 1/2$ Not dark energy Violation of DEC $\omega \to 0, \frac{\bar{M}}{\bar{r}_\Sigma} \ll 1$ Dark energy $\omega \to 0, \frac{\bar{M}}{\bar{r}_\Sigma} \to 1/2$ Not dark energy Violation of DEC	Schwarzschild

We have standard energy core and envelope, but in all the cases we have at least one of thin shells made of dark energy

DEC dominant energy condition

Acknowledgments We would like to thank Dr. Anzhong Wang for the helpful suggestions. The financial assistance from UERJ (JFVdaR) and FAPERJ/UERJ (MFAdaS) is gratefully acknowledged. The author (RC) acknowledges the financial support from FAPERJ (no. E-26/171.754/2000, E-26/171.533/2002 and E-26/170.951/2006). The authors (RC and MFAdaS) also acknowledge the financial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq - Brazil. The author (MFAdaS) acknowledges the financial support from Financiadora de Estudos e Projetos - FINEP - Brazil (Ref. 2399/03).

References

1. Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
2. Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
3. Riess, A.G., et al.: Astrophys. J. **607**, 665 (2004)
4. Astier, P., et al.: Astron. Astrophys. **447**, 31 (2006)
5. Spergel, D.N., et al.: Astrophys. J. Suppl. **170**, 335 (2007) [ArXiv:astro-ph/0603449]
6. Wood-Vasey, W.M., et al.: Astrophys. J. **666**, 694 (2007) [ArXiv:astro-ph/0701041]
7. Davis, T.M., et al.: Astrophys. J. **666**, 716 (2007) [ArXiv:astro-ph/0701510]
8. Bennett, C.L., et al.: Astrophys. J. Suppl. **148**, 1 (2003) [arXiv:astro-ph/0302207]

9. Spergel, D.N., et al.: [WMAP Collaboration]. Astrophys. J. Suppl. **148**, 175 (2003) [arXiv:astro-ph/0302209]
10. Tegmark, M., et al.: [SDSS Collaboration]. Phys. Rev. D **69**, 103501 (2004) [arXiv:astro-ph/0310723]
11. Abazajian, K., et al.: Astron. J. **129**, 1755 (2005) [arXiv:astro-ph/0410239]
12. Abazajian, K., et al.: [SDSS Collaboration]. Astron. J. **128**, 502 (2004) [arXiv:astro-ph/0403325]
13. Abazajian, K., et al.: [SDSS Collaboration]. Astron. J. **126**, 2081 (2003) [arXiv:astro-ph/0305492]
14. Hawkins, E., et al.: Mon. Not. R. Astron. Soc. **346**, 78 (2003) [arXiv:astro-ph/0212375]
15. Verde, L., et al.: Mon. Not. R. Astron. Soc. **335**, 432 (2002) [arXiv:astro-ph/0112161]
16. Caldwell, R.R., Dave, R., Steinhardt, P.J.: Phys. Rev. Lett. **80**, 1582 (1998)
17. Liddle, A.R., Scherrer, R.J.: Phys. Rev. D **59**, 023509 (1999)
18. Steinhardt, P.J., Wang, L.M., Zlatev, I.: Phys. Rev. D **59**, 123504 (1999)
19. Dvali, G.R., Gabadadze, G., Porrati, M.: Phys. Lett. B **484**, 112 (2000)
20. Deffayet, C.: Phys. Lett. B **502**, 199 (2001)
21. Capozziello, S., Carloni, S., Troisi, A.: (2003) [arXiv:astro-ph/0303041]
22. Carroll, S.M., et al.: Phys. Rev. D **70**, 043528 (2003)
23. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **68**, 123512 (2003)
24. Townsend, P.K., Wohlfarth, N.R.: Phys. Rev. Lett. **91**, 061302 (2003)
25. Gibbons, G.W.: In: de Aguila, F., et al. (eds.) Supersymmetry, Supergravity and Related Topics, p. 124. World Scientific, Singapore (1985)
26. Maldacena, J.M., Nuñez, C.: Int. J. Mod. Phys. A **16**, 822 (2001)
27. Ohta, N.: Phys. Rev. Lett. **91**, 061303 (2003)
28. Wohlfarth, N.R.: Phys. Lett. B **563**, 1 (2003)
29. Roy, S.: Phys. Lett. B **567**, 322 (2003)
30. Webb, J.K., et al.: Phys. Rev. Lett. **87**, 091301 (2001)
31. Cline, J.M., Vinet, J.: Phys. Rev. D **68**, 025015 (2003)
32. Ohta, N.: Prog. Theor. Phys. **110**, 269 (2003)
33. Ohta, N.: Int. J. Mod. Phys. A **20**, 1 (2005)
34. Chen, C.M., et al.: JHEP **10**, 058 (2003)
35. Bergshoeff, E.: Class. Quantum Grav. **21**, 1947 (2004)
36. Gong, Y., Wang, A.: Class. Quantum Grav. **23**, 3419 (2006)
37. Neupane, I.P., Wiltshire, D.L.: Phys. Lett. B **619**, 201 (2005)
38. Neupane, I.P., Wiltshire, D.L.: Phys. Rev. D **72**, 083509 (2005)
39. Maeda, K., Ohta, N.: Phys. Rev. D **71**, 063520 (2005)
40. Neupane, I.P.: Phys. Rev. Lett. **98**, 061301 (2007)
41. Gong, Y., Wang, A., Wu, Q.: Phys. Lett. B **663**, 147 (2008) [arXiv:gr-qc/0711.1597]
42. Pereira, P.R.C.T., da Silva, M.F.A., Chan, R.: IJMPD **15**, 991 (2006)
43. Brandt, C.F.C., Chan, R., da Silva, M.F.A., Villas da Rocha, J.F.: Gen. Relat. Grav. **39**, 1675 (2007)
44. Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D **15**, 1753 (2006)
45. Padmanabhan, T.: Gen. Rel. Grav. **40**, 529 (2008) [arXiv:gr-qc/0705.2533]
46. Ma, C., Caldwell, R.R., Bode, P., Wang, L.: Astrophys. J. **521**, L1 (1999)
47. Ferreira, P.G., Joyce, M.: Phys. Rev. Lett. **79**, 4740 (1997)
48. Mota, D.F., van de Bruck, C.: Astron. Astrophys. **421**, 71 (2004)
49. Mazur, P.O., Mottola, E.: (2002) [arXiv:gr-qc/0109035]
50. Visser, M., Wiltshire, D.L.: Class. Quantum Grav. **21**, 1135 (2004)
51. http://www.nature.com/news/2005/050328/full/050328-8.html
52. http://www.bioon.com/TILS/news/200504/97002.html
53. Babichev, E., Dokuchaev, V., Eroshenko, Yu.: Phys. Rev. Lett. **93**, 021102 (2004)
54. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **70**, 103522 (2004) [arXiv:hep-th/0408170]
55. Li, Z.-H., Wang, A.: Mod. Phys. Lett. A **22**, 1663–1676 (2007) [arXiv:astro-ph/0607554]
56. Cai, R.-G., Wang, A.: Phys. Rev. D **73**, 063005 (2006) [arXiv:astro-ph/0505136]
57. Nath, S., Chakraborty, S., Debnath, U.: Int. J. Mod. Phys. D **15**, 1225 (2006) [arXiv:gr-qc/0512120]
58. Debnath, U., Chakraborty, S.: (2006) [arXiv:gr-qc/0601049]
59. Horvat, D., Ilijic, S.: Class. Quantum Grav. **24**, 5637 (2007)
60. Chirenti, C.B.M.H., Rezzolla, L.: Class. Quantum Grav. **24**, 4191 (2007)
61. Broderick, A.E., Narayan, R.: Class. Quantum Grav. **24**, 659 (2007)
62. Rocha, P., Miguelote, A.Y., Chan, R., da Silva, M.F.A., Santos, N.O., Wang, A.Z.: J. Cosmol. Astropart. Phys. **06**, 025 (2008)

63. Rocha, P., Chan, R., da Silva, M.F.A., Wang, A.: J. Cosmol. Astropart. Phys. **11**, 10 (2008) [arXiv:gr-qc/08094879]
64. Bertolami, O., Páramos, J.: Phys. Rev. D **72**, 123512 (2005) [arXiv:astro-ph/0509547]
65. Lobo, F.: Class. Quantum Grav. **24**, 1069 (2007) [arXiv:gr-qc/0611083]
66. Cattoen, C., Faber, T., Visser, M.: Class. Quantum Grav. **22**, 4189 (2005)
67. Lobo, F.: Class. Quantum Grav. **23**, 1525 (2006)
68. Chan, R., da Silva, M.F.A., Villas da Rocha, J.F.: Mod. Phys. Lett. A (2009, in press) [arXiv:gr-qc/08032508]
69. Hawking, S.W., Ellis, G.F.R.: The Large Scale Structure of Spacetime. Cambridge University Press, Cambridge (1973)