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RESEARCH ARTICLE

Dark energy and gravity

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Abstract I review the problem of dark energy focussing on cosmological constant as the candidate and discuss what it tells us regarding the nature of gravity. Section 1 briefly overviews the currently popular "concordance cosmology" and summarizes the evidence for dark energy. It also provides the observational and theoretical arguments in favour of the cosmological constant as a candidate and emphasizes why no other approach really solves the conceptual problems usually attributed to cosmological constant. Section 2 describes some of the approaches to understand the nature of the cosmological constant and attempts to extract certain key ingredients which must be present in any viable solution. In the conventional approach, the equations of motion for matter fields are invariant under the shift of the matter Lagrangian by a constant while gravity breaks this symmetry. I argue that until the gravity is made to respect this symmetry, one cannot obtain a satisfactory solution to the cosmological constant problem. *Hence cosmological constant problem essentially has to do with our understanding of the nature of gravity*. Section 3 discusses such an alternative perspective on gravity in which the gravitational interaction—described in terms of a metric on a smooth spacetime—is an emergent, long wavelength phenomenon, and can be described in terms of an effective theory using an action associated with normalized vectors in the spacetime. This action is explicitly invariant under the shift of the matter energy momentum tensor $T_{ab} \to T_{ab} + \Lambda g_{ab}$ and any bulk cosmological constant can be gauged away. Extremizing this action leads to an equation *determining the background geometry* which gives Einstein's theory at the lowest order with Lanczos–Lovelock type corrections. In this approach, the observed value of the cosmological constant has to arise from the energy fluctuations of degrees of freedom located in the boundary of a spacetime region.

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1 Cosmological constant as the dark energy

1.1 The cosmological paradigm

A host of different observations, which became available in the last couple of decades, have thrusted upon us a preposterous composition for the energy density of different components in the universe which defies any simple explanation. The energy densities of the different species which drive the expansion of the universe, can be measured in terms of a *critical energy density* $\rho_c = 3H_0^2/8\pi G$ where $H_0 = (\dot{a}/a)_0$ is the rate of expansion of the universe at present. The variables $\Omega_i = \rho_i/\rho_c$ will then give the fractional contribution of different components of the universe (i denoting baryons, dark matter, radiation, etc.) to the critical density required to close the universe. Observations suggest that the universe has $0.98 \lesssim \Omega_{\text{tot}} \lesssim 1.08$ with radiation (R), baryons (B), dark matter, made of weakly interacting massive particles (DM) and dark energy (DE) contributing $\Omega_R \simeq 5 \times 10^{-5}$, $\Omega_B \simeq 0.04$, $\Omega_{\text{DM}} \simeq 0.26$, $\Omega_{\text{DE}} \simeq 0.7$, respectively. All known observations [1–15] are consistent with such an—admittedly weird—composition for the universe.[1]

The conventional cosmological paradigm—which is remarkably successful—is based on these numbers and can be summarized (for recent reviews of cosmological paradigm, see, e.g., [16,17]) as follows: The key idea is that if there existed small fluctuations in the energy density in the early universe, then gravitational instability can amplify them in a well-understood manner leading to structures like galaxies, etc., today. The most popular model for generating these fluctuations is based on the idea that if the very early universe went through an inflationary phase [18–25], then the quantum fluctuations of the field driving the inflation can lead to energy density fluctuations [26–33].[2] It is possible to construct models of inflation such that these fluctuations are described by a Gaussian random field and are characterized by a power spectrum of the form $P(k) = Ak^n$ with $n \simeq 1$. The inflationary models cannot predict the value of the amplitude A in an unambiguous manner. But it can be determined from CMBR observations and the inflationary model parameters can be fine-tuned to reproduce the observed value. The CMBR observations are consistent with the inflationary model for the generation of perturbations and gives $A \simeq (28.3\,\text{h}^{-1}\,\text{Mpc})^4$ and $n \lesssim 1$. (The first results were from COBE [34–36] and WMAP has re-confirmed them with far greater accuracy). When the perturbation is small, one can use well defined linear perturbation theory to study its growth. But when $\delta \approx (\delta\rho/\rho)$ is comparable to unity the perturbation theory breaks down. Since there is more power at small scales, smaller scales go non-linear first and structure forms hierarchically. The non-linear evolution of the *dark matter halos* can be understood by simulations as well as theoretical models based on approximate ansatz ([37–43]; this is essentially an example of statistical mechanics of self gravitating systems; see e.g., [44–46]) and non-linear scaling relations [47–54]. The baryons in the halo will cool and undergo collapse in a fairly complex manner because of gas dynamical processes. It seems unlikely that

[1] For a review of BBN, see [13].

[2] For a recent discussion with detailed set of references, see [31].

the baryonic collapse and galaxy formation can be understood by analytic approximations; one needs to do high resolution computer simulations to make any progress (For a pedagogical description, see [55,56]). The results obtained from all these attempts are broadly consistent with observations but the summary given above demonstrates that modelling the universe and comparing the theory with observations is a rather involved affair.

So, to the zeroth order, the universe is characterized by just seven numbers: $h \approx 0.7$ describing the current rate of expansion; $\Omega_{\rm DE} \simeq 0.7$, $\Omega_{\rm DM} \simeq 0.26$, $\Omega_B \simeq 0.04$, $\Omega_R \simeq 5 \times 10^{-5}$ giving the composition of the universe; the amplitude $A \simeq (28.3\, h^{-1}\,\text{Mpc})^4$ and the index $n \simeq 1$ of the initial perturbations. The remaining challenge, of course, is to make some sense out of these numbers themselves from a more fundamental point of view. Among all these components, the dark energy, which exerts negative pressure, is probably the weirdest and—since non-cosmologists often wonder how strong is the evidence for it—it is useful keep the following points in mind:

- The rapid acceptance of dark energy by the community is partially due to the fact that—even before the supernova data came up—there were strong indications for the existence of dark energy. Early analysis of several observations [57–59] indicated that this component is unclustered and has negative pressure. This is, of course, confirmed dramatically by the supernova observations [60–65]. (For a critical look at the current data, see [66–69]; a sample of recent (2007) work in SN data analysis papers and related topics can be found in refs. [70–77].)
- The WMAP-CMBR data with a reasonable prior on Hubble constant implies $\Omega_{\rm tot} \approx 1$ while a host of other astronomical observations show that the clustered matter contributes only about $\Omega_{\rm DM} \approx 0.25 - -0.4$. Together, they require a unclustered (negative pressure) component in the universe independent of SN data. *It, therefore, seems very unlikely that dark energy will "go away"*.

The key observational feature of dark energy is that—treated as a fluid with a stress tensor $T^a_b = \text{dia}\,(\rho, -p, -p, -p)$—it has an equation state $p = w\rho$ with $w \lesssim -0.8$ at the present epoch. The spatial part $\mathbf{g}$ of the geodesic acceleration (which measures the relative acceleration of two geodesics in the spacetime) satisfies an *exact* equation in general relativity given by:

$$\nabla \cdot \mathbf{g} = -4\pi G(\rho + 3p). \tag{1}$$

This shows that the source of geodesic acceleration is $(\rho + 3p)$ and not ρ. As long as $(\rho + 3p) > 0$, gravity remains attractive while $(\rho + 3p) < 0$ can lead to "repulsive" gravitational effects. In other words, dark energy with sufficiently negative pressure will accelerate the expansion of the universe, once it starts dominating over the normal matter. This is precisely what is established from the study of high redshift supernova, which can be used to determine the expansion rate of the universe in the past [60–65].

The simplest model for a fluid with negative pressure is the cosmological constant (for a sample of recent reviews, see refs. [78–87]) with $w = -1$, $\rho = -p =$ constant. If dark energy is indeed the cosmological constant, then it introduces a fundamental length scale in the theory $L_\Lambda \equiv H_\Lambda^{-1}$, related to the constant dark energy density $\rho_{\rm DE}$ by $H_\Lambda^2 \equiv (8\pi G\rho_{\rm DE}/3)$. In classical general relativity, based on G, c and L_Λ, it is not possible to construct any dimensionless combination from these constants. But when one introduces the Planck constant, $\hbar$, it is possible to form the dimensionless combination $H_\Lambda^2(G\hbar/c^3) \equiv (L_P^2/L_\Lambda^2)$. Observations then require $(L_P^2/L_\Lambda^2) \lesssim 10^{-123}$. This will require enormous fine tuning. What is more, in the past, the energy density of normal matter and radiation would have been higher while the energy density contributed by the cosmological constant does not change. Hence we need to adjust the energy densities of normal matter and cosmological constant in the early epoch very carefully so that $\rho_\Lambda \gtrsim \rho_{\rm NR}$ around the current epoch. This raises the second of the two cosmological constant problems: Why is $(\rho_\Lambda/\rho_{\rm NR}) = \mathcal{O}(1)$ at the *current* phase of the universe? These are the two conventional conceptual difficulties associated with the cosmological constant and have been discussed extensively in literature.

1.2 The "denial" approach to the cosmological constant

Because of these conceptual problems associated with the cosmological constant, people have explored a large variety of alternative possibilities. The most popular among them uses a scalar field ϕ with a suitably chosen potential $V(\phi)$ so as to make the vacuum energy vary with time. The hope then is that, one can find a model in which the current value can be explained naturally without any fine tuning. A simple form of the source with variable w are scalar fields with Lagrangians of different forms, of which we will discuss two possibilities:

$$\mathcal{L}_{\rm quin} = \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi); \quad \mathcal{L}_{\rm tach} = -V(\phi)[1-\partial_a\phi\partial^a\phi]^{1/2}. \tag{2}$$

Both these Lagrangians involve one arbitrary function $V(\phi)$. The first one, $\mathcal{L}_{\rm quin}$, which is a natural generalization of the Lagrangian for a non-relativistic particle, $L = (1/2)\dot{q}^2 - V(q)$, is usually called quintessence (For a sample of recent ($\gtrsim$ 2003) papers covering time varying w in different guises, see [88–122]). When it acts as a source in Friedman universe, it is characterized by a time dependent $w(t)$ with

$$\rho_q(t) = \frac{1}{2}\dot{\phi}^2 + V; \quad p_q(t) = \frac{1}{2}\dot{\phi}^2 - V; \quad w_q = \frac{1-(2V/\dot{\phi}^2)}{1+(2V/\dot{\phi}^2)}. \tag{3}$$

The structure of the second Lagrangian in Eq. (2) (which arises in string theory) can be understood by a simple analogy from special relativity. A relativistic particle with (1-dimensional) position $q(t)$ and mass m is described by the Lagrangian $L = -m\sqrt{1-\dot{q}^2}$. It has the energy $E = m/\sqrt{1-\dot{q}^2}$ and momentum $k = m\dot{q}/\sqrt{1-\dot{q}^2}$ which are related by $E^2 = k^2 + m^2$. As is well known, this allows the possibility of having *massless* particles with finite energy for which $E^2 = k^2$. This is achieved by taking the limit of $m \to 0$ and $\dot{q} \to 1$, while keeping the ratio in $E = m/\sqrt{1-\dot{q}^2}$ finite. The momentum acquires a life of its own, unconnected with the velocity $\dot{q}$, and the energy is expressed in terms of the momentum (rather than in terms of $\dot{q}$) in the Hamiltonian formulation. We can now construct a field theory by upgrading $q(t)$ to a field ϕ. Relativistic invariance now requires ϕ to depend on both space and time [$\phi = \phi(t, \mathbf{x})$] and $\dot{q}^2$ to be replaced by $\partial_i\phi\partial^i\phi$. It is also possible now to treat the mass parameter m as a function of ϕ, say, $V(\phi)$ thereby obtaining a field theoretic Lagrangian $L = -V(\phi)\sqrt{1-\partial^i\phi\partial_i\phi}$. The Hamiltonian structure of this theory is algebraically very similar to the special relativistic example we started with. In particular, the theory allows solutions in which $V \to 0$, $\partial_i\phi\partial^i\phi \to 1$ simultaneously, keeping the energy (density) finite. Such solutions will have finite momentum density (analogous to a massless particle with finite momentum k) and energy density. Since the solutions can now depend on both space and time (unlike the special relativistic example in which q depended only on time), the momentum density can be an arbitrary function of the spatial coordinate. The structure of this Lagrangian is similar to those analysed in a wide class of models called *K-essence* (for a small sample of recent ($\gtrsim$ 2003) papers, see [123–133]) and provides a rich gamut of possibilities in the context of cosmology [134–161].

Since the quintessence field (or the tachyonic field) has an undetermined free function $V(\phi)$, it is possible to choose this function in order to produce a given expansion history of the universe characterized by the function $H(a) = \dot{a}/a$ expressed in terms of a. To see this explicitly, let us assume that the universe has two forms of energy density with $\rho(a) = \rho_{\rm known}(a) + \rho_\phi(a)$ where $\rho_{\rm known}(a)$ arises from any known forms of source (matter, radiation,...) and $\rho_\phi(a)$ is due to a scalar field. Let us first consider quintessence. Here, the potential is given implicitly by the form [134–136, 162, 163]

$$V(a) = \frac{1}{16\pi G} H(1-Q)\left[6H + 2aH' - \frac{aHQ'}{1-Q}\right], \tag{4}$$

$$\phi(a) = \left[\frac{1}{8\pi G}\right]^{1/2} \int \frac{da}{a}\left[aQ' - (1-Q)\frac{d\ln H^2}{d\ln a}\right]^{1/2}, \tag{5}$$

where $Q(a) \equiv [8\pi G\rho_{\rm known}(a)/3H^2(a)]$ and prime denotes differentiation with respect to a. Given any $H(a)$, $Q(a)$, these equations determine $V(a)$ and $\phi(a)$ and thus the potential $V(\phi)$. *Every quintessence model studied in the literature can be obtained from these equations.*

Similar results exists for the tachyonic scalar field as well [134–136]. For example, given any $H(a)$, one can construct a tachyonic potential $V(\phi)$ so that the scalar field is the source for the cosmology. The equations determining $V(\phi)$ are now given by:

$$\phi(a) = \int \frac{da}{aH}\left(\frac{aQ'}{3(1-Q)} - \frac{2}{3}\frac{aH'}{H}\right)^{1/2}, \tag{6}$$

$$V(a) = \frac{3H^2}{8\pi G}(1-Q)\left(1 + \frac{2}{3}\frac{aH'}{H} - \frac{aQ'}{3(1-Q)}\right)^{1/2}. \tag{7}$$

Equations (6) and (7) completely solve the problem. Given any $H(a)$, these equations determine $V(a)$ and $\phi(a)$ and thus the potential $V(\phi)$. A wide variety of phenomenological models with time dependent cosmological constant have been considered in the literature; all of these can be mapped to a scalar field model with a suitable $V(\phi)$.

While the scalar field models enjoy considerable popularity (one reason being they are easy to construct!) it is very doubtful whether they have helped us to understand

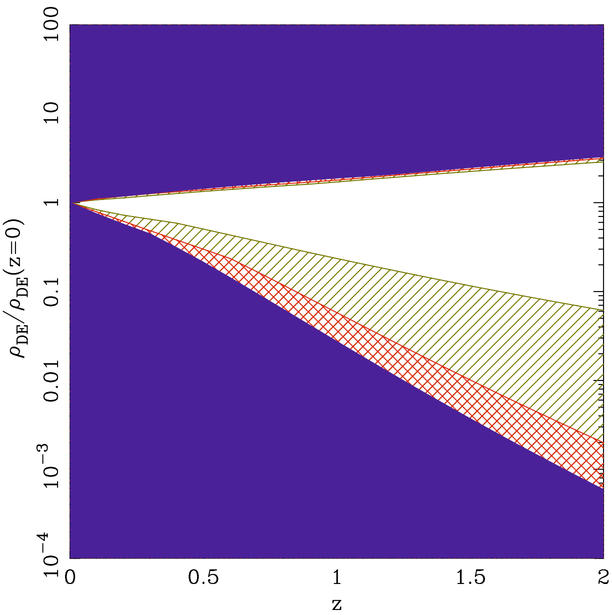

Fig. 1 The observational constraints on the variation of dark energy density as a function of redshift from WMAP and SNLS data (see [164]). The *green/hatched* region is excluded at 68% confidence limit, *red/cross-hatched* region at 95% confidence level and the *blue/solid* region at 99% confidence limit. The *white* region shows the allowed range of variation of dark energy at 68% confidence limit

the nature of the dark energy at any deeper level. These models, viewed objectively, suffer from several shortcomings:

- They have no predictive power. As explicitly demonstrated above, virtually every form of $a(t)$ can be modelled by a suitable "designer" $V(\phi)$.
- These models are degenerate in another sense. The previous discussion illustrates that even when $w(a)$ is known/specified, it is not possible to proceed further and determine the nature of the scalar field Lagrangian. The explicit examples given above show that there are *at least* two different forms of scalar field Lagrangians—corresponding to the quintessence or the tachyonic field—which could lead to the same $w(a)$. (See the first paper in refs. [66–69] for an explicit example of such a construction.)
- By and large, the potentials used in the literature have no natural field theoretical justification. All of them are non-renormalizable in the conventional sense and have to be interpreted as a low energy effective potential in an ad hoc manner.
- One key difference between cosmological constant and scalar field models is that the latter lead to a $w(a)$ which varies with time. If observations have demanded this, or even if observations have ruled out $w = -1$ at the present epoch, then one would have been forced to take alternative models seriously. However, all available observations are consistent with cosmological constant ($w = -1$) and—in fact—the possible variation of w is strongly constrained [164] as shown in Fig. 1.

- While on the topic of observational constraints on $w(t)$, the following point needs to be stressed: One should be careful about the hidden assumptions in the statistical analysis of these data. Claims regarding the value of w depends crucially on the data sets used, priors which are assumed and possible parameterizations which are adopted. (For more details related to these issues, see the last reference in [164].) It is fair to say that all currently available data is consistent with $w = -1$. Further, there is some amount of tension between WMAP and SN-Gold data with the recent SNLS data [65] being more concordant with WMAP than the SN Gold data.
- The most serious problem with the scalar field models is the following: All the scalar field potentials require fine tuning of the parameters in order to be viable. This is obvious in the quintessence models in which adding a constant to the potential is the same as invoking a cosmological constant. So to make the quintessence models work, *we first need to assume the cosmological constant is zero.* These models, therefore, merely push the cosmological constant problem to another level, making it somebody else's problem!.

The last point makes clear that if we shift $\mathcal{L} \to \mathcal{L}_{\text{matt}} - 2\lambda_m$ in an otherwise successful scalar field model for dark energy, we end up "switching on" the cosmological constant and raising the problems again. It is therefore important to address this issue, which we will discuss in Sect. 3.

Given this situation, we shall first take a more serious look at the cosmological constant as the source of dark energy in the universe.

2 Aspects of the cosmological constant

2.1 Facing up to the challenge

The observational and theoretical features described above suggests that one should consider cosmological constant as the most natural candidate for dark energy. Though it leads to well known problems, it is also the most economical [just one number] and simplest explanation for all the observations.

Once we invoke the cosmological constant, classical gravity will be described by the three constants G, c and $\Lambda \equiv L_\Lambda^{-2}$. Since $\Lambda(G\hbar/c^3) \equiv (L_P/L_\Lambda)^2 \approx 10^{-123}$, it is obvious that the cosmological constant is telling us something regarding *quantum gravity*, indicated by the combination $G\hbar$. *An acid test for any quantum gravity model will be its ability to explain this value;* needless to say, all the currently available models—strings, loops, etc.—flunk this test. Even assuming that this is more of an issue in semiclassical gravity rather than quantum gravity, one cannot help noticing that several different approaches to semiclassical gravity [165–172] are silent about cosmological constant.

In terms of the energy scales, the cosmological constant problem is an infra red problem *par excellence*. At the same time, the occurrence of $\hbar$ in $\Lambda(G\hbar/c^3)$ shows that it is a relic of a quantum gravitational effect (or principle) of unknown nature. One is envisaging here a somewhat unusual possibility of a high energy phenomenon leaving a low energy relic and an analogy will be helpful to illustrate this idea [173]. Suppose we solve the Schrodinger equation for the Helium atom for the quantum states of the two electrons $\psi(x_1, x_2)$. When the result is compared with observations, we will find that only half the states—those in which $\psi(x_1, x_2)$ is antisymmetric under $x_1 \longleftrightarrow x_2$ interchange—are realized in nature. But the low energy Hamiltonian for electrons in the Helium atom has no information about this effect! Here is a low energy (IR) effect which is a relic of relativistic quantum field theory (spin-statistics theorem) that is totally non-perturbative, in the sense that writing corrections to the Hamiltonian of the Helium atom in some $(1/c)$ expansion will *not* reproduce this result. I suspect the current value of cosmological constant is related to quantum gravity in a similar spirit. There must exist a deep principle in quantum gravity which leaves its non-perturbative trace even in the low energy limit that appears as the cosmological constant.

2.1.1 Cosmology with two length scales

Given the two length scales L_P and L_Λ, one can construct two energy scales $\rho_{\rm UV} = 1/L_P^4$ and $\rho_{\rm IR} = 1/L_\Lambda^4$ in natural units ($c = \hbar = 1$). There is sufficient amount of justification from different theoretical perspectives to treat L_P as the zero point length of spacetime [174–185], giving a natural interpretation to $\rho_{\rm UV}$. The second one, $\rho_{\rm IR}$ also has a natural interpretation. Since the universe dominated by a cosmological constant at late times will be asymptotically DeSitter with $a(t) \propto \exp(t/L_\Lambda)$ at late times, it will have a horizon and associated thermodynamics [186,187] with a temperature $T = H_\Lambda/2\pi$. The corresponding thermal energy density is $\rho_{\rm thermal} \propto T^4 \propto 1/L_\Lambda^4 = \rho_{\rm IR}$. Thus L_P determines the *highest* possible energy density in the universe while L_Λ determines the *lowest* possible energy density in this universe. As the energy density of normal matter drops below this value, $\rho_{\rm IR}$, the thermal ambience of the DeSitter phase will remain constant and provide the irreducible "vacuum noise". The observed dark energy density is the geometric mean

$$\rho_{\rm DE} = \sqrt{\rho_{\rm IR}\,\rho_{\rm UV}} = \frac{1}{L_P^2 L_\Lambda^2} \tag{8}$$

of these two energy densities. If we define a dark energy length scale $L_{\rm DE}$ such that $\rho_{\rm DE} = 1/L_{\rm DE}^4$ then $L_{\rm DE} = \sqrt{L_P L_\Lambda}$ is the geometric mean of the two length scales in the universe.[3]

Figure 2 describes some peculiar features in such a universe [188,189]. Using the characteristic length scale of expansion, the Hubble radius $d_H \equiv (\dot{a}/a)^{-1}$, we can distinguish between three different phases of such a universe. The first phase is when the universe went through a inflationary expansion with $d_H =$ constant; the second phase is the radiation/matter dominated phase in which most of the standard cosmology operates and d_H increases monotonically; the third phase is that of re-inflation (or accelerated expansion) governed by the cosmological constant in which d_H is again a constant. The first and last phases are time translation invariant; that is, $t \to t+$ constant is an (approximate) invariance for the universe in these two phases.

[3] Incidentally, $L_{\rm DE} \approx 0.04$ mm is macroscopic; it is also pretty close to the length scale associated with a neutrino mass of 10^{-2} eV; another intriguing coincidence ?!

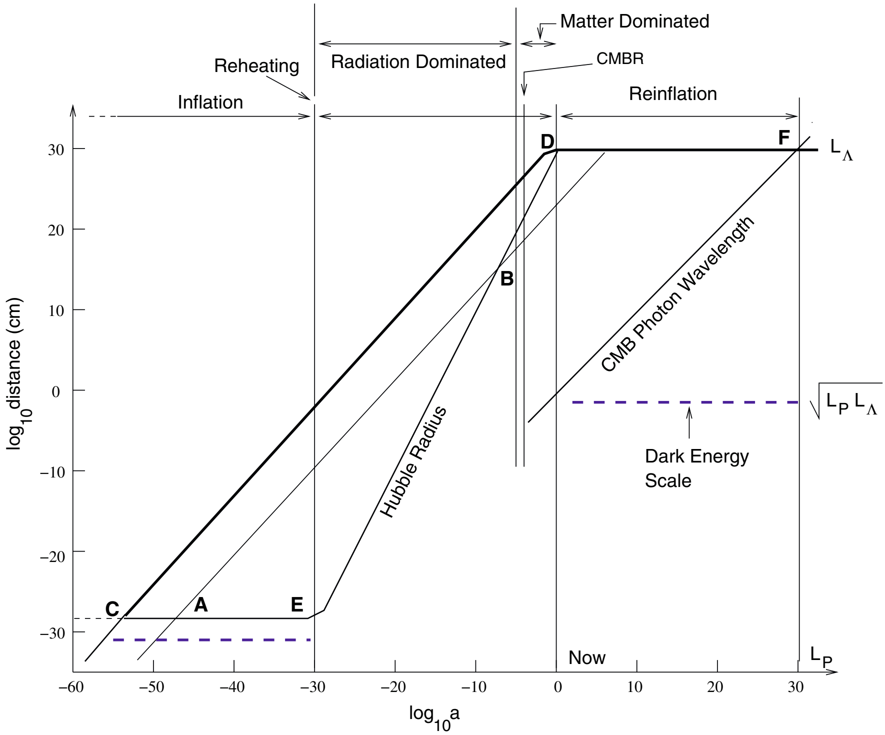

Fig. 2 The geometrical structure of a universe with two length scales L_P and L_Λ corresponding to the Planck length and the cosmological constant [188,189]. Such a universe spends most of its time in two DeSitter phases which are (approximately) time translation invariant. The first DeSitter phase corresponds to the inflation and the second corresponds to the accelerated expansion arising from the cosmological constant. Most of the perturbations generated during the inflation will leave the Hubble radius (at some A, say) and re-enter (at B). However, perturbations which exit the Hubble radius earlier than C will never re-enter the Hubble radius, thereby introducing a specific dynamic range CE during the inflationary phase. The epoch F is characterized by the redshifted CMB temperature becoming equal to the DeSitter temperature $(H_\Lambda/2\pi)$ which introduces another dynamic range DF in the accelerated expansion after which the universe is dominated by vacuum noise of the DeSitter spacetime

The universe satisfies the perfect cosmological principle and is in steady state during these phases!

In the most natural scenario, the two DeSitter phases (first and last) can be of arbitrarily long duration [188]. If $\Omega_\Lambda \approx 0.7$, $\Omega_{\rm DM} \approx 0.3$ the final DeSitter phase *does* last forever; as regards the inflationary phase, nothing prevents it from lasting for arbitrarily long duration. Viewed from this perspective, the in between phase—in which most of the "interesting" cosmological phenomena occur—is of negligible measure in the span of time. It merely connects two steady state phases of the universe. The Fig. 2 also shows the variation of $L_{\rm DE}$ by broken horizontal lines.

While the two DeSitter phases can last forever in principle, there is a natural cutoff length scale in both of them which makes the region of physical relevance to be finite [188]. Let us first discuss the case of re-inflation in the late universe. As the universe grows exponentially in the phase 3, the wavelength of CMBR photons are being redshifted rapidly. When the temperature of the CMBR radiation drops below the DeSitter temperature (which happens when the wavelength of the typical CMBR photon is stretched to the L_Λ) the universe will be essentially dominated by the vacuum thermal noise of the DeSitter phase. This happens at the point marked F when the expansion factor is $a = a_F$ determined by the equation $T_0(a_0/a_F) = (1/2\pi L_\Lambda)$. Let $a = a_\Lambda$ be the epoch at which cosmological constant started dominating over matter, so that $(a_\Lambda/a_0)^3 = (\Omega_{\rm DM}/\Omega_\Lambda)$. Then we find that the dynamic range of DF is

$$\frac{a_F}{a_\Lambda} = 2\pi T_0 L_\Lambda \left(\frac{\Omega_\Lambda}{\Omega_{\rm DM}}\right)^{1/3} \approx 3\times 10^{30}. \tag{9}$$

One can also impose a similar bound on the physically relevant duration of inflation. We know that the quantum fluctuations generated during this inflationary phase could act as seeds of structure formation in the universe [26–31].[2] Consider a perturbation at some given wavelength scale which is stretched with the expansion of the universe as $\lambda \propto a(t)$. (See the line marked AB in Fig. 2.) During the inflationary phase, the Hubble radius remains constant while the wavelength increases, so that the perturbation will "exit" the Hubble radius at some time (the point A in Fig. 2). In the radiation dominated phase, the Hubble radius $d_H \propto t \propto a^2$ grows faster than the wavelength $\lambda \propto a(t)$. Hence, normally, the perturbation will "re-enter" the Hubble radius at some time (the point B in Fig. 2). If there was no re-inflation, this will make *all* wavelengths re-enter the Hubble radius sooner or later. But if the universe undergoes re-inflation, then the Hubble radius "flattens out" at late times and some of the perturbations will *never* reenter the Hubble radius. The limiting perturbation which just "grazes" the Hubble radius as the universe enters the re-inflationary phase is shown by the line marked CD in Fig. 2. If we use the criterion that we need the perturbation to reenter the Hubble radius, we get a natural bound on the duration of inflation which is of direct astrophysical relevance. This portion of the inflationary regime is marked by CE and its dynamic range can be calculated to be:

$$\left(\frac{a_{\rm end}}{a_i}\right) = \left(\frac{T_0 L_\Lambda}{T_{\rm reheat} H_{in}^{-1}}\right)\left(\frac{\Omega_\Lambda}{\Omega_{\rm DM}}\right)^{1/3} = \left(\frac{a_F}{a_\Lambda}\right)(2\pi T_{\rm reheat} H_{in}^{-1})^{-1} \cong 10^{25} \tag{10}$$

for a GUTs scale inflation with $E_{\rm GUT} = 10^{14}$GeV, $T_{\rm reheat} = E_{\rm GUT}$, $\rho_{in} = E_{\rm GUT}^4$ we have $2\pi H_{in}^{-1} T_{\rm reheat} \approx 10^5$. If we consider a quantum gravitational, Planck scale, inflation with $2\pi H_{in}^{-1} T_{\rm reheat} = \mathcal{O}(1)$, the phases CE and DF are approximately equal. The region in the quadrilateral CEDF is the most relevant part of standard cosmology, though the evolution of the universe can extend to arbitrarily large stretches in both directions in time. This figure is telling us something regarding the duality between Planck scale and Hubble scale or between the infrared and ultraviolet limits of the theory and is closely related to the fact that $\rho_{\rm DE}^2 = \rho_{UV}\rho_{\rm IR}$.

2.1.2 Area scaling for energy fluctuations

The geometrical mean relation described above can also be presented in a different manner which allows us to learn something significant. Consider a 3-dimensional region of size L with a bounding area which scales as L^2. Let us assume that we associate with this region N microscopic cells of size L_P each having a Poissonian fluctuation in energy of amount $E_P \approx 1/L_P$. Then the mean square fluctuation of energy in this region will be $(\Delta E)^2 \approx N L_P^{-2}$ corresponding to the energy density $\rho = \Delta E/L^3 = \sqrt{N}/L_P L^3$. If we make the usual assumption that $N = N_{\rm vol} \approx (L/L_P)^3$, this will give

$$\rho = \frac{\sqrt{N_{\rm vol}}}{L_P L^3} = \frac{1}{L_P^4}\left(\frac{L_P}{L}\right)^{3/2} \quad \text{(bulk fluctuations).} \tag{11}$$

On the other hand, if we assume that (for reasons which are unknown), the relevant degrees of freedom scale as the surface area of the region, then $N = N_{\rm sur} \approx (L/L_P)^2$ and the relevant energy density is

$$\rho = \frac{\sqrt{N_{\rm sur}}}{L_P L^3} = \frac{1}{L_P^4}\left(\frac{L_P}{L}\right)^{2} = \frac{1}{L_P^2 L^2} \quad \text{(surface fluctuations).} \tag{12}$$

If we take $L \approx L_\Lambda$, the surface fluctuations in Eq. (12) give precisely the geometric mean in Eq. (8) which is observed. On the other hand, the bulk *fluctuations* lead to an energy density which is larger by a factor $(L/L_P)^{1/2}$. Of course, if we do not take fluctuations in energy but coherently add them, we will get $N/L_P L^3$ which is $1/L_P^4$ for the bulk and $(1/L_P)^4(L_P/L)$ for the surface. In summary, we have the hierarchy:

$$\rho = \frac{1}{L_P^4} \times \left[1, \left(\frac{L_P}{L}\right), \left(\frac{L_P}{L}\right)^{3/2}, \left(\frac{L_P}{L}\right)^{2}, \left(\frac{L_P}{L}\right)^{4} \cdots\right] \tag{13}$$

in which the first one arises by coherently adding energies $(1/L_P)$ per cell with $N_{\rm vol} = (L/L_P)^3$ cells; the second arises from coherently adding energies $(1/L_P)$ per cell with $N_{\rm sur} = (L/L_P)^2$ cells; the third one is obtained by taking *fluctuations* in energy and using $N_{\rm vol}$ cells; the fourth from energy fluctuations with $N_{\rm sur}$ cells; and finally the last one is the thermal energy of the DeSitter space if we take $L \approx L_\Lambda$ and clearly the further terms are irrelevant due to this vacuum noise. Of all these, the only viable possibility is the one that is obtained if we assume that

- The number of active degrees of freedom in a region of size L scales as $N_{\rm sur} = (L/L_P)^2$.
- It is the *fluctuations* in the energy that contributes to the cosmological constant [190–199] and the bulk energy does not gravitate.[4]

[4] For earlier attempts in similar spirit, see [192].

Recently, it has been shown—in a series of papers, see refs. [200–205]—that it is possible to obtain classical relativity from purely thermodynamic considerations in which the surface term of the gravitational actions play a crucial role. The area scaling is familiar from the usual result that entropy of horizons scale as area. In fact, one can argue from general considerations that the entropy associated with *any* null surface should be $(1/4)$ per unit area and will be observer dependent. Further, in cases like Schwarzschild black hole, one cannot even properly define the volume inside a horizon. A null surface, obtained as a limit of a sequence of timelike surfaces (like the $r = 2M$ obtained from $r = 2M + k$ surfaces with $k \to 0^+$), "loses" one dimension in the process (e.g., $r = 2M + k$ is 3-dimensional and timelike for $k > 0$ but is 2-dimensional and null for $k = 0$) suggesting that the scaling of degrees of freedom has to change appropriately. It is difficult to imagine that these features are unconnected and accidental and we will discuss these ideas further in Sect. 3.

2.2 Attempts on the life of Λ

Let us now turn our attention to few of the many attempts to understand the cosmological constant with the choice dictated by personal bias. There is extensive literature on different paradigms for solving the cosmological constant problem, like e.g., those based on new symmetries: [206–208]; those based on QFT in CST: [209–216]. Non-ideal fluids mimicking cosmological constant, like e.g., [217]; Quantum cosmological considerations: [218–223]. Holographic dark energy: [224–229]. Those based on renormalization group, running coupling constants and more general time dependent decay schemes: [230–240] and many more.

2.2.1 Conservative explanations of dark energy

One of the *least* esoteric ideas regarding the dark energy is that the cosmological constant term in the FRW equations arises because we have not calculated the energy density driving the expansion of the universe correctly. The motivation for such a suggestion arises from the following fact: The energy momentum tensor of the real universe, $T_{ab}(t, \mathbf{x})$ is inhomogeneous and anisotropic and will lead to a complicated metric g_{ab} if only we could solve the exact Einstein's equations $G_{ab}[g] = \kappa T_{ab}$. The metric describing the large scale structure of the universe should be obtained by averaging this exact solution over a large enough scale, to get $\langle g_{ab}\rangle$. But what we actually do is to average the stress tensor *first* to get $\langle T_{ab}\rangle$ and *then* solve Einstein's equations. But since $G_{ab}[g]$ is non-linear function of the metric, $\langle G_{ab}[g]\rangle \neq G_{ab}[\langle g\rangle]$ and there is a discrepancy. This is most easily seen by writing

$$G_{ab}[\langle g\rangle] = \kappa \left[\langle T_{ab}\rangle + \kappa^{-1}(G_{ab}[\langle g\rangle] - \langle G_{ab}[g]\rangle)\right] \equiv \kappa \left[\langle T_{ab}\rangle + T^{\text{corr}}_{ab}\right]. \quad (14)$$

If—based on observations—we take the $\langle g_{ab}\rangle$ to be the standard Friedman metric, this equation shows that it has, as its source, *two* terms: The first is the standard average stress tensor and the second is a purely geometrical correction term $T^{\text{corr}}_{ab} = \kappa^{-1}(G_{ab}[\langle g\rangle] - \langle G_{ab}[g]\rangle)$ which arises because of non-linearities in the Einstein's theory that leads to $\langle G_{ab}[g]\rangle \neq G_{ab}[\langle g\rangle]$. If this term can mimic the cosmological constant at large scales there will be no need for dark energy and—as a bonus—one will solve the coincidence problem!

The approach requires us to identify an effective expansion factor $a_{\text{eff}}(t)$ of an inhomogeneous universe after suitable averaging, to be sourced by terms which will lead to $\ddot{a}_{\text{eff}}(t) > 0$ while the standard matter [with $(\rho+3p) > 0$] leads to deceleration of standard expansion factor $a(t)$. Since correct averaging of positive quantities in $(\rho + 3p)$ will not lead to a negative quantity, the real hope is in defining $a_{\text{eff}}(t)$ and obtaining its dynamical equation such that $\ddot{a}_{\text{eff}}(t) > 0$. In spite of some recent attention this idea has received [241–252] it is doubtful whether it will lead to the correct result when implemented properly. The reasons for my skepticism are the following:

- Any calculation in linear theory or any calculation in which special symmetries are invoked are inconclusive in settling the issue. The key question, of identifying a suitable analogue of expansion factor from an averaged geometry, is non-trivial and it is not clear that the answer will be unique. To stress the point by an extreme (and a bit silly) example, suppose we decide to call $a(t)^n$ with, say $n > 2$ as the effective expansion factor $a_{\text{eff}}(t) = a(t)^n$; obviously $\ddot{a}_{\text{eff}}$ can be positive ("accelerating universe") even with $\ddot{a}$ being negative. So, unless one has a *unique* procedure to identify the expansion factor of the average universe, it is difficult to settle the issue.
- It is obvious that T_{ab}^{corr} is non-zero (for an explicit example, in a completely different context of electromagnetic plane wave, see [253]); the question that needs to be settled is how big is it compared to T_{ab}. It seems unlikely that when properly done, we will get a large effect for the simple reason that the amount of mass which is contained in the non-linear regimes in the universe today is subdominant.
- This approach is too strongly linked to explaining the acceleration as observed by SN. Even if we decide to completely ignore all SN data, we still have reasonable evidence for dark energy and it is not clear how this approach can tackle such evidence.

Another equally conservative explanation of the cosmic acceleration will be that we are located in a large underdense region in the universe; so that, locally, the underdensity acts like negative mass and produces a repulsive force. While there has been some discussion in the literature [254,255] as to whether observations indicate such a local "Hubble bubble", this does not seem to be a tenable explanation that one can take seriously at this stage. Again, CMBR observations indicating dark energy, for example, will not be directly affected by this feature though one does need to take into account the effect of the local void.

Finally, one should not forget that a *vanishing* cosmological constant is still a problem that needs an explanation. So even if all the evidence for dark energy disappears within a decade, we still need to understand why cosmological constant is zero and much of what I have to say in the sequel will remain relevant. I stress this because there is a recent tendency to forget the fact that the problem of the cosmological constant existed (and was recognized as a problem) long before the observational evidence for dark energy, accelerating universe, etc., cropped up. In this sense, cosmological constant problem has an important theoretical dimension which is distinct from what has been introduced by the observational evidence for dark energy.

2.2.2 Cosmic Lenz law

The second simplest possibility which has been attempted in the literature several times in different guises is to try and "cancel out" the cosmological constant by some process, usually quantum mechanical in origin. One can, for example, ask whether switching on a cosmological constant will lead to a vacuum polarization with an effective energy momentum tensor that will tend to cancel out the cosmological constant. A less subtle way of doing this is to invoke another scalar field (here we go again!) such that it can couple to cosmological constant and reduce its effective value [256–262]. Unfortunately, none of this could be made to work properly. By and large, these approaches lead to an energy density which is either $\rho_{\rm UV} \propto L_P^{-4}$ or to $\rho_{\rm IR} \propto L_\Lambda^{-4}$. The first one is too large while the second one is too small!

2.2.3 Unimodular gravity

One possible way of addressing the issue of cosmological constant is to simply eliminate from the gravitational theory those modes which couple to cosmological constant. If, for example, we have a theory in which the source in Eq. (1) is $(\rho + p)$ rather than $(\rho + 3p)$, then cosmological constant will not couple to gravity at all. Unfortunately it is not possible to develop a covariant theory of gravity using $(\rho + p)$ as the source. But we can probably gain some insight from the following considerations. Any metric g_{ab} can be expressed in the form $g_{ab} = f^2(x)q_{ab}$ such that $\det q = 1$ so that $\det g = f^4$. From the action functional for gravity

$$A = \frac{1}{16\pi G}\int \sqrt{-g}\, d^4x\, (R - 2\Lambda) = \frac{1}{16\pi G}\int \sqrt{-g}\, d^4x\, R - \frac{\Lambda}{8\pi G}\int d^4x f^4(x) \tag{15}$$

it is obvious that the cosmological constant couples *only* to the conformal factor f. So if we consider a theory of gravity in which $f^4 = \sqrt{-g}$ is kept constant and only q_{ab} is varied, then such a model will be oblivious of direct coupling to cosmological constant. If the action (without the Λ term) is varied, keeping $\det g = -1$, say, then one is lead to a *unimodular theory of gravity* that has the equations of motion

$$R_{ab} - (1/4)g_{ab}R = \kappa(T_{ab} - (1/4)g_{ab}T) \tag{16}$$

with zero trace on both sides. Using the Bianchi identity, it is now easy to show that this is equivalent to the usual theory with an *arbitrary* cosmological constant. That is, cosmological constant arises as an undetermined integration constant in this model [263–268].

While this is all very interesting, we still need an extra physical principle to fix the value (even the sign) of cosmological constant. One possible way of doing this, suggested in Eq. (15), is to interpret the Λ term in the action as a Lagrange multiplier for the proper volume of the spacetime. Then it is reasonable to choose the cosmological constant such that the total proper volume of the universe is equal to a specified number. While this will lead to a cosmological constant which has the correct order of magnitude, it has an obvious problem because the proper four volume of the universe is infinite unless we make the spatial sections compact and restrict the range of time integration.

Amongst all approaches, this one has some valuable ingredients for a solution to the cosmological constant problem because it directly eliminates the coupling between gravity and bulk cosmological constant. But it needs to be remodelled considerably to be made viable. We will discuss in the next section how this can be done in a completely different approach to gravity which holds promise.

3 Gravity as an emergent phenomenon and the cosmological constant

3.1 The necessary ingredients of a new perspective

In conventional approach to gravity, one derives the equations of motion from a Lagrangian $\mathcal{L}_{\rm tot} = \mathcal{L}_{\rm grav}(g) + \mathcal{L}_{\rm matt}(g, \phi)$ where $\mathcal{L}_{\rm grav}$ is the gravitational Lagrangian dependent on the metric and its derivative and $\mathcal{L}_{\rm matt}$ is the matter Lagrangian which depends on both the metric and the matter fields, symbolically denoted as ϕ. This total Lagrangian is integrated over the spacetime volume with the covariant measure $\sqrt{-g}d^4x$ to obtain the action. In such an approach, the cosmological constant can be introduced via two different routes which are conceptually different but operationally the same.

First, one may decide to take the gravitational Lagrangian to be $\mathcal{L}_{\rm grav} = (2\kappa)^{-1}(R - 2\Lambda_g)$ where Λ_g is a parameter in the (low energy effective) action just like the Newtonian gravitational constant κ. This is equivalent to assuming that, even in the absence of matter, flat spacetime is *not* a solution to the field equations. The second route through which the cosmological constant can be introduced is by shifting the matter Lagrangian by $\mathcal{L}_{\rm matt} \to \mathcal{L}_{\rm matt} - 2\lambda_m$. The equations of motion for matter are invariant under such a transformation which implies that—in the absence of gravity—we cannot determine the value of λ_m. But such a shift is clearly equivalent to adding a cosmological constant $2\kappa\lambda_m$ to the $\mathcal{L}_{\rm grav}$. In general, what can be observed through gravitational interaction is the combination $\Lambda_{\rm tot} = \Lambda_g + 2\kappa\lambda_m$.

It is clear that there are two distinct aspects to the so called cosmological constant problem. The first question is why $\Lambda_{\rm tot}$ is very small when expressed in natural units. Second, since $\Lambda_{\rm tot}$ could have had two separate contributions from the gravitational and matter sectors, why does the *sum* remain so fine tuned? This question is particularly relevant because it is believed that our universe went through several phase transitions in the course of its evolution, each of which shifts the energy momentum tensor of matter by $T^a_b \to T^a_b + L^{-4}\delta^a_b$ where L is the scale characterizing the transition. For example, the GUT and Weak Interaction scales are about $L_{\rm GUT} \approx 10^{-29}\,$cm, $L_{SW} \approx 10^{-16}\,$cm respectively which are tiny compared to L_Λ. Even if we take a more pragmatic approach, the observation of Casimir effect in the lab sets a bound that $L < \mathcal{O}(1)$ nanometer, leading to a ρ which is about 10^{12} times the observed value

[269]. Given all these, it seems reasonable to assume that gravity is quite successful in ignoring most of the energy density in the vacuum.

The transformation $\mathcal{L} \to \mathcal{L}_{\rm matt} - 2\lambda_m$ is a symmetry of the matter sector (at least at scales below the scale of supersymmetry breaking; we shall ignore supersymmetry in what follows). The matter equations of motion do not care about constant λ_m. In the conventional approach, gravity breaks this symmetry. *This is the root cause of the so called cosmological constant problem.* As long as gravitational field equations are of the form $E_{ab} = \kappa T_{ab}$ where E_{ab} is some geometrical quantity (which is G_{ab} in Einstein's theory) the theory cannot be invariant under the shifts of the form $T^a_b \to T^a_b + \rho\delta^a_b$. Since such shifts are allowed by the matter sector, it is very difficult to imagine a definitive solution to cosmological constant problem within the conventional approach to gravity.

If metric represents the gravitational degree of freedom that is varied in the action and we demand full general covariance (unlike in the unimodular theory of gravity), we cannot avoid $\mathcal{L}_{\rm matter}\sqrt{-g}$ coupling and cannot obtain of the equations of motion which are invariant under the shift $T_{ab} \to T_{ab} + \Lambda g_{ab}$. Clearly a new, drastically different, approach to gravity is required.

Even if we manage to obtain a theory in which gravitational action is invariant under the shift $T_{ab} \to T_{ab} + \Lambda g_{ab}$, we would have only succeeded in making gravity is decouple from the bulk vacuum energy. While this is considerable progress, there still remains the second issue of explaining the observed value of the cosmological constant. Once the bulk value of the cosmological constant (or vacuum energy) decouples from gravity, classical gravity becomes immune to cosmological constant; that is, the bulk classical cosmological constant can be gauged away. Any observed value of the cosmological constant has to be necessarily a quantum phenomenon arising as a relic of microscopic spacetime fluctuations. This is a non-trivial issue to address at least for two reasons: First, even the structure of *matter* vacuum in the presence of non-trivial metric is far from simple; for example, it is well known that the vacuum state depends on the class of observers we are considering [270–275] and it is not clear whether this aspect has any fundamental significance. Second, and more important, we have no clue as to what is the substructure from which the spacetime arises as an excitation. The concept of gravitons is fairly useless [276] in providing an answer to this—inherently non-perturbative—question.

Nevertheless, in an approach in which the surface degrees of freedom play the dominant role, rather than bulk degrees of freedom, we have a hope for obtaining the correct value for the cosmological constant. We have already seen that, in this case one obtains the correct result if the relevant degrees of freedom are scales as the surface area of a region rather as volume. Hence, to be considered plausible, any model should single out surface degrees of freedom in some suitable manner. To summarize the above discussion, we are looking for an approach which has the following ingredients [277,278]:

- The field equations must remain invariant under the shift $\mathcal{L}_{\rm matt} \to \mathcal{L}_{\rm matt} + \lambda_m$ of the matter Lagrangian $\mathcal{L}_{\rm matt}$ by a constant λ_m. That is, we need to have some kind of "gauge freedom" to absorb any λ_m. Once we have succeeded in decoupling gravity from bulk vacuum energy, we have won more than half the battle.

- General covariance requires using the integration measure $\sqrt{-g}d^D x$ in actions. Since we do not want to restrict general covariance but at the same time do not want this coupling to metric tensor via $\sqrt{-g}$, it follows that *metric cannot be the dynamical variable in our theory.*
- The discussion in Sect. 2.1.2, especially Eq. (12), shows that the relevant degrees of freedom should be linked to surfaces in spacetime rather than bulk regions. This is important because—after we eliminate the coupling between the bulk cosmological constant and gravity—we still need to address the observed value of cosmological constant. This is a relic of quantum gravitational physics and should arise from degrees of freedom which scale as the surface area.
- In such a approach, one should naturally obtain a theory of gravity which is more general than Einstein's theory with the latter emerging as a low energy approximation.

We will now describe how this can be achieved in a model in which gravity arises as an emergent phenomenon like elasticity.

3.2 Micro-structure of the spacetime

For reasons described above, we abandon the usual picture of treating the metric as the fundamental dynamical degrees of freedom of the theory and treat it as providing a coarse grained description of the spacetime at macroscopic scales, somewhat like the density of a solid—which has no meaning at atomic scales [279–289]. The unknown, microscopic degrees of freedom of spacetime (which should be analogous to the atoms in the case of solids), will play a role only when spacetime is probed at Planck scales (which would be analogous to the lattice spacing of a solid [174–185]).

Moreover, in the study of ordinary solids, one can distinguish between three levels of description. At the macroscopic level, we have the theory of elasticity which has a life of its own and can be developed purely phenomenologically. At the other extreme, the microscopic description of a solid will be in terms of the statistical mechanics of a lattice of atoms and their interaction. Both of these are well known; but interpolating between these two limits is the thermodynamic description of a solid at finite temperature *which provides a crucial window into the existence of the corpuscular substructure of solids.* As Boltzmann taught us, heat is a form of motion and we will not have the thermodynamic layer of description if matter is a continuum all the way to the finest scales and atoms did not exist! *The mere existence of a thermodynamic layer in the description is proof enough that there are microscopic degrees of freedom.*

Move on from a solid to the spacetime. Again we should have three levels of description. The macroscopic level is the smooth spacetime continuum with a metric tensor $g_{ab}(x^i)$ and the equations governing the metric have the same status as the phenomenological equations of elasticity. At the microscopic level, we expect a quantum description in terms of the "atoms of spacetime" and some associated degrees of freedom q_A which are still elusive. But what is crucial is the existence of an interpolating layer of thermal phenomenon associated with null surfaces in the spacetime. Just as a solid cannot exhibit thermal phenomenon if it does not have microstructure, *thermal nature of horizon, for example, cannot arise without the spacetime having a microstructure.*

In such a picture, we normally expect the microscopic structure of spacetime to manifest itself only at Planck scales or near singularities of the classical theory. However, in a manner which is not fully understood, the horizons—which block information from certain classes of observers—link [290,291] certain aspects of microscopic physics with the bulk dynamics, just as thermodynamics can provide a link between statistical mechanics and (zero temperature) dynamics of a solid. The reason is probably related to the fact that horizons lead to infinite redshift, which probes *virtual* high energy processes; it is, however, difficult to establish this claim in mathematical terms.

The above paradigm, in which the gravity is an emergent phenomenon, is anchored on a fundamental relationship between the dynamics of gravity and thermodynamics of horizons [292,293] and the following three results are strongly supportive of the above point of view:

- There is a deep connection between the dynamical equations governing the metric and the thermodynamics of horizons. An explicit example was provided in ref. [294], in the case of spherically symmetric horizons in four dimensions in which it was shown that, Einstein's equations can be interpreted as a thermodynamic relation $TdS = dE + PdV$ arising out of virtual radial displacements of the horizon. Further work showed that this result is valid in *all* the cases for which explicit computation can be carried out—like in the Friedmann models [295–298] as well as for rotating and time dependent horizons in Einstein's theory [299].
- The Hilbert Lagrangian has the structure $\mathcal{L}_{EH} \propto R \sim (\partial g)^2 + \partial^2 g$. In the usual approach the surface term arising from $\mathcal{L}_{\rm sur} \propto \partial^2 g$ has to be ignored or cancelled to get Einstein's equations from $\mathcal{L}_{\rm bulk} \propto (\partial g)^2$. But there is a peculiar (unexplained) relationship between $\mathcal{L}_{\rm bulk}$ and $\mathcal{L}_{\rm sur}$:

$$\sqrt{-g}\mathcal{L}_{\rm sur} = -\partial_a \left(g_{ij} \frac{\partial \sqrt{-g}\mathcal{L}_{\rm bulk}}{\partial(\partial_a g_{ij})} \right). \tag{17}$$

  This shows that the gravitational action is "holographic" with the same information being coded in both the bulk and surface terms and one of them is sufficient. One can indeed obtain Einstein's equations from an action principle which uses *only* the surface term and the virtual displacements of horizons [277,278,300,301]. Since the surface term has the thermodynamic interpretation as the entropy of horizons, this establishes a direct connection between spacetime dynamics and horizon thermodynamics.
- Most importantly, recent work has shown that *all the above results extend far beyond Einstein's theory.* The connection between field equations and the thermodynamic relation $TdS = dE + PdV$ is not restricted to Einstein's theory alone, but is in fact true for the case of the generalized, higher derivative Lanczos–Lovelock gravitational theory in D dimensions as well [302–306]. The same is true [307] for the holographic structure of the action functional: the Lanczos–Lovelock action has the same structure and—again—the entropy of the horizons is related to the surface term of the action. *These results show that the thermodynamic description*

is far more general than just Einstein's theory and occurs in a wide class of theories in which the metric determines the structure of the light cones and null surfaces exist blocking the information.

The conventional approach to gravity fails to provide any clue on these results just as Newtonian continuum mechanics—without corpuscular, discrete, substructure for matter—cannot explain thermodynamic phenomena. A natural explanation for these results requires a different approach to spacetime dynamics which I will now outline.

3.3 Gravity from normalized vector fields

Suppose there are certain microscopic—as yet unknown—degrees of freedom q_A, analogous to the atoms in the case of solids, described by some microscopic action functional $A_{\text{micro}}[q_A]$. In the case of a solid, the relevant long-wavelength elastic dynamics is captured by the *displacement vector field* which occurs in the equation $x^a \to x^a + \xi^a(x)$. In the case of spacetime, we no longer want to use metric as a dynamical variable; so we need to introduce some other degrees of freedom, analogous to ξ^a in the case of elasticity, and an effective action functional based on it. Normally, varying an action functional with respect certain degrees of freedom will lead to equations of motion determining *those* degrees of freedom. But we now make an unusual demand that varying our action principle with respect to some (non-metric) degrees of freedom should lead to an equation of motion *determining the background metric* which remains non-dynamical.

Based on the role expected to be played by surfaces in spacetime, we shall take the relevant degrees of freedom to be the normalized vector fields $n_i(x)$ in the spacetime [308][5] with a norm which is fixed at every event but might vary from event to event: (i.e., $n_i n^i \equiv \epsilon(x)$ with $\epsilon(x)$ being a fixed function; one can choose the norm to be $0, \pm 1$ at each event by our choice of the vector fields but its nature can vary from event to event.). That is, just as the displacement vector ξ^a captures the macro-description in case of solids, the normalized vectors (e.g., normals to surfaces) capture the essential macro-description in case of gravity in terms of an effective action $S[n^a]$. More formally, we expect the coarse graining of microscopic degrees of freedom to lead to an effective action in the long wavelength limit:

$$\sum_{q_A} \exp(-A_{\text{micro}}[q_A]) \longrightarrow \exp(-S[n^a]). \tag{18}$$

To proceed further we need to determine the nature of $S[n^a]$. The general form of $S[n^a]$ in such an effective description, at the quadratic order, will be:

$$S[n^a] = \int_{\mathcal{V}} d^D x \sqrt{-g} \left(4P_{ab}{}^{cd} \nabla_c n^a \nabla_d n^b - T_{ab} n^a n^b \right), \tag{19}$$

[5] This is a generalisation of the ideas presented in an earlier work, which only considered null normals.

where $P_{ab}{}^{cd}$ and T_{ab} are two tensors and the signs, notation, etc., are chosen with hindsight. We will see that T_{ab} can be identified with the matter stress-tensor. The full action for gravity plus matter will be taken to be $S_{\text{tot}} = S[n^a] + S_{\text{matt}}$ with:

$$S_{\text{tot}} = \int_{\mathcal{V}} d^D x \sqrt{-g} \left(4P_{ab}{}^{cd} \nabla_c n^a \nabla_d n^b - T_{ab} n^a n^b \right) + \int_{\mathcal{V}} d^D x \sqrt{-g} \mathcal{L}_{\text{matter}} \tag{20}$$

with an important extra prescription: Since the gravitational sector is related to spacetime microstructure, we must *first* vary the n^a and *then* vary the matter degrees of freedom. (In the language of path integrals, we should integrate out the gravitational degrees of freedom n^a first and use the resulting action for the matter sector.) We shall comment more fully on this point at the end of this section.

We next address the crucial conceptual difference between the dynamics in gravity and elasticity, say, which we mentioned earlier. In the case of solids, one will write a similar functional [say, for entropy or free energy] in terms of the displacement vector ξ^a and extremizing it will lead to an equation *which determines* ξ^a. In the case of spacetime, we expect the variational principle to hold for all vectors n^a with constant norm and lead to a condition on the *background metric*. Obviously, the action functional in Eq. (19) must be rather special to accomplish this and one need to impose two restrictions on the coefficients $P_{ab}{}^{cd}$ and T_{ab} to achieve this. First, the tensor P_{abcd} should have the algebraic symmetries similar to the Riemann tensor R_{abcd} of the D-dimensional spacetime. Second, we need:

$$\nabla_a P^{abcd} = 0 = \nabla_a T^{ab}. \tag{21}$$

In a complete theory, the explicit form of P^{abcd} will be determined by the long wavelength limit of the microscopic theory just as the elastic constants can—in principle—be determined from the microscopic theory of the lattice. In the absence of such a theory, we can take a cue from the renormalization group theory and expand P^{abcd} in powers of derivatives of the metric [300,301,308]. That is, we expect,

$$P^{abcd}(g_{ij}, R_{ijkl}) = c_1 \overset{(1)}{P}{}^{abcd}(g_{ij}) + c_2 \overset{(2)}{P}{}^{abcd}(g_{ij}, R_{ijkl}) + \cdots \tag{22}$$

where $c_1, c_2, \ldots$ are coupling constants and the successive terms progressively probe smaller and smaller scales. The lowest order term must clearly depend only on the metric with no derivatives. The next term depends (in addition to metric) linearly on curvature tensor and the next one will be quadratic in curvature, etc. It can be shown that the mth order term which satisfies our constraints is *unique* and is given by

$$\overset{(m)}{P}{}_{ab}{}^{cd} \propto \delta^{cda_3\ldots a_{2m}}_{abb_3\ldots b_{2m}} R^{b_3b_4}_{a_3a_3} \cdots R^{b_{2m-1}b_{2m}}_{a_{2m-1}a_{2m}} = \frac{\partial \mathcal{L}^{(D)}_m}{\partial R^{ab}{}_{cd}}, \tag{23}$$

where $\delta^{cda_3\ldots a_{2m}}_{abb_3\ldots b_{2m}}$ is the alternating tensor and the last equality shows that it can be expressed as a derivative of the m th order Lanczos–Lovelock Lagrangian [300,301, 309–311], given by

$$\mathcal{L}^{(D)} = \sum_{m=1}^{K} c_m \mathcal{L}_m^{(D)}; \quad \mathcal{L}_m^{(D)} = \frac{1}{16\pi} 2^{-m} \delta^{a_1 a_2 \ldots a_{2m}}_{b_1 b_2 \ldots b_{2m}} R^{b_1 b_2}_{a_1 a_2} R^{b_{2m-1} b_{2m}}_{a_{2m-1} a_{2m}}, \tag{24}$$

where the c_m are arbitrary constants and $\mathcal{L}_m^{(D)}$ is the mth order Lanczos–Lovelock term and we assume $D \geq 2K + 1$. (See Appendix for a brief description of Lanczos–Lovelock gravity.) The lowest order term (which leads to Einstein's theory) is

$$\overset{(1)}{P}{}^{ab}_{cd} = \frac{1}{16\pi} \frac{1}{2} \delta^{a_1 a_2}_{b_1 b_2} = \frac{1}{32\pi} (\delta^a_c \delta^b_d - \delta^a_d \delta^b_c) \tag{25}$$

while the first order term (which gives the Gauss–Bonnet correction) is:

$$\overset{(2)}{P}{}^{ab}_{cd} = \frac{1}{16\pi} \frac{1}{2} \delta^{a_1 a_2 a_3 a_4}_{b_1\, b_2\, b_3\, b_4} R^{b_3 b_4}_{a_3 a_4} = \frac{1}{8\pi} \left(R^{ab}_{cd} - G^a_c \delta^b_d + G^b_c \delta^a_d + R^a_d \delta^b_c - R^b_d \delta^a_c \right), \tag{26}$$

where the fourth order alternating tensor is

$$\delta^{a_1 a_2 a_3 a_4}_{b_1\, b_2\, b_3\, b_4} = \frac{-1}{(D-4)!} \epsilon^{c_1 \cdots c_{D-4} a_1 a_2 a_3 a_4} \epsilon_{c_1 \cdots c_{D-4} b_1 b_2 b_3 b_4}. \tag{27}$$

The alternating tensors are totally antisymmetric in both sets of indices and take values $+1$, -1 and 0. They can be written in any dimension as an appropriate contraction of the Levi–Civita tensor density with itself. All higher orders terms are obtained in a similar manner (see Appendix).

In our paradigm based on Eq. (18), the field equations for gravity arises from extremizing S with respect to variations of the vector field n^a, with the constraint $\delta(n_a n^a) = 0$, and demanding that the resulting condition holds for *all normalized vector fields*. Varying the normal vector field n^a after adding a Lagrange multiplier function $\lambda(x)$ for imposing the constant norm condition $n_a \delta n^a = 0$, we get

$$\delta S = 2 \int_{\mathcal{V}} d^D x \sqrt{-g} \left(4 P_{ab}{}^{cd} \nabla_c n^a \left(\nabla_d \delta n^b \right) - T_{ab} n^a \delta n^b - \lambda(x) g_{ab} n^a \delta n^b \right), \tag{28}$$

where we have used the symmetries of $P_{ab}{}^{cd}$ and T_{ab}. An integration by parts and the condition $\nabla_d P_{ab}{}^{cd} = 0$, leads to

$$\delta S = 2 \int_{\mathcal{V}} d^D x \sqrt{-g} \left[-4 P_{ab}{}^{cd} \left(\nabla_d \nabla_c n^a \right) - (T_{ab} + \lambda g_{ab}) n^a \right] \delta n^b$$
$$+ 8 \int_{\partial \mathcal{V}} d^{D-1} x \sqrt{h} \left[k_d P_{ab}{}^{cd} \left(\nabla_c n^a \right) \right] \delta n^b, \tag{29}$$

where k^a is the D-vector field normal to the boundary $\partial\mathcal{V}$ and h is the determinant of the intrinsic metric on $\partial\mathcal{V}$. As usual, in order for the variational principle to be well defined, we require that the variation δn^a of the vector field should vanish on the boundary. The second term in Eq. (29) therefore vanishes, and the condition that $S[n^a]$ be an extremum for arbitrary variations of n^a then becomes

$$2P_{ab}{}^{cd}\,(\nabla_c\nabla_d - \nabla_d\nabla_c)\,n^a - (T_{ab} + \lambda g_{ab})n^a = 0, \tag{30}$$

where we used the antisymmetry of $P_{ab}{}^{cd}$ in its upper two indices to write the first term. The definition of the Riemann tensor in terms of the commutator of covariant derivatives reduces the above expression to

$$\left(2P_b{}^{ijk}R^a{}_{ijk} - T^a_b + \lambda\delta^a_b\right) n_a = 0, \tag{31}$$

and we see that the equations of motion *do not contain* derivatives with respect to n^a which is, of course, the crucial point. This peculiar feature arose because of the symmetry requirements we imposed on the tensor $P_{ab}{}^{cd}$. We further require that the condition in Eq. (31) hold for *arbitrary* vector fields n^a. A simple argument based on local Lorentz invariance then implies that

$$2P_b{}^{ijk}R^a{}_{ijk} - T^a_b = -\lambda\delta^a_b. \tag{32}$$

The scalar λ is arbitrary so far and we will now show how it can be determined in the physically interesting cases. To see what is involved, consider the lowest order approximation (viz. Einstein gravity) in which we take $P_{ab}{}^{cd}$ to be given in Eq. (25) so that the above equation reduces to:

$$\frac{1}{8\pi}R^a_b - T^a_b = -\lambda\delta^a_b, \tag{33}$$

where $-\lambda$ can be an arbitrary function of the metric. Writing this equation as $(G^a_b - 8\pi T^a_b) = Q(g)\delta^a_b$ with $Q = -8\pi\lambda - (1/2)R$ and using $\nabla_a G^a_b = 0$, $\nabla_a T^a_b = 0$ we get $\partial_b Q = \partial_b[-8\pi\lambda - (1/2)R] = 0$; so that Q is an undetermined integration constant, say Λ, and λ must have the form $8\pi\lambda = -(1/2)R - \Lambda$. The resulting equation is

$$R^a_b - (1/2)R\delta^a_b = 8\pi T^a_b + \Lambda\delta^a_b, \tag{34}$$

which leads to Einstein's theory if we identify T_{ab} as the matter energy momentum tensor using the standard Newtonian limit of the theory. *Clearly, the cosmological constant appears as an integration constant*. The mathematical similarity with unimodular gravity is apparent; keeping the function $n_a n^a = \epsilon(x)$ fixed while varying n_a is equivalent to keeping $\sqrt{-g}$ fixed in unimodular gravity. Taking the trace of Eq. (33) will lead, for example, to Eq. (16), etc. But the conceptual structure is quite different and we maintain full general covariance.

The crucial feature of the coupling between matter and gravity through $T_{ab}n^a n^b$ is that, under the shift $T_{ab} \to T_{ab} + \rho_0 g_{ab}$ the ρ_0 term in the action in Eq. (19) decouples from n^a and becomes irrelevant:

$$\int\limits_{\mathcal{V}} d^D x \sqrt{-g} T_{ab} n^a n^b \to \int\limits_{\mathcal{V}} d^D x \sqrt{-g} T_{ab} n^a n^b + \int\limits_{\mathcal{V}} d^D x \sqrt{-g} \epsilon \rho_0. \tag{35}$$

Since ϵ is not varied when n_a is varied there is no coupling between ρ_0 and the dynamical variables n_a the theory is invariant under the shift $T_{ab} \to T_{ab} + \rho_0 g_{ab}$. We see that the condition $n_a n^a =$ constant on the dynamical variables have led to a "gauge freedom" which allows an arbitrary integration constant to appear in the theory which can absorb the bulk cosmological constant. This was our key objective.

The same procedure works with the more general structure in the family of theories starting with Einstein's GR, Gauss–Bonnet gravity, etc., and—in the general case—one obtains the field equations:

$$16\pi \left[P_b^{\ ijk} R^a_{\ ijk} - \frac{1}{2} \delta^a_b \mathcal{L}_m^{(D)} \right] = 8\pi T^a_b + \Lambda \delta^a_b. \tag{36}$$

These are identical to the field equations for Lanczos–Lovelock gravity with a cosmological constant arising as an undetermined integration constant. To the lowest order, when we use Eq. (25) for $P_b^{\ ijk}$, the Eq. (36) reproduces Einstein's theory. More generally, we get Einstein's equations with higher order corrections which are to be interpreted as emerging from the derivative expansion of the action functional as we probe smaller and smaller scales. Remarkably enough, we can derive not only Einstein's theory but even Lanczos–Lovelock theory from a dual description in terms on the normalized vectors in spacetime, *without varying g_{ab} in an action functional!*

To gain a bit more insight into what is going on, let us consider the on-shell value of the action functional. Manipulating the covariant derivatives in Eq. (19) and using the field equation Eq. (36) we can write

$$\begin{aligned} S_{\rm tot}|_{\rm on-shell} &= S[n] + S_{\rm mat} \\ &= \int\limits_{\mathcal{V}} d^D x \sqrt{-g} \left[4\nabla_d \left(P_{ab}^{\ \ cd} \left(\nabla_c n^a\right) n^b \right) - 4 P_{ab}^{\ \ cd} \left(\nabla_d \nabla_c n^a \right) n^b - T_{ab} n^a n^b \right] + S_{\rm mat} \\ &= 4 \int\limits_{\partial\mathcal{V}} d^{D-1} x \sqrt{h} k_d \left(P_{ab}^{\ \ cd} n^b \nabla_c n^a \right) + \int\limits_{\mathcal{V}} d^D x \sqrt{-g} \epsilon \left(\mathcal{L}_m^{(D)} + \frac{\Lambda}{8\pi} \right) \\ &\quad + \int\limits_{\mathcal{V}} d^D x \sqrt{-g} \mathcal{L}_{\rm matter}, \end{aligned} \tag{37}$$

where $\epsilon \equiv n_a n^a$. We see that, on shell, the only dependence on n_a is through a surface term. Since the metric tensor is not dynamical, second term is irrelevant and we can now vary the matter Lagrangian with respect to matter variables to determine the behaviour of matter in a given curved spacetime, which, of course is sourced by the matter stress tensor through Eq. (36) obtained earlier.

The key new feature, which survives and depends on our original variables n_a is the surface term which we shall now explore further. Explicitly, this surface term is given by:

$$\begin{aligned} S|_{\text{on-shell}} &= 4\int_{\partial\mathcal{V}} d^{D-1}x\sqrt{h}\,k_a\left(P^{abcd}n_c\nabla_b n_d\right) \\ &\longrightarrow \frac{1}{8\pi}\int_{\partial\mathcal{V}} d^{D-1}x\sqrt{h}\,k_a\left(n^a\nabla_b n^b - n^b\nabla_b n^a\right) \\ &= -\frac{1}{8\pi}\int_{\partial\mathcal{V}} d^{D-1}x\sqrt{h}\,k_i\left(n^i K + a^i\right), \end{aligned} \tag{38}$$

where we have manipulated a few indices using the symmetries of P^{abcd}. The expression in the second line, after the arrow, is the result for general relativity. Note that the integrand has the familiar structure of $k_i(n^i K + a^i)$ where $a^i = n^b\nabla_b n^i$ is the acceleration associated with the vector field n^a and $K \equiv -\nabla_b n^b$ is the trace of extrinsic curvature in the standard context. If we restrict to a series of surfaces foliating the spacetime with n_i representing their unit normals and take the boundary to be one of them, we can identify k_i with n_i; then $a_i n^i = 0$ and the surface term is just

$$S|_{\text{on-shell}} = \mp\frac{1}{8\pi}\int_{\partial\mathcal{V}} d^{D-1}x\sqrt{h}K, \tag{39}$$

which is the York–Gibbons–Hawking boundary term in general relativity [312,313] if we normalize ϵ to ± 1 depending on the nature of the surface.

It is now obvious that this term in the on-shell action will lead to the entropy of the horizons (which will be 1/4 per unit transverse area) in the case of general relativity. More formally, we treat the horizon surface as a limit of a sequence of timelike surfaces; for example, in the case of Schwarschild metric we consider surfaces with $r = 2M + \delta$ with $\delta \to 0$. In fact, the result is far more general. Even in the case of a more general P^{ab}_{cd} it can be shown that the on-shell value of the action reduces to [308][5] the entropy of the horizons. The general expression is:

$$S|_{\mathcal{H}} = \sum_{m=1}^{K} 4\pi m c_m \int_{\mathcal{H}} d^{D-2}x_\perp\sqrt{\sigma}\mathcal{L}^{(D-2)}_{(m-1)} = \frac{1}{4}[\text{Area}]_\perp + \text{corrections}, \tag{40}$$

where $x_\perp$ denotes the transverse coordinates on the horizon $\mathcal{H}$, σ is the determinant of the intrinsic metric on $\mathcal{H}$ and we have restored a summation over m thereby giving the result for the most general Lanczos–Lovelock case obtained as a sum of individual Lanczos–Lovelock lagrangians. The expression in Eq. (40) *is precisely the entropy of a general Killing horizon in Lanczos–Lovelock gravity* based on the general prescription given by Wald [314] and Iyer and Wald [315] and computed by several authors. Further, in any spacetime, if we take a local Rindler frame around any event we will obtain an entropy for the locally defined Rindler horizon. In the case of GR, this entropy per unit transverse area is just 1/4 as expected.

This result shows that, in the semiclassical limit in which the action can possibly be related to entropy, we reproduce the conventional entropy which scales as the area in Einstein's theory. Since the entropy counts the relevant degrees of freedom, this shows that the degrees of freedom which survives and contributes in the long wave length limit scales as the area. The quantum fluctuations in these degrees of freedom can then lead to the correct, observed, value of the cosmological constant. The last aspect can be made more quantitative and we will briefly describe in the next section how this can be done.

Our action principle is somewhat peculiar compared to the usual action principles in the sense that we have varied n_a and demanded that the resulting equations hold for *all* vector fields of constant norm. Our action principle actually stands for an infinite number of action principles, one for each vector field of constant norm! This class of *all* n^i allows an effective, coarse grained, description of some (unknown) aspects of spacetime micro physics. This is why we need to first vary n_a, obtain the equations constraining the background metric and then use the action in Eq. (37) to obtain the equations of motion for matter. (If, instead, we vary matter terms first the coupling $T_{ab}n^a n^b$ will couple matter to n^a which will remain undetermined since we have no equation for n_a.) Of course, in most contexts, $\nabla_a T^a_b = 0$ will take care of the dynamical equations for matter and these issues are irrelevant.[6]

At this stage, it is not possible to proceed further and relate n^i to some microscopic degrees of freedom q^A. This issue is conceptually similar to asking one to identify the atomic degrees of freedom, given the description of an elastic solid in terms of a displacement field ξ^a—which we know is impossible. However, the same analogy tells us that the relevant degree of freedom in the long wavelength limit (viz. ξ^a or n^i) can be completely different from the microscopic degrees of freedom and it is best to proceed phenomenologically.

3.4 Gravity as detector of the vacuum fluctuations

The description of gravity using the action principle given above provides a natural back drop for gauging away the bulk value of the cosmological constant since it decouples from the dynamical degrees of freedom in the theory. Once the bulk term is eliminated, what is observable through gravitational effects, in the correct theory of quantum gravity, should be the *fluctuations* in the vacuum energy. These fluctuations will be non-zero if the universe has a DeSitter horizon which provides a confining volume. In this paradigm the vacuum structure can readjust to gauge away the bulk energy density $\rho_{\rm UV} \simeq L_P^{-4}$ while quantum *fluctuations* can generate the observed value $\rho_{\rm DE}$.

[6] On shell, the last two terms in the action in Eq. (37) is the same as the conventional one for gravity coupled matter, if $\epsilon = 1$ but the surface term in, for example, Eq. (39) has the wrong sign.

The role of energy fluctuations contributing to gravity also arises, more formally, when we study the question of *detecting* the energy density using gravitational field as a probe. Recall that an Unruh–DeWitt detector with a local coupling $\mathcal{L}_I = M(\tau)\phi[x(\tau)]$ to the *field* ϕ actually responds to $\langle 0|\phi(x)\phi(y)|0\rangle$ rather than to the field itself [270–275]. Similarly, one can use the gravitational field as a natural "detector" of energy momentum tensor T_{ab} with the standard coupling $L = \kappa h_{ab}T^{ab}$. Such a model was analysed in detail in ref. [316] and it was shown that the gravitational field responds to the two point function $\langle 0|T_{ab}(x)T_{cd}(y)|0\rangle$. In fact, it is essentially this fluctuations in the energy density which is computed in the inflationary models [18–25] as the *source* for gravitational field, as stressed in refs. [32,33]. All these suggest treating the energy fluctuations as the physical quantity "detected" by gravity, when one incorporates quantum effects.

If the cosmological constant arises due to the fluctuations in the energy density of the vacuum, then one needs to understand the structure of the quantum gravitational vacuum at cosmological scales. Quantum theory, especially the paradigm of renormalization group has taught us that the concept of the vacuum state depends on the scale at which it is probed. The vacuum state which we use to study the lattice vibrations in a solid, say, is not the same as vacuum state of the QED and it is not appropriate to ask questions about the vacuum without specifying the scale. If the spacetime has a cosmological horizon which blocks information, the natural scale is provided by the size of the horizon, L_Λ, and we should use observables defined within the accessible region. The operator $H(< L_\Lambda)$, corresponding to the total energy inside a region bounded by a cosmological horizon, will exhibit fluctuations ΔE since vacuum state is not an eigenstate of *this* operator. The corresponding fluctuations in the energy density, $\Delta\rho \propto (\Delta E)/L_\Lambda^3 = f(L_P, L_\Lambda)$ will now depend on both the ultraviolet cutoff L_P as well as L_Λ. To obtain $\Delta\rho_{\rm vac} \propto \Delta E/L_\Lambda^3$ which scales as $(L_P L_\Lambda)^{-2}$ we need to have $(\Delta E)^2 \propto L_P^{-4}L_\Lambda^2$; that is, the square of the energy fluctuations should scale as the surface area of the bounding surface which is provided by the cosmic horizon. Remarkably enough, a rigorous calculation [80] of the dispersion in the energy shows that for $L_\Lambda \gg L_P$, the final result indeed has the scaling

$$(\Delta E)^2 = c_1 \frac{L_\Lambda^2}{L_P^4}, \tag{41}$$

where the constant c_1 depends on the manner in which ultraviolet cutoff is imposed. Similar calculations have been done (with a completely different motivation, in the context of entanglement entropy) by several people and it is known that the area scaling found in Eq. (41), proportional to L_Λ^2, is a generic feature [317–320] (this result can also be obtained from those in ref. [316]). For a simple exponential UV-cutoff, $c_1 = (1/30\pi^2)$ but cannot be computed reliably without knowing the full theory. We thus find that the fluctuations in the energy density of the vacuum in a sphere of radius L_Λ is given by

$$\Delta\rho_{\rm vac} = \frac{\Delta E}{L_\Lambda^3} \propto L_P^{-2}L_\Lambda^{-2} \propto \frac{H_\Lambda^2}{G}. \tag{42}$$

The numerical coefficient will depend on c_1 as well as the precise nature of infrared cutoff radius; but it is a fact of life that a fluctuation of magnitude $\Delta\rho_{\rm vac} \simeq H_\Lambda^2/G$ will exist in the energy density inside a sphere of radius H_Λ^{-1} if Planck length is the UV cutoff. On the other hand, since observations suggest that there is a $\rho_{\rm vac}$ of similar magnitude in the universe it seems natural to identify the two. Our approach explains why there is a *surviving* cosmological constant which satisfies $\rho_{\rm DE} = \sqrt{\rho_{\rm IR}\rho_{\rm UV}}$.

We stress that the computation of energy fluctuations is completely meaningless in the conventional models of gravity in which the metric couples to the bulk energy density. Once a UV cutoff at Planck scale is imposed, one will always get a bulk contribution $\rho_{UV} \approx L_P^{-4}$ with the usual problems. It is only because we have a way of decoupling the bulk term from contributing to the dynamical equations that, we have a right to look at the subdominant term $L_P^{-4}(L_P/L_\Lambda)^2$. Approaches in which the sub-dominant term is introduced by an ad hoc manner are technically flawed since the bulk term cannot be ignored in these usual approaches to gravity. Getting the correct value of the cosmological constant from the energy fluctuations is not as difficult as understanding why the bulk value (which is larger by 10^{120}!) can be ignored. Our approach provides a natural backdrop for ignoring the bulk term—and as a bonus—we get the right value for the cosmological constant from the fluctuations. It is small because it is a purely quantum effect.

4 Conclusions

It is obvious that the existence of a component with negative pressure constitutes a major challenge in theoretical physics. The simplest choice for this component is the cosmological constant; other models based on scalar fields [as well as those based on branes, etc., which I have not discussed] do not alleviate the difficulties faced by cosmological constant and—in fact—makes them worse. The key point I want to stress is that the cosmological constant is most likely to be a low energy relic of a quantum gravitational effect or principle and its explanation will require a radical shift in our current paradigm.

I have tried to advertize a new approach to gravity as a possible broad paradigm to understand the cosmological constant. On the negative side, there are some very obvious difficulties with the ideas that I have outlined. The most serious objections are the following:

- The normalized vectors n_i were introduced in a totally ad hoc manner and does not relate to anything we know about gravity and hence the motivation for the condition the $n_i n^i$ = constant is unclear. The unusual nature of this variable and the action $S[n_a]$ makes it difficult to construct a quantum theory via path integrals.
- While we have fairly attractive scheme to eliminate the bulk cosmological constant term, the arguments given in the last section to obtain the observed value is, at best, tentative. The area scaling for surviving degrees of freedom emerges naturally but it is unclear how to connect up the energy fluctuations in these degrees of freedom to the source of gravity.

Against this, one should compare the attractive features of the approach in a broader context. The conceptual basis for this approach rests on the following logical ingredients.

1. It is impossible to solve the cosmological constant problem unless the gravitational sector of the theory is invariant under the shift $T_{ab} \to T_{ab} + \lambda_m g_{ab}$. Any approach which does not address this issue cannot provide a comprehensive solution to the cosmological constant problem.
2. General covariance requires us to use the measure $\sqrt{-g}d^D x$ in D-dimensions in the action. This will couple the metric (through its determinant) to the matter sector. Hence, as long as we insist on metric as the fundamental variable describing gravity, one cannot address the issue in (1) above. So we need to introduce some other degrees of freedom and an effective action which, however, is capable of constraining the background metric.
3. We found an action principle, based on the normalized vector fields in spacetime, that satisfies all these criteria mentioned above. The new action does not couple to the bulk energy density and maintains invariance under the shift $T_{ab} \to T_{ab} + \lambda_m g_{ab}$. What is more, the on shell value of the action is related to the entropy of horizons showing the relevant degrees of freedom scales as the area of the bounding surface.
4. Since our formalism ensures that the bulk energy density does not contribute to gravity—and only because of that—it makes sense to compute the next order correction due to fluctuations in the energy density. This is impossible to do rigorously with the machinery available but a plausible case can be made as how this will lead to the correct, observed, value of the cosmological constant.
5. In the long wavelength limit, the relevant physics is captured in terms of an effective theory related to the degrees of freedom contained in the fluctuations of the normalized vectors. The resulting theory is far more general than Einstein gravity since the thermodynamic interpretations should transcend classical considerations and incorporate some of the microscopic corrections. Einstein's equations provide the lowest order description of the dynamics and *calculable*, higher order, corrections arise as we probe smaller scales. The mechanism for ignoring the bulk cosmological constant is likely to survive quantum gravitational corrections which are likely to bring in additional, higher derivative, terms to the action.

Taking stock, I strongly believe there is no way out of the points mentioned in (1) and (2) above and a tenable description of gravity must be based on variables other than the metric. Such a theory is very likely to have most of the ingredients I have outlined here.

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Appendix: A primer on Lanczos–Lovelock gravity

The Lanczos–Lovelock Lagrangian is a specific example from a general class of Lagrangians which describes a (possibly semiclassical) theory of gravity and are given by

$$\mathcal{L} = Q_a^{\ bcd} R^a_{\ bcd}, \tag{43}$$

where $Q_a^{\ bcd}$ is the most general fourth rank tensor sharing the algebraic symmetries of the Riemann tensor $R^a_{\ bcd}$ and further satisfying the criterion $\nabla_b Q_a^{\ bcd} = 0$ (Several general properties of this class of Lagrangians are discussed in ref. [307]). It can be shown that (see, e.g., [307]) the equations of motion for a general theory of gravity derived from the Lagrangian in Eq. (43) using the standard variational principle with g^{ab} as the dynamical variables, are given by

$$E_{ab} = \frac{1}{2}T_{ab}; \quad E_{ab} \equiv \frac{1}{\sqrt{-g}}\frac{\partial}{\partial g^{ab}}\left(\sqrt{-g}\mathcal{L}\right) - 2\nabla^m\nabla^n P_{amnb}. \tag{44}$$

Here T_{ab} is the energy–momentum tensor for the matter fields. The tensor P_{abcd} defined through $P_a^{\ bcd} \equiv (\partial\mathcal{L}/\partial R^a_{\ bcd})$. The partial derivatives are taken treating g^{ab}, $\Gamma^a_{\ bc}$ and $R^a_{\ bcd}$ as independent quantities.

The D-dimensional Lanczos–Lovelock Lagrangian is given by [309–311] a polynomial in the curvature tensor:

$$\mathcal{L}^{(D)} = \sum_{m=1}^{K} c_m \mathcal{L}_m^{(D)}; \quad \mathcal{L}_m^{(D)} = \frac{1}{16\pi}2^{-m}\delta^{a_1a_2\ldots a_{2m}}_{b_1b_2\ldots b_{2m}} R^{b_1b_2}_{a_1a_2} R^{b_{2m-1}b_{2m}}_{a_{2m-1}a_{2m}}, \tag{45}$$

where the c_m are arbitrary constants and $\mathcal{L}_m^{(D)}$ is the mth order Lanczos–Lovelock term. Here the generalized alternating tensor $\delta^{\cdots}_{\cdots}$ is the natural extension of the one defined in Eq. (27) for $2m$ indices, and we assume $D \geq 2K + 1$. The mth order Lanczos–Lovelock term $\mathcal{L}_m^{(D)}$ given in Eq. (45) is a homogeneous function of the Riemann tensor of degree m. For each such term, the tensor $Q_a^{\ bcd}$ defined in Eq. (43) carries a label m and becomes

$${}^{(m)}Q_{ab}^{\ \ cd} = \frac{1}{16\pi}2^{-m}\delta^{cda_3\ldots a_{2m}}_{abb_3\ldots b_{2m}} R^{b_3b_4}_{a_3a_3}\cdots R^{b_{2m-1}b_{2m}}_{a_{2m-1}a_{2m}}. \tag{46}$$

The full tensor $Q_{ab}^{\ \ cd}$ is a linear combination of the ${}^{(m)}Q_{ab}^{\ \ cd}$ with the coefficients c_m. Einstein's GR is a special case of Lanczos–Lovelock gravity in which only the coefficient c_1 is non-zero. Since the tensors ${}^{(m)}Q_{ab}^{\ \ cd}$ appear linearly in the Lanczos–Lovelock Lagrangian and consequently in all other tensors constructed from it, for most applications it is sufficient to concentrate on the case where a single coefficient c_m is non-zero. All the results that follow can be easily extended to the case where more than one of the c_m are non-zero, by taking suitable linear combinations of the tensors involved.

For the mth order Lanczos–Lovelock Lagrangian $\mathcal{L}_m^{(D)}$, since P^{abcd} is divergence-free, the expression for the tensor E_{ab} in Eq. (44) becomes

$$E_{ab} = \frac{\partial\mathcal{L}_m^{(D)}}{\partial g^{ab}} - \frac{1}{2}\mathcal{L}_m^{(D)} g_{ab}\,, \tag{47}$$

where we have used the relation $\partial(\sqrt{-g})/\partial g^{ab} = -(1/2)\sqrt{-g}\, g_{ab}$. The first term in the expression for E_{ab} in Eq. (47) can be simplified to give

$$\frac{\partial \mathcal{L}_m^{(D)}}{\partial g^{ab}} = m Q_a^{\ ijk} R_{bijk} = P_a^{\ ijk} R_{bijk}, \tag{48}$$

where the expressions in Eq. (48) can be verified by direct computation, or by noting that $\mathcal{L}_m^{(D)}$ is a homogeneous function of the Riemann tensor $R^a_{\ bcd}$ of degree m. To summarize, the Lanczos–Lovelock field equations are given by

$$16\pi \left[P_b^{\ ijk} R^a_{\ ijk} - \frac{1}{2}\delta^a_b \mathcal{L}_m^{(D)} \right] = 8\pi T^a_b, \tag{49}$$

where we have included a possible cosmological constant in the definition of T^a_b. Taking the trace of this equation, we find that that $\mathcal{L}_m^{(D)} = (2m - D)^{-1} T$. In other words, the on-shell value of the Lagrangian is proportional to the trace of the stress tensor in all Lanczos–Lovelock theories, just like in GR.

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