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RESEARCH ARTICLE

Le Châtelier–Braun principle in cosmological physics

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Abstract Assuming that dark energy may be treated as a fluid with a well-defined temperature, close to equilibrium, we argue that if nowadays there is a transfer of energy between dark energy and dark matter, it must be such that the latter gains energy from the former and not the other way around.

Keywords Cosmology · Dark energy · Thermodynamics

Recently, there have been some proposals in the literature to the effect that the dark energy and dark matter should not conserve separately but interact with each other [1–8]. As is well known, the interaction may substantially alleviate the coincidence problem [9–16], explain why observationally the equation of state parameter of dark energy, $w_x = p_x/\rho_x < -1/3$, may seem of phantom type (i.e., lower than -1) [17,18,22], and account for the age of the quasar APM0879+5255 at redshift $z = 3.91$ [19]. However, there is not consensus about whether the overall transfer of energy should go from dark energy to dark matter [9–21] or viceversa [22–27]. In this short communication, we assume that both components (dark matter and dark energy) are amenable to a thermo-fluid description and resort to the second law of thermodynamics to discern the sense in which the transfer of energy proceeds. Obviously, this approach does not apply if dark energy is a scalar field in a pure quantum state since its entropy vanishes and no temperature can be defined. However, this is not the more general

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case. One may consider dark energy as the effective manifestation of a mixture of scalar fields. Generally, the mixture would not be in a pure quantum state whereby it would be entitled to an entropy and a global temperature.

If these two components conserved separately in an expanding Friedmann–Lemaître–Robertson–Walker universe we would have

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \tag{1}$$

$$\dot{\rho}_x + 3H(1 + w_x)\rho_x = 0, \tag{2}$$

where the equation of state of dark matter can be approximately written in parametric form as [28,29]

$$\rho_m = n_m\, M + \tfrac{3}{2}\, n_m\, T_m\,, \quad p_m = n_m\, T_m \quad (k_B = 1) \tag{3}$$

so long as $T_m \ll M$. As a consequence, $\rho_m \sim a^{-3}$ and $\rho_x \propto \exp \int -3(1 + w_x)\, da/a$.

The temperatures dependence on the scale factor

$$T_m \propto a^{-2}, \quad T_x \propto \exp \int -3w_x\, da/a \tag{4}$$

follow from integrating the evolution equation, $\dot{T}/T = -3H(\partial p/\partial \rho)_n$. The latter is straightforwardly derived from Gibbs' equation, $T dS = d(\rho/n) + p\, d(1/n)$, and the condition for dS to be a differential expression, $\partial^2 S/(\partial T\, \partial n) = \partial^2 S/(\partial n\, \partial T)$.

Equations (4) suggest that currently $T_m \ll T_x$, and viceversa at very early times (modulo, $T_m \ll M$ still at that times).

When both components interact, Eqs. (1) and (2) generalize to

$$\dot{\rho}_m + 3H(\rho_m + p_m) = Q\,, \tag{5}$$

$$\dot{\rho}_x + 3H(1 + w_x)\rho_x = -Q\,, \tag{6}$$

respectively, where Q denotes the interaction term (we note in passing that the overall energy density $\rho_m + \rho_x$ is conserved). Obviously, for $Q > 0$ the energy proceeds from dark energy to dark matter.

Assuming

$$Q = 3H\lambda\, \rho_x\,, \tag{7}$$

with λ a small constant, one follows $\dot{\rho}_x + 3H(1 + w_x + \lambda)\, \rho_x = 0$. Thereby, $\dot{T}_x/T_x = -3(w_x + \lambda)\, \dot{a}/a$ and

$$T_x \propto \exp \int -3\,(w_x + \lambda)\, \frac{da}{a}\,. \tag{8}$$

Consequently, if $\lambda > 0$ (i.e., $Q > 0$), T_x will increase more slowly as the Universe expands than in the absence of interaction and correspondingly T_m will also decrease more slowly. This is fully consistent with the second law of thermodynamics. The latter implies that when two systems that are not in mutual equilibrium (thermal or otherwise) interact with one another, the interaction tends to drive the systems to a final common equilibrium. If they are left to themselves, they will eventually reach equilibrium. In the case contemplated here the equilibrium is never achieved because the expansion of the Universe (that can be viewed as an external agent) acts in the opposite sense. If λ were negative ($Q < 0$), the temperature difference would augment, something at variance with the second law. Clearly, our conclusion critically rests on the validity of Eq. (7). Next, we will argue that this expression approximately holds, at least piecewise.

In view of any of the two Eqs. (5) and (6), Q must be a function of the energy densities multiplied by a quantity with units of inverse of time. For the latter the obvious choice is the Hubble factor H; thus, $Q = Q(H\rho_m\,, H\rho_x)$. By power law expanding Q and retaining just the first term we may write $Q \simeq \lambda_m\, H\rho_m + \lambda_x\, H\rho_x$. In the scaling regime, i.e., when the ratio ρ_m/ρ_x stays constant, Eq. (7) is readily recovered. Outside this particular regime—at any rate, not at very early times—one expects that the said ratio varies slowly (i.e., not much faster than the scale factor $a(t)$) whence it might be considered piecewise constant. Thus, we recover again Eq. (7) though this time λ is constant only piecewise.

The foregoing scenario can be also understood in the light of the Le Châtelier–Braun principle: when a system is perturbed out of its equilibrium state it reacts trying to restore it or achieve a new one [30–32]. At sufficiently early times $T_m > T_x$, and the Universe expansion rapidly drives both systems to a common -equilibrium-temperature $T^{(\rm eq)}$ at say, $a = a^{(\rm eq)}$. However, subsequently $a > a^{(\rm eq)}$ and the thermal equilibrium is lost, $T_m < T^{(\rm eq)} < T_x$. In our case, the answer of the system to the equilibrium loss is a continuous transfer of energy from dark energy to dark matter. Whereas this does not bring the system to any equilibrium state it certainly slows down the rate at which it moves away from equilibrium. Notice that when $a < a^{(\rm eq)}$ the system is approaching equilibrium thereby it "sees" no reason for any energy transfer.

We now turn to a very short reasoning, independent of expression (7), leading to $Q > 0$ as well. Following Zimdahl [33] the entropy production associated to our two interacting fluids (modulo their chemical potentials vanish) reads[1]

$$S^a_{m;a} + S^a_{x;a} = \left(\frac{1}{T_m} - \frac{1}{T_x}\right) Q \tag{9}$$

(see also [34]), where S^a_i, with $i = m, x$, stand for the entropy flux of each fluid. From the second law, $S^a_{m;a} + S^a_{x;a} \geq 0$, and the fact that nowadays $T_m < T_x$ is expected, the desired result follows. At this point, a remark is in order. In the special—and oftentimes—assumed case $w_x =$ constant the sound speed of dark energy, $c_{s,x}$, is not longer given by $c^2_{s,x} = \dot{p}_x/\dot{\rho}_x$, see however [35], whereby the dark energy does not behave adiabatically and a further term should be added to the right hand side of Eq. (9). This difficulty can be avoided in the more general case $w_x \neq$ constant.

[1] See Eq. (24) of Ref. [33].

In spite of this reassuring outcome, owing to the fact that the lower ρ_x, the larger T_x the thermo-fluid description might look suspicious. However, it is worth recalling that this is very often the case for systems in which gravity plays a leading role (as in Schwarzschild black holes). At any rate, it is reasonable to expect that the said description breaks down at some point, both when $a \ll 1$ and when $a \gg 1$. It must be added that very likely a clear-cut answer to the validity of the thermo-fluid approach will not be available until the nature of dark energy gets elucidated. In the meantime the best we can do is to explore all possible venues connected with this ingredient of the cosmic budget.

Altogether, our analysis seems to indicate that, so long as dark energy is amenable to a fluid description with a well defined temperature not far from equilibrium, the overall energy transfer should go from dark energy to dark matter if the second law of thermodynamics and Le Châtelier–Braun principle are to be fulfilled. Interestingly enough, the result $Q > 0$ guarantees that the ratio ρ_m/ρ_x asymptotically tends to a constant [9–16,36], thus greatly alleviating the coincidence problem.

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